12F002
Stochastic Processes

6 ECTS

Overview and Objectives

Develop good skills and intuition in rigorous probability. Understand and prove important results in martingale theory. Develop foundations of mathematical finance, linking asset-pricing theory to martingales in both complete and incomplete markets. .

Course Outline

Motivation: why martingales in finance?

Part A: Introduction to rigorous probability

Probability spaces, random variables, expectation, limit theorems, independence, change of measure and Radon-Nikodym theorem; the Esscher transform;

Running examples: infinite coin toss space, Cox-Ross-Rubinstein model, trinomial model;

Introduction to conditional expectation.

Part B: conditional expectation, martingales, stopping times

Definitions, main properties, conditional probability, independence lemma;

Martingales and examples, martingale transforms, investment strategies and theorems;

Running examples: Cox-Ross-Rubinstein model, trinomial model, Radon-Nikodym derivative process;

Stopping times, stopped martingales, optional stopping theorems.

Part C: applications of martingale theory to mathematical finance

Pricing path dependent European derivatives in the C-R-R model: arbitrage and martingale properties;

Equivalence of measures, fundamental theorem of asset pricing, complete markets;

Markov processes and computational considerations; exotic options;

State-density process;

Pricing American derivatives: definitions, supermartingale properties, hedging and pricing, optimal exercise, American call options;

12F002

Stochastic Processes

Incomplete markets: guided tour through the trinomial model.

Required Activities

Weekly exercises, final project (consisting of three parts, covering both martingales and finance)

Evaluation

Final exam, weekly homework, project

Materials

Parts A and B:

a. Shreve Stochastic Calculus for Finance volume II, material from chapters 1-3

b. Williams, Probability with martingales, material from Chapters 1-10 (more advanced material)

Part C:

a. Shreve Stochastic Calculus for Finance volume I, material from chapters 1-5

b. Baxter and Rennie, Financial Calculus