Ideological Consistency and Valence

BSE Working Paper 1383| November 2022

Enriqueta Aragonès, Dimitrios Xefteris

Ideological Consistency and Valence*

Enriqueta Aragonès[†]
Institut d'Anàlisi Econòmica, CSIC and Barcelona School of Economics

Dimitrios Xefteris[‡]
University of Cyprus

November 21, 2022

Abstract

We study electoral competition between two win-motivated candidates, considering that voters care both about the valence and the ideological consistency of the competing candidates. When valence asymmetries are not too large we find a unique pure strategy Nash equilibrium in which (i) platform polarization (i.e. the distance between the candidates' policy proposals) is solely determined by the strength of preferences for consistency, and (ii) the expected policy outcome may move to the right as the valence of the leftist candidate increases. When valence differences are large, a mixed equilibrium emerges: the high-valence left-wing candidate chooses a moderate right policy and the low-valence right-wing candidate responds, usually, with an extreme right position and, occasionally, with a moderate left one. Our analysis provides novel insights regarding candidates' flip-flopping incentives, and parties' motives to nominate low-quality candidates.

Keywords: valence; ideology; consistency; flip-flopping; electoral competition; mixed equilibrium.

JEL classification: D72

---

*We thank participants at seminars at Stanford Graduate School of Business, University of California, San Diego, Columbia University, IAE-CSIC, and University of Quebec at Montreal for their useful comments. Aragonès acknowledges financial support by the Generalitat de Catalunya grant number 2017-SGR-1359, the Spanish Ministry of Science, Innovation and Universities grant number PGC2018-097898-B-100, and the Barcelona School of Economics through the Severo Ochoa Programme for Centres of Excellence in R&D CEX2019-000915-S.

[†]Institut d'Anàlisi Econòmica, CSIC, Campus UAB, 08193 Bellaterra (Spain). Email: enriqueta.aragones@iae.csic.es

[‡]Department of Economics, University of Cyprus, PO Box, 20537, 1678, Cyprus. Email: xefteris@ucy.ac.cy

1 Introduction

In representative democracies, the ability of a candidate to uphold positions and values over a long period of time is often viewed as a virtue. Voters elect politicians in offices on the basis of their current electoral platforms but also taking into account past platforms and behaviors. When a candidate flip-flops from one position to another, then his reputation is damaged and becomes, arguably, less appealing as a potential leader. Indeed, voters seem to value a politician's consistency independently of whether they are ideologically aligned with him or not (Kreps et al. 2017). For this reason politicians are very careful and try to adhere to the image of a consistent and moral statesman.

Of course, voters do not only care about the political platform and the ideological consistency of a candidate, but also about his qualitative (i.e. non-policy) characteristics like valence, experience, etc. Numerous empirical studies have established that such considerations crucially determine the voting decisions and, to a large degree, shape electoral outcomes (see Stone and Simas 2010 and references therein).

The theoretical literature on electoral competition has tried to complement the empirical findings by uncovering relevant insights regarding the effect of preferences for ideological consistency and valence on electoral competition outcomes. Hummel (2010a), Agranov (2016) and Andreottola (2021) consider contexts where candidates can change policy positions between primaries and general elections, and establish the polarization effect of the voters' preferences for consistency: candidates are punished by voters if they deviate from their primaries' platforms, and this keeps their general election platforms apart. On the other hand, Groseclose (2001), Aragonès and Palfrey (2002) and Bernhardt et al. (2020) have shown that valence asymmetries affect both platform polarization and the policy outcome of the election.[1]

[1] Several other papers have complemented this literature, considering alternative valence magnitudes (Hummel 2010b), strategy spaces (Aragonès and Xefteris 2012), electoral institutions (Hummel 2013), and voting behavior (Schofield 2004). The literature on electoral competition between heterogeneous candidates is vast. For a more complete literature review see Aragonès and Xefteris (2017).

To our knowledge, there is no theoretical study that considers both preferences for consistency and valence asymmetries at the same time. Understanding the dynamics of electoral competition when both these elements are present is important because –when addressed separately– each of them affects common features of the electoral outcome (e.g. platform polarization). Hence, it is not clear at all how these forces interact when both are present in an electoral context. From an empirical point of view, this makes it hard to identify, for instance, whether the observed increase in political polarization (see McCarthy et al. 2016 and Poole and Rosenthal 1984) is mainly due to valence asymmetries or stronger preferences for consistency. In this paper we try to bridge these branches of the literature and map this so far unexplored territory.

We analyze a model of electoral competition between two win-motivated candidates, nominated by two parties with distinct values/ideologies, considering that voters care both about the valence and the consistency of each candidate's platform with the ideology of the party that nominates him. When valence asymmetries are small then a preference for ideological consistency is found to exert a stabilizing force on electoral competition dynamics (i.e. a unique pure equilibrium exists). This is a first key difference with models that focus only on valence, since when voters do not value the candidates' attachment to previously expressed views or to their party values, then even the slightest asymmetry in valence level precludes the existence of pure equilibria, and, hence, the possibility of stability in electoral competition (Aragonès and Palfrey 2002). The identified pure equilibrium is divergent, and the degree of platform polarization that it exhibits is increasing in the importance that voters assign to candidates' ideological consistency. This is quite intuitive, as such preferences induce a trade-off to candidates' attempting to propose more popular/centrist policies: more popular platforms are appealing to more voters, but being proposed by a candidate that has previously expressed other views is not likeable to anybody. Therefore, as the preferences for ideological consistency become more intense, the candidates prefer to stay closer to their reference policies and polarization increases.

What is, perhaps, less intuitive is the fact that, in such cases, platform polarization is not sensitive at all to changes in valence asymmetries. Indeed, an advance-retreat phenomenon is observed: as valence asymmetries increase, the advantaged candidate moves towards the direction of her opponent –proposing more popular policies– while the disadvantaged candidate backs away. It is noteworthy that similar phenomena have been detected also by Groseclose (2001) and Bernhardt et al. (2020) who analyze electoral competition between (partially) policy-motivated candidates and legislative elections respectively. In those models, though, an increase in valence asymmetries also induces changes in the degree of platform polarization, and perhaps more importantly, as valence asymmetries increase the expected policy outcome moves towards the ideal policy of the high valence candidate. Instead, in our case –i.e. when preferences for consistency are brought in the standard Downsian model with win-motivated candidates– the advance of the advantaged candidate and the retreat of the disadvantaged candidate, as a consequence of an increase in valence asymmetries, happen exactly at the same pace, and, hence, the degree of ideological differentiation remains invariant.

While platform polarization is determined solely by the strength of preferences for ideological consistency, the ideological leaning of the election –i.e. the extent at which the platforms' midpoint tilts to the left/right– is found to depend both on valence differences (as the valence of the leftist candidate increases, the platforms' midpoint moves to the right) and on the strength of the preferences for consistency (as the preferences for ideological consistency become stronger, the relevance of the valence asymmetries in determining the ideological leaning of the election decreases –i.e. the platforms' midpoint moves towards the center of the policy space). Interestingly, when there is high uncertainty about the voters' policy preferences, the expected policy outcome also moves in the same direction as the platform's midpoint: *a higher valence advantage of the leftist candidate moves the expected policy outcome towards the right (not the left!)*.[2]

These observations are completely novel and pin down a, so far, uncharted dilemma that

[2]Otherwise, that is, when candidates have relatively good information regarding the ideal policy of the median voter, the expected policy outcome moves closer to the reference policy of the advantaged candidate as his advantage increases, and after valence asymmetries cross a certain level, it starts to tilt towards the reference policy of the disadvantaged candidate.

parties face between when nominating candidates: if voters care about candidates' adherence to party values, a party increases its chances of winning the election by selecting a high-valence candidate, but this may come at the cost of worse policy outcome. Therefore, the degree at which party elites care about winning vs. policy, can influence their preferences on the optimal valence level of their nominee: it might be preferable for a policy-motivated party elite to nominate a low-valence candidate, even when higher-valence candidates are available, in order to secure a more appealing policy outcome. Notice that this stands in stark contrast to earlier studies (e.g. Groseclose 2001) which find that –when candidates are policy-motivated– the expected policy outcome always comes closer to the reference policy of the advantaged candidate as his advantage increases, suggesting that ideology-driven party elites are unambiguously better off by nominating higher-valence candidates. While we do not aspire to model and solve such a complicated candidate-selection game between parties, the comparative statics analysis of our equilibrium clearly shows that forces pushing parties to nominate lower-quality candidates are also present and might be relevant in explaining why parties do not always nominate the candidate deemed "best" in terms of quality and/or valence.[3]

Notice that when either valence asymmetries or preferences for ideological consistency are absent from the analysis, the election does not have any ideological leaning (i.e. the expected policy outcome coincides with the expected ideal policy of the median voter – see, for instance, Aragonès and Palfrey 2002, and Hummel 2010b). Hence, while neither of these elements has the ability to bias the policy outcome when it is present in the model alone, we find that when they operate in tandem they interact in creating a non-neutral policy outcome. Among others, this reaffirms the relevance of studying electoral competition with both valence heterogeneity and consistency concerns.

When the valence asymmetries are large, then a mixed equilibrium emerges. The advantaged candidate locates to the side of the median voter where the reference policy of her opponent lies, and the disadvantaged candidate mixes between a very leftist and a very rightist

[3]For alternative theories regarding the emergence of low-quality candidates one is referred to Castanheira et al. (2020) and references therein.

platform with uneven probability. Namely, a candidate of a leftist party that enjoys a large advantage may find herself propagating a moderate-right platform, while the disadvantage candidate from the rightist party might try to survive by pushing, more likely, an extremely conservative agenda, or, with some smaller probability, she might completely flip-flop and adopt a progressive rhetoric. When we are in this mixed equilibrium, both platform polarization, and the ideological mark of the election are affected by valence asymmetries and the strength of preferences for consistency.

These results provide new insights regarding flip-flopping incentives (i.e. departure away from a reference policy) of advantaged and disadvantaged candidates. When valence asymmetries are moderate, in line with Groseclose (2001) and Bernhardt et al. (2020), we find that advantaged candidates have greater interest to move away from their reference policies than disadvantaged ones. When valence asymmetries are large, though, we show that hopeless candidates might engage in large scale platform revisions in order to survive. That is, flip-flopping incentives follow a non-trivial non-monotonic pattern: the benefits from moving away from the reference policy are larger when one enjoys a moderate valence advantage or when one is faced with a large valence disadvantage. This finding is, arguably, of independent interest as it provides a dual reasoning behind policy revisions that has not been proposed by earlier studies: advantaged candidates engage in moderate, yet aggressive, revisions –in the sense that they move away from their reference policies to capture more voters–, while severely disadvantaged candidates may employ dramatic in magnitude, but, defensive in essence flip-flopping behaviors to preserve some chances of being elected.

Our work has links with additional sections of the electoral competition literature, apart from the theoretical studies on flip-flopping incentives and exogenous valence asymmetries. For instance, it connects in a genuine manner to the endogenous valence literature (see, e.g., Carrillo and Castanheira 2008, Zakharov 2009, Ashworth and Bueno de Mesquita 2009). Indeed, while valence is a policy-independent candidate characteristic commonly valued by all voters, ideological consistency is a policy-relevant feature of a candidate that still enjoys a uniform appeal. That is, candidates when choosing their policy platforms they are also endogenously influencing the degree of common appeal that they enjoy. To be sure, investing in quality or valence is not the same like building a reputation of a consistent politician. But since the non-policy component of the voters' utilities depends on a candidate's choices, preference for consistency endogenize commonly valued features of a candidate, in an alternative way to that employed by the aforementioned endogenous valence studies.

Moreover, the trade-off between sticking to a potentially unpopular reference policy and proposing a more centrist platform that is introduced by the preference for consistency, is reminiscent of the way preferences for candidate character are modelled in relevant studies (Kartik and McAfee 2007, Callander and Wilkie 2007, Bernheim and Kartik 2014). In that literature, some candidates simply wish to propose the platform that maximizes their chance of being elected, while others have character: they prefer to stick to policies that they deem correct for the society, even if they are unpopular. When voters value candidate character but do not know whether a candidate has this feature, then the remaining candidates want also to mimic having a character and do not fully pander to the median voter. While preferences for character and consistency are quite similar, the remaining behavioral and informational assumptions of our study, do not align with those of the candidate character studies. In those papers, the central behavioral assumption is that some candidates do propose their ideal policy without engaging in any strategic reasoning, and hence, the question is about how candidates that lack character (and, thereafter, a relevant ideal policy) behave. In our context all candidates are instrumental, their reference policies –induced by their promises during primary elections, by their party ideology etc.– are common information, and, crucially, they differ in non-policy characteristics.

In what follows, we first build our model (section 2), then we derive the results (section 3), and, finally, we conclude by discussing empirical implications and directions for future research (section 4).

2 The model

There are two candidates, A and B, that compete for votes over a unidimensional policy space represented by the real line $\mathbb{R}$. We assume that candidates have reference points in the policy space that are equal to -1 for A and equal to 1 for B. Each candidate's objective is to maximize his probability of winning the election.

Voters have a utility function with three components: a policy component, a candidate image component and an ideological disadvantage component. The policy component is characterized by an ideal point in the policy space, with utility of alternatives in the policy space determined by a quadratic function of the distance between the ideal point and the location of the policy. The image component is captured by an additive constant to the utility a voter gets if candidate A wins the election. The ideological disadvantage component is represented by the quadratic distance between the candidate's policy choice and his reference policy. Notice that this set up can be interpreted as if candidates can affect the value of their advantage through their policy choice: the overall advantage of a candidate decreases with the distance between his policy proposal and the candidate's reference point.

Let x_A denote the policy position chosen by candidate A, and let x_B denote the policy position chosen by candidate B. Then, the utility that a voter with ideal point x_i obtains if candidate A wins the election is given by $U_i(A) = \delta - \gamma(-1 - x_A)^2 - (x_i - x_A)^2$ and his utility if candidate B wins is given by $U_i(B) = -\gamma(1 - x_B)^2 - (x_i - x_B)^2$, where $\delta \geq 0$ denotes the size of candidate A's valence advantage and $\gamma \geq 0$ represents the magnitude of the ideological consistency effect. The assumption that $\delta \geq 0$ is without loss of generality, because otherwise it would imply that candidate B is the one that enjoys the valence advantage and the analysis would be completely symmetric. The assumption that $\gamma \geq 0$ is introduced because it is plausible that if a candidate proposes a policy that diverges from his reference point (e.g. the ideology of the nominating party, his primary platform, etc.), voters may punish him because of his lost credibility. But it would be hardly the case that voters reward a candidate for such behavior.

The game takes place in two stages. In the first stage, candidates simultaneously choose positions in $\mathbb{R}$. In the second stage, voters vote for the candidate whose election would give them the highest utility. In case of indifference, a voter is assumed to vote for each candidate with even probability.[4]

Candidates do not know the distribution of the voters' ideal points, but they know that a unique median voter exists. They have beliefs about the ideal point of the median voter that are common and common knowledge and they are represented by a twice differentiable probability distribution function $F(x)$, with a bell-shaped density function that is symmetric with respect to 0 (i.e. $F'(x) = F'(-x)$ for every $x \in \mathbb{R}$, and $F''(x) > 0$ for every x<0).[5] The assumption that both F and the reference policies of the two candidates are symmetric about zero is very helpful for tackling issues with respect to mixed strategies. As we will argue, though, several of the novel insights that we uncover (e.g. the existence of pure equilibria when valence asymmetries are not large, and the corresponding comparative statics) may hold even when the reference policies of the candidates or/and F are not symmetric about zero.

Candidates are office motivated, thus they maximize their probability of winning the election. Let $\Pi_A(x_A, x_B)$ and $\Pi_B(x_A, x_B)$ denote the respective candidates' payoffs, with $\Pi_A(x_A, x_B) = 1 - \Pi_B(x_A, x_B)$.

Since the behavior of the voters is unambiguous in this model, we define an equilibrium of the game only in terms of the location strategies of the two candidates in the first round.[6] The solution concept we consider is Nash equilibrium. A pure strategy equilibrium is a pair of candidate locations (x_A, x_B) such that both candidates are maximizing the probability of winning, given the choice of the other candidate. A mixed strategy equilibrium is a pair of probability distributions (σ^A, σ^B) over $\mathbb{R}$ such that there is no mixed strategy for A that guarantees higher

[4]We assume that if all voters are indifferent between the two candidates, then each candidate is elected with probability $1/2$.

[5]In this model, it is easy to show that if the median voter prefers a candidate over another, then the former is preferred by a majority of the voters to the latter. Thus, to compute the probability of election of each candidate, there is no need to make further assumptions regarding the number of other voters, and their preferences.

[6]We consider that voter i votes for candidate A if $u_i(A) > u_i(B)$, for candidate B if $u_i(B) > u_i(A)$, and splits her vote between the two otherwise. Indeed, such a behavior is dominant even if there is a finite number of voters, and, hence, individual choices are consequential.

probability of winning than σ^A, given σ^B, and there is no mixed strategy for B that guarantees higher probability of winning than σ^B, given σ^A.

3 Equilibrium analysis

We first explain why a unique indifferent voter exists for every pair of distinct platforms chosen by the candidates, and investigate the preferences of such a voter for alternative configurations of candidates' platforms.[7] Then, we argue that in our setup, convergent pure equilibria are precluded when voters care about the candidates' ideological consistency (i.e. when $\gamma > 0$), and characterize the unique pure strategy equilibrium of the game, whenever such an equilibrium exists. Finally, we turn our attention to the cases in which pure equilibria do not obtain and we characterize a unique mixed strategy equilibrium of the game.

3.1 Indifferent voter

To better grasp the intuition behind the subsequent indifferent voter analysis, it is useful to first explain how the net candidate advantage –understood as the candidate's valence minus the ideological loss from deviating from its reference policy– is determined by the candidates' platform choices. Indeed, candidate A is ex-ante the advantaged candidate, in the sense that he has higher valence than B. But since the candidate's characteristics that are commonly valued by all voters include, beyond valence, the candidates' ideological consistency, there are platform pairs such that candidate B is the real advantaged candidate (e.g. when A proposes a platform very far away from his reference policy, while B is close to his).

First, notice that for $x_B = x_A$ we have that all voters vote for candidate A if $x_B = x_A < \frac{\delta}{4\gamma}$; all voters vote for candidate B if $x_B = x_A > \frac{\delta}{4\gamma}$; and all voters are indifferent if $x_B = x_A = \frac{\delta}{4\gamma}$. The

[7] In fact, several indifferent voters may co-exist in our model with identical ideal policies, or even none. When we say that a unique indifferent voter exists, we essentially mean that there always exists a unique ideal policy that may characterize such a voter.

threshold $\frac{\delta}{4\gamma}$ denotes the limit in which the overall advantage changes: when both candidates' policies converge to $x < \frac{\delta}{4\gamma}$ candidate A is able to maintain his advantage because the policy choice is not too far from his reference point, and therefore his ideological cost is not enough to cancel his valence advantage. This changes when the candidates' policies converge to $x > \frac{\delta}{4\gamma}$. In this case candidate A has to pay a large ideological cost, that more than compensates his valence advantage. In fact, his final net advantage is negative and larger in absolute terms than the one that candidate B enjoys. Therefore, for $x_B = x_A > \frac{\delta}{4\gamma}$ candidate B becomes the actual advantaged candidate (see figure 1).

Figure 1: Subpolicy space where a candidate has an overall advantage.

Now we consider the case of $x_B \neq x_A$. In this case the indifferent voter, uniquely determined by $u_i(A) = u_i(B)$, is given by

$$\widetilde{x}_i = \frac{\delta + [(1+\gamma)(x_B - x_A) - 2\gamma](x_A + x_B)}{2(x_B - x_A)}$$

We have that for $x_B > x_A$ all voters with $x_i < \widetilde{x}_i$ vote for candidate A and for $x_B < x_A$ all voters with $x_i > \widetilde{x}_i$ vote for candidate A.

Since $\frac{\partial \widetilde{x}_i}{\partial x_A} = \frac{\delta - 4\gamma x_B}{2(x_B - x_A)^2} + 1 + \gamma$ we have that for $x_B < \frac{\delta}{4\gamma}$ the indifferent voter increases with x_A. And since $\frac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2} = \frac{\delta - 4\gamma x_B}{(x_B - x_A)^3}$ we have that for $x_B < \frac{\delta}{4\gamma}$ the indifferent voter diverges to infinity when x_A increases towards x_B; and the indifferent voter diverges to minus infinity when x_A decreases towards x_B. In other words, when candidate B proposes a policy in A's side of the policy space, then, naturally A has incentives to mimic B's proposal and sweep all the votes.

Conversely, for $x_B > \frac{\delta}{4\gamma}$ –that is, when candidate B proposes a policy in the side of the policy space that favors him– for values of x_A close enough to x_B, the indifferent voter decreases with x_A. It diverges to minus infinity when x_A increases towards x_B; and it diverges to infinity when x_A decreases towards x_B.

Turning our attention to candidate B, we observe that $\frac{\partial \widetilde{x}_i}{\partial x_B} = \frac{4\gamma x_A - \delta}{2(x_B - x_A)^2} + 1 + \gamma$. Therefore, for $x_A < \frac{\delta}{4\gamma}$ –that is, when candidate A proposes a platform at her privileged side– for values of x_A close enough to x_B, the indifferent voter decreases with x_B. And since $\frac{\partial^2 \widetilde{x}_i}{\partial (x_B)^2} = \frac{4\gamma x_A - \delta}{(x_B - x_A)^3}$ we have that for $x_A < \frac{\delta}{4\gamma}$ the indifferent voter diverges to minus infinity when x_B increases towards x_A; and it diverges to infinity when x_B decreases towards x_A. For $x_A > \frac{\delta}{4\gamma}$, the indifferent voter increases with x_B. It diverges to infinity when x_B increases towards x_A; and it diverges to minus infinity when x_B decreases towards x_A. (see figure 2)

Since $\frac{\partial \widetilde{x}_i}{\partial \delta} = \frac{1}{2(x_B - x_A)}$, we have that the indifferent voter increases with the valence only for $x_B > x_A$. Thus, we have that candidate A's probability of winning always increases with his valence if $x_B > x_A$.

We also have that $\frac{\partial \widetilde{x}_i}{\partial \gamma} = \frac{(x_B - x_A - 2)(x_A + x_B)}{2(x_B - x_A)}$. This implies that for $x_B > x_A$, candidate A's probability of winning increases with the importance of the ideological cost only if the two policies are sufficiently distant. Otherwise, when candidate's policies are close enough candidate A's probability of winning decreases with the importance of the ideological cost.

3.2 Impossibility of a convergent equilibrium

Regarding possible stable configurations, we first analyze the case in which both candidates choose the same policy, and we find that it cannot be an equilibrium. This finding can be interpreted as an extension of the impossibility result that is well known in the literature about two candidate electoral competition with an advantaged candidate. The advantaged candidate always prefers to move as close as possible to the disadvantaged candidate, and the disadvantaged candidate always prefers to be not too close to the advantaged candidate.

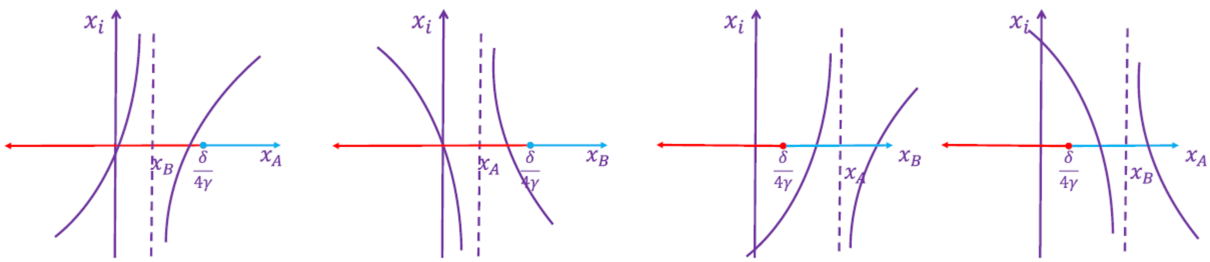

Figure 2: The ideal policy of the indifferent voter, x_i, as a function of a candidates policy platform (in the first and third graph, as a function of A's platform, and in the second and the forth graph as a function of B's platform), for different values of her opponent's policy platform (in the first and second graph, the two candidates' platforms can coincide only on the red part of the axis and hence candidate A is advantaged, while in the third and fourth graph coincidence of platform can occur only on the blue part of the axis, so candidate B is advantaged).

Here instead, no candidate is universally advantaged, or disadvantaged: the identity of the favored candidate depends on the exact policy platform that both candidates offer in a convergent strategy profile, and there exists a policy platform such that no candidate enjoys any form of advantage over the other. We still find, though, that offering the same platform does not constitute a Nash equilibrium, for any given policy platform.

Proposition 1: *There is no equilibrium in which $x_A = x_B$*, unless $\delta = \gamma = 0$.[8]

We have already stated that when the two candidates choose convergent policies, the candidate that enjoys the net advantage is determined by how far apart the policy chosen is from candidate A's reference point. For policies close enough to candidate A's reference point ($x < \frac{\delta}{4\gamma}$), candidate A is the overall advantaged candidate and thus candidate B wants to deviate away from a converging policy. Similarly, for policies far enough from candidate A's reference point ($x > \frac{\delta}{4\gamma}$), candidate B is the overall advantaged candidate and thus candidate A wants to deviate away from a converging policy. Finally, when the two candidates converge on $x = \frac{\delta}{4\gamma}$ their probabilities of winning are tied, and we find that candidate A always can increase his probability of winning by moving to a policy slightly closer to its reference point. Thus we conclude

[8]All the proofs can be found in the Appendix.

that an equilibrium in which candidates' choose the same policy is not possible.

3.3 Pure strategy equilibrium

Next we will show that the introduction of ideological consistency considerations in a setup of two candidate electoral competition with a valence advantage allows the existence of a pure strategy equilibrium. We know from the literature of electoral competition with a valence advantage that, in absence of preferences for consistency, only mixed strategy equilibria obtain in such settings. Here we introduce ideological costs, which can be interpreted as a form of endogenous valence. This new feature changes completely the strategic structure of the electoral competition, by determining different policy subspaces in which different candidates become the overall advantaged ones.

Indeed, the candidates in this model have the choice of playing it safe: candidate A, who enjoys the initial valence advantage, can avoid his opponent to become the overall advantaged candidate by choosing policies not too far from his reference point; similarly, candidate B can avoid his opponent to benefit fully from his advantage by choosing policies far away from his opponent's reference policy. As we show, such a conservative behavior often constitutes an equilibrium. The next proposition characterizes the unique Nash equilibrium in pure strategies, and the parameter values for which it exists.

Proposition 2: *The policies $x_A = \frac{\delta}{4\gamma} - \frac{\gamma}{1+\gamma}$ and $x_B = \frac{\delta}{4\gamma} + \frac{\gamma}{1+\gamma}$ are the unique pure strategy equilibrium for $\delta < \frac{8\gamma^2}{1+\gamma}$.*

In this pure strategy equilibrium candidates' policies are symmetric around $\frac{\delta}{4\gamma}$. In proposition 1 we have seen that if candidate A chooses a policy larger than $\frac{\delta}{4\gamma}$, candidate B becomes the overall advantaged candidate and can obtain all votes by mimicking candidate A's policy. Similarly, we saw that if candidate B chooses a policy smaller than $\frac{\delta}{4\gamma}$, candidate A becomes the overall advantaged candidate and can obtain all votes by mimicking candidate B's policy.

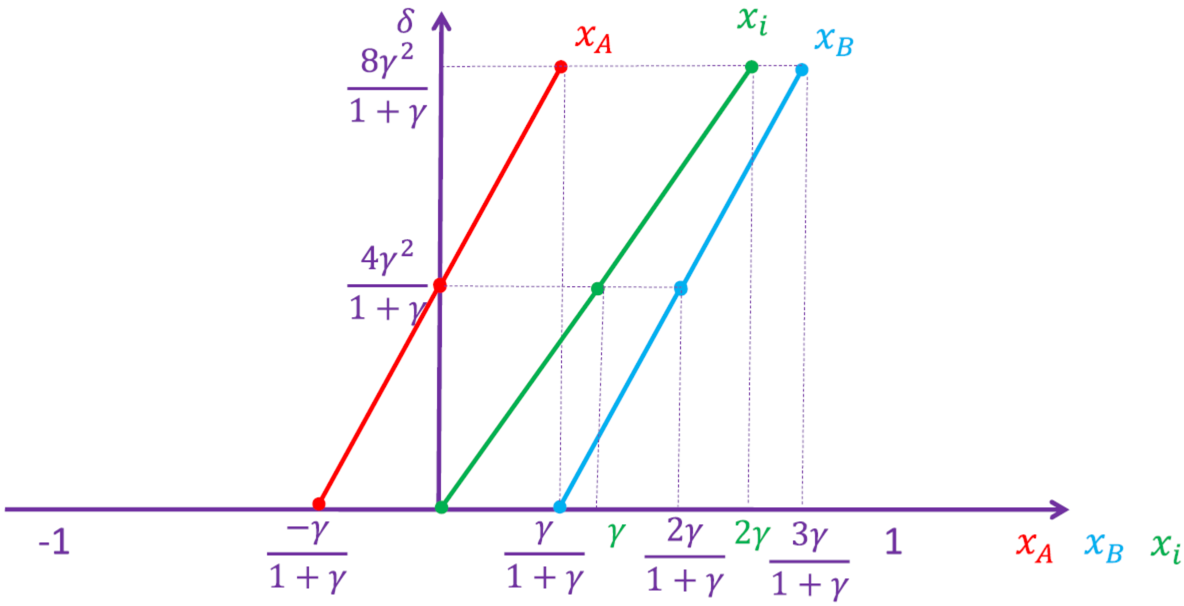

Figure 3: Comparative statics of the pure strategy Nash equilibrium with respect to δ for $\gamma \leq \frac{1}{2}$.

Thus, it is with no surprise that we find that in the pure strategy equilibrium each candidate chooses a policy in his own safe policy subspace. In particular we find that x_B is always positive, and x_A is only positive for $\delta > \frac{4\gamma^2}{1+\gamma}$ (see figure 3).

The ideal point of the indifferent voter at the equilibrium strategies is given by $\widetilde{x}_i = \frac{\delta(1+\gamma)}{4\gamma}$. Thus it is always positive, it increases with the valence advantage and it decreases with the ideological costs. The indifferent voter's ideal policy is between the two equilibrium policy choices for small values of γ ($\gamma \leq \frac{1}{2}$). Otherwise, for larger values of γ, the ideal point of the indifferent voter at the equilibrium strategies is larger than candidate B's equilibrium policy for large values of δ ($\delta > \frac{4\gamma}{1+\gamma}$). This presents a very interesting phenomenon in which only right wing extremists support the rightist candidate, while moderate rightist voters –including those that fully agree with the policy proposal of the rightist candidate– prefer to support the advantaged leftist candidate who proposes a moderate policy.

We find that candidate A's equilibrium policy choice is always larger than his reference point, and candidate B's equilibrium policy choice can be smaller or larger than his reference point, depending on whether δ is smaller or larger than $\frac{4\gamma}{1+\gamma}$. We also find that the distance between candidate A's equilibrium policy choice and his reference point is larger than his opponent's whenever candidate B's policy choice is smaller than 1. Otherwise, the distance between candidate A's equilibrium policy choice and his reference point is larger than his opponent's whenever $\gamma < \frac{1}{2}$.

The larger is the valence advantage the larger are both equilibrium policy choices: thus candidate A moves away from his reference policy and candidate B moves away from candidate A. When the valence advantage goes to zero we obtain an equilibrium in pure strategies that are symmetric around zero: $x_A = -\frac{\gamma}{1+\gamma}$ and $x_B = \frac{\gamma}{1+\gamma}$. Notice that in this case we obtain a unique symmetric divergent equilibrium, such that, with the range of positive values of γ, the equilibrium policies cover all the policy space that lies between the two reference policy points of the two candidates.

Finally, we note that neither the symmetry of F, nor the fact that the candidates' reference points are equidistant to zero is a necessary requirement for the existence of this equilibrium. As one can easily check, the described strategy profile is an equilibrium for every F –symmetric or asymmetric– and any pair of reference policies in the restriction of the game in which the leftist (rightist) candidate is constrained to propose a platform to the left (right) of the strategy of her competitor. That is, the symmetry conditions that we imposed ensure that no candidate finds it profitable to "jump over" her competitor (e.g. that the leftist candidate proposes an ultra-right platform –which could be profitable if F assigned most of the probability to ideologies that are further to the right, compared to the reference policy of the rightist candidate). Therefore, if F assigns sufficiently high probability to ideologies in between the two reference policies, then a pure equilibrium will still exist in the unrestricted version of our game (again, conditional on the valence asymmetries not being very large), and the comparative statics that we presented, will still hold. This observation generalizes the relevance of our analysis substantially, as it ensures that the corollaries and conclusions that follow do not require symmetry, but can be valuable to (mildly) asymmetric settings as well.[9]

[9]Notice, also, that the robustness of our analysis to asymmetric settings strengthens the potential interpretation

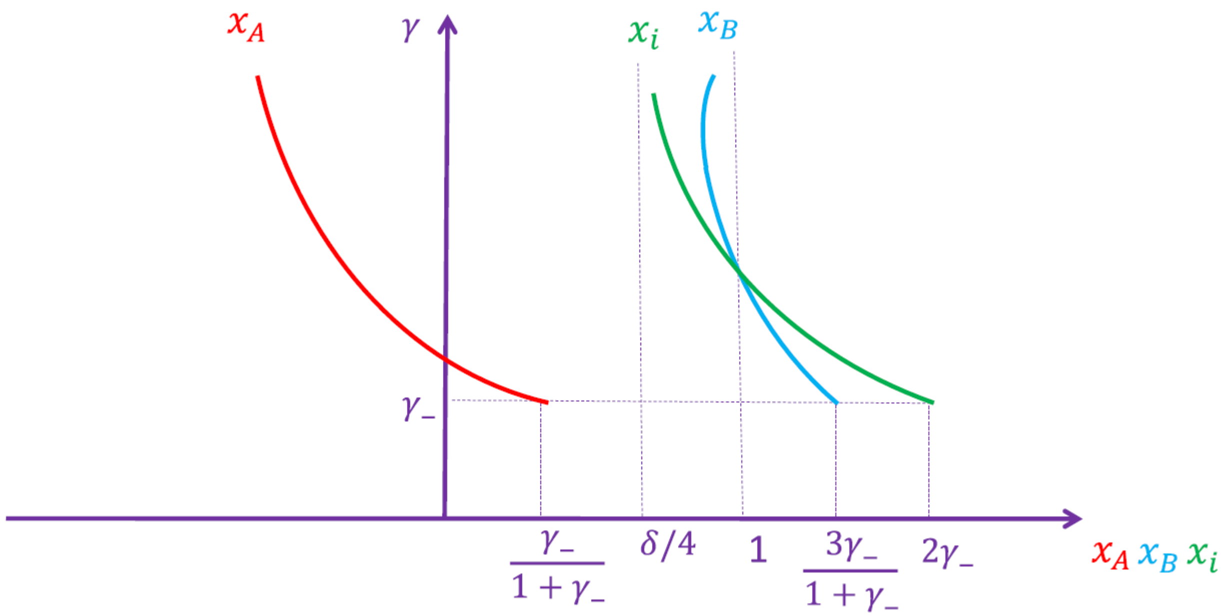

Figure 4: Comparative statics of the pure strategy Nash equilibrium with respect to γ for $\delta < 4$.

Regarding the degree of polarization of the equilibrium platforms and the ideological leaning of the election we state the following corollary (see figure 4).

Corollary 1: *In the unique pure-strategy equilibrium, the degree of polarization of the candidates' policy platforms,* $x_B - x_A = \frac{2\gamma}{1+\gamma}$*, is increasing in* γ *and invariant in* δ*; the ideological leaning of the election,* $\frac{x_A+x_B}{2} = \frac{\delta}{2\gamma}$*, is increasing in* δ *and decreasing in* γ*.*

The first observation relates to polarization: The distance between the two equilibrium policies is only a function of the importance of the ideological costs (γ) and it increases with it. Increasing ideological costs always induces candidate A to choose policies that are closer to his reference point. Instead, the equilibrium policy of candidate B increases with the ideological costs only when the valence advantage is not very large $\delta < \frac{4\gamma^2}{1+\gamma}$. Otherwise it decreases.

The second observation refers to the midpoint of the two platforms, which we interpret as the ideological leaning of the election (i.e. whether the middle-ground of the electoral debate

of the reference policies as the winning platforms of primary elections, since the platforms of primary elections' winners are not guaranteed to be symmetric about the expected ideal point of the median voter.

lies to the left or to the right of the policy space, and by how much). Since the equilibrium platforms are symmetric about $\frac{\delta}{4\gamma}$, that point represents the platforms' midpoint. Evidently, this policy is increasing in valence asymmetries and decreasing in the importance of ideological consistency. That is, in an election between a high-valence leftist candidate and a low-valence rightist one, the candidates' platforms are biased to the right and this bias is pronounced when valence asymmetries are large and mitigated when voters care a lot about the ideological consistency of the competing candidates.

While similar results have also been identified in earlier studies, in which the source of platform asymmetry depended on candidates having policy preferences (e.g. Groseclose 2002), in our case they are driven purely by the voters' preferences and the candidates' office motivation. Perhaps more importantly, what we find is that –according to the current mechanism– increasing valence differences can push not only the platforms of the two candidates towards the reference policy of the low-valence candidate, but also the expected policy outcome. This is not the case when candidates are policy motivated: while an increase in valence asymmetry pushes both platforms towards the side of the weaker candidate, the expected policy outcome moves towards the opposite direction favouring the high-valence candidate. Instead, when candidates care solely about winning the election and they differentiate only because voters punish flip-flopping behavior, then the expected policy outcome can move towards the weaker candidate when valence asymmetries increase.

Proposition 3: *In the unique pure strategy equilibrium if the uncertainty about the preferences of the median voter is high (i.e. $F'(0) < \frac{1}{2\gamma}$) the expected policy outcome, $x_e = x_A F(\widetilde{x}_i) + x_B(1 - F(\widetilde{x}_i))$, is increasing in δ and is decreasing in γ.*

To our knowledge, this constitutes a completely new insight and uncovers a potential force pushing ideologically-motivated party elites to nominate candidates of low quality in certain electoral races. Undeniably, the fact that in the identified equilibrium the expected policy outcome may move towards the reference policy of the weaker candidate as valence differences increase follows from our specific assumptions. This may not hold in more general cases in which candidates have strong policy or other concerns, and voters care about additional candidate features. Notice, though, that the aim of a formal analysis like ours is not to generate results that align perfectly with real world observations –which are generated in complex and, occasionally, chaotic environments–, but, rather, to uncover important forces that, under certain conditions, push the outcome in a given direction. From this perspective, our work identifies a force that urges policy-driven party elites to nominate low-valence candidates. Whether this force dominates other forces that push for higher-quality candidates is an obviously interesting question, but it is clearly beyond the scope of the current analysis.

Finally, at the edges of the parameter space for which the identified unique equilibrium exists, the model converges to the known cases in which either valence asymmetries do not exist, or preferences for ideological consistency are absent. When the ideological costs go to zero we obtain a standard set up of two candidate electoral competition with a valence advantage and thus there is no pure strategy equilibrium. If both the valence advantage and the ideological costs become smaller, we approach a standard Downsian model and the equilibrium strategies approach the median voter result: $x_A = x_B = 0$.

Regarding equilibrium payoffs, since candidates are win-motivated, the comparative statics are as expected. The indifferent voter in equilibrium is given by $\widetilde{x}_i = \frac{\delta(1+\gamma)}{4\gamma}$ and we have that the equilibrium payoffs are: $\Pi_A = F\left(\frac{\delta(1+\gamma)}{4\gamma}\right)$ and $\Pi_B = 1 - F\left(\frac{\delta(1+\gamma)}{4\gamma}\right)$. Therefore, they increase for candidate A with the valence advantage; and they increase for candidate B with the importance of the ideological costs, reinforcing the idea that preferences for ideological consistency mitigate the impact of valence asymmetries. And since $\frac{\delta(1+\gamma)}{4\gamma} > 0$ the payoffs are always larger for candidate A.

3.4 Mixed strategy equilibrium

In the previous section we analyzed the case in which the value of the valence advantage was not large relative to the importance of the ideological costs. In that case we showed that the effect of the ideological costs could be used as a counterbalance for the incentives of the advantaged candidate to imitate his opponent's policies and win all votes. The strategic use of the ideological costs in that case allowed for existence of a pure strategy equilibrium. When the value of the valence advantage becomes larger, this argument does not hold any longer: the ideological costs are not enough to counterbalance the valence advantage, and therefore the strategic structure of the electoral competition becomes very similar to the models of electoral competition that only exhibit a valence advantage. In those models only mixed equilibria can exist.

In a sense, the pure equilibrium that we have analyzed in the previous section predicts a cautious electoral-campaign from both candidates: each candidate chooses a platform in the side of the policy space where he enjoys the net advantage (i.e. if the other candidate attempts to imitate his platform, he will win with certainty). If valence asymmetries are very large vis-a-vis the strength of preferences for ideological consistency, then this cautious behavior on behalf of both candidates cannot be sustained in equilibrium. If candidates use the platforms described in Proposition 2 and the corresponding condition fails (i.e., if $\delta > \frac{8\gamma^2}{1+\gamma}$), then, candidate B (the disadvantaged rightist candidate) is better off by proposing a very progressive platform to the left of x_A. That is, he is willing to penetrate the enemy's ground and become de facto the leftist candidate.

This is true because as valence differences increase and B is pushed farther away to the right, he proposes an increasingly unpopular rightist platform and drifts away from his reference policy. So at some point, the rightist candidate is squeezed so much to the right, that he finds it profitable to adopt a popular moderate left policy platform. Of course, by doing so he incentivizes the leftist candidate to mimic his position and hence a pure equilibrium cannot exist. But this reasoning is enough to explain why in some cases a candidate employs a more risky campaign, and proposes policies out of his privileged space.

In this section we investigate the possibilities of equilibrium exactly for these cases; when pure equilibria do not obtain (see figure 5). We show that a mixed strategy equilibrium exists

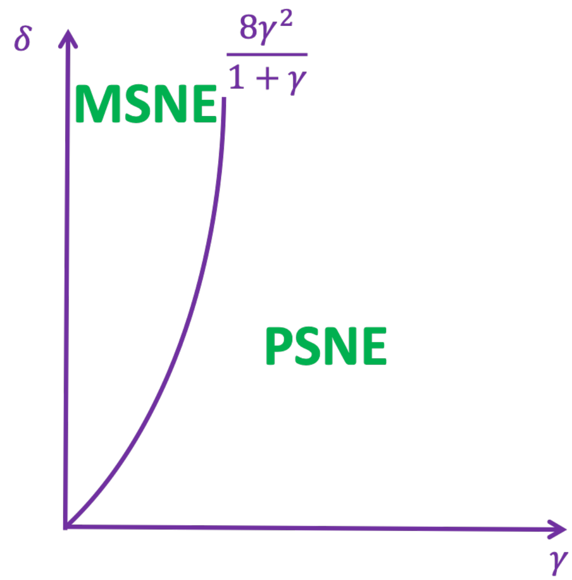

Figure 5: Characterization of the Nash equilibria according to the parameter values.

for large values of the valence advantage relative to the importance of the ideological costs, and we characterize the conditions that guarantee its uniqueness. We first present conditions for the existence of a local equilibrium in mixed strategies (i.e. a strategy profile such that marginal deviations away from it are not profitable for any player) for different parameter values, and then we present conditions under which this local equilibrium is a global equilibrium (i.e. no deviations are profitable).

3.4.1 Large, but not too large, differences in valence

Our approach is as follows: we propose a set of candidate strategies for a mixed Nash equilibrium, and set out to investigate when players have any profitable deviations. In the next proposition we show that when the value of the valence advantage is large enough so that a pure strategy equilibrium does not exist, but it is not extremely large, then a local equilibrium in mixed strategies is guaranteed to exist.

Proposition 4: *The policies* $x_A = \frac{\gamma}{1+\gamma}$ *and candidate* B *mixing as follows:* $x_B = \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ *with probability* $p = \frac{1}{2} + \frac{\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}$ *and* $x_B = \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ *with probability* $1-p$ *satisfy the conditions for a local equilibrium for* $\frac{8\gamma^2}{1+\gamma} \leq \delta \leq \frac{12\gamma^2}{1+\gamma}$.

Notice that the two actions that candidate B uses in this strategy profile, are the unique two best responses to the pure strategy used by candidate A. Thus, there are no profitable deviations for candidate B. We can further show that the payoff function of candidate A evaluated at the mixed strategy proposed for candidate B exhibits a local maximum at $x_A = \frac{\gamma}{1+\gamma}$. Thus, the posited profile is a local equilibrium. In order to guarantee that it is also a global equilibrium, we have to show that there are no profitable deviations for candidate A from the strategy $x_A = \frac{\gamma}{1+\gamma}$, that is, we have to show that the payoff function of candidate A evaluated at the mixed strategy proposed for candidate B exhibits a global maximum at $x_A = \frac{\gamma}{1+\gamma}$. In the next subsection we deal with this issue and we characterize the conditions that guarantee that the strategies stated in proposition 4 constitute a unique Nash equilibrium of the game.

The identified equilibrium is reminiscent of existing results in that the advantaged candidate chooses a moderate pure strategy and the disadvantaged candidate mixes between two strategies that are equidistant to that of the opponent (see, e.g., Aragonès and Xefteris, 2012), but it also features significant differences. Most importantly, the advantaged leftist candidate chooses a moderate right platform –and not an unbiased central position as in models with no preferences for ideological consistency– and the disadvantaged rightist candidate chooses more likely an extreme right position, and less likely a (less extreme) left platform (when ideological costs are absent, he mixes between two equally extreme policies with even probability). Candidate A's equilibrium policy does not change with the values of the valence advantage and it increases with the ideological costs. On the other hand, candidate B's equilibrium policies become more extreme for larger values of the valence advantage and for larger ideological costs. Moreover, the probability with which candidate B chooses the rightist policy is always larger than $1/2$ and it decreases with the value of the valence advantage and increases with importance of the ideological costs (see figure 6).

This equilibrium predicts a cautious but aggressive behavior on behalf of the advantaged leftist candidate –he proposes a moderate right policy, which squeezes the rightist candidate out of the popular moderate policies, but still locates within his privileged space– and an occa-

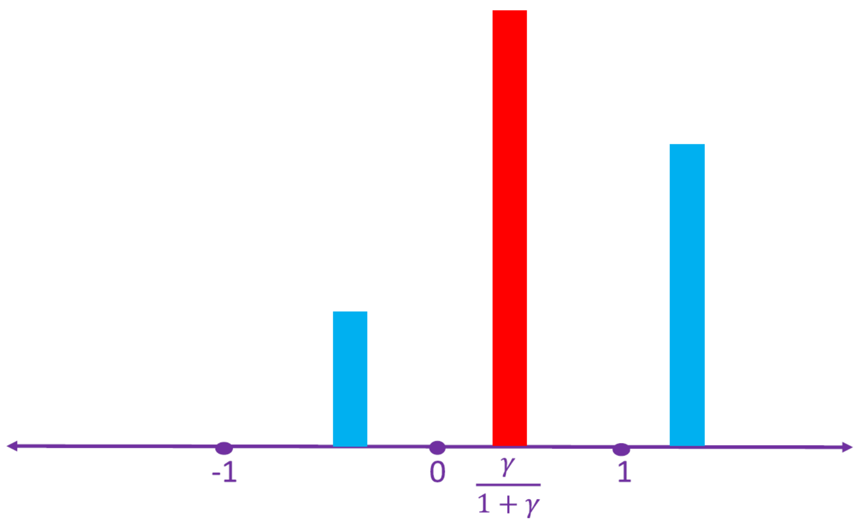

Figure 6: Probability distributions corresponding to the mixed strategy Nash equilibrium.

sionally risky behavior from the disadvantaged rightist candidate –he locates in his privileged space with high probability, but with some non-negligible probability he enters the privileged space of his opponent and proposes a leftist policy.

While both candidates flip-flop, in the sense that they significantly depart from their reference policies, they do not flip-flop in the same style and for the same reasons. The advantaged leftist candidate flip-flops with probability one in a quite moderate way to dominate his opponent, while the disadvantaged candidate flip-flops occasionally to a leftist position just to "survive." It is precisely this credible threat of penetration of the rightist candidate to the privileged space of the leftist candidate, that keeps the leftist candidate from further advancing to the right when his valence advantage increases. Evidently, in this case –and unlike when we have a pure strategy equilibrium– candidate differentiation in realized policy platforms increases with the size of valence asymmetries.

The distance between the strategies that candidate B chooses in equilibrium is larger for larger values of the valence advantage and it decreases with the importance of the ideological costs. On the one hand, choosing policies that are further apart when the valence increases reproduces the existing results of the standard the models of electoral competition with a valence advantage. On the other hand, choosing policies that are closer to candidate A's policy

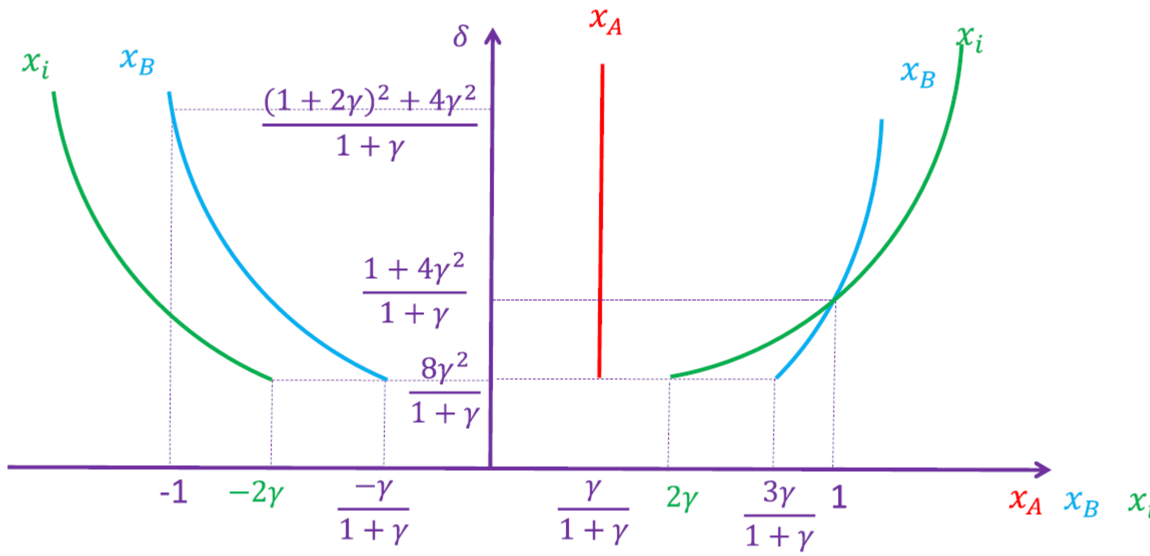

Figure 7: Comparative statics of the mixed strategy Nash equilibrium with respect to δ for $\gamma \leq \frac{1}{2}$.

choice when the importance of the ideological costs increases can be explained by the fact that increasing ideological costs reduce the effects of the valence advantage. The distance between each pure strategy of candidate B and the equilibrium strategy of candidate A increases with the value of the valence advantage and it decreases with the importance of the ideological costs (see figures 7 and 8).

The equilibrium strategy of candidate A depends only on the importance of the ideological costs and it increases with them. Notice that in the pure strategy equilibrium we found that candidate A's policy approaches his reference point for increasing importance of the ideological costs. Instead, in this mixed strategy equilibrium we find that candidate A's policy choice becomes more aggressive when the importance of the ideological costs increases. This is due to the fact that for larger values of the valence advantage the negative effect of increasing ideological costs is less important for both candidates: thus it makes candidate A more aggressive, which reduces his overall advantage and it allows candidate B to move closer to his opponent policy choice.

The locations of the two indifferent voters in equilibrium are given by $\widetilde{x}_i^l = -\sqrt{\delta\,(1+\gamma) - 4\gamma^2}$

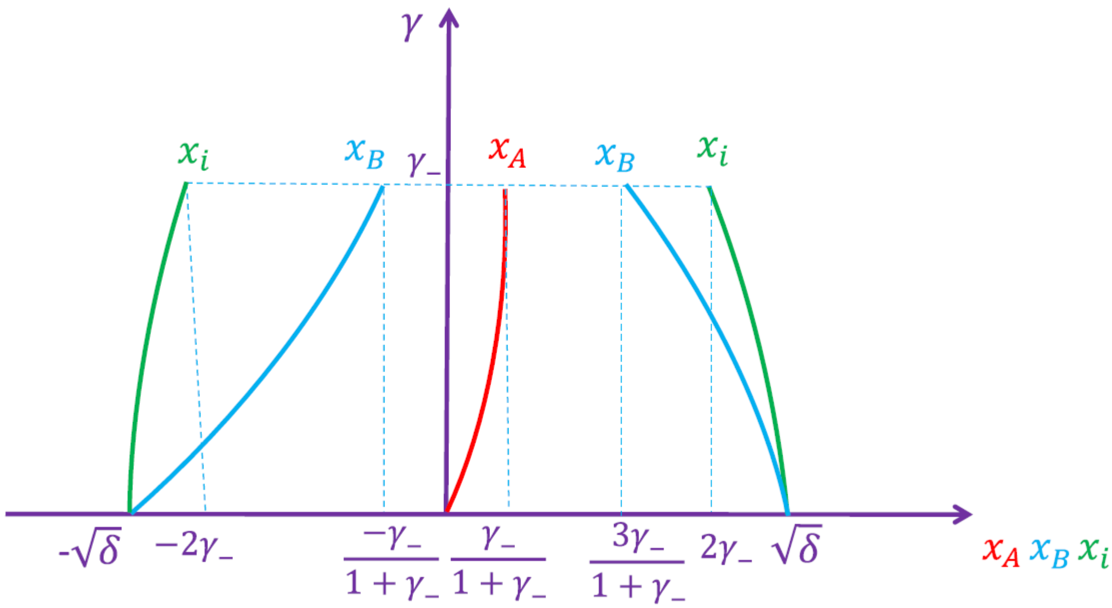

Figure 8: Comparative statics of the mixed strategy Nash equilibrium with respect to γ for $\delta < 4$.

and $\widetilde{x}_i^h = \sqrt{\delta\,(1+\gamma) - 4\gamma^2}$. Thus, in equilibrium payoffs are given by $\Pi_A = F\left(\sqrt{\delta\,(1+\gamma) - 4\gamma^2}\right)$ and $\Pi_B = 1 - \Pi_A$. Therefore, they increase for candidate A with the valence advantage; and since $\frac{\partial \widetilde{x}_i^h}{\partial \gamma} = \frac{\delta - 8\gamma}{2\sqrt{\delta(1+\gamma)-4\gamma^2}} < 0$ if and only if $8\gamma > \delta$ we have that candidate A's payoffs decrease with γ for large values of γ, and they increase with γ otherwise. And since $\sqrt{\delta\,(1+\gamma) - 4\gamma^2} > 0$ the equilibrium payoffs are always larger for candidate A.

As in most similar models, we can interpret F as the distribution of voter's ideal policies, and candidates' objective function as vote-share maximization when voters' preferences are known. These are precisely the assumptions in the original model of Downs (1957). Observe that, under this alternative framing of the model, while this is a mixed equilibrium, there is no uncertainty regarding payoffs. That is, a candidate's expected vote share coincides with his realized vote share. This is due to the fact that the equilibrium is hybrid: candidate A uses a pure strategy and candidate B uses a mixed one. This increases the empirical relevance of the prediction unambiguously, since it implies that the equilibrium is resilient to alternative and, potentially, asymmetric risk attitudes on behalf of the candidates. Indeed, a candidate might be risk averse (i.e. his utility might be concave in his vote-share) and another might be risk loving (i.e. his utility might be convex in his vote-share). As long as a candidate's utility is an increasing function of his vote share, the identified hybrid strategy profile remains an equilibrium of the game.

Finally, the hybrid nature of the identified equilibrium, along with the fact that the game is zero-sum, suggest that its predictions are robust to an alternative and, in some cases, more plausible timing of events. In many instances incumbents commit to policy platforms much earlier than challengers do –either by implementing policies, or simply by positioning themselves on the various issues during their period in office. Challengers, who sometimes emerge as candidates only a few months before the elections, are practically second-movers: they best respond to whatever the incumbent has already promised. This alternative and often more natural timing of events, has attracted the attention of electoral competition research, and has been shown to lead in many cases to different equilibria compared to the ones derived in settings of simultaneous decisions (see, e.g., Carter and Patty 2015).

Interestingly, our predictions remain qualitatively identical if we consider such a sequential timing according to which the advantaged candidate moves first:[10] in the, essentially, unique Subgame Perfect Equilibrium of the game the leftist candidate locates at $x_A = \frac{\gamma}{1+\gamma}$, and the rightist candidate locates either at $x_B = \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ or at $x_B = \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ (or mixes between these two platforms). Notice that the above interpretation of our mixed equilibrium would not be possible if the advantaged candidate also employed a mixed strategy in the simultaneous game or if the game were not zero sum (e.g. if candidates had also policy motives). This observation strengthens the empirical relevance of our analysis as mixed equilibria in such games are often viewed as theoretical artifacts. In our case, this is clearly not true: the mixed equilibrium that we have identified aligns perfectly with the deterministic behavior that the players would follow if they were simply choosing their platforms in turns, following the most commonly observed order.[11]

[10]Indeed, the fact that the advantaged candidate is almost always the incumbent is a well-established empirical fact (see, e.g., Cox and Katz 1996 and references therein).

[11]Mixed strategies can also be purified by considering the limit of the variation of our game in which players

3.4.2 Very large differences in valence

But what happens when the valence asymmetries are very large? This case represents, arguably, the less interesting of all, as when one candidate is deemed by the voters as very superior to his opponent, then the tension of the electoral competition is low and the, other, secondary forces that are absent from the model –e.g. expressive concerns, or even concerns regarding future candidacy– might prescribe the candidates' behavior. Despite that, we try to characterize an equilibrium for this case as well and we find that it follows the same functional form, conditional on the uncertainty regarding voters preferences being sufficiently low.

Proposition 5: *The policies $x_A = \frac{\gamma}{1+\gamma}$ and candidate B mixing as follows: $x_B = \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ with probability $p = \frac{1}{2} + \frac{\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}$ and $x_B = \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ with probability $1-p$ satisfy the conditions for a local equilibrium for $\delta \geq \frac{12\gamma^2}{1+\gamma}$ if $\frac{F''\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right)}{F'\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right)} \leq -\frac{1}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\left(1-\frac{4\gamma^2}{\delta(1+\gamma)-8\gamma^2}\right)$.*

As before, the proof of the proposition shows that the payoff function of candidate A evaluated at the mixed strategy proposed for candidate B exhibits a local maximum at $x_A = \frac{\gamma}{1+\gamma}$, and in the next section we show sufficient conditions for this equilibrium to be global.

Since $\delta > \frac{12\gamma^2}{1+\gamma}$ we have that $\frac{1}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\left(1-\frac{4\gamma^2}{\delta(1+\gamma)-8\gamma^2}\right) > 0$. Given that $F''(x) \leq 0$ for $x > 0$, the equilibrium condition only imposes a restriction on how fast the probability density decreases on the positive support: it has to decrease fast enough, relative to its density.

3.4.3 Equilibrium existence and uniqueness

Up to now we have shown that the pure strategies in the support of candidate B's strategy are best responses to the pure strategy chosen by candidate A, and we have also shown that the payoff function of candidate A evaluated at the mixed strategy of candidate B exhibits a local maximum at $x_A = \frac{\gamma}{1+\gamma}$. In this section we characterize conditions that guarantee that

have also policy preferences and private information about them, when the weight assigned to those preferences becomes arbitrarily small (see, e.g., Aragonès and Palfrey 2005).

candidate A does not have a profitable deviation, and thus that the mixed strategies proposed are the unique Nash equilibrium if $\delta \geq \frac{8\gamma^2}{1+\gamma}$. For this purpose, the probability distribution F that represents the candidates' beliefs about the location of the ideal point of the median voter will now be represented by a Normal distribution with mean equal to zero and standard deviation denoted by σ.

This assumption implies that the local equilibrium characterized in Proposition 4 for moderate values of are δ $\left(\frac{8\gamma^2}{1+\gamma} \leq \delta \leq \frac{12\gamma^2}{1+\gamma}\right)$ exists for all values of σ because $F''(x) \leq 0$ for $x > 0$. And the local equilibrium characterized in Proposition 5 for larger values of δ $\left(\delta \geq \frac{12\gamma^2}{1+\gamma}\right)$ exists as long as $\sigma^2 \leq \frac{\delta(1+\gamma)-4\gamma^2}{1-\frac{4\gamma^2}{\delta(1+\gamma)-8\gamma^2}} = \frac{\left(\delta(1+\gamma)-4\gamma^2\right)\left(\delta(1+\gamma)-8\gamma^2\right)}{\delta(1+\gamma)-12\gamma^2}$. Thus it holds for small enough values of the standard deviation. Overall we have that the proposed equilibrium strategies are a local maximum of candidate A's payoff function for all $\delta \geq \frac{8\gamma^2}{1+\gamma}$ if the probability that the ideal point of the median voter is close to zero is large enough. The next proposition shows that such a restriction on the amount of uncertainty about the location of the ideal point of the median voter guarantees that the proposed mixed strategy equilibrium is indeed a global equilibrium, and it is further unique.

Proposition 6: *The policies $x_A = \frac{\gamma}{1+\gamma}$ and candidate B mixing as follows: $x_B = \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ with probability $p = \frac{1}{2} + \frac{\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}$ and $x_B = \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ with probability $1-p$ constitute the unique mixed strategy equilibrium for $\delta \geq \frac{8\gamma^2}{1+\gamma}$* if *$F(x) = N(0,\sigma)$ and σ is small enough.*

When the uncertainty about the voters' preferences is very high, both candidates use proper mixed strategies in equilibrium. Therefore, their exact behavior will depend on their attitudes towards risk and/or losses/gains with respect to expected payoffs (in the sense of Koszegi and Rabin 2006). That is, for such parameter values, equilibrium conditions are very sensitive to additional unobserved candidate characteristics and cannot lead to robust predictions regarding the electoral competition outcome.

4 Concluding remarks

In this paper we have introduced preferences for ideological consistency of the competing candidates to their reference policies (primary platforms and/or party principles), in a canonical Downsian model with an advantaged candidate. Our results break new ground in several ways, and uncover how preferences for valence and ideological consistency shape electoral competition and policy outcomes.

A novel and, quite, unexpected finding of our study was that parties might not lose on all fronts when nominating a lower quality candidate. In certain situations, they have policy-related gains by doing so. As discussed above, other forces pushing for the nomination of a higher valence candidate might dominate, and lead parties have a positive net preference for candidate quality. In any case, studying a nomination game between competing parties appears as the natural next step. Moreover, it might be interesting to check whether different nomination processes and degrees of intraparty democracy, lead to higher or lower quality nominees. For instance, if win-motivated party elites nominate the candidates, then the quality might be higher compared to when a policy-motivated base makes the appointment.

In our analysis we have assumed that both, the reference policies of the candidates and the distribution of the voters' preferences, are symmetric around zero. Extending the model by including an asymmetry in the candidates' reference policies would introduce an additional source of strategic advantage that would affect the candidates' equilibrium policy choices. Furthermore, we have assumed that the intensity with which voters punish the ideological inconsistency of the candidates is equal for both parties. It is plausible that voters decide to punish candidates that belong to different parties with different intensities. For instance, candidates that belong to older parties may be punished more heavily than candidates that belong to younger parties, since the reference policies of the former ones may be considered to have a larger relevance for voters because of their persistence over time. These are extensions of our model that may produce interesting results.

Finally, since the introduction of preferences for ideological consistency in a model which does not admit pure equilibria, allowed for stable outcomes for a wide range of parameter values, one would potentially like to study whether it exerts the same stabilizing force in alternative models that admit only mixed strategy equilibria. For instance, we know that multidimensional electoral competition between vote-share motivated candidates rarely admits pure strategy equilibria (Plott 1967). Would preferences for ideological consistency lead to more stable outcomes, and if so, how would these outcomes depend on the strength of these preferences and the voters' policy preferences?[12] Answering these questions is clearly beyond the scope of the present analysis, but, undeniably, the prospect of restoring stability in this important model by introducing preferences for consistency seems quite promising.

5 References

Agranov, M., 2016. Flip-flopping, primary visibility, and the selection of candidates. American Economic Journal: Microeconomics, 8(2), pp.61-85.

Andreottola, G., 2021. Flip-flopping and electoral concerns. Journal of Politics, forthcoming.

Aragonès, E. and Palfrey, T.R., 2002. Mixed equilibrium in a Downsian model with a favored candidate. Journal of Economic Theory, 103(1), pp.131-161.

Aragonés, E. and Palfrey, T.R., 2005. Electoral competition between two candidates of different quality: The effects of candidate ideology and private information. In Social Choice and Strategic Decisions (pp. 93-112). Springer, Berlin, Heidelberg.

Aragonès, E. and Xefteris, D., 2012. Candidate quality in a Downsian model with a continuous policy space. Games and Economic Behavior, 75(2), pp.464-480.

[12]Existence of a pure strategy equilibrium in multidimensional settings can be restored by probabilistic voting (see, e.g., Enelow and Hinich 1989 and Coughlin 1992), policy-motivated candidates (see, e.g., Calvert 1985 and Duggan and fey 2005), and by candidate differentiation in fixed dimensions (see, e.g., Krasa and Polborn 2010 and Xefteris 2017).

Aragonès, E. and Xefteris, D., 2017. Voters' private valuation of candidates' quality. Journal of Public Economics, 156, pp.121-130.

Ashworth, S. and De Mesquita, E.B., 2009. Elections with platform and valence competition. Games and Economic Behavior, 67(1), pp.191-216.

Bernhardt, D., Buisseret, P. and Hidir, S., 2020. The Race to the Base. American Economic Review, 110(3), pp.922-42.

Bernheim, B.D. and Kartik, N., 2014. Candidates, character, and corruption. American Economic Journal: Microeconomics, 6(2), pp.205-46.

Callander, S. and Wilkie, S., 2007. Lies, damned lies, and political campaigns. Games and Economic Behavior, 60(2), pp.262-286.

Calvert, R.L., 1985. Robustness of the multidimensional voting model: Candidate motivations, uncertainty, and convergence. American Journal of Political Science, pp.69-95.

Carrillo, J.D. and Castanheira, M., 2008. Information and strategic political polarisation. The Economic Journal, 118(530), pp.845-874.

Carter, J. and Patty, J.W., 2015. Valence and campaigns. American Journal of Political Science, 59(4), pp.825-840.

Castanheira, M., Huck, S. and Leutgeb, J., 2020. How Trump Triumphed: Multi-candidate Primaries with Buffoons. ULB working paper.

Coughlin, P.J., 1992. Probabilistic voting theory. Cambridge University Press.

Cox, G.W. and Katz, J.N., 1996. Why did the incumbency advantage in US House elections grow?. American journal of political science, pp.478-497.

Downs, A., 1957. An economic theory of democracy. HarperCollins Publishers Inc.

Duggan, J. and Fey, M., 2005. Electoral competition with policy-motivated candidates. Games and Economic Behavior, 51(2), pp.490-522.

Enelow, J.M. and Hinich, M.J., 1989. A general probabilistic spatial theory of elections. Public Choice, 61(2), pp.101-113.

Groseclose, T., 2001. A model of candidate location when one candidate has a valence advantage. American Journal of Political Science, pp.862-886.

Hummel, P., 2010a. Flip-flopping from primaries to general elections. Journal of Public Economics, 94(11-12), pp.1020-1027.

Hummel, P., 2010b. On the nature of equilibria in a Downsian model with candidate valence. Games and Economic Behavior, 70(2), pp.425-445.

Hummel, P., 2013. Candidate strategies in primaries and general elections with candidates of heterogeneous quality. Games and Economic Behavior, 78, pp.85-102.

Kartik, N. and McAfee, R.P., 2007. Signaling character in electoral competition. American Economic Review, 97(3), pp.852-870.

Koszegi, B. and Rabin, M., 2006. A model of reference-dependent preferences. The Quarterly Journal of Economics, 121(4), pp.1133-1165.

Krasa, S. and Polborn, M., 2010. The binary policy model. Journal of Economic Theory, 145(2), pp.661-688.

Kreps, T.A., Laurin, K. and Merritt, A.C., 2017. Hypocritical flip-flop, or courageous evolution? When leaders change their moral minds. Journal of Personality and Social Psychology, 113(5), p.730.

McCarty, N., Poole, K.T. and Rosenthal, H., 2016. Polarized America: The dance of ideology and unequal riches. MIT Press.

Plott, C.R., 1967. A notion of equilibrium and its possibility under majority rule. The American Economic Review, 57(4), pp.787-806.

Poole, K.T. and Rosenthal, H., 1984. The polarization of American politics. The Journal of Politics, 46(4), pp.1061-1079.

Schofield, N., 2004. Equilibrium in the spatial valence model of politics. Journal of Theoretical Politics, 16(4), pp.447-481.

Stone, W.J. and Simas, E.N., 2010. Candidate valence and ideological positions in US House elections. American Journal of Political Science, 54(2), pp.371-388.

Xefteris, D., 2017. Multidimensional electoral competition between differentiated candidates. Games and Economic Behavior, 105, pp.112-121.

Zakharov, A.V., 2009. A model of candidate location with endogenous valence. Public Choice, 138(3-4), pp.347-366.

6 Appendix

Proof of Proposition 1: The fact that there is a convergent equilibrium if $\delta = \gamma = 0$ is known since Downs (1957). The non-existence of a convergent equilibrium if $\delta > 0$ and $\gamma = 0$ follows from Groseclose (2001). Hence, the remaining proof focuses on the case in which $\delta \geq 0$ and $\gamma > 0$.

Suppose that $x_A = x_B = x$, then we have that $u_i(A) > u_i(B)$ if and only if $x < \frac{\delta}{4\gamma}$. Therefore we must have that $\Pi_A(x,x) = 1$ and $\Pi_B(x,x) = 0$ for $x < \frac{\delta}{4\gamma}$. In this case, candidate B has a profitable deviation by diverging from A. Similarly, we have that $\Pi_A(x,x) = 0$ and $\Pi_B(x,x) = 1$ for $x > \frac{\delta}{4\gamma}$. In this case, A has a profitable deviation by diverging from B. Finally, we have that $\Pi_A(x,x) = \Pi_B(x,x) = \frac{1}{2}$ for $x = \frac{\delta}{4\gamma}$. Now consider a deviation for A such that $x_A < x_B = \frac{\delta}{4\gamma}$. In this case we have that this deviation is profitable whenever

$$\Pi_A(x_A, \tfrac{\delta}{4\gamma}) = F(\widetilde{x}_i(x_A, \tfrac{\delta}{4\gamma})) > \tfrac{1}{2}.$$

That is, we need $\widetilde{x}_i(x_A, \frac{\delta}{4\gamma}) > 0$ which holds if and only if

$$(1+\gamma)(x_A)^2 + 2\gamma x_A - (1+\gamma)\left(\tfrac{\delta}{4\gamma}\right)^2 - \tfrac{\delta}{2} < 0.$$

This expression is satisfied for all values of x_A such that $-\frac{2\gamma}{1+\gamma} - \frac{\delta}{4\gamma} < x_A < \frac{\delta}{4\gamma}$. Thus, these values are profitable deviations for A. This is sufficient to establish that $x_A = x_B = x = \frac{\delta}{4\gamma}$ is not an equilibrium. ⧫

Proof of Proposition 2: We first compute the best responses for both candidates. For candidate A we solve two maximization problems, indexed by (A.I) and (A.II) respectively. The former (latter) attempts to find the best possible action of candidate A to the left (right) of x_B.

$$\text{(A.I)} \max_{x_A : x_A \leq x_B} \Pi_A(x_A, x_B) = F(\widetilde{x}_i)$$

This maximization problem admits a first order condition (FOC),

$$F'(\widetilde{x}_i)\tfrac{\partial \widetilde{x}_i}{\partial x_A} = 0 \text{ if and only if } \tfrac{\partial \widetilde{x}_i}{\partial x_A} = 0$$

and a second order condition (SOC),

$$F''(\widetilde{x}_i)\tfrac{\partial \widetilde{x}_i}{\partial x_A} + F'(\widetilde{x}_i)\tfrac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2} \leq 0.$$

The SOC, when evaluated at x_A^* is given by

$$F'(\widetilde{x}_i)\tfrac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2} \leq 0 \text{ if and only if } \tfrac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2} \leq 0.$$

$$\text{(A.II)} \max_{x_A : x_A \geq x_B} \Pi_A(x_A, x_B) = 1 - F(\widetilde{x}_i)$$

This maximization problem admits a FOC,

$$-F'\left(\widetilde{x}_i\right)\frac{\partial \widetilde{x}_i}{\partial x_A}=0 \text{ if and only if } \frac{\partial \widetilde{x}_i}{\partial x_A}=0$$

and a SOC,

$$-F''\left(\widetilde{x}_i\right)\frac{\partial \widetilde{x}_i}{\partial x_A}-F'\left(\widetilde{x}_i\right)\frac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2}\leq 0,$$

which, evaluated at x_A^*, becomes

$$-F'\left(\widetilde{x}_i\right)\frac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2}\leq 0 \text{ if and only if } \frac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2}\geq 0,$$

In both cases, for the FOC we need

$$\frac{\partial \widetilde{x}_i}{\partial x_A}=\frac{\delta-4\gamma x_B+(1+\gamma)(x_B-x_A)^2}{2(x_B-x_A)^2}=0,$$

which is equivalent to

$$x_A=x_B\pm\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}} \text{ as long as } x_B\geq\frac{\delta}{4\gamma}.\text{[13]}$$

For the SOC we have that $\frac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2}=\frac{\delta-4x_B\gamma}{(x_B-x_A)^3}$, and thus $\frac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2}\leq 0$ if $x_B>\frac{\delta}{4\gamma}$ and $x_B>x_A$; and $\frac{\partial^2 \widetilde{x}_i}{\partial (x_A)^2}\geq 0$ if $x_B>\frac{\delta}{4\gamma}$ and $x_B<x_A$. This implies that for $x_B\geq\frac{\delta}{4\gamma}$ the maximum of $\Pi_A(x_A,x_B)=F\left(\widetilde{x}_i\right)$ is $x_A=x_B-\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}}$, and the maximum of $\Pi_A(x_A,x_B)=1-F\left(\widetilde{x}_i\right)$ is $x_A=x_B+\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}}$.

Therefore, we have that $x_A=x_B-\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}}$ is a best response for A if and only if

$$F(\widetilde{x}_i\left(x_B-\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}},x_B\right))\geq 1-F(\widetilde{x}_i\left(x_B+\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}},x_B\right))$$

[13] For $x_B<\frac{\delta}{4\gamma}$ we have that candidate A's best response is $x_A=x_B$.

Since $1-F(\widetilde{x}_i\left(x_B+\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}},x_B\right))=F(-\widetilde{x}_i\left(x_B+\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}},x_B\right))$, we have to check whether $\widetilde{x}_i\left(x_B-\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}},x_B\right) > -\widetilde{x}_i\left(x_B+\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}},x_B\right))$, which holds if and only if $x_B > -\frac{\gamma}{1+\gamma}$. Given that $\frac{\delta}{4\gamma} > -\frac{\gamma}{1+\gamma}$, we have that for $x_B \geq \frac{\delta}{4\gamma}$ the best response of candidate A is $x_A = x_B-\sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}}$; and for $x_B < \frac{\delta}{4\gamma}$ the best response of candidate A is $x_A = x_B$.

For candidate B we solve two maximization problems, indexed by (B.I) and (B.II) respectively. The former (latter) attempts to find the best possible action of candidate B to the left (right) of x_A.

(B.I) $\max_{x_B:x_B\geq x_A}\Pi_B(x_A,x_B)=1-F\left(\widetilde{x}_i\right)$

This maximization problem admits a FOC,

$$-F'\left(\widetilde{x}_i\right)\frac{\partial\widetilde{x}_i}{\partial x_B}=0 \text{ if and only if } \frac{\partial\widetilde{x}_i}{\partial x_B}=0$$

and a SOC,

$$-F''\left(\widetilde{x}_i\right)\frac{\partial\widetilde{x}_i}{\partial x_B}-F'\left(\widetilde{x}_i\right)\frac{\partial^2\widetilde{x}_i}{\partial(x_B)^2}\leq 0,$$

which, evaluated at x_B^*, becomes

$$-F'\left(\widetilde{x}_i\right)\frac{\partial^2\widetilde{x}_i}{\partial(x_B)^2}\leq 0 \text{ if and only if } \frac{\partial^2\widetilde{x}_i}{\partial(x_B)^2}\geq 0.$$

(B.II) $\max_{x_B:x_B\leq x_A}\Pi_B(x_A,x_B)=F\left(\widetilde{x}_i\right)$

This maximization problem admits a FOC,

$$F'\left(\widetilde{x}_i\right)\frac{\partial\widetilde{x}_i}{\partial x_B}=0 \text{ if and only if } \frac{\partial\widetilde{x}_i}{\partial x_B}=0$$

and a SOC,

$$F''\left(\widetilde{x}_i\right)\frac{\partial\widetilde{x}_i}{\partial x_B}+F'\left(\widetilde{x}_i\right)\frac{\partial^2\widetilde{x}_i}{\partial(x_B)^2}\leq 0,$$

which, evaluated at x_B^* becomes,

$$F'(\widetilde{x}_i)\frac{\partial^2 \widetilde{x}_i}{\partial (x_B)^2} \leq 0 \text{ if and only if } \frac{\partial^2 \widetilde{x}_i}{\partial (x_B)^2} \leq 0.$$

In both cases, for the FOC we need

$$\frac{\partial \widetilde{x}_i}{\partial x_B} = \frac{(1+\gamma)\left((x_B)^2-(x_A)^2\right)-2x_A[(1+\gamma)(x_B-x_A)-2\gamma]-\delta}{2(x_B-x_A)^2} = 0$$

which is equivalent to

$$x_B = x_A \pm \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}} \text{ as long as } x_A \leq \frac{\delta}{4\gamma}.$$ [14]

For the SOC we have that $\frac{\partial^2 \widetilde{x}_i}{\partial (x_B)^2} = \frac{\delta-4x_A\gamma}{(x_B-x_A)^3}$, and thus $\frac{\partial^2 \widetilde{x}_i}{\partial (x_B)^2} \leq 0$ if $x_A < \frac{\delta}{4\gamma}$ and $x_B < x_A$; and $\frac{\partial^2 \widetilde{x}_i}{\partial (x_B)^2} \geq 0$ if $x_A < \frac{\delta}{4\gamma}$ and $x_B > x_A$. This implies that for $x_A \leq \frac{\delta}{4\gamma}$ the maximum of $\Pi_B(x_A, x_B) = F(\widetilde{x}_i)$ is $x_B = x_A - \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$, and the maximum of $\Pi_B(x_A, x_B) = 1 - F(\widetilde{x}_i)$ is $x_B = x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$.

Therefore, we have that $x_B = x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$ is a best response for B if and only if

$$F(\widetilde{x}_i\left(x_A, x_A - \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)) < 1 - F(\widetilde{x}_i\left(x_A, x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right))$$

Since $1 - F(\widetilde{x}_i\left(x_A, x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)) = F(-\widetilde{x}_i\left(x_A, x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right))$, we have to see whether $\widetilde{x}_i\left(x_A, x_A - \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right) < -\widetilde{x}_i\left(x_A, x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right))$, which holds if and only if $\frac{\gamma}{1+\gamma} > x_A$. Thus, for $x_A < \frac{\delta}{4\gamma}$ the best response of candidate B is $x_B = x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$ for all $x_A < \frac{\gamma}{1+\gamma}$; and the best response of candidate B is $x_B = x_A - \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$ whenever $x_A > \frac{\gamma}{1+\gamma}$.

Notice that if $\frac{4\gamma^2}{1+\gamma} > \delta$ we have that $\frac{\gamma}{1+\gamma} > \frac{\delta}{4\gamma}$, thus candidate B's best response is $x_B = x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$ for all $x_A \leq \frac{\delta}{4\gamma}$, and it is $x_B = x_A$ for all $x_A \geq \frac{\delta}{4\gamma}$. And for $\frac{4\gamma^2}{1+\gamma} < \delta$ we have that $\frac{\gamma}{1+\gamma} < \frac{\delta}{4\gamma}$, thus candidate B's best response is $x_B = x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$ for all $\frac{\gamma}{1+\gamma} < x_A < \frac{\delta}{4\gamma}$; candidate B's best response is $x_B = x_A - \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$ for all $x_A \leq \frac{\gamma}{1+\gamma} < \frac{\delta}{4\gamma}$, and it is $x_B = x_A$ for all $x_A \geq \frac{\delta}{4\gamma}$

Since, a pure strategy equilibrium must be such that $x_A < \frac{\delta}{4\gamma} < x_B$, we have a unique plausible combination of the best responses, given by

[14] For $x_A > \frac{\delta}{4\gamma}$ we have that candidate B's best response is $x_B = x_A$.

$$x_A = x_B - \sqrt{\tfrac{4x_B\gamma - \delta}{1+\gamma}} \text{ and } x_B = x_A + \sqrt{\tfrac{\delta - 4x_A\gamma}{1+\gamma}}.$$

Therefore, in a pure strategy equilibrium, we have

$$x_A + x_B = \tfrac{\delta}{2\gamma} \text{ and } x_B - x_A = \sqrt{\tfrac{4x_B\gamma - \delta}{1+\gamma}} = \sqrt{\tfrac{\delta - 4x_A\gamma}{1+\gamma}}.$$ [15]

Given that $x_A = x_B - \sqrt{\frac{4x_B\gamma-\delta}{1+\gamma}}$ and $x_A + x_B = \frac{\delta}{2\gamma} \Leftrightarrow x_B = \frac{\delta}{2\gamma} - x_A$ and substituting the latter in the first equation, we get (after some algebraic manipulations) that

$$x_A = \tfrac{\delta}{4\gamma} \text{ or } x_A = \tfrac{\delta}{4\gamma} - \tfrac{\gamma}{(1+\gamma)}.$$

Suppose that $x_A = \frac{\delta}{4\gamma}$. In this case, candidate B's best response is given by $x_B = \frac{\delta}{2\gamma} - x_A = \frac{\delta}{4\gamma}$. But we know from Proposition 1 that there is no equilibrium with convergent policies unless $\delta = \gamma = 0$.

Now suppose that $x_A = \frac{\delta}{4\gamma} - \frac{\gamma}{1+\gamma}$. In this case, candidate B's best response is given by $x_B = \frac{\delta}{4\gamma} + \frac{\gamma}{1+\gamma}$ for $x_A < \frac{\gamma}{1+\gamma}$. Since $x_A = \frac{\delta}{4\gamma} - \frac{\gamma}{1+\gamma} < \frac{\delta}{4\gamma} < x_B = \frac{\delta}{4\gamma} + \frac{\gamma}{1+\gamma}$ we have that $x_A = \frac{\delta}{4\gamma} - \frac{\gamma}{1+\gamma}$ and $x_B = \frac{\delta}{4\gamma} + \frac{\gamma}{1+\gamma}$ is a pure strategy equilibrium whenever $\frac{4\gamma^2}{1+\gamma} > \delta$. Because in this case candidate B's best response is $x_B = x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$ for all $x_A < \frac{\delta}{4\gamma}$. And for $\frac{4\gamma^2}{1+\gamma} < \delta$ we have that candidate B's best response is $x_B = x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}$ for all $\frac{\gamma}{1+\gamma} > x_A$. Therefore, whenever $\frac{\gamma}{1+\gamma} > \frac{\delta}{4\gamma} - \frac{\gamma}{1+\gamma}$, that is, $\frac{8\gamma^2}{1+\gamma} > \delta$ we have that $x_A = \frac{\delta}{4\gamma} - \frac{\gamma}{1+\gamma}$ and $x_B = \frac{\delta}{4\gamma} + \frac{\gamma}{1+\gamma}$ is a pure strategy equilibrium.

Thus we have that $x_A = \frac{\delta}{4\gamma} - \frac{\gamma}{1+\gamma}$ and $x_B = \frac{\delta}{4\gamma} + \frac{\gamma}{1+\gamma}$ is an equilibrium for $\delta < \frac{8\gamma^2}{1+\gamma}$. By the constant-sum nature of the game (i.e., by the fact that equilibrium strategies are interchangeable), and by the fact that the described best responses are unique given that the other player uses the described equilibrium strategy; it follows that for $\delta < \frac{8\gamma^2}{1+\gamma}$ this equilibrium is unique. ♦

Proof of Proposition 3: The expected policy of the pure strategy equilibrium is given by

[15] These expressions support Corollary 1.

$$x_e = F\left(\widetilde{x}_i\right)\left(\frac{\delta}{4\gamma} - \frac{\gamma}{1+\gamma}\right) + \left(1 - F\left(\widetilde{x}_i\right)\right)\left(\frac{\delta}{4\gamma} + \frac{\gamma}{1+\gamma}\right) = \frac{\delta}{4\gamma} + \frac{\gamma}{1+\gamma}\left(1 - 2F\left(\widetilde{x}_i\right)\right),$$

with $\widetilde{x}_i = \frac{\delta(1+\gamma)}{4\gamma}$.

The derivative with respect to the valence advantage is given by $\frac{\partial x_e}{\partial \delta} = \frac{1}{2}\left(\frac{1}{2\gamma} - F'\left(\widetilde{x}_i\right)\right)$, and $\frac{\partial x_e}{\partial \delta} > 0$ if and only if $\frac{1}{2\gamma} > F'\left(\widetilde{x}_i\right)$. Thus, $\frac{1}{2\gamma} > F'(0)$ is a sufficient condition for $\frac{\partial x_e}{\partial \delta} > 0$.

The derivative with respect to the ideological cost is given by

$$\frac{\partial x_e}{\partial \gamma} = -\frac{\delta}{4\gamma^2}\left(1 - \frac{\gamma}{1+\gamma} 2F'\left(\widetilde{x}_i\right)\right) + \frac{1}{(1+\gamma)^2}\left(1 - 2F\left(\widetilde{x}_i\right)\right).$$

Since $F\left(\widetilde{x}_i\right) > \frac{1}{2}$, we have that $\frac{1}{(1+\gamma)^2}\left(1 - 2F\left(\widetilde{x}_i\right)\right) < 0$. We also have that $1 - \frac{\gamma}{1+\gamma} 2F'\left(\widetilde{x}_i\right) > 0$ if and only if $\frac{1+\gamma}{2\gamma} > F'\left(\widetilde{x}_i\right)$. Therefore, $\frac{1+\gamma}{2\gamma} > F'(0)$ is a sufficient condition for $\frac{\partial x_e}{\partial \gamma} < 0$.

Proof of Proposition 4: We consider the following equilibrium candidate: candidate A uses the pure strategy $x_A = \frac{\gamma}{1+\gamma}$, and candidate B mixes as follows: $x_B = \frac{\gamma + \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma}$ with probability p and $x_B = \frac{\gamma - \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma}$ with probability $1 - p$.

We know that B does not have a profitable deviation, because in Proposition 2 we found that if $x_A = \frac{\gamma}{1+\gamma}$ then the only candidates to best response of B are

$$x_B = \frac{\gamma + \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma} \text{ and } x_B = \frac{\gamma - \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma}.$$

Notice that $\delta(1+\gamma) - 4\gamma^2 > 0$ if and only if $\delta > \frac{4\gamma^2}{1+\gamma}$, which holds now that we assume $\delta > \frac{8\gamma^2}{1+\gamma}$. We also have that

$$\Pi_B\left(\frac{\gamma}{1+\gamma}, \frac{\gamma + \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma}\right) = 1 - F\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma + \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma}\right)\right) = F\left(-\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma + \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma}\right)\right),$$

and

$$\Pi_B\left(\frac{\gamma}{1+\gamma}, \frac{\gamma - \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma}\right) = F\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma - \sqrt{\delta(1+\gamma) - 4\gamma^2}}{1+\gamma}\right)\right).$$

Since,

$$\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)=-\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)=\sqrt{\delta\left(1+\gamma\right)-4\gamma^2},$$

it follows that $\Pi_B\left(\frac{\gamma}{1+\gamma},\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)=\Pi_B\left(\frac{\gamma}{1+\gamma},\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)$. Thus, we know that candidate B is indifferent between these two strategies and therefore B does not have a profitable deviation.

Now we have to check that candidate A does not have a profitable deviation. In this case we have that

$$\Pi_A(x_A,\left(x_A+\sqrt{\tfrac{\delta-4x_A\gamma}{1+\gamma}},p;x_A-\sqrt{\tfrac{\delta-4x_A\gamma}{1+\gamma}},1-p\right)=$$
$$pF\left(\widetilde{x}_i\left(x_A,x_A+\sqrt{\tfrac{\delta-4x_A\gamma}{1+\gamma}}\right)\right)+\left(1-p\right)F\left(-\widetilde{x}_i\left(x_A,x_A-\sqrt{\tfrac{\delta-4x_A\gamma}{1+\gamma}}\right)\right)$$

and

$$\frac{\partial\Pi_A}{\partial x_A}=pF'\left(\widetilde{x}_i\left(x_A,x_A+\sqrt{\tfrac{\delta-4x_A\gamma}{1+\gamma}}\right)\right)\frac{\partial\widetilde{x}_i\left(x_A,x_A+\sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)}{\partial x_A}-$$
$$\left(1-p\right)F'\left(-\widetilde{x}_i\left(x_A,x_A-\sqrt{\tfrac{\delta-4x_A\gamma}{1+\gamma}}\right)\right)\frac{\partial\widetilde{x}_i\left(\left(x_A,x_A-\sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)\right)}{\partial x_A}.$$

We evaluate this derivative at $x_A=\frac{\gamma}{1+\gamma}$ and obtain

$$\frac{\partial\Pi_A}{\partial x_A}=F'\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\left[p\frac{\partial\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}-\left(1-p\right)\frac{\partial\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right],$$

which is equal to zero if and only if

$$p\left[\frac{\partial\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}+\frac{\partial\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right]=\frac{\partial\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}.$$

Since,

$$\frac{\partial\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}=\left(1-\frac{2\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\right)\left(1+\gamma\right)\text{ and}$$
$$\frac{\partial\widetilde{x}_i\left(\frac{\gamma}{1+\gamma},\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}=\left(1+\frac{2\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\right)\left(1+\gamma\right),$$

we have that $p = \frac{1}{2} + \frac{\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}$ satisfies the FOC at the equilibrium candidate $x_A = \frac{\gamma}{1+\gamma}$.

Next we have to check the SOC,

$$\frac{\partial^2 \Pi_A}{\partial (x_A)^2} = p\left[F''\left(\widetilde{x}_i\left(x_A, x_A + \sqrt{\frac{\delta - 4x_A\gamma}{1+\gamma}}\right)\right)\left(\frac{\partial \widetilde{x}_i\left(x_A, x_A+\sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)}{\partial x_A}\right)^2 + F'\left(\widetilde{x}_i\left(x_A, x_A + \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)\right)\frac{\partial^2 \widetilde{x}_i\left(x_A, x_A+\sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)}{\partial (x_A)^2}\right] +$$

$$+(1-p)\left[F''\left(-\widetilde{x}_i\left(x_A, x_A - \sqrt{\frac{\delta - 4x_A\gamma}{1+\gamma}}\right)\right)\left(\frac{\partial \widetilde{x}_i\left(x_A, x_A-\sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)}{\partial x_A}\right)^2 - F'\left(-\widetilde{x}_i\left(x_A, x_A - \sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)\right)\frac{\partial^2 \widetilde{x}_i\left(x_A, x_A-\sqrt{\frac{\delta-4x_A\gamma}{1+\gamma}}\right)}{\partial (x_A)^2}\right] \leq 0$$

Since,

$$\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right) = -\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right),$$

we have that

$$F''\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right) = F''\left(-\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)$$

and

$$F'\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right) = F'\left(-\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right).$$

Thus, in order to have the SOC satisfied at the equilibrium candidate it suffices to show that

$$\frac{\partial^2 \Pi_A}{\partial (x_A)^2} = F''\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\left[p\left(\frac{\partial \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right)^2 + (1-p)\left(\frac{\partial \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right)^2\right] +$$

$$+F'\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\left[p\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} - (1-p)\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2}\right] \leq 0$$

Since, $F''\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right) \leq 0$, it follows that

$$F''\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\left[p\left(\frac{\partial \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right)^2 + (1-p)\left(\frac{\partial \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right)^2\right] \leq 0.$$

Given that $F'\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right) > 0$, for the SOC to be satisfied, it suffices to show that

$$p\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} - (1-p)\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} \leq 0.$$

Since,

$$\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} = \frac{-4\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{\delta(1+\gamma)-4\gamma^2}(1+\gamma)^2 \text{ and}$$
$$\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} = -\frac{4\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{\delta(1+\gamma)-4\gamma^2}(1+\gamma)^2,$$

it follows that

$$\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} + \frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} = -\frac{8\gamma}{\delta(1+\gamma)-4\gamma^2}(1+\gamma)^2 < 0.$$

By substituting $p = \frac{1}{2} + \frac{\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}$ and the above in

$$p\left[\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} + \frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2}\right] - \frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} \leq 0$$

we obtain $-\left(\frac{1}{2} + \frac{\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\right)\frac{8\gamma}{\delta(1+\gamma)-4\gamma^2}(1+\gamma)^2 + \frac{4\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{\delta(1+\gamma)-4\gamma^2}(1+\gamma)^2 \leq 0$ if and only if $-\left(\frac{2\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\right)4\gamma + 4\gamma + \sqrt{\delta(1+\gamma)-4\gamma^2} \leq 0$. The latter is true if and only if $\delta < \frac{12\gamma^2}{1+\gamma}$.

Thus we have shown that for $\frac{8\gamma^2}{1+\gamma} < \delta < \frac{12\gamma^2}{1+\gamma}$ and $F''\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right) \leq 0$ the posited profile is robust to local deviations for any admissible distribution. ♦

Proof of Proposition 5: For larger values of δ (that is, for $\delta > \frac{12\gamma^2}{1+\gamma}$) we need to investigate under which conditions on F, candidate A's SOC (as presented in the proof of Proposition 4) is still satisfied. We have that $\frac{\partial^2 \Pi_A}{\partial (x_A)^2} \leq 0$ evaluated at the posited profile, if and only if

$$\frac{F''\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)}{F'\left(\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)} \leq \frac{\left[-p\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} + (1-p)\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2}\right]}{\left[p\left(\frac{\partial \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right)^2 + (1-p)\left(\frac{\partial \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right)^2\right]}.$$

for $p = \frac{1}{2} + \frac{\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}$

Since,

$$\left(\frac{\partial \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right)^2 - \left(\frac{\partial \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}\right)^2 = -\frac{8\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\left(1+\gamma\right)^2$$

and

$$\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} = \frac{-4\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{\delta(1+\gamma)-4\gamma^2}\left(1+\gamma\right)^2 = \frac{(1+\gamma)^2}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\left(1-\frac{4\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\right),$$

$$\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} = -\frac{4\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{\delta(1+\gamma)-4\gamma^2}\left(1+\gamma\right)^2 = -\frac{(1+\gamma)^2}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\left(1+\frac{4\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\right),$$

it follows that

$$\frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} + \frac{\partial^2 \widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial (x_A)^2} = -\frac{8\gamma(1+\gamma)^2}{\delta(1+\gamma)-4\gamma^2} < 0.$$

Therefore, the SOC is satisfied if and only if

$$\frac{F''\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right)}{F'\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right)} \leq -\frac{1}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\left(1-\frac{4\gamma^2}{\delta(1+\gamma)-8\gamma^2}\right). \blacklozenge$$

Proof of Proposition 6: Notice that if F is a Normal distribution with mean zero and standard error $\sigma > 0$, then, for any $x \in \mathbb{R}$, we have

$$\frac{F''(x)}{F'(x)} = -\frac{x}{\sigma^2}.$$

By the fact that $\sqrt{\delta\left(1+\gamma\right)-4\gamma^2} > 0$ for the parameter space under consideration, it follows that when $\sigma > 0$ is sufficiently small the SOC of candidate A at the posited profile (as presented at the end of the proof of Proposition 5)

$$\frac{F''\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right)}{F'\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right)} = -\frac{\sqrt{\delta(1+\gamma)-4\gamma^2}}{\sigma^2} \leq -\frac{1}{\sqrt{\delta(1+\gamma)-4\gamma^2}}\left(1-\frac{4\gamma^2}{\delta(1+\gamma)-8\gamma^2}\right)$$

holds for $\delta \geq \frac{12\gamma^2}{1+\gamma}$ if and only if $\sigma^2 \leq \frac{\left(\delta(1+\gamma)-4\gamma^2\right)\left(\delta(1+\gamma)-8\gamma^2\right)}{\delta(1+\gamma)-12\gamma^2}$.

And we already have from Proposition 4 that the SOC of candidate A at the posited profile for $\delta < \frac{12\gamma^2}{1+\gamma}$ holds for any value of $\sigma > 0$. Thus, we have that for $\sigma > 0$ sufficiently small $x_A = \frac{\gamma}{1+\gamma}$ is a local maximum for Π_A for all $\delta \geq \frac{8\gamma^2}{1+\gamma}$.

To conclude that the posited profile is a Nash equilibrium when $\sigma > 0$ is sufficiently small, we further need to argue that any deviation of candidate A is not profitable in such cases –not just the marginal ones about $x_A = \frac{\gamma}{1+\gamma}$.[16]

We split the problem in different cases.

First, observe that it is easy to see that for any $x_A < \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ we have $\frac{\partial \Pi_A}{\partial x_A} > 0$ for all values of σ since both $\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}, \frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A} > 0$ and therefore

$\frac{\partial \Pi_A}{\partial x_A} > 0$ if and only if

$$\frac{pF'\left(\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}}{(1-p)F'\left(\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}} \geq -1$$

which holds since the LHS is positive.

When $x_A > \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ there are two possible sub-cases. The first one is when $\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma} < \frac{\delta}{4\gamma}$ (or $\frac{20\gamma^2}{1+\gamma} < \delta$); that is when the highest possible action used by B in the posited mixed strategy, is in the privileged space of candidate A. In this case for any $x_A > \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ we have that $\frac{\partial \Pi_A}{\partial x_A} < 0$ for all values of σ, because if x_A increases it moves away from the policies chosen by B, which both lie on the subspace where A's has the overall advantage, and it also moves away from A's reference point. In fact in this case we have that $\frac{\partial \Pi_A}{\partial x_A} < 0$ if and only if

$$\frac{-pF'\left(-\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}}{(1-p)F'\left(-\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}} \leq 1$$

[16]For candidate B we have already established in the beginning of this proof that the posited strategy is a best response.

and it holds since both $\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}, \frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A} > 0.$

The second sub-case is when $\frac{\delta}{4\gamma} \leq \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ (or $\frac{20\gamma^2}{1+\gamma} \geq \delta$); that is when the highest possible action used by B in the posited mixed strategy is either in the privileged space of candidate B, or it does not belong to the privileged space of any candidate. Then, $\Pi_A \to 0$ for any $x_A > \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ as $\sigma \to 0$, while the payoff of candidate B at the posited profile converges to one. This is so because if $\frac{\delta}{4\gamma} \leq \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ for any $x_A > \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$, we have that the two indifferent voters are always strictly larger than zero, for any realization of candidate B's strategy.

Notice that

$$\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right) = \frac{\delta+\left[(1+\gamma)\left(\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}-x_A\right)-2\gamma\right]\left(x_A+\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{2(x_B-x_A)} > 0$$

if and only if

$$\delta < \left[2\gamma + (1+\gamma)\left(x_A - \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right]\left(x_A + \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right),$$

and we have that the RHS increases with x_A and at $x_A = \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ it holds if and only if $\frac{\delta}{4\gamma} < \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$.

Similarly,

$$\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right) = \frac{\delta+\left[(1+\gamma)\left(\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}-x_A\right)-2\gamma\right]\left(x_A+\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{2(x_B-x_A)} > 0$$

if and only if

$$\delta < \left[2\gamma + (1+\gamma)\left(x_A - \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right]\left(x_A + \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)$$

and we have that the RHS increases with x_A and at $x_A = \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ it holds because $\frac{\delta}{4\gamma} < \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$.

Finally, we have to show that for $\sigma > 0$ sufficiently small, we have that $\frac{\partial \Pi_A}{\partial x_A} > 0$ for $x_A \in [\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}, \frac{\gamma}{1+\gamma})$ and $\frac{\partial \Pi_A}{\partial x_A} < 0$ for $x_A \in (\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}]$.

First, notice that if $\delta \leq \frac{20\gamma^2}{1+\gamma}$ we have that $\frac{\partial \Pi_A}{\partial x_A} < 0$ for $x_A \in (\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}]$ for all values of $\sigma > 0$ since in this case if x_A increases it moves away from the leftist strategy of candidate B, which lies on the subspace where A has the overall advantage, it also moves away from A's reference point, and it moves close to the rightist strategy of candidate B, which lies on the subspace where B has the overall advantage.

If $\delta > \frac{20\gamma^2}{1+\gamma}$we have to show that $\frac{\partial \Pi_A}{\partial x_A} < 0$ for $x_A \in (\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}]$, that is

$$\frac{pF'\left(\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}}{(1-p)F'\left(-\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}} \leq 1$$

or

$$\left(\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)^2 - \left(\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)^2 \geq -2\sigma^2 \ln \frac{(1-p)\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}}{p\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}}.$$

We have that

$$\frac{(1-p)\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}}{p\frac{\partial \widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}} < 1$$

since it holds if and only if

$$\frac{\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right)^2-8\gamma^2+2\gamma\sqrt{\delta(1+\gamma)-4\gamma^2}}{\left(\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}-x_A\right)^2} < \frac{\left(\sqrt{\delta(1+\gamma)-4\gamma^2}\right)^2-8\gamma^2-2\gamma\sqrt{\delta(1+\gamma)-4\gamma^2}}{\left(\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}-x_A\right)^2} + \frac{4\gamma(1+\gamma)^2}{\sqrt{\delta(1+\gamma)-4\gamma^2}}.$$

Notice that for $\frac{\gamma}{1+\gamma} \leq x_A \leq \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ the LHS decreases with x_A and the the RHS increases with x_A. Since at $x_A = \frac{\gamma}{1+\gamma}$ it holds with equality, it must be the case that for $\frac{\gamma}{1+\gamma} \leq x_A \leq \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$ it holds.

Furthermore, we have that

$$\left|\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right| < \left|\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right|$$

since

$$\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right) = \sqrt{\delta\left(1+\gamma\right)-4\gamma^2} \leq \widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right) \leq \widetilde{x}_i\left(\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right) = +\infty$$

and

$$\widetilde{x}_i\left(\frac{\gamma}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right) = -\sqrt{\delta\left(1+\gamma\right)-4\gamma^2} \leq \widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)$$
$$\leq \widetilde{x}_i\left(\frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right) = \frac{4\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{4} < \sqrt{\delta\left(1+\gamma\right)-4\gamma^2}$$

if and only if $\frac{\frac{16}{25}\gamma^2+4\gamma^2}{1+\gamma} < \delta$.

Thus

$$\left|\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right| < \left|\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right|$$

for $\frac{\gamma}{1+\gamma} \leq x_A \leq \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}$, since $\frac{\partial\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A} \geq 0$ and $\frac{\partial\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A} \geq 0$.

Therefore, we need

$$\frac{\left(\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)^2 - \left(\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)\right)^2}{2\ln\frac{(1-p)\frac{\partial\widetilde{x}_i\left(x_A, \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}}{p\frac{\partial\widetilde{x}_i\left(x_A, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}\right)}{\partial x_A}}} \geq \sigma^2$$

to hold for $x_A \in (\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}]$.

The LHS converges to $\frac{(\delta\gamma+\delta-8\gamma^2)(\delta\gamma+\delta-4\gamma^2)}{\delta\gamma+\delta-12\gamma^2}$ when $x_A \to \frac{\gamma}{1+\gamma}$, which is strictly positive if $\delta > \frac{12\gamma^2}{1+\gamma}$ and bounded strictly above zero otherwise. Therefore, for small enough values of $\sigma > 0$, $\frac{\partial \Pi_A}{\partial x_A} > 0$ for every $x_A \in (\frac{\gamma}{1+\gamma}, \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}]$ when $\delta > \frac{8\gamma^2}{1+\gamma}$.

By applying similar arguments, we can establish that, for small enough values of $\sigma > 0$, $\frac{\partial \Pi_A}{\partial x_A} > 0$ for every $x_A \in [\frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma}, \frac{\gamma}{1+\gamma})$ when $\delta > \frac{8\gamma^2}{1+\gamma}$. Hence, the posited strategy profile is an equilibrium for every $\delta > \frac{8\gamma^2}{1+\gamma}$.

To establish the uniqueness of this equilibrium we utilize the constant sum nature of the game and the fact that candidate A has a unique best response when candidate B uses the posited equilibrium mixed strategy. In a constant sum game equilibrium strategies are interchangeable, hence in every equilibrium candidate A uses $x_A = \frac{\gamma}{1+\gamma}$. We know from the proof of Proposition 4, that when candidate A uses $x_A = \frac{\gamma}{1+\gamma}$, then candidate B has two pure best responses,

$$x_B = \frac{\gamma+\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma} \text{ and } x_B = \frac{\gamma-\sqrt{\delta(1+\gamma)-4\gamma^2}}{1+\gamma},$$

and, hence, any mixing between these two policies is a best response. But we also know from the proof of Proposition 4 that $x_A = \frac{\gamma}{1+\gamma}$ is a best response to a mixing between these two policies, only if the former policy is chosen with probability $p = \frac{1}{2} + \frac{\gamma}{\sqrt{\delta(1+\gamma)-4\gamma^2}}$. Therefore, if the posited profile is an equilibrium, then it is also unique. ⧫