

Exclusionary Pricing and Rebates When Scale Matters*

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April 9, 2009

Abstract

We consider an incumbent firm and a more efficient entrant, both offering a network good to several asymmetric buyers, and both being able to price discriminate. The incumbent disposes of an installed base, while the entrant has a network of size zero, and needs to attract a critical mass of buyers to operate. We analyze different price schemes (uniform pricing, implicit price discrimination - or rebates, explicit price discrimination) and show that the schemes which - for given market structure - induce lower equilibrium prices are also those under which the incumbent is more likely to exclude the rival.

JEL classification: L11, L14, L42.

*We are indebted to two anonymous referees, Daniel Cerquera, Claude Crampes, Joe Farrell, Carl Shapiro, Yossi Spiegel, Jeanine Thal, Helder Vasconcelos, John Vickers, Christian Wey, and especially Chiara Fumagalli and Patrick Rey, for helpful comments and suggestions. We are also grateful to seminar participants at Hitotsubashi University, Tokyo; CORE, Université Catholique de Louvain; University of Vienna; University of Linz; CEPR Conference on Applied Industrial Organisation, Madeira; 2nd CSEF Symposium on Economics and Institutions, Capri; 1st CRESSE Conference on 'Abuse of Dominance', Corfu; CCP Workshop 2005, University of East Anglia; Kiel-Munich Workshop on Economics of Internet and Network Industries 2005; EMNet Conference 2005, Budapest; 2nd Workshop on Empirical IO, University of Innsbruck; European Economic Association Meeting 2006, Vienna; IDEI Conference on the Economics of the Software and Internet Industries 2007, Toulouse; EUI, Florence; RNIC Conference, ZEW Mannheim; Economics of ICT Conference, ENST, Paris, EARIE 2007, Valencia; ASSET 2007, Padova. The usual disclaimer applies.

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1 Introduction

This paper deals with exclusionary pricing practices. One such practice which has recently received renewed attention is *rebates*, i.e. discounts applicable where a customer exceeds a specified target for sales in a defined period.

There are different types of rebates. They can be made contingent on the buyer making most or all of its purchases from the same supplier ("fidelity" or "loyalty" rebates), on increasing its purchases relative to previous years, or on purchasing certain quantity thresholds specified in absolute terms. It is on this last category of rebates that we focus here.

Under US case law, rebates are generally said to promote competition on the merits, and the (high) burden of demonstrating their anticompetitive effect is on the plaintiff.¹ In the EU, instead, the European Commission and the Community Courts have systematically imposed large fines on dominant firms applying different forms of rebates.² The recent *Michelin II* judgment has established that even standardized quantity discounts (that is, standardized rebates given to any buyer whose purchases exceed a predetermined number of units) are anticompetitive if used by a dominant firm.³

One of the objectives of this paper is to study whether rebates, in the form of pure quantity discounts, can have anticompetitive effects. In an industry exhibiting network effects, we find that if rebates are allowed, an incumbent firm having a critical customer base is more likely to exclude a more efficient entrant that can use the same rebate schemes but does not have a customer base yet. Rebates are a form of implicit discrimination, and the incumbent can use them to make more attractive offers to some crucial group of consumers, thereby depriving the entrant of the critical mass of consumers it needs (in our model, network externalities imply that consumers will want to consume a network product only if demand has reached a critical threshold). Only very efficient entrants will be able to overcome the entry barriers that incumbents can raise in this manner.

To give an example of the type of industry that we have in mind, consider the *Microsoft Licensing Case* of 1994-95 (Civil Action No. 94-1564). Microsoft markets its PC operating systems (Windows and MS-DOS) primarily through original equipment manufacturers ("OEMs"), which manufacture PCs. When discussing the substantial barriers to entry for potential rivals of Microsoft, the Complaint explicitly mentions "the difficulty in convincing OEMs to offer and promote a non-Microsoft PC operating system, particularly one with a small installed base".

¹See Kobayashi (2005) for a review of the US case law on rebates.

²For a review of the EU case law on rebates, see e.g. Gyselen (2003).

³The (almost) per se illegality of exclusive contracts, rebates and discriminatory prices by dominant firms has led to a hot debate on the EU policy towards abuse of dominance. See Gual et al. (2006) for a contribution to the debate.

The US Department of Justice alleged that Microsoft designed its pricing policy to deter OEMs from entering into licensing agreements with competing operating system providers, thereby reinforcing the entry barriers raised by the network effects that are inherent in this industry.

Although rebates may have exclusionary effects, it is far from clear that they should be presumed to be welfare-detrimental, even if used by a dominant firm. As John Vickers, then Chairman of the UK Office of Fair Trading, put it:

“These cases about discounts and rebates, on both sides of the Atlantic, illustrate sharply a fundamental dilemma for the competition law treatment of abuse of market power. A firm with market power that offers discount or rebate schemes to dealers is likely to sell more, and its rivals less, than in the absence of the incentives. But that is equally true of low pricing generally.” (Vickers, 2005: F252)

Discriminatory pricing has similar contrasting effects. Consider for instance an oligopolistic industry. On the procompetitive side, it allows firms to decrease prices to particular customers, thereby intensifying competition: each firm can be more aggressive in the rival’s customer segments while maintaining higher prices with the own customer base, but since each firm will do the same, discriminatory pricing will result in fiercer competition than uniform pricing, and consumers will benefit from it.⁴ On the anticompetitive side, though, in asymmetric situations discriminatory pricing may allow a dominant firm to achieve cheaper exclusion of a weaker rival: prices do not need to be decreased for all customers but only for the marginal customers.⁵

This fundamental dilemma between the efficiency effects created by discriminatory pricing and their potential exclusionary effects is one of the main themes of the paper. We show that explicit price discrimination is the pricing scheme with the highest exclusionary potential (and hence the worst welfare outcome if exclusion does occur), followed by implicit price discrimination (i.e., rebates, or pure quantity discounts) and then uniform pricing. However, for given market structure (i.e., when we look at equilibria where entry does occur), the welfare ranking is exactly reversed: the more aggressive the pricing scheme the lower the prices (and thus the higher the surplus) at equilibrium. This trade-off between maximizing the entrant’s chances to enter and maximizing consumer welfare for given market structure, illustrates the difficulties that antitrust agencies and courts find in practice: a tough stance against discounts and other aggressive pricing strategies may well increase the likelihood that monopolies or dominant positions are successfully contested, but may also deprive consumers of the possibility to enjoy lower prices, if entry did occur.

Although it deals with pricing schemes rather than contracts, our paper is closely related to the literature on anticompetitive exclusive dealing. Segal and

⁴ See Thisse and Vives (1988). For a survey on discriminatory pricing, see e.g. Stole (2005).

⁵ See e.g. Armstrong and Vickers (1993).

Whinston (2000) is probably the closest work to ours.⁶ Building on Rasmusen et al. (1991), they show the exclusionary potential of exclusive contracts when the incumbent can discriminate on the compensatory offers it makes to buyers. Our study differs from theirs in several respects: (i) in their game the incumbent has a (first-mover) strategic advantage in that it is allowed to contract with buyers before entry occurs; (ii) if buyers accept the exclusivity offer of the incumbent, they commit to it and cannot renegotiate it even if entry occurs; (iii) buyers are symmetric and only linear pricing is considered. In our paper, instead, (i) the incumbent and the entrant choose price schedules simultaneously, (ii) buyers simply observe prices and decide which firm to buy from (therefore avoiding any problems related to assumptions on commitment and renegotiation); (iii) we explore the role of rebates and quantity discounts in a world where buyers differ in size. Yet, the mechanisms which lead to exclusion in the two papers are very similar: both papers present issues of buyers' miscoordination, and scale economies which are created by fixed costs in their model are created instead by network effects in ours.

Our paper is also related to the literature on divide-and-conquer strategies, in particular to Innes and Sexton (1993, 1994) and Segal (2003). A major innovation of our work relative to theirs is that we allow the entrant to use the same discriminatory tools available to the incumbent. Also, contrary to Innes and Sexton's (1994) finding, in our case a ban on discrimination cannot prevent inefficient outcomes: in our setting, exclusion can arise also under uniform linear pricing.⁷

Finally, our paper is related to the literature on incompatible entry in network industries. The very nature of network effects provides a strong incumbency advantage, shielding dominant firms against competitors even in the absence of any anticompetitive conduct (Farrell and Klemperer, 2006). Crémer et al. (2000) show that an incumbent can strategically use compatibility decisions so as to deter entry.⁸ More closely related to our paper, Jullien (2001) studies how an entrant can use divide-and-conquer strategies to induce buyers to coordinate on the entrant instead of the incumbent. This insight reappears in an extension (Section 5.2), where we show that negative prices (i.e. usage subsidies) may indeed break miscoordination equilibria, thus making successful

⁶Bernheim and Whinston (1998) analyze the possible exclusionary effects of exclusive dealing when firms make simultaneous offers (as in our paper), but in *non-coincident markets*: first, exclusivity is offered to a buyer in a first market; afterwards, offers are made to a buyer in a second market. In their terminology, our paper is looking at *coincident market* effects, which makes our analysis closer to Aghion and Bolton (1985), Rasmusen et al. (1991), Segal and Whinston (2000) and Fumagalli and Motta (2006). All these papers, however, study only exclusive dealing arrangements and assume that the entrant can enter the market (if at all) only *after* the incumbent and the buyers have negotiated an exclusive contract.

⁷Innes and Sexton (1993, 1994) consider a very different contracting environment, strategic variables, and timing of the game. In particular, after the incumbent made its offers, they allow the buyers to contract with the entrant (or to enter themselves), so as to create countervailing power to the incumbent's.

⁸Where incompatibility could be overcome through multi-homing, Shapiro (1999) argues that incumbents can use exclusive dealing contracts to block multi-homing, thus excluding a technologically superior firm.

entry more likely.

The paper continues in the following way. Section 2 describes the model, Section 3 solves the model under the assumption that prices have to be non-negative. Three cases are analyzed: uniform pricing, explicit price discrimination and implicit price discrimination (that is, rebates). Section 4 studies the effects of the different pricing schemes on consumer surplus. Section 5 shows how our results are affected when relaxing the assumptions of the basic model. Section 6 concludes the paper.

2 The setup

Consider an industry composed of two firms, the incumbent I , and an entrant E . The incumbent supplies a network good, and has an installed consumer base of size $\beta_I > 0$. (The network good is durable: “old” buyers will continue to consume it but no longer need to buy it.) I incurs constant marginal cost $c_I \in (0, 1)$ for each unit it produces of the network good.

The entrant can supply a competing network good at marginal cost $c_E < c_I$, i.e. it is more cost-efficient than the incumbent. Since it has not been active in the market so far, it has installed base $\beta_E = 0$. To focus on the role of network externalities, we assume away any fixed costs of entry.

The good can be sold to $m + 1$ different “new” buyers, indexed by $j = 1, \dots, m + 1$. There are $m \geq 1$ identical small buyers, and 1 large buyer.⁹ Goods acquired by one buyer cannot be resold to another buyer, but they can be disposed of at no cost by the buyer who bought them (in case the latter cannot consume them). Side payments of any kind between buyers are ruled out. Define firm i ’s network size s_i (where $i = I, E$) as $s_i = \beta_i + q_i^1 + \dots + q_i^{m+1}$, i.e. the firm’s installed base plus its total sales to all “new” buyers.

To simplify the analysis, we assume that demands are inelastic. A buyer will either buy from the incumbent, or from the entrant (but not from both). The large buyer can consume at most $Q^l = 1 - k$ units, while any small buyer can consume at most $Q^s = \frac{k}{m}$ units. Total market size is normalized to 1: $m(k/m) + (1 - k) = 1$.¹⁰ The parameter $k \in (0, 1)$ measures the market share of the group of small buyers, $1 - k$ measures the large buyer’s share. Assume that $1 - k > k/m$, so that the large buyer’s demand is always larger than a small buyer’s demand (provided they both demand strictly positive quantities). This implies that:

$$k < \frac{m}{m+1} \in \left[\frac{1}{2}, 1 \right). \quad (1)$$

⁹We assume $m \geq 1$ so as to allow for the large buyer to be smaller than the set of all small buyers (which in turn allows for the large buyer to receive better price offers) and to show that prices under rebates depend on the degree of fragmentation of small buyers (and converge to prices under explicit discrimination as $m \rightarrow \infty$).

¹⁰These quantities apply for general (positive or negative) prices. In the base model we restrict prices to be non-negative. Section 5 considers the case where prices can be negative.

Buyers exert positive consumption externalities on each other: If firm i 's network size s_i is below the threshold level \bar{s} , consumption of i 's good gives zero surplus to its buyer. The goods produced by the two firms are incompatible, so that buyers of firm i do not exert network externalities on buyers of firm j . For a network good of sufficient size, large and small buyers have the same maximum willingness to pay of $\bar{p} = 1$.

The assumption that a buyer's utility from consuming is positive only if the network in question reaches the threshold size \bar{s} is designed to capture in an admittedly simple way the presence of network effects.¹¹ We will deal with the case of continuous utility functions in Section 5.3.

We assume that $\beta_I \geq \bar{s}$: the incumbent has already reached the critical size, while the entrant will have to attract enough buyers to reach \bar{s} .¹²

Let the unit prices offered by the two firms to a buyer of type $j = l, s$ be $p_I^j \leq 1$ and $p_E^j \leq 1$. A buyer's net consumer surplus is given by gross consumer surplus minus total expenditure, $CS^j(q_i^j, p_i^j, s_i) = \text{gross}CS^j(q_i^j, s_i) - p_i^j q_i^j$, where gross consumer surplus is defined as:

$$\begin{aligned} \text{gross}CS^l(q_i^l, s_i) &= \begin{cases} \min\{q_i^l, 1 - k\} & \text{if } s_i \geq \bar{s} \\ 0 & \text{otherwise} \end{cases} \\ \text{gross}CS^s(q_i^s, s_i) &= \begin{cases} \min\{q_i^s, k/m\} & \text{if } s_i \geq \bar{s} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (2)$$

Since both types of buyers have the same prohibitive price $\bar{p} = 1$, a monopolist who could price discriminate would set a uniform unit price $p_i^m = 1$. Thus, discriminatory pricing can arise only as a result of strategic interaction.

We assume that neither demand of the large buyer alone, nor demand of all small buyers taken together, is sufficient for the entrant to reach the minimum size:

$$\bar{s} > \max\{1 - k, k\}. \quad (3)$$

In other words, in order to reach the minimum size, the entrant has to serve the large buyer plus at least one (and possibly more than one) small buyer.¹³ We relax this assumption in Section 5.1.

We also assume that the threshold level \bar{s} is such that if the entrant sells to all $m + 1$ new buyers, then it will reach the minimum size: $\bar{s} \leq 1$. This, together with the assumption $c_E < c_I$, implies that the social planner would want the entrant (and not the incumbent) to serve all buyers.

¹¹ It also has the advantage that the old generation of buyers can be ignored when studying welfare effects: since we assume that they have already attained the highest level of utility, new buyers' decisions will never affect old buyers' utility. Of course, this means that we cannot formalise here the possibility that entry may hurt the old generation of buyers, but this is a well-known effect which does not need to be emphasised again.

¹² Note that if the entrant manages to reach the minimum size \bar{s} , then consumers will consider I 's and E 's networks as being of homogenous quality, even if $s_I \neq s_E$.

¹³ Note that only units which are actually consumed count towards firm i 's network size.

The game. Play occurs in the following sequence: At time $t = 0$, the incumbent and the entrant simultaneously announce their prices, which will be binding at $t = 1$. At time $t = 1$, each of the $m + 1$ buyers decides whether to patronize the incumbent or the entrant. We also assume that offers are observable to everyone.

As for the prices that firms can offer in $t = 0$, in the base model (Section 3) we consider three different possibilities: uniform prices (Section 3.2.1); explicit (or third-degree) price discrimination (Section 3.2.2); and implicit (or second-degree) price discrimination, i.e. the case of standardized quantity discounts or “rebates” (Section 3.2.3).

3 Equilibrium solutions

In this Section, we assume that firms set non-negative prices, and we find the equilibria under the three different price regimes. In line with Segal and Whinston (2000), we find that in each regime our game has two types of pure-strategy Nash equilibria: an exclusionary (miscoordination) equilibrium where all buyers buy from the incumbent, and an entry equilibrium where all buyers buy from the entrant. For each type of equilibria, we shall focus on the highest prices that can be sustained.

Since miscoordination equilibria are the same independently of the pricing regime, we first state a general miscoordination result which holds for any regime (Section 3.1), and we then analyze entry equilibria under the different regimes (Section 3.2).

3.1 Miscoordination equilibria

Proposition 1 (*Miscoordination equilibrium under all price regimes*) *If firms can only set non-negative prices, the following pure-strategy Nash equilibrium exists: I sets $p_I^s = p_I^l = p_I^m = 1$, and in all continuation equilibria where $p_E^j \leq p_I^j$, with $j = s, l$, all buyers buy from I . The prices identified are the highest that can be sustained in a miscoordination equilibrium.*

Proof: see Appendix A

To understand this Proposition, note that when $p_E^j \leq p_I^j$ (with $j = s, l$) there is a miscoordination equilibrium where all buyers buy from the incumbent: despite the higher price p_I , no buyer has a unilateral incentive to deviate, since - given that all other buyers buy from I - the entrant’s network would be below the critical size, and buying from the entrant would then give zero (gross) utility.

Continuation equilibria play a role for the equilibrium at the firms’ decision stage. Consider the candidate miscoordination equilibrium where $p_E = p_I = 1$ and all buyers buy from the incumbent. This equilibrium is sustained by having that when $p_E \leq p_I$ the chosen continuation equilibria are those where all buyers

will buy from the incumbent.¹⁴ (As we shall see later, when $p_E \leq p_I$ there are also continuation equilibria where all buyers buy from the entrant). Otherwise, a deviation by the entrant could attract all the buyers, undermining the candidate equilibrium.

The equilibrium characterized in this Proposition represents an extreme case, in the sense that the underlying continuation equilibrium is the most favorable one for firm I . *This equilibrium is by no means the only miscoordination equilibrium that can arise in our game.* For instance, there are other equilibria where all buyers miscoordinate on the incumbent, but the latter can at most charge some price $\tilde{p}_I^j < p_I^n = 1$. Such an equilibrium is sustained by continuation equilibria where buyers buy from I as long as $p_E^j \leq p_I^j \leq \tilde{p}_I^j$, but would switch to E if p_I^j exceeded \tilde{p}_I^j . *For the rest of the paper, for both exclusionary and entry equilibria we will focus on those continuation equilibria which are the most profitable ones for the firm that eventually serves the buyers.* The motivation for this choice is two-fold: First, these equilibria are the Pareto-dominant ones from the point of view of the firms. Second, from a policy point of view, the equilibria with the highest profits are those which cause most concern.

Finally, note that the miscoordination equilibrium identified here does not depend on the price regime, as long as prices are non-negative. The driving force behind this equilibrium is just that a unilateral deviation by a buyer - given that all others buy from the incumbent - is not sufficient to give the entrant the threshold size it needs.¹⁵

3.2 Entry equilibria

In this Section, we look for the entry equilibria of the game. The conditions for their existence depend on the price regimes assumed, as we show below.

3.2.1 Uniform pricing

Assume that firms can only use uniform linear prices, p_i with $i = I, E$. Recall that any buyer's demand for E 's good, $q_E^j(\dots, s_E)$, depends on the size of E 's network, s_E , which in turn depends on E 's sales to the buyers, $\{q_E^1, \dots, q_E^{m+1}\}$. Thus, the following can be proved.

Proposition 2 (*Entry equilibria under uniform prices*) *If firms can only use uniform flat prices, the following pure-strategy Nash equilibrium exists:*

E sets $p_E = c_I$, I sets $p_I = c_I$, and in all continuation equilibria where $p_E \leq p_I$, all buyers buy from E .

¹⁴In this situation, the entrant is indifferent among all prices $p_E \geq 0$ it could charge, and might as well offer the monopoly price, which weakly dominates all other possible equilibrium prices.

¹⁵This also implies that the equilibria identified in Proposition 1 are not Perfectly Coalition Proof (PCP): In the second stage of the game, a collective deviation by a *group* of buyers sufficiently large to generate critical mass for the entrant could always disrupt a miscoordination equilibrium. However, we show in Section 5 that there are other ways to break miscoordination equilibria: for instance, allowing the entrant to offer negative prices and/or to discriminate perfectly.

Proof: see Appendix A

We have seen in Section 3.1 that when $p_E \leq p_I$, there is a miscoordination equilibrium where all buyers buy from the incumbent. However, there is also an equilibrium where all buyers buy from the entrant: no buyer has an incentive to deviate given that all others buy from the entrant, since he would pay a (weakly) higher price p_I for a product which is as good as the entrant's (if all buy, the entrant reaches critical size).¹⁶

Continuation equilibria are chosen to prevent deviations in the firms' stage of the game. Consider the candidate entry equilibrium where $p_E = c_I = p_I$ and all buyers buy from the entrant. Because of the multiplicity of equilibria at the buyers' stage, when the incumbent deviates by increasing its price, there might also be a continuation equilibrium where $p_E < p_I$ and all buyers buy from the incumbent. To eliminate such counter-intuitive deviations, it is required that in all continuation equilibria where $p_E \leq p_I$ all buyers buy from the entrant.

As in the case of miscoordination equilibria, there is a continuum of entry equilibria, and the particular equilibrium chosen in Proposition 2 is the most favorable for firm E .¹⁷

3.2.2 Explicit (3rd degree) discrimination

Under explicit price discrimination, each firm can set one price for the large buyer, and a different price for the small buyers (all buyers of the same type will be charged the same price). When firms can price discriminate, entry equilibria do not necessarily exist, unlike the uniform pricing case. To fix ideas, start with the candidate entry equilibrium where both firms charge c_I and all buyers buy from the entrant (a natural candidate, as this was an entry equilibrium under uniform pricing). This equilibrium can be disrupted by the incumbent setting a price $c_I - \epsilon$ to one category of buyers and the monopoly price to the other category: the loss made on the former would be outweighed by the profits made on the latter. Indeed, under this deviation the former category strictly prefers to buy from I , thus preventing the entrant from reaching the minimum size. Anticipating that for the former buyers it is a dominant strategy to buy from the incumbent, the latter category of buyers would also prefer to buy from I rather than from the entrant, since they would derive zero utility from buying from E .

Therefore, an entry equilibrium can exist only if it is immune to the deviations outlined above, i.e. if the entrant's prices to *both* large and small buyers are so low that the incumbent cannot profitably undercut either of the two prices while charging the monopoly price to the other group. This implies that the highest prices that the entrant can charge in any entry equilibrium will be

¹⁶To be precise, if $p_I = p_E$, there is also a buyers' equilibrium where some buyers (in a sufficient number for the entrant to reach the critical size) buy from the entrant and the remaining buyers buy from the incumbent.

¹⁷Under different continuation equilibria, there are also entry equilibria where the entrant must charge a strictly lower price than c_I to induce buyers to coordinate on E .

strictly below c_I to *both* sets of buyers. Thus, for an entry equilibrium to exist, the efficiency gap between entrant and incumbent must be large enough.

Proposition 3 (*Entry equilibria under explicit discrimination*) Under explicit price discrimination, entry equilibria only exist if $c_I \geq \min \left\{ \frac{1+c_E}{2}, k + c_E, 1 - k + c_E \right\}$. The highest prices that the entrant can charge in any such entry equilibrium are $p_E^s = \max \left\{ \frac{c_I - (1-k)}{k}, 0 \right\} < c_I$ and $p_E^l = \max \left\{ \frac{c_I - k}{1-k}, 0 \right\} < c_I$.

Proof: see Appendix A

Figure 1 illustrates the results of Proposition 3 (recall that miscoordination equilibria exist for all parameter values). The figure shows that, for given k , the

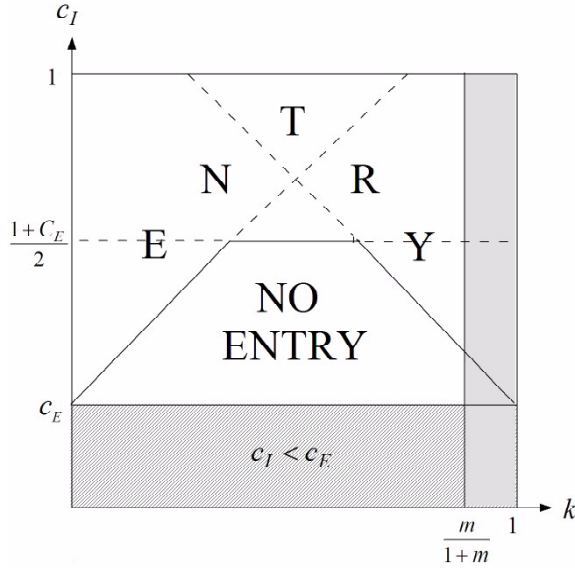


Figure 1: Existence of entry equilibria under explicit price discrimination (the grey areas are outside of the parameter space)

larger c_I with respect to c_E (that is, the larger the efficiency gap) the more likely for entry to be an equilibrium of the game. The intuition is straightforward: if the incumbent is less efficient, it will find it more difficult to profitably make low (discriminatory) price offers, which in turn makes it possible for the entrant to sustain higher (more profitable) prices which are immune to incumbent's deviations. The effect of k on equilibrium outcomes is slightly more complex.

Corollary 4 *The more asymmetry there is between the large buyer and the group of small buyers, the more likely are entry equilibria to exist.*

Proof:

Follows immediately from our existence condition $c_I \geq \min \left\{ \frac{1+c_E}{2}, k + c_E, 1 - k + c_E \right\}$. Minimizing the expression in brackets with respect to k , we find that it converges to its global minimum c_E both as k goes to zero and as k goes to 1. In other words, entry equilibria will exist even for arbitrarily small differences between c_I and c_E , provided the two buyer groups are sufficiently asymmetric. \square

To understand why entry is more likely at very low levels and very high levels of k , consider for instance a candidate entry equilibrium (p_E^s, p_E^l) when k is very small. In order to disrupt this equilibrium, the incumbent could discriminate across buyers, by offering the large buyer a very low price and recovering losses by setting a high price to the small buyers, and vice versa. However, since k is very small, the incumbent cannot offer the large buyer a price (much) below c_I , since the profits it could make on the small buyers are very small (they account for a tiny part of the total market). In contrast, it could use the profits it makes on the (very) large buyer to decrease the price offered to the small buyers. But since prices are restricted to be non-negative here, the incumbent's best offer to the small buyers will be $p_I^s = 0$. In order to avoid deviations, the entrant will therefore have to set $p_E^s = 0$ and p_E^l slightly lower than c_I . As small buyers account for a small proportion of demand (k is very small), the entrant will make positive profits at these prices, and the entry equilibrium will exist. The same argument can be used symmetrically to explain why entry equilibria are more likely to exist if k is sufficiently large. Of course, what drives this result is that prices cannot go below zero. We shall see in Section 5.2 that when prices may be negative, k will affect results monotonically.

To summarize this Section, note that relative to uniform pricing, explicit price discrimination: (a) on the one hand, makes entry equilibria less likely to exist (they always exist under uniform pricing, but under discriminatory pricing they only exist only if c_I is high enough relative to c_E); (b) on the other hand, for given market structure, it results in (weakly) lower prices.¹⁸

3.2.3 Implicit (2nd degree) discrimination (or rebates)

Explicit discrimination may not always be feasible, for instance because of informational constraints (firms cannot observe buyer types), or because of policy constraints. Let us then consider the case where firms cannot condition their offers directly on the type of buyer (large or small), but have to make uniform

¹⁸ For parameter values such that entry equilibria exist under both regimes, prices are strictly lower than c_I under discrimination, while they equal c_I under uniform pricing. The exclusionary equilibrium always exists - and the highest sustainable prices are the same - under both regimes.

offers to both types which may only depend on the quantity bought by buyer $j = 1, \dots, m + 1$:

$$T_i(q_i^j) = \begin{cases} p_{i,1}q_i^j & \text{if } q_i^j \leq \bar{q}_i \\ p_{i,2}q_i^j & \text{if } q_i^j \geq \bar{q}_i \end{cases} \quad (4)$$

(If the buyer buys exactly the threshold quantity, $q_i^j = \bar{q}_i$, the firm may either charge $p_{i,1}$ or $p_{i,2}$.) Each buyer can now choose his tariff from this price menu by buying either below the sales target \bar{q}_i or above it.¹⁹

If this menu is designed appropriately, buyers will self-select into different tariffs: small buyers will buy below the threshold, while the large buyer will buy above the threshold, thus paying a different price than the small buyers. We say that firm i 's offer satisfies the "self-selection conditions" if neither of the two buyer types wants to masquerade as the other, i.e. if

$$\begin{aligned} CS^l(p_{i,2}, 1 - k) &\geq CS^l(p_{i,1}, \bar{q}_i) \\ \text{and } CS^s(p_{i,1}, k/m) &\geq CS^s(p_{i,2}, \bar{q}_i) \end{aligned} \quad (5)$$

For any offer that satisfies the self-selection condition, denote $(p_{i,1})$ by (p_i^s) , and $(p_{i,2})$ by (p_i^l) , for $i = I, E$.

We now look for the equilibria that arise in this game when both firms can use quantity discounts.

Entry equilibria under implicit discrimination The implicitly discriminatory effect of rebates gives rise to an exclusionary mechanism similar to the one under explicit discrimination. Since buyers are asymmetric, they can be induced to self-select either into the high-quantity or the low-quantity bracket of the price menu, thus allowing the incumbent to de facto price-discriminate between them. This in turn enables the incumbent to offer a below-cost price to one group, thus winning their orders, while making up for the resulting losses by charging a high price (possibly the monopoly price) to the other group.

The major difference between explicit and implicit discrimination lies in the self-sorting conditions, which reduce the range of prices that the incumbent can offer. Consider for instance the case where, under explicit discrimination, the incumbent charges the monopoly price $p_I^s = 1$ to the small buyers, and $p_I^l = 0$ to the large buyer. Clearly, this offer does not satisfy the small buyers' self-sorting condition: At a zero price, the small buyers would always prefer to "buy" above the quantity threshold (i.e. receive a large quantity for free, and dispose of the

¹⁹Each buyer is allowed only one transaction. This rules out the possibility that a large buyer makes "multiple small purchases" so as to buy a large amount of units at the lower price. Important transaction costs may be invoked to justify this assumption, which in a way is the counterpart of the assumption that a small buyer cannot buy a large quantity and then resell it to others. In both cases, it is arbitrage which is prevented. Recall also that we exclude reselling of units between buyers (while allowing for free disposal), so the only thing a small buyer can do with units he cannot consume is to throw them away.

units they cannot consume) rather than paying $p_I^s = 1$ (or any other positive price) for a small quantity.

Likewise, an offer where $p_I^s < c_I$ and $p_I^l = 1$ cannot be replicated through a rebate tariff: in this case, it is the large buyer who would prefer to buy below the threshold and enjoy a positive surplus on the (few) units he consumes, rather than buying above the threshold and being left with zero surplus.²⁰

Thus, while rebates still have exclusionary potential, the incumbent's deviation offers will be less aggressive under rebates than under explicit discrimination, allowing for entry equilibria to be sustained in some regions where they do not exist if firms can explicitly price discriminate.

Proposition 5 (*entry equilibria under implicit discrimination*) *Under rebates as defined in (4), entry equilibria only exist if*

- (i) $c_E < \frac{1}{2(m+1)}$ and $c_I \geq \min \left\{ c_E(1+m), k + c_E, \frac{m}{1+m} + c_E - k \right\}$
- (ii) or if $c_E \geq \frac{1}{2(m+1)}$ and $c_I \geq \min \left\{ \frac{m+(1+m)c_E}{1+2m}, k + c_E, \frac{m}{1+m} + c_E - k \right\}$

The highest prices that the entrant can charge in any such entry equilibrium are

$$p_E^s = \begin{cases} 1 - \frac{m(1-c_I)}{k(m+1)} & \text{if } c_I \geq 1 - k - k/m \\ 0 & \text{if } c_I < 1 - k - k/m \end{cases} \quad p_E^l = \begin{cases} \frac{c_I - k}{1-k} & \text{if } c_I \geq \frac{k(1+m)}{m} \\ \frac{c_I}{(1-k)(m+1)} & \text{if } c_I < \frac{k(1+m)}{m} \end{cases}$$

These prices will satisfy the self-sorting conditions if the quantity threshold is $\bar{q}_E = k/m$ if $p_E^s \leq p_E^l$; $\bar{q}_E = 1 - k$ if $p_E^s > p_E^l$.

Proof: see Appendix A

Corollary 6 *The parameter space for which entry equilibria exist under explicit discrimination is a proper subset of the parameter space for which entry equilibria exist under rebates.*

Proof: see Appendix A

Figure 2 illustrates the results of the analysis of entry equilibria under rebates and non-negative prices for the case where $c_E \geq \frac{1}{2(m+1)}$ (recall that miscoordination equilibria exist for all parameter values). We see that the region where entry equilibria do not exist is smaller under rebates than under explicit discrimination. While nothing changes for low values of k (rebates exactly replicate the outcome under explicit discrimination), exclusion becomes more difficult for intermediate and high values of k . Intuitively, given m , the large buyer becomes smaller and smaller the higher k is, and so he becomes more and more similar

²⁰Such a rebate scheme may appear as somewhat unorthodox, since buyers are “rewarded” for buying little and “penalized” for buying a lot. However, this is a deviation offer which will never be made in equilibrium.

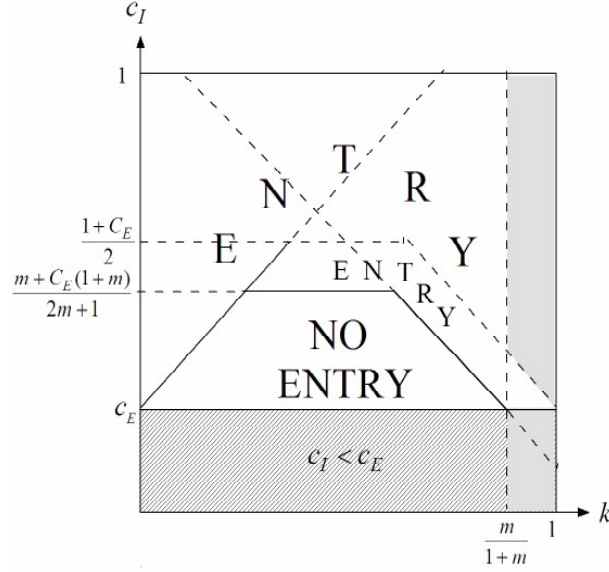


Figure 2: Regions where entry equilibria exist and do not exist under rebates (i.e. implicit price discrimination), compared to explicit discrimination

to the small buyers, making it difficult to discriminate between them through rebates without violating any of the self-sorting conditions.

Note that as m grows (that is, a single small buyer becomes smaller), both the efficiency thresholds and prices under rebates converge to the values under explicit discrimination. In the limit case where $m \rightarrow \infty$, the self-selection constraints play no role: the large buyer will never want to behave like a small buyer whose demand is infinitely small, and vice versa for the small buyer, and so the implicit and explicit discrimination cases coincide.

Let us take stock of the results obtained in this section. One of the motivations for this paper was to investigate whether rebates, in the particular form of quantity discounts, can be exclusionary. Our analysis shows that indeed an incumbent firm could use rebates to exclude a more efficient rival, even if the latter can also make use of rebates. The main intuition is that by relying on quantity discounts the incumbent can (implicitly) discriminate across buyers by making attractive offers to some of them, thus subtracting to the rival firm buyers that it critically needs in order to reach the minimum viable size. Therefore, rebates reduce the likelihood that successful entry takes place.

Nevertheless, precisely because they imply competing aggressively for each group of buyers, rebates might also have a procompetitive function: for given market structures (that is, if one compares regions where entry occurs), prices are lower when rebates are allowed than when prices are uniform. It is to explore more formally this basic trade-off between exclusion and lower prices that we

now turn to the analysis of consumer welfare under the different price schemes.

4 Consumer welfare

In our model, entry is always socially efficient, because the entrant produces at a lower marginal cost than the incumbent. Thus, all miscoordination equilibria entail a productive inefficiency, which is the only source of inefficiency due to the simplifying assumption of inelastic demands.

Yet, prices do matter, firstly because they determine consumer surplus, which is usually considered the objective function of antitrust agencies; secondly, because if we used an elastic demand function, exclusion would also cause an allocative inefficiency. Comparing equilibrium prices across different price regimes is not straightforward because each price regime gives rise to multiple equilibria, both entry and miscoordination equilibria, and each of these can be sustained by a broad range of prices. The approach we take here is to compare the "worst case scenarios" *given* market structure, i.e. the highest sustainable prices under each price regime given that either the incumbent or the entrant serves the buyers.

Proposition 7 (*Consumer surplus*)

- (i) Miscoordination equilibria: Under all three price regimes the highest equilibrium price is the monopoly price, and so consumer surplus is always zero.
- (ii) Entry equilibria: At the highest sustainable prices under each regime, consumer surplus is maximal under explicit discrimination, intermediate under rebates, and minimal under uniform pricing.

Proof: see Appendix A

By combining the results about the conditions under which entry exists and about the price comparisons, we identify the fundamental dilemma we mentioned in the introduction. The more aggressive the price regime the less likely entry will take place (entry equilibria always exist under uniform pricing, and they exist under rebates for a larger region of the parameter space than under explicit discrimination). But when entry equilibria exist - that is, for given market structure - the more aggressive the price regime the higher consumer welfare (in regions where entry exists, prices are the highest under uniform pricing, followed by rebates and then by explicit discrimination).

This explains the difficult task faced by competition policy: by banning price discrimination - in its possible forms - one would reduce the risk of anti-competitive exclusion, but at the risk of chilling competition, and ending up with higher prices. By allowing it, one would foster competition but at the risk of exclusionary outcomes.

5 Extensions

In this Section we deal with a number of extensions of the basic model in order to explore the role of each of the key elements of our base model in generating the results obtained in the previous section. In Section 5.1 we modify the threshold level \bar{s} . In Section 5.2 we consider the possibility that firms subsidize consumption, i.e. can charge negative prices. In Section 5.3, we study the case where buyers' utility increases continuously in the network size. Section 5.4 discusses the case of perfect price discrimination, where firms are allowed to discriminate even across units sold to the same buyer. As it turns out, each of these modifications allows us to generate cases where discrimination also plays *against* the incumbent (not only in its favor), because they open the possibility for the entrant to use price discrimination in order to disrupt miscoordination equilibria (recall that this was not possible so far).

5.1 The role of the threshold (comparative statics)

In the base model, we assume that $\bar{s} > \max\{k, 1 - k\}$: neither by serving all the small buyers nor by serving the large buyer alone would the entrant be able to reach the critical threshold base \bar{s} . This assumption is at the heart of the mechanism of exclusion highlighted by this paper, and the following Proposition studies the case where alternative assumptions on \bar{s} are made.²¹

Proposition 8 (*Varying levels of critical thresholds*)

(a) *If $k/m < (1 - k) < \bar{s} \leq k$ then the equilibrium outcomes are exactly as in the base model.*

(b) *If $k/m < \bar{s} \leq 1 - k$, then under uniform pricing there exists no exclusionary equilibrium, but only an entry equilibrium where all buyers buy from the entrant at $p_E = c_I$. Under discriminatory pricing, if $c_I \leq k$ there is an exclusionary equilibrium where $(p_I^l = 0, p_I^s = 1)$ and all buyers buy from the incumbent. If $c_I > k$, the exclusionary equilibrium does not exist. If $c_I \geq \min\{(1 + c_E)/2, c_E + k\}$ there exists an entry equilibrium where $p_E^l = \max\{(c_I - k)/(1 - k), 0\}$. Otherwise, the entry equilibrium does not exist.*

(c) *If $\bar{s} < k/m < 1 - k$, there exists a unique entry equilibrium where $p_E^l = p_E^s = c_I$.*

Proof. See Appendix A

²¹For shortness we focus on the cases of uniform prices and of explicit discrimination. The case of implicit discrimination - being 'intermediate' among these two - would not give rise to any additional insight.

Comments. This Proposition stresses the importance of network externalities, in the sense that - if consumers value the good only if a network has reached a certain minimum size - the higher that minimum threshold the more difficult for entrants to challenge an incumbent firm.

It also highlights the role of buyers' concentration. For given minimum size \bar{s} , part (b) of Proposition 8 tells us that the existence of a very large buyer is sufficient to avoid miscoordination equilibria, and part (c) that if each buyer commands a large enough demand, then network effects become irrelevant. In other words, we would expect industries with fragmented buyers to be more prone to the type of exclusionary mechanism we have highlighted here. Buyer power would increase the size of the orders an individual buyer would bring, and make it less likely that a dominant incumbent firm may exclude a more efficient but new rival.²²

5.2 Allowing for usage subsidies

In this section, we relax the assumption that prices must be non-negative. Recall that we assume free disposal of the good. Thus, a buyer could exploit negative prices by buying an infinite amount of the good. Therefore, we have to assume that firms can monitor consumption, and that the subsidy is only paid for units that are actually consumed, thus limiting sales to a maximum of $1 - k$ for the large buyer, and k/m for any small buyer.

5.2.1 Uniform prices

Under uniform price offers, the results are the same as in the base model. The *miscoordination equilibrium* cannot be disrupted by negative price offers, because the entrant cannot profitably offer negative prices to all buyers. For the same reason, the *entry equilibrium* will also exist for all parameter values. Therefore, *Propositions 1 and 2 still hold good.*

5.2.2 Explicit price discrimination

We consider first miscoordination equilibria and then entry equilibria.

Miscoordination equilibria The possibility to offer negative prices changes dramatically the analysis of miscoordination equilibria. Consider for instance a natural candidate equilibrium, that is the miscoordination equilibrium prevailing under uniform (non-negative) prices: $(p_I^s = 1, p_I^l = 1)$ and all buyers buy from the incumbent. If firm E sets $p_E^l = p_I^l - \varepsilon = 1 - \varepsilon$ and $p_E^s < 0$, then all buyers will buy from the entrant. Indeed, by buying from the entrant each small buyer would receive a strictly positive surplus $(k/m)(-p_E^s) > 0$ even if nobody else consumed the product. Therefore, they will want to consume

²²The role of buyer power in preventing exclusion is also stressed by Fumagalli and Motta (2008), in a model where - however - scale economies are on the supply-side and discriminatory prices are not considered.

in order to receive the payment. But since it is a dominant strategy for the small buyers to consume the product, the large buyer will now prefer to buy from the entrant as well, since the critical network size will be met, and since $CS^l(p_E^l) = (1-k)(1-p_E^l) > CS^l(p_I^l) = 0$.

More generally, a miscoordination equilibrium with prices (p_I^s, p_I^l) will not exist if the entrant can offer a negative price $p_E^s < 0$ to the small buyers such that $CS^s(p_E^s, s_E < \bar{s}) > CS^s(p_I^s, s_I \geq \bar{s})$ while slightly undercutting the incumbent's offer to the large buyer, $p_E^l = p_I^l - \varepsilon$.²³

Proposition 9 (*Exclusionary equilibria under negative prices*) *Let $\bar{s} > (1-k) + \frac{k}{m}$. Then, if both firms charge negative prices, a miscoordination equilibrium will only exist if $c_I \leq k + c_E$.*

- (i) *If $c_E \leq 1-k$, the equilibrium is characterized by $(p_I^l = 1, p_I^s = 1 - \frac{1}{k}[1-k-c_E])$, $p_E^l \in [0, 1]$, $p_E^s = -\frac{1-k-c_E}{k}$.*
- (ii) *If instead $c_E > 1-k$, the equilibrium is characterized by $p_I^l = p_I^s = 1$, and $p_E^l = p_E^s = 1$ and it exists for all c_I .*

Proof: See Appendix A

Figure 3 illustrates in the space (k, c_I) the region where the miscoordination equilibrium arises, for the case $c_E < 1/2$. It shows that this equilibrium exists only if c_I is sufficiently close to c_E .

The main conclusions from the analysis are as follows. Firstly, when negative prices are possible, allowing for explicit discrimination disrupts miscoordination equilibria when c_I is sufficiently high. Secondly, when a miscoordination equilibrium exists under explicit discrimination (with linear prices which can be negative), the incumbent will not be able to enjoy the monopoly outcome $(p_I^s = 1, p_I^l = 1)$, unless $c_E > 1-k$; the incumbent needs to lower its prices to prevent the entrant from stealing its buyers.

Relative to uniform pricing regimes, where a miscoordination equilibrium which reproduces the monopolistic outcome is always possible, allowing for negative prices has the effect of both rendering miscoordination equilibria less likely, and, where such equilibria survive, of reducing the equilibrium prices. Note that in this case, p_I^s may even be below-cost, i.e. $p_I^s < c_I$!

Entry equilibria The analysis of entry equilibria when we allow for negative prices requires just a small modification of the problem already analyzed in

²³In the case where $\bar{s} \leq (1-k) + \frac{k}{m}$, the entrant might as well charge a negative price to the *large* buyer, while matching *I*'s offer to the small buyers. In this case, as soon as *E* attracted the large buyer, *E* needs just one more buyer to reach the minimum size. Thus, any small buyer will find it optimal to buy from *E* as well, and the miscoordination equilibrium is broken. This is not the case if $\bar{s} > (1-k) + \frac{k}{m}$, where the entrant needs more than one small buyer to reach the minimum size, so that attracting the large buyer is not sufficient to solve the coordination problem among the small buyers. For shortness, we focus on this case.

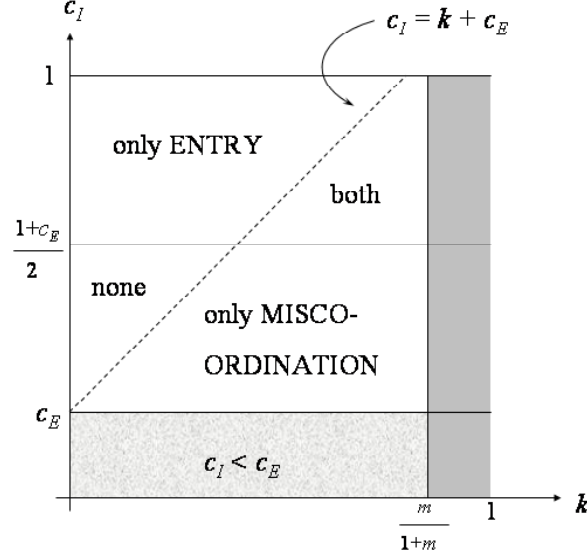


Figure 3: Regions where miscoordination equilibria and/or entry equilibria (or none) exist under negative prices, for $c_E < 1/2$

Section 3.2.2 above, i.e. allowing for p_I^s and p_I^l to take negative values, which was not possible before.

Proposition 10 (*Entry equilibria under negative prices*) *If both firms can use explicit price discrimination and charge negative prices, entry equilibria only exist if $c_I \geq \frac{1+c_E}{2}$. The highest prices that the entrant can charge in any such entry equilibrium are $p_E^s = \frac{c_I - (1-k)}{k}$ and $p_E^l = \frac{c_I - k}{1-k}$.*

Proof: see Appendix A

Figure 3 illustrates entry equilibria. Note that under negative pricing, the incumbent can prevent entry for a larger region of parameter values than under non-negative prices: for values such as $c_I < (1 + c_E)/2$, entry may occur under non-negative prices, but not under negative ones.

The figure also shows that under explicit discrimination, there might be a situation where, for given c_E and k , for c_I sufficiently close to c_E a miscoordination equilibrium exists, for intermediate values of c_I no equilibrium in pure strategies exists, and for high values of c_I only the entry equilibrium will exist. (To be precise, such a situation exists if $c_E < 1/3$). For high values of k ,

there exists an area of parameter values where both miscoordination and entry equilibria will coexist.

To compare results, recall that under uniform pricing both entry and miscoordination equilibria exist under all parameter values. This multiplicity of equilibria in the base case makes it difficult to identify precise policy implications. However incomplete (depending on the values of c_E , there may also exist other regions where no equilibria exist under explicit discrimination, or where multiple equilibria exist also under explicit discrimination), the following Table allows to fix ideas. It shows that for relatively high efficiency gaps between incumbent and entrant, if explicit discrimination schemes are allowed consumer welfare will always be (weakly) higher than under uniform pricing (miscoordination equilibria never exist, and entry equilibria are characterized by (weakly) lower prices). For relatively low efficiency gaps between incumbent and entrant, though, the impact on consumer welfare is not unambiguous: at equilibrium, the incumbent will always serve, and the desirability of explicit discrimination schemes depends on which equilibrium would prevail under uniform pricing: if under uniform pricing a miscoordination equilibrium is played, then explicit discrimination will increase consumer welfare, but if under uniform pricing an entry equilibrium is played, then explicit discrimination leads to exclusion and higher prices. We would then find again the same tension between exclusion and low prices that we have stressed in the main Section above, although it is to be noticed that - apart from very specific cases ($c_E > 1 - k$) - exclusion can be achieved by the incumbent only by decreasing equilibrium prices.

	Uniform pricing	Explicit discrim. (neg. prices)
$c_I > \max \left\{ \frac{1+c_E}{2}, k + c_E \right\}$	$\left\{ \begin{array}{l} I \text{ serves: } p_I^l = p_I^s = 1 \\ \implies CS = 0 \\ E \text{ serves: } p_E^l = p_E^s = c_I \\ \implies CS = 1 - c_I \end{array} \right.$	$E \text{ serves: } \hat{p}_E^l \leq c_I; \hat{p}_E^s \leq c_I \\ \implies CS \geq 1 - c_I$
$c_I < \min \left\{ \frac{1+c_E}{2}, k + c_E \right\}$	$\left\{ \begin{array}{l} I \text{ serves: } p_I^l = p_I^s = 1 \\ \implies CS = 0 \\ E \text{ serves: } p_E^l = p_E^s = c_I \\ \implies CS = 1 - c_I \end{array} \right.$	$I \text{ serves: } \hat{p}_I^l = 1; \hat{p}_I^s \leq c_I \\ \implies CS \leq 1 - c_I$

5.2.3 Implicit price discrimination (rebates)

It would be tedious to characterize all the equilibrium solutions for the case of rebates as well. Like for the case of explicit discrimination, the possibility to set negative prices allows the incumbent to make more aggressive offers, eliminating entry equilibria which would have existed under uniform prices; also, and again like for explicit discrimination, it allows the entrant to subsidize a group of buyers and induce them to use the product independently of what other buyers do, thus leading to the disruption of miscoordination equilibria. The fact that the self-selection constraint needs to be satisfied does not therefore eliminate

the possibility to disrupt some of the equilibria;²⁴ however, it does imply that competition is softer under rebates than under explicit discrimination. Even in this case, therefore, we find the result that rebates are less exclusionary than explicit discrimination, but lead to higher prices when similar equilibrium market structures are compared.

5.3 Continuous utility function

In this Section, we study the case where consumers' gross utility increases continuously in network size, and neither a lower threshold \bar{s} is required to have a positive utility from consumption, nor is there an upper bound on the network externality.

Assume that a buyer of type $j = l, s$ buying from firm $i = I, E$ has a net surplus $CS_i^j = (v(s_i) - p_i) Q^j$, where $v(s_i)$ is continuous, monotone increasing and concave, and where $v(0) = 0$. For simplicity, we restrict attention to the case where there is only one large and one small buyer. Hence, $Q^l = 1 - k$ and $Q^s = k$, with $k < 1/2$. The incumbent has already an established base $\beta_I > 1 - k$.²⁵ Also assume that: $c_I < v(\beta_I)$, which guarantees that the market was viable when the incumbent served 'old' consumers; and that:

$$v(\beta_I + 1)(1 + \beta_I) - c_I < \beta_I v(\beta_I) + v(1) - c_E, \quad (6)$$

which implies that social efficiency is the highest when the entrant serves the new cohort of buyers.²⁶

We shall look for both exclusionary and entry equilibria, under uniform pricing and price discrimination. As a preliminary remark, note that the miscoordination equilibrium characterized in Proposition 1 does not apply here. When network effects are continuous and there is no minimum threshold size for consumers to reach positive utility, it is no longer true that the entrant necessarily needs both large and small buyers. Accordingly, miscoordination results do not arise, even though an exclusionary equilibrium may still arise as an effect of the established base advantage of the incumbent.

5.3.1 Uniform pricing

Proposition 11 (*Continuous network effects, uniform prices*) *If firms set uniform prices and there is no minimum threshold base:*

²⁴At first sight, one may wonder why a buyer may want to buy at positive prices when it could mimic a buyer who is offered a negative price. But recall that a large buyer may get more surplus from buying $1 - k$ units at a positive price than a smaller number of units k/m at a negative price. However, we have seen in Section 3.2.3 that small buyers will never be willing to buy at positive price if they have the chance to buy more units than they need at zero price. A fortiori, this is true when the price offered for a large number of units is negative.

²⁵This is consistent with the assumptions made in the base model, where $\beta_I \geq \bar{s} > \max\{1 - k, k\}$.

²⁶Note that not only does the entrant have to offer higher net surplus to the new cohort, but this extra surplus must outweigh the loss in surplus that the "stranded" old cohort experiences if the new cohort is served by the entrant rather than the incumbent.

(a) If $c_I \leq c_E + [v(\beta_I + 1) - v(1 - k)]$, exclusion arises with the Incumbent selling to buyers at $p_I = c_E + [v(\beta_I + 1) - v(1 - k)]$.

Otherwise, the exclusionary equilibrium does not exist.

(b) The entry equilibrium always exists, with the entrant selling to buyers at $p_E = c_I - [v(\beta_I + 1 - k) - v(1)]$.

Proof. See Appendix A

Contrary to the results in the base model, where both equilibria always exist, here the exclusionary equilibrium exists only when the efficiency gap is small enough or when - for given efficiency gap - the incumbency advantage is sufficiently important (i.e., if the established base β_I of the incumbent is sufficiently important and the network externality is not 'too flat').

Clearly, the fact that there was a minimum threshold that the entrant had to reach for customers to derive utility from consumption was an important element to the advantage of the incumbent. The existence of a smoother function makes it less likely that exclusion will arise.

5.3.2 Explicit price discrimination

Although not conceptually more difficult, the case of continuous network effects does make it more lengthy to find the equilibrium solutions. For this reason, rather than fully characterising the equilibrium solutions, we limit ourselves to stating the following result.

Proposition 12 (*Continuous network effects, explicit discrimination*) *If firms set discriminatory prices and there is no minimum threshold base, relative to uniform pricing:*

(a) *Exclusionary equilibria exist under a narrower region of parameter values.*

(b) *Entry equilibria exist under a narrower region of parameter values.*

Proof. See Appendix A

Modelling network effects as continuous we obtain similar results as when allowing for usage subsidies: for certain parameters, the entrant can overcome the coordination problem by targeting individual buyers, making it a dominant strategy for them to buy from the entrant, thus inducing other buyers to switch as well. This means that price discrimination also allows the entrant to break some exclusionary equilibria, a result in contrast to our benchmark model with minimum threshold and non-negative prices.

However, price discrimination also reduces the scope for entry equilibria (again similar to the model with usage subsidies), because it also allows the incumbent to play a "divide-and-conquer" strategy, making very favorable (below-cost) offers to one group of buyers while recouping the losses on the other group.

As usual, the welfare effects are complex because of multiplicity of equilibria, but it remains true that price discrimination has an ambiguous effect: it makes entry (as well as exclusionary) equilibria less likely, but - if comparing regions where entry equilibria exist both under uniform and under discriminatory prices - it lowers the prices that consumers would have to pay for the good.

5.4 Perfect Price Discrimination

Suppose that the firms can set a different price on each unit sold, i.e. they can discriminate even across units, and restrict attention to non-negative prices.

Assume, without loss of generality, that $1 - k > k$. For simplicity, also assume that to sell the number of units necessary to prevent the entrant from reaching critical size, the incumbent does not have to split orders among buyers:

$$1 - \bar{s} + \epsilon = \bar{m} \frac{k}{m}, \quad (7)$$

where $n \leq m$, and $n, m, \bar{m} \in N$. This assumption will be discussed below.

Proposition 13 (*Perfect discrimination*) *If the firms can discriminate by units, the following describe existence of entry equilibria:*

- (i) *If $c_I \geq c_E + \bar{s}(1 - c_E)$, then $p_E^* = \frac{c_I - \bar{s}}{1 - \bar{s}}$ and all buyers buy from E.²⁷*
- (ii) *If $c_I \geq c_E / (1 - \bar{s})$, then E sells \bar{s} units at $p_E^{\bar{s}} = 0$ and $(1 - \bar{s})$ units at $p_E^{1 - \bar{s}} = c_I$ and all buyers buy from E.*
- (iii) *For lower values of c_I , no entry equilibria exist.*

Proof: see Appendix A

Lemma 14 (*Explicit vs. perfect discrimination*) *Relative to the case of explicit price discrimination, perfect discrimination may either reduce or increase the parameter space where entry equilibria exist.*

Proof: see Appendix A

To understand the logic behind these results, note that there are two effects at play here: On the one hand, perfect price discrimination allows the incumbent to make more aggressive offers, because it can concentrate more rent on fewer buyers; on the other hand, if the entrant can discriminate even among buyers of the same type, making targeted zero-price offers may allow the entrant to secure enough small buyers so that - added to the large buyer - they give the entrant sufficient size \bar{s} . Since prices must be non-negative, a zero price cannot be undercut by the incumbent. Once the entrant reached minimum size, it can

²⁷For simplicity, we focus on equilibria where the entrant sets the same price p_E^* for all units, but there also exist other equilibria where the entrant charges different prices on different units, but the average price it receives equals p_E^* .

then engage in a Bertrand-style competition for the remaining small buyers, thus recovering losses made on the other buyers. This strategy was not available under explicit discrimination (i.e. discrimination by buyer type), since the entrant could not give lower prices to some small buyers but not to others.

Discussion of assumption 7. In general, the size of orders needed to prevent E from reaching critical size will not correspond precisely to an integer multiple of k/m . Define instead $\bar{m} \leq m$ as the lowest integer number of buyers that firm I needs to secure to prevent E from reaching critical size; that is, \bar{m} will satisfy:

$$(\bar{m} - 1) \frac{k}{m} \leq 1 - \bar{s} < \bar{m} \frac{k}{m}. \quad (8)$$

In words, to implement the deviation identified above, the incumbent would have to offer to one buyer some units at a price $p_E - \epsilon$ and other units at a price 1. Obviously, this buyer would not buy from the incumbent as this would deliver him a lower utility than the entrant's offer (which guarantees a price p_E on all units). Hence, to induce this pivotal buyer to buy from it, the incumbent will have to offer the price $p_E - \epsilon$ for *all* units demanded by him.

As a result, the incumbent's profitability condition becomes $\pi_I = \bar{m} p_E k/m + (1 - \bar{m} k/m) - c_I \geq 0$. For the entry equilibrium to be immune from this deviation, it must therefore be: $p_E^* = \max \left\{ 0, \frac{c_I - 1 + \bar{m} k/m}{\bar{m} k/m} \right\}$. From $\bar{m} k/m > 1 - \bar{s}$, it follows that the entry equilibrium will be more likely to exist, and that the price which can be sustained at such equilibrium are higher.

Random coupons One possible instrument of price discrimination is to use discount coupons which are randomly sent by firms. One may think that such coupons are another efficient discriminatory strategy to achieve exclusion, because - similar to the case of perfect price discrimination - they allow for different units to sell at different prices (depending on whether the buyer received a coupon for his purchase or not). It turns out, however, that random coupons cannot reproduce the exclusionary results of perfect discrimination. Suppose for instance the incumbent sent \bar{m} coupons entitling their recipients to buy k/m units at the price $p_E^* - \epsilon$. If all the coupons reached small buyers, this strategy would replicate the optimal deviation under perfect discrimination. However, there is a positive probability that one or more coupons will end up in the hands of the large buyer. Understanding he is pivotal, he will not use such coupons, resulting in the incumbent making losses (he will sell some units at a price $p_E^* - \epsilon < c_I$) without preventing E from reaching critical size. In essence, the randomness of these coupons prevents the incumbent from carefully targeting price cuts to the pivotal buyers, which makes random coupons a less effective tool of discrimination than perfect price discrimination (which is itself, as Lemma

14 shows, does not dominate third-degree discrimination as an exclusionary device).

6 Conclusions, and a policy discussion

Our paper demonstrates the exclusionary potential of price discrimination and rebates in a model where - relative to the literature on exclusionary practices - the entrant is in a fairly good initial position: it is more efficient than the incumbent, it does not have to pay any set-up cost, it can approach buyers at the same time as the incumbent, and it can use the same pricing schemes. However, the incumbent does enjoy an incumbency advantage (when the game starts, its network has already reached the minimum threshold size to be viable, whereas the entrant's has not), and this turns out to be crucial.

We show that - if buyers are sufficiently fragmented and/or the threshold size is sufficiently high and prices are non-negative - both exclusionary equilibria and entry equilibria exist under uniform pricing, and that both explicit and implicit discrimination (that is, rebates) increase the likelihood of an exclusionary outcome: while discrimination does not prevent miscoordination, it makes it easier for the incumbent to disrupt entry equilibria. This is done by a "divide and rule" strategy where some buyers are offered a below-cost price, thereby depriving the entrant of the critical mass it needs, and allowing the incumbent to recover losses from the remaining buyers, who become captive to it.

On the other hand, if we look at regions where entry equilibria exist under all pricing regimes, we find that consumers would be better off when discrimination is allowed: to counter aggressive price cuts from the incumbent, the entrant has to reduce prices, resulting in lower prices for consumers.

These results emphasize a fundamental dilemma that is at the origin of the difficulties of dealing with price abuses in competition law. If antitrust agencies and courts pursued a policy of forbidding discriminatory pricing they might avoid exclusion of efficient entry, but at the cost of having higher prices whenever entry-deterrence is not an issue.

One might be tempted to think that an *asymmetric policy which prohibits below-cost or discriminatory prices by dominant incumbent firms*, while letting the entrant free to choose its pricing policy, might be an appropriate policy option. In fact, such a policy would have two limits. First, as showed by Proposition 1, price discrimination does not enable the entrant to break miscoordination equilibria (unless subsidies could be used, which is not always feasible). Second, it is true that if the incumbent cannot engage in below-cost (or discriminatory) prices, then entry equilibria will exist for the entire parameter space. However, in such entry equilibria consumers would pay the entrant a price equal to the incumbent's marginal cost, which is higher than the price they would pay under explicit discrimination (when the incumbent cannot price below cost, it suffices to set a price slightly below c_I for the entrant to get all buyers). Thus, an asymmetric anti-discrimination policy would have the same effects as a symmetric

imposition of uniform pricing, making exclusion less likely, but raising prices to consumers in the case of entry.

If anything, an asymmetric rule preventing dominant firms from giving usage subsidies may be a more promising road: on the one hand, the incumbent can still discriminate (as long as all prices it offers are non-negative), which preserves some (if not all) of the beneficial effects of discrimination on equilibrium prices in the entry equilibrium; on the other hand, an entrant who can offer usage subsidies is able to break some miscoordination equilibria that could not be broken otherwise, thus facilitating coordination on the socially desirable entry equilibrium.

We have also identified the conditions under which the exclusionary issues studied here may arise at all. In particular, if buyers were sufficiently concentrated, or critical threshold size was sufficiently small, the game would resemble the standard Bertrand model with asymmetric cost, and only entry equilibria would emerge. Also, we have seen that if network effects are modelled in a continuous way, exclusionary outcomes are somewhat less likely, but it would still be true that discrimination - by making competition fiercer - makes it more likely for the incumbent to prevent entry equilibria, while at the same time resulting in lower prices in case an entry equilibrium does emerge.

Allowing for subsidies (i.e. negative prices) does not fundamentally change this insight: while subsidies might allow the entrant to disrupt miscoordination equilibria, they also allow the incumbent to prevent entry equilibria for an even wider region of parameter values. Furthermore, they reduce further the maximal prices that can be sustained in any entry equilibrium. Overall, usage subsidies (i) make exclusion most likely, but (ii) given market structure, result in the lowest prices. Therefore, the trade-off between exclusionary potential and (for given market structure) lower equilibrium prices reappears even when negative prices are allowed.

The possibility of exclusionary outcomes is intimately linked with the assumption that the incumbent has already reached the minimum threshold size (in the base model), or that it has in any case a strong initial customer base (in the model with continuous network effects). For this reason, the mechanism identified in our paper seems well suited to industries (such as those of recent liberalisation or those where a firm's dominant position is built upon intellectual property rights whose protection is about to expire), where entrants can challenge an incumbent firm only after the latter has developed a strong customer base.

Given the trade-off between exclusion and consumer welfare, and given the fairly specific conditions under which the exclusionary mechanism identified here would take place, it would be difficult to advise a policy *prohibiting* price discrimination (even if such a policy was limited to dominant firms, as discussed above). This is not to say that our analysis favours a *laissez-faire* policy. Indeed, we have offered here a possible anti-competitive rationale for price discrimination and rebates. In any abuse of dominance (or, in the US, monopolisation) case,

Antitrust Agencies and Courts have to formulate a theory of harm. What our paper suggests is that - if network or more generally scale effects are at work, the dominant firm has a strong incumbency advantage, and buyers are sufficiently fragmented - the incumbent might use rebates and discriminatory prices in order to exclude as- or more efficient new rivals. Hence, when facing a case with such features, there would be a strong rationale for agencies and courts to argue the anti-competitive effects of discriminatory practices.

In this paper, we have chosen to model scale effects as a demand-side variable, by using network effects and by considering a network's installed base as the incumbency advantage. However, our results would be identical if we assumed there are scale economies on the supply side, and that there is a firm which has already paid its sunk costs, as the incumbency advantage.

Consider the following game. At time 1, firms I and E simultaneously set prices (according to the different price regimes, prices can be uniform or differentiated); at time 2, all buyers decide which firm they want to buy from and make firm orders; at time 3, firm E decides on entry (if it does enter, it has to pay sunk cost $f > 0$); at time 4, payoffs are realized. Like in Section 2, continue to assume that there are m small buyers and 1 large buyer, and let the sunk cost f be large enough so that entry is profitable only if firm E serves the large buyer plus at least one small buyer. With these modifications, results will be of the same nature as those obtained in this paper, and even the calculations will be to a large extent the same.²⁸

Finally, one may wonder how the existence of switching costs (which play an important role in shaping entry in the real world) would change our model. First of all, consider our basic model with network effects. One simple way to take switching costs into account would be to assume that all buyers repeat their purchases, but there are some buyers who (equivalently to the 'old' buyers in our basic model) have arbitrarily large switching costs and therefore would never buy from the entrant, and others who (equivalently to our 'new' buyers) have switching costs σ which are small enough, so that the entrant's effective marginal cost, $c_E + \sigma \equiv \tilde{c}_E$, is still lower than the incumbent's: $c_E + \sigma < c_I$. Provided that there are both large and small buyers among the latter category of buyers, and after replacing c_E with the effective marginal cost \tilde{c}_E , the analysis would be the same as in our model, and the comparative statics on the switching costs would be straightforward. An increase in switching costs σ would be equivalent to an increase in the marginal cost of the entrant, \tilde{c}_E , and would thus lead to more likely exclusionary equilibria. At the other extreme, if all buyers repeat their purchases but switching costs are very small for all of them, then exclusion would be unlikely.

Of course, one could find more sophisticated and interesting ways to incorporate switching costs in the analysis, but it is clear that the basic mechanisms illustrated in this paper would still be at work and would be exacerbated by the existence of switching costs. Both under consumption externalities and un-

²⁸Fumagalli and Motta (2001) study a similar model with economies of scale in production. However, they focus on the role of buyer power and downstream competition, and do not consider price discrimination and rebates (buyers are identical in their model).

der economies of scale, switching costs would add to the incumbency advantage provided by the installed base and the sunk cost, respectively. Note, however, that in our framework, switching costs alone (i.e. without installed base or sunk cost) would not be sufficient to obtain the results.

Similarly, one could think of a model where products have a life of, say, two periods, so that old buyers would not buy today but would buy again tomorrow. Anticipating that the market will include them in the future (absent switching costs and with buyers attaching enough weight to future consumption), only entry equilibria will arise with the current (and the future) cohort of buyers sponsoring the entrant firm. This suggests that the frequency of the renewal of the purchases might have an effect on the structure of the market. Of course, one would need a dynamic model to deal properly with such a situation.

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7 Appendix A - Proofs

Proof of Proposition 1 (*Miscoordination equilibrium, all price regimes*):

Consider the candidate equilibrium where $p_I^s = p_I^l = 1$ and all buyers buy from I . Recall that $\bar{s} > \max \{1 - k, k\}$: none of the individual buyers alone is sufficient for E to reach the minimum size. Thus, no buyer $j = s, k$ will want to deviate and buy from E , even if $p_E^j = 0$, as E 's product would have zero value for the deviating buyer. Firm I has no incentive to increase or decrease its price as it is getting the monopoly profits. Since in all continuation equilibria buyers will not switch to E no matter how low p_E^j is, E has no incentive to decrease its price either.

More generally, there exists a continuum of miscoordination equilibria with any price $p_I^j \in [c_I, 1]$ and buyers $j = s, l$ all buying from I , sustained by the appropriate continuation equilibria. The proof is analogous. First, no buyer has an incentive to deviate and buy from the entrant as the latter would not reach size \bar{s} . Firm I would not have an incentive to increase its price to $p_I^{j'}$ if in the continuation equilibrium where $p_E^j < p_I^{j'}$ buyers would all buy from the entrant (recall that for any pair $p_E^j < p_I^j$ there exist two types of equilibria); firm E would have no incentive to change its prices provided in all continuation equilibria where $p_E^j < p_I^j$ all buyers buy from the incumbent. \square

Proof of Proposition 2 (*Entry equilibria, uniform prices*)

With all buyers buying from E at $p_E = c_I$, total demand is $1 \geq \bar{s}$: E will reach the minimum size. Since E 's product has the same value to the buyers as I 's, and the price is the same, no buyer has an incentive to deviate and buy from I . Firm I will not want to deviate either: To attract buyers, it would have to set $p_I < c_I$, i.e. sell at a loss; and increasing p_I above c_I will not attract any buyers under the appropriate continuation equilibria. Firm E has no incentive to change its price either: increasing p_E would imply losing the buyers to I , and decreasing p_E will just reduce profits.

Note also that, following the same logic, there exists a continuum of entry equilibria with any price $p_E^j \in [c_E, c_I]$ and buyers $j = s, l$ all buying from E , sustained by the appropriate continuation equilibria.

Finally, note that there can be no equilibrium where E serves all buyers at a price $p_E > c_I$: In this case, I could profitably undercut E , and all buyers would switch to I . \square

Proof of Proposition 3 (*Entry equilibria, explicit discrimination*)

Consider a candidate equilibrium where (p_E^s, p_E^l) and all buyers buy from E . For this to be an equilibrium, it must be immune from deviations by the incumbent, which could set $p_I^j < p_E^j$ to buyers of type j to deprive the entrant of the critical scale, and then charge monopoly price $p_I^{-j} = 1$ to the other group of buyers $-j$ ($j = s, l$).

The offer $(p_I^s, 1)$ to attract the small buyers is feasible as long as $\pi_I(p_I^s, 1) = m \frac{k}{m} (-c_I + p_I^s) + (1 - k)(1 - c_I) \geq 0$. Likewise, the offer $(1, p_I^l)$ to attract the large buyer is feasible as long as $\pi_I(1, p_I^l) = (1 - k)(-c_I + p_I^l) + m \frac{k}{m} (1 - c_I) \geq 0$.

Call \hat{p}_I^s and \hat{p}_I^l the prices that solve the equations associated with the two profitability conditions above:

$$\hat{p}_I^s = \frac{c_I - (1 - k)}{k} < c_I; \quad \hat{p}_I^l = \frac{c_I - k}{1 - k} < c_I.$$

The lowest possible deviation prices of the incumbent are identified by $p_I^s = \max(\hat{p}_I^s, 0)$ and $p_I^l = \max(\hat{p}_I^l, 0)$, since prices are non-negative.

The entrant can avoid the incumbent's deviations if it can set prices (p_E^s, p_E^l) such that the incumbent will not find it profitable to undercut either the small or the large buyers: $p_E^s = \max(\hat{p}_I^s, 0)$ and $p_E^l = \max(\hat{p}_I^l, 0)$, while making positive profits: $\pi_E(p_E^s, p_E^l) \geq 0$. By substitution, the entry equilibrium exists if:

$$k \left(\max\left(\frac{c_I - (1 - k)}{k}, 0\right) - c_E \right) + (1 - k) \left(\max\left(\frac{c_I - k}{1 - k}, 0\right) - c_E \right) \geq 0.$$

This identifies four regions, according to values of k and c_I :

$$\begin{aligned} (2) & \text{ if } c_I \in [k, 1 - k] \text{ and } k < 1/2: \quad \pi_E(0, \hat{p}_I^l) \geq 0 \\ (3) & \text{ if } c_I \in [1 - k, k] \text{ and } k \geq 1/2: \quad \pi_E(\hat{p}_I^s, 0) \geq 0 \\ (4) & \text{ if } c_I < \min\{k, 1 - k\}: \quad \pi_E(0, 0) \geq 0 \\ (1) & \text{ else: } \pi_E(\hat{p}_I^s, \hat{p}_I^l) \geq 0 \end{aligned} \tag{9}$$

After replacing, we can then find that:

$$(1) \quad \pi_E(\hat{p}_I^s, \hat{p}_I^l) = k \left(\frac{c_I - (1 - k)}{k} - c_E \right) + (1 - k) \left(\frac{c_I - k}{1 - k} - c_E \right) \geq 0, \text{ satisfied for } c_I \geq (1 + c_E)/2 \equiv \bar{c}_{I1}.$$

$$(2) \quad \pi_E(0, \hat{p}_I^l) = -c_E k + (1 - k) \left(\frac{c_I - k}{1 - k} - c_E \right) \geq 0, \text{ which holds for } c_I \geq k + c_E \equiv \bar{c}_{I2}.$$

$$(3) \quad \pi_E(\hat{p}_I^s, 0) = k \left(\frac{c_I - (1 - k)}{k} - c_E \right) - c_E(1 - k) \geq 0, \text{ which holds for } c_I \geq 1 + c_E - k \equiv \bar{c}_{I3}.$$

$$(4) \quad \pi_E(0, 0) = -c_E \geq 0, \text{ which never holds, apart from the knife-edge case where } c_E = 0.^{29}$$

Finally, straightforward algebra shows that if $c_I \geq \max\{k, 1 - k\}$, so that threshold $\bar{c}_{I1} = \frac{1 + c_E}{2}$ applies, we have that $\bar{c}_{I1} = \min\{\bar{c}_{I1}, \bar{c}_{I2}, \bar{c}_{I3}\}$, and the analogous relation holds for the other two threshold values of c_I : in the parameter region where \bar{c}_{Ii} applies, $\bar{c}_{Ii} = \min\{\bar{c}_{I1}, \bar{c}_{I2}, \bar{c}_{I3}\}$. \square

Proof of Proposition 5 (*Entry equilibria, implicit discrimination*)

Any equilibrium where the entrant serves the buyers must satisfy two conditions:

²⁹Since prices cannot go below zero in this basic model, the best that the incumbent can offer to buyers is to give them the good for free; but when $c_E = 0$, the entrant could match that offer without making losses, and entry equilibria would always exist. Clearly, though, this is a very special case.

(i) the entrant's prices (p_E^s, p_E^l) must be immune to all profitable deviations by the incumbent. The important difference to the case of explicit discrimination is that the incumbent's offers now have to satisfy the self-sorting constraints, either (SSlarge) or (SSsmall), in addition to the break-even constraint (BE).

(ii) the entrant's prices (p_E^s, p_E^l) must themselves satisfy the self-sorting constraints. We will show below that this is implied by the price pairs that are constructed to satisfy condition (i).

Ad (i): When stealing the small buyers at the expense of the large buyer, the incumbent can no longer charge the large buyer $p_I^l = 1$ (as under explicit discrimination): at this price the large buyer is left with zero surplus, and so the large buyer's self-sorting constraint is bound to be violated. Thus, the price pair giving maximum surplus to the small buyers is fully determined by the following two constraints:

$$(p_I^s - c_I)k + (p_I^l - c_I)(1 - k) \geq 0 \quad (\text{BE})$$

$$(1 - p_I^l)(1 - k) \geq \frac{k}{m} (1 - p_I^s) \quad (\text{SSlarge})$$

Note that (SSlarge) will always be binding, and that p_I^s must be non-negative. Call the solution to this problem $(\tilde{p}_I^s, \tilde{p}_I^l)$.

Likewise, if the incumbent wants to steal the large buyer at the expense of the small buyers, the price offered to the small buyers must satisfy their self-sorting constraint. The price pair that gives maximum surplus to the large buyer solves:

$$(p_I^s - c_I)k + (p_I^l - c_I)(1 - k) \geq 0 \quad (\text{BE})$$

$$\frac{k}{m} (1 - p_I^s) \geq \frac{k}{m} - p_I^l(1 - k) \quad (\text{SSsmall})$$

If the small buyers are sufficiently fragmented, i.e. if m is high enough, then (SSsmall) may not be binding, i.e. the incumbent can charge price $p_I^s = 1$ as under explicit price discrimination (and price $p_I^l = \frac{c_I - k}{1 - k}$ to the large buyer). Note that the non-negativity constraint on p_I^l will never be binding (a price of zero is incompatible with self-sorting by small buyers). Call the solution to this problem $(\bar{p}_I^s, \bar{p}_I^l)$.

Then, the incumbent's optimal (deviation) offers to both small and large buyers can be summarized as follows:

$$\tilde{p}_I^s = \begin{cases} 1 - \frac{m(1-c_I)}{k(m+1)} & \text{if } c_I \geq 1 - k - k/m \\ 0 & \text{if } c_I < 1 - k - k/m \end{cases} \quad \tilde{p}_I^l = \begin{cases} \frac{c_I - k}{1 - k} & \text{if } c_I \geq \frac{k(1+m)}{m} \\ \frac{c_I}{(1-k)(m+1)} & \text{if } c_I < \frac{k(1+m)}{m} \end{cases}$$

Again, these prices represent the upper bound on the prices that the entrant can charge in any entry equilibrium. For entry to be feasible, $(\tilde{p}_I^s, \tilde{p}_I^l)$ must be high enough to allow the entrant to break even. The functions $(\tilde{p}_I^s, \tilde{p}_I^l)$ identify four regions, and for each of them we have to verify whether the entrant's break-even condition holds or not:

- (i) if $c_I \in \left[1 - k - k/m, \frac{k(1+m)}{m}\right]$ and $k \geq \frac{m}{2(1+m)}$: $\pi_E(1 - \frac{m(1-c_I)}{k(m+1)}, \frac{c_I}{(1-k)(m+1)}) \geq 0$
- (ii) if $c_I \in \left[\frac{k(1+m)}{m}, 1 - k - k/m\right]$ and $k < \frac{m}{2(1+m)}$: $\pi_E(0, \frac{c_I - k}{1-k}) \geq 0$
- (iii) if $c_I < \min\left\{\frac{k(1+m)}{m}, 1 - k - k/m\right\}$: $\pi_E(0, \frac{c_I}{(1-k)(m+1)}) \geq 0$
- (iv) else: $\pi_E(1 - \frac{m(1-c_I)}{k(m+1)}, \frac{c_I - k}{1-k}) \geq 0$

After replacing, we can then find that:

- (i) $\pi_E(1 - \frac{m(1-c_I)}{k(m+1)}, \frac{c_I}{(1-k)(m+1)}) \geq 0$ holds for $c_I \geq \frac{m}{1+m} + c_E - k$
- (ii) $\pi_E(0, \frac{c_I - k}{1-k}) \geq 0$ holds for $c_I \geq k + c_E$
- (iii) $\pi_E(0, \frac{c_I}{(1-k)(m+1)}) \geq 0$ holds for $c_I \geq c_E(1 + m)$
- (iv) $\pi_E(1 - \frac{m(1-c_I)}{k(m+1)}, \frac{c_I - k}{1-k}) \geq 0$ is satisfied for $c_I \geq \frac{m+(1+m)c_E}{1+2m}$

If $c_E < \frac{1}{2(m+1)}$, then we have that $c_E(1 + m) < \frac{m+(1+m)c_E}{1+2m} < \frac{1}{2}$. Tedious algebra shows that in this case, $c_I \geq \frac{m+(1+m)c_E}{1+2m}$ is redundant, and that each of the remaining thresholds is the minimum of all thresholds in the parameter region where it applies. Conversely, if $c_E \geq \frac{1}{2(m+1)}$, then we have that $\frac{m+(1+m)c_E}{1+2m} < c_E(1 + m)$ and $c_E(1 + m) \geq \frac{1}{2}$. In this case, $c_I \geq c_E(1 + m)$ is redundant, and each of the remaining thresholds is the minimum of all thresholds in the parameter region where it applies.

Ad (ii): If firms practice implicit rather than explicit discrimination, then the entrant's equilibrium offers must satisfy the self-sorting constraints. As it turns out, the latter are always satisfied whenever the entrant's offers are constructed to be immune against the incumbent's deviations, i.e. if $(p_E^s, p_E^l) = (\tilde{p}_I^s, \tilde{p}_I^l)$: If $\tilde{p}_I^s < \tilde{p}_I^l$, then only the large buyer's self-selection constraint could be violated (but not the small buyers'). But recall that \tilde{p}_I^s satisfies the large buyer's self-selection constraint (SSlarge) by construction. Now, the price that the large buyer is charged in the entry equilibrium is of course lower than the one it would be charged if the incumbent were to steal the small buyers and to recover the losses on the large buyer, i.e. we have that $\bar{p}_I^l > \tilde{p}_I^l = p_E^l$. But that implies that $(\tilde{p}_I^s, \tilde{p}_I^l)$ must also satisfy the large buyer's self-selection condition. The reasoning is exactly analogous for the case where $\tilde{p}_I^s > \tilde{p}_I^l$. \square

Proof of Corollary 6

Under explicit discrimination, the lower bound on c_I for entry equilibria to exist is $\min\left\{\frac{1+c_E}{2}, k + c_E, 1 - k + c_E\right\}$. Now, if $c_E < \frac{1}{2(m+1)}$, the corresponding condition under rebates reads $c_I \geq \min\left\{c_E(1 + m), k + c_E, \frac{m}{1+m} + c_E - k\right\}$. Comparing the components of the two sets, we see that the second component is the same, $k + c_E = k + c_E$. The third component is lower under rebates, $\frac{m}{1+m} +$

$c_E - k < 1 - k + c_E$. Finally, $c_E < \frac{1}{2(m+1)}$ implies that $c_E(1+m) < \frac{1+c_E}{2}$, i.e. the first component is lower under rebates as well. If instead $c_E \geq \frac{1}{2(m+1)}$, the first component under rebates is $\frac{m+(1+m)c_E}{1+2m}$, which is always smaller than $\frac{1+c_E}{2}$. Thus, we can conclude that the parameter space for which entry equilibria exist under rebates fully includes the corresponding parameter space under explicit discrimination. \square

Proof of Proposition 7 (*Consumer surplus*)

Under all three price regimes, buyers consume the same quantities. Thus, their consumer surplus is solely determined by the price they pay: the higher the price, the lower is consumer surplus.

(i) It follows immediately from Proposition 1.

(ii) The following table shows the prices buyers pay under each of the three price regimes. The inequalities follow from simple algebra.

Table 1: Highest Sustainable Prices in Entry Equilibria					
	Uniform		Implicit		Explicit
Large Buyer:					
$c_I < k$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I}{(1-k)(m+1)}$	$>$	$p_E^l = 0$
$c_I \in \left[k, \frac{k(1+m)}{m}\right)$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I}{(1-k)(m+1)}$	$>$	$p_E^l = \frac{c_I - k}{1-k}$
$c_I \geq \frac{k(1+m)}{m}$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I - k}{1-k}$	$=$	$p_E^l = \frac{c_I - k}{1-k}$
Small Buyers:					
$c_I < 1 - k - k/m$	$p_E^s = c_I$	$>$	$p_E^s = 0$	$=$	$p_E^s = 0$
$c_I \in [1 - k - k/m, 1 - k)$	$p_E^s = c_I$	$>$	$p_E^s = 1 - \frac{m(1-c_I)}{k(m+1)}$	$>$	$p_E^s = 0$
$c_I \geq 1 - k$	$p_E^s = c_I$	$>$	$p_E^s = 1 - \frac{m(1-c_I)}{k(m+1)}$	$>$	$p_E^s = \frac{c_I - (1-k)}{k}$

The ranking of consumer surplus is the reverse of the ranking of prices, and can be summarized thus:

- $CS_{\text{expl}}^l \geq CS_{\text{impl}}^l > CS_{\text{unif}}^l > 0$ with strict inequality if $c_I < \frac{k(1+m)}{m}$;
 - $CS_{\text{expl}}^s \geq CS_{\text{impl}}^s > CS_{\text{unif}}^s > 0$ with strict inequality if $c_I \geq 1 - k - k/m$.
- \square

Proof of Proposition 8 (*Varying levels of critical threshold*)

(a) Suppose $k > 1 - k$, and $k/m < (1 - k) < \bar{s} \leq k$, (this implies that $m \geq 2$). In this case, miscoordination would still arise, given that no single buyer would bring enough size to the entrant. As usual, there will also exist the entry equilibrium. Qualitatively, the results of Propositions 1 to 3 still hold good. The only difference is that now, the region where entry equilibria exist is larger: If $(1 - k) < \bar{s} \leq k$, the incumbent can lock in the large buyer after stealing the small buyers, but not the other way round: the group of small buyers is sufficient to generate critical size, even if the large buyer does not join E 's

network. This implies that the large buyer will never be offered a price below c_I : $p_I^l \geq c_I$. To prevent the incumbent from stealing the small buyers, the entrant must set p_E^s such that $\pi_I = (p_E^s - c_I)k + (1 - c_I)(1 - k) \leq 0$. For the entrant to break even at candidate equilibrium prices ($p_E^l = c_I, p_E^s = (c_I - 1 + k)/k$), we must have $\pi_E = p_E^s k + c_I(1 - k) - c_E \geq 0$, which can be rearranged to read $c_I \geq (c_E + 1 - k)/(2 - k)$.

(b) Suppose $k < \bar{s} \leq 1 - k$. Consider first *uniform pricing*. A *miscoordination equilibrium* with $p_I \geq c_I$ and all buyers buying from I cannot exist. If E sets $c_I - \epsilon$, the large buyer would buy from it and get positive utility. Knowing that, small buyers would buy from E as well. It is easy to see that the *entry equilibrium* always exists, with $p_E = p_I = c_I$ and all buyers buying from E .

Consider next *discriminatory* (non-negative) pricing. Consider a *miscoordination equilibrium* where I sets ($p_I^l < c_I, p_I^s = 1$) and all buyers buy from the incumbent. The entrant could break this equilibrium by setting ($p_E^l = p_I^l - \epsilon, p_E^s = 1 - \epsilon$), thus making it a dominant strategy for the large buyer to buy from E , and in turn making the small buyers buy from E as well. The incumbent could prevent this deviation only by setting ($p_I^l = 0, p_I^s = 1$). Under the assumption that prices are non-negative, the entrant cannot attract the large buyer, and the miscoordination equilibrium cannot be broken. The equilibrium is feasible as long as $\pi_I \geq -c_I(1 - k) + (1 - c_I)k \geq 0$, or $c_I \leq k$.

As for the *entry equilibrium*, the only deviation which could threaten it is the one where the incumbent attracts the large buyer, thus preventing the entrant from reaching its minimum base. Therefore, the candidate equilibrium must be of the type ($p_E^l < c_I, p_E^s = c_I$). To avoid the incumbent's deviation, it must be: $\pi_I = (p_E^l - c_I)(1 - k) + (1 - c_I)k \leq 0$. Hence, $p_E^l = \max\{(c_I - k)/(1 - k), 0\}$. This is profitable for E as long as $c_I \geq \min\{(1 + c_E)/2, c_E + k\}$. (Unlike the base model, here there is no need to lower the price for small buyers, as they are not needed to reach the minimum customer base.)

(c) If $\bar{s} < k/m < 1 - k$. In this case, any buyer would guarantee enough scale to the entrant, and everything will be as in the standard Bertrand game with asymmetric firms. Suppose there is an exclusionary equilibrium with $p_I^j \geq c_I$, ($j = l, s$) and all buyers buy from I . Clearly, the entrant could undercut the incumbent and profitably get buyer of type j , and this deviation cannot be prevented. It is also straightforward to check that the entry equilibrium with $p_E^j = c_I$ and all buyers buying from E cannot be disrupted. If the incumbent undercuts the entrant on the type- j buyer and set $p_I^j = c_I - \epsilon$ it would just get that buyer; clearly, it would get negative profits and the deviation would not be profitable. In order to obtain enough buyers to prevent entry, it should get all the buyers (recall that discrimination within the same group of buyers is not possible), which is not profitable. \square

Proof of Proposition 9 (*Exclusionary equilibria under negative prices*)

To make it a dominant strategy for a small buyer to buy from E , E must offer a price p_E^s that yields a (weakly) higher net surplus as I 's offer to the small buyers: $-p_E^s \frac{k}{m} \geq \frac{k}{m}(1 - p_I^s)$, whence $p_E^s \leq -(1 - p_I^s) < 0$. If the small buyers

consume E 's product for sure, then the large buyer will switch to E whenever $p_E^l \leq p_I^l$.

To check whether E will find it profitable to carry out this deviation, insert $p_E^s = -(1 - p_I^s)$ and $p_E^l = p_I^l$ into $\pi_E(p_E^s, p_E^l) \geq 0$, to obtain $\pi_E = -k(1 - p_I^s) - c_E + p_I^l(1 - k) \geq 0$.

This implies that, for a candidate exclusionary equilibrium to be immune from the deviation of the entrant, given p_I^l , the incumbent should solve the following problem:

$$\begin{aligned} \max_{p_I^s, p_I^l} \pi_I &= (p_I^s - c_I)k + (p_I^l - c_I)(1 - k) \\ \text{s.t. (1) } p_I^l &\leq 1; \text{ (2) } p_I^s \leq \min \left\{ 1 - \frac{1}{k} [p_I^l(1 - k) - c_E], 1 \right\} \end{aligned}$$

and obtain positive profits. It is easy to see that:

(i) If $c_E \leq 1 - k$, the programme is solved by $p_I^l = 1$ and $p_I^s = \frac{1}{k}[1 - k - c_E]$. By substitution, $\pi_I = k + c_E - c_I$, which entails that the equilibrium exists only if $c_I \leq k + c_E$.

(ii) If $c_E > 1 - k$, the programme is solved by $p_I^s = p_I^l = 1$, and π_I will always be positive. Therefore, the equilibrium exists for all values of c_I . \square

Proof of Proposition 10 (*Entry equilibria under negative prices*)

By following the same steps as in the proof of Proposition 3, one can check that the lowest deviation prices that the incumbent can profitably set are: $\hat{p}_I^s = \frac{c_I - (1 - k)}{k}$ and $\hat{p}_I^l = \frac{c_I - k}{1 - k}$. An entry equilibrium will exist only if the entrant is able to set $p_E^s = \hat{p}_I^s$, and $p_E^l = \hat{p}_I^l$ so as to prevent deviations on both large and small buyers. Therefore, such an equilibrium exists if and only if:

$$\pi_E(\hat{p}_I^s, \hat{p}_I^l) = k \left(\frac{c_I - (1 - k)}{k} - c_E \right) + (1 - k) \left(\frac{c_I - k}{1 - k} - c_E \right) \geq 0,$$

which is satisfied for $c_I \geq (1 + c_E)/2$. \square

Proof of Proposition 11 (*Continuous network effects, uniform pricing*)

a) Consider an exclusionary equilibrium where firm I sets p_I and both buyers buy from it. For the entrant to break this equilibrium, it should set a price p_E at which either the small or the large buyer (or both) get higher surplus than at the candidate equilibrium. To win the small buyer, the entrant should set $p_E < p_I - [v(\beta_I + 1) - v(k)] \equiv p_E^s$ and to win the large buyer, it should set $p_E < p_I - [v(\beta_I + 1) - v(1 - k)] \equiv p_E^l$.³⁰ Since $p_E^l > p_E^s$ the latter deviation is more profitable. For the exclusionary equilibrium to be immune from deviations, the entrant should find it unprofitable to charge p_E^l , that is,

³⁰ The key difference relative to the threshold case is as follows. When a single buyer of type $j = s, l$ unilaterally deviates to the entrant, his surplus will be proportional to $v(j) - p_E$. In this Section, $v(1 - k) > v(k) > 0$, whereas in the base model, $v(k) = v(1 - k) = 0$, so there was no non-negative price firm E could charge that would induce a buyer to unilaterally deviate from the exclusionary equilibrium.

it must be $p_E^l < c_E$, or $p_I = c_E + [v(\beta_I + 1) - v(1 - k)]$. Of course, this price can be an equilibrium only if $\pi_I = c_E + [v(\beta_I + 1) - v(1 - k)] - c_I \geq 0$, or $c_I \leq c_E + [v(\beta_I + 1) - v(1 - k)]$.

(b) Consider an entry equilibrium where firm E sets p_E and both buyers buy from it. Similarly to the analysis above, the incumbent's most profitable deviation would be to attract the large buyer, that is, to set $p_I < p_E + v(\beta_I + 1 - k) - v(1) \equiv p_I^l$. The deviation is profitable as long as $p_I^l > c_I$. Hence, if firm E is able to set the price $p_E \leq c_I - [v(\beta_I + 1 - k) - v(1)]$ the deviation will be avoided. This amounts to requiring that $p_E = c_I - [v(\beta_I + 1 - k) - v(1)] \geq c_E$, which is always verified under the assumption of efficient entry. \square

Proof of Proposition 12 (*Continuous network effects, explicit discrimination*)

a) At the candidate equilibrium, buyers' surplus from buying from I is respectively $CS_I^l = [v(\beta_I + 1) - p_I^l](1 - k)$ and $CS_I^s = [v(\beta_I + 1) - p_I^s]k$. In order to induce a unilateral deviation from the large buyer, the entrant should set a price p_E^l such that $CS_E^l = [v(1 - k) - p_E^l](1 - k) > [v(\beta_I + 1) - p_I^l](1 - k)$, i.e. it should offer a price $p_E^l < p_I^l - [v(\beta_I + 1) - v(1 - k)]$. Given such a price p_E^l , the large buyer will buy from E no matter who the small buyer buys from. But then, the small buyer will anticipate that E 's network will have at least size $1 - k$. If the small buyer decides to switch as well, E 's network will have a size of 1. If the small buyer instead stays with I , then I 's network will have size $\beta_I + k$. Therefore, to induce the small buyer to switch, E must offer a price p_E^s such that $CS_E^s = [v(1) - p_E^s]k > [v(\beta_I + k) - p_I^s]k$, i.e. E 's offer must satisfy $p_E^s < p_I^s - [v(\beta_I + k) - v(1)]$. This deviation is profitable if $\pi_E = (1 - k)p_E^l + kp_E^s - c_E \geq 0$ which after substituting becomes: $(1 - k)(p_I^l - [v(\beta_I + 1) - v(1 - k)]) + k(p_I^s - [v(\beta_I + k) - v(1)]) - c_E \geq 0$. Therefore, for the pair of prices set by the incumbent to be immune from deviations, it must be:

$$(1 - k)p_I^l + kp_I^s \leq c_E + (1 - k)([v(\beta_I + 1) - v(1 - k)]) + k[v(\beta_I + k) - v(1)].$$

It will be profitable for I to set this pair of prices if $\pi_I' = (1 - k)p_I^l + kp_I^s - c_I \geq 0$, or:

$$c_I \leq c_E + (1 - k)([v(\beta_I + 1) - v(1 - k)]) + k[v(\beta_I + k) - v(1)] \equiv c_I^{pdl}.$$

This is a *necessary* condition for an exclusionary equilibrium to exist under discriminatory pricing. The condition for the equilibrium under uniform pricing was $c_I \leq c_E + v(\beta_I + 1) - v(1 - k) \equiv c_I^{up}$. It is easy to check that $c_I^{pdl} < c_I^{up}$, implying that price discrimination makes this equilibrium less likely to exist.

For a miscoordination equilibrium to exist at all, it must also be immune to another deviation, whereby the entrant first tries to induce a unilateral deviation by the small buyer. In that case, E would offer $p_E^s < p_I^s - [v(\beta_I + 1) - v(k)]$. Since for the *small* buyer it would be a dominant strategy to buy from E , the large buyer will anticipate that E 's network will have at least size k . Then, to induce the large buyer to switch, it would be sufficient for E to offer a price p_E^l

such that $CS_E^l = [v(1) - p_E^l] (1 - k) > [v(\beta_I + 1 - k) - p_I^l] (1 - k)$, i.e. E 's offer must satisfy $p_E^l < p_I^l - [v(\beta_I + 1 - k) - v(1)]$.

For a miscoordination equilibrium (p_I^l, p_I^s) to be immune from this deviation as well, it must be:

$$(1 - k)p_I^l + kp_I^s - c_I \leq c_E + (1 - k) ([v(\beta_I + 1 - k) - v(1)]) + k [v(\beta_I + 1) - v(k)] - c_I,$$

which is profitable for the incumbent if:

$$c_I \leq c_E + (1 - k) ([v(\beta_I + 1 - k) - v(1)]) + k [v(\beta_I + 1) - v(k)] \equiv c_I^{pds}.$$

Therefore, the necessary and sufficient condition for a miscoordination equilibrium to exist are as follows: $c_I \leq \min \{c_I^{pds}, c_I^{pdl}\}$. Simple (albeit tedious) algebra shows that indeed there exist parameter values where $c_E + (1 + \beta_I) v(\beta_I + 1) - \beta_I v(\beta_I) - v(1) - c_E \leq c_I \leq \min \{c_I^{pds}, c_I^{pdl}\}$, the first inequality being the assumption of efficient entry.

b) Proceed analogously to prove the conditions for the entry equilibrium. To induce a unilateral deviation from an entry equilibrium by the large buyer, the incumbent should set a price such that $CS_I^l = [v(\beta_I + 1 - k) - p_I^l] (1 - k) > CS_E^l = [v(1) - p_E^l] (1 - k)$; then, to "steal" the small buyer as well, the offer to the small buyer must satisfy $CS_I^s = [v(\beta_I + 1) - p_I^s] k > CS_E^s = [v(k) - p_E^s] k$. The deviation is profitable if $\pi_I = (1 - k)p_I^l + kp_I^s - c_I \geq 0$, where $p_I^l = p_E^l + v(\beta_I + 1 - k) - v(1)$, and $p_I^s = p_E^s + v(\beta_I + 1) - v(k)$. Hence, an entry equilibrium would be immune to such a deviation if $(1 - k)p_I^l + kp_I^s - c_I = 0$, and satisfies the break-even condition if $(1 - k)p_I^l + kp_I^s - c_I \geq c_E$. Equivalently, we can write:

$$c_I \leq c_E + (1 - k) [v(\beta_I + 1 - k) - v(1)] + k [v(\beta_I + 1) - v(k)] \equiv c_1.$$

The other possible deviation is to first "target" the small buyer, and then the large buyer, i.e. offer $p_I^s < p_E^s + (v(\beta_I + k) - v(1))$ and $p_I^l < p_E^l + (v(\beta_I + 1) - v(1 - k))$. Proceeding as above, one obtains that this amounts to requiring:

$$c_I \leq c_E + (1 - k) (v(\beta_I + 1) - v(1 - k)) + k (v(\beta_I + k) - v(1)) \equiv c_2.$$

For an entry equilibrium to exist, it must be that $c_I \leq \min \{c_1, c_2\}$. Simple algebra shows that there exist values which satisfy this condition (while simultaneously satisfying the condition for efficient entry), and under which then an entry equilibrium would exist. However, under uniform pricing the entry equilibrium always existed, whereas here it exists only for some values of the parameter space. \square

Proof of Proposition 13 (*Perfect discrimination*)

(i) Consider a candidate equilibrium where E sells all units at a price p_E . The incumbent may deviate by selling $1 - \bar{s} + \epsilon$ units at the price $p_E - \epsilon$ (thereby

securing enough units to prevent E from reaching critical size), and the remaining $\bar{s} - \epsilon$ units at the monopoly price 1.³¹ This deviation is profitable if $\pi_I = (1 - \bar{s})p_E + \bar{s} - c_I \geq 0$. For the entry equilibrium to be immune from this deviation, it must therefore be: $p_E^* = \max\left\{0, \frac{c_I - \bar{s}}{1 - \bar{s}}\right\}$. This equilibrium exists if $\pi_E(p_E^*) = \max\left\{0, \frac{c_I - \bar{s}}{1 - \bar{s}}\right\} - c_E \geq 0$. Clearly, a necessary condition for the profitability condition to hold must be that $p_E^* \geq 0$, i.e. that $c_I \geq \bar{s}$. Further, it must be that $c_I \geq c_E + \bar{s}(1 - c_E)$.

(ii) Another natural candidate equilibrium is one where the entrant sells the \bar{s} units it needs to secure at a price $p_E^{\bar{s}}$, and the remaining units at a higher price $p_E^{1-\bar{s}}$, with all units sold by it.

Consider first the case where $0 < p_E^{\bar{s}} \leq p_E^{1-\bar{s}}$. In this case, the incumbent's optimal deviation would be to set the price $p_E^{1-\bar{s}} - \epsilon$ for $1 - \bar{s} + \epsilon$ units, thereby securing enough units to make sure the entrant does not reach critical size, and set the price 1 for the remaining $\bar{s} - \epsilon$ units. This deviation is profitable if $\pi_I = (1 - \bar{s})p_E^{1-\bar{s}} + \bar{s} - c_I \geq 0$, which is the same condition as above. It follows that the entrant should set the price for all units at $p_E^{\bar{s}} = p_E^{1-\bar{s}} = p_E^*$: we fall back to the case analysed under (i).³²

But consider now the case where $0 = p_E^{\bar{s}} \leq p_E^{1-\bar{s}} = c_I$. In this case, due to the assumption that prices are non-negative, the incumbent cannot subtract any of the units sold by the entrant at the zero price. Since the entrant has secured the \bar{s} units it needs, the equilibrium cannot be broken by an incumbent's deviation. The pair $(0, c_I)$ must guarantee positive profits to the entrant: $\pi_E(0, c_I) = -c_E\bar{s} + (c_I - c_E)(1 - \bar{s}) \geq 0$. The equilibrium then exists if $c_I \geq c_E/(1 - \bar{s})$. \square

Proof of Lemma 14 (*Explicit vs. perfect discrimination*)

By combining the existence conditions obtained so far, we conclude that an entry equilibrium exists if

$$c_I \geq \min\{c_E/(1 - \bar{s}), c_E + \bar{s}(1 - c_E)\}.$$

Recall that the analogous condition for entry under explicit discrimination reads:

$$c_I \geq \min\left\{\frac{1 + c_E}{2}, k + c_E, 1 - k + c_E\right\}.$$

It is possible to show that $c_E + \bar{s}(1 - c_E) > \min\left\{\frac{1 + c_E}{2}, k + c_E, 1 - k + c_E\right\}$, but also that there are values for which $c_E/(1 - \bar{s}) < \min\left\{\frac{1 + c_E}{2}, k + c_E, 1 - k + c_E\right\}$.

This implies that perfect discrimination, may either reduce or increase the parameter space for which pure strategy entry equilibria exist. \square

³¹It may be useful to recall that by assumption $\bar{s} > \max(k, 1 - k)$, from which it follows that $\bar{s} > 1/2$.

³²The same result would occur if $0 < p_E^{1-\bar{s}} \leq p_E^{\bar{s}}$. The incumbent would set $p_E^{\bar{s}} - \epsilon$ for $1 - \bar{s} + \epsilon$ units, and 1 for the remaining $\bar{s} - \epsilon$ units. This deviation is profitable if $\pi_I = (1 - \bar{s})p_E^{\bar{s}} + \bar{s} - c_I \geq 0$. Hence, the entrant should sell all units at $p_E^{\bar{s}} = p_E^{1-\bar{s}} = p_E^*$.