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Reflections on Gains and Losses: A 2?2.7 Experiment Antoni Bosch-Domènech and Joaquim Silvestre

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# ABSTRACT <br> "Reflections on Gains and Losses: A 2?2?7 Experiment" 

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What determines risk attraction or aversion? We experimentally examine three factors: the gain-loss dichotomy, the probabilities ( 0.2 vs .0 .8 ), and the money at risk ( 7 amounts).

We find that, for both gains and losses and for low and high probabilities, the majority display risk attraction for small amounts of money, and risk aversion for larger amounts. Thus, when examining the risk attitudes of the majority, what matters is the amount of money at risk, and not the gain-loss dichotomy, or the probabilities.

Yet the frequency of risk-attraction behavior does vary according to the gain-loss dichotomy and to the probabilities involved. Since Kahneman and Tversky, the literature has studied gain-loss reflections. We submit that a reflection can be decomposed into a "translation" and a probability "switch." We find (a) a translation effect for low probabilities of the bad outcome, but not for high ones; (b) a strong switch effect for gains, but not for losses, and (c) a strong reflection effect for high probabilities of gains, but not for low ones. We also argue that, while both the translation effect and the switch effect contradict the expected utility hypothesis, the translation effect implies a deeper violation of preference theory, invalidating non-paternalistic welfare economics.

Keywords: Reflection Effect, Risk Attraction, Risk Aversion, Gains, Losses, Experiments, House Money, Paternalistic Welfare Economics.

JEL Classification Numbers: C91, D81

# Reflections on Gains and Losses: A 2? 2? 7 Experiment $^{1}$ 

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## 1. Introduction

What determines risk attraction or aversion? Present conventional wisdom, no doubt inspired by the pioneer work of Nobel laureate Daniel Kahneman and his late coauthor Amos Tversky, views individuals as risk averse for gains and risk seeking for losses. They asked "What happens when the signs of the outcomes are reversed so that gains are replaced by losses?" (Kahneman and Tversky, 1979, p. 268), and answered, "... the preference between negative prospects is the mirror image of the preference between positive prospects. Thus the reflection of prospects around zero reverses the preference order. We label this pattern the reflection effect" and continued, "...the reflection effect implies that risk aversion in the positive domain is accompanied by risk seeking in the negative domain."

In addition to the gain-loss dichotomy, the present paper experimentally examines the role of probabilities ( 0.2 vs. 0.8 ) and of the amount of money at stake (seven amounts, from $\$ 3$ to a relatively substantial $\$ 100$ ). Our subjects make real, not hypothetical, choices between simple uncertain money prospects and their expected money values. This yields a $2 ? 2 ? 7$ experimental design, implemented in four treatments named $G, G^{\prime}, L$ and $L^{\prime}$, each dealing with the seven amounts of money at stake.

A first result is that the majority of our subjects display risk attraction for small amounts of money ( $\$ 3$ to $\$ 7$ ), and risk aversion for larger amounts ( $\$ 33$ to $\$ 100$ ). The implications are noteworthy: when examining the risk attitudes of the majority, what matters is the amount of money at risk, and not the gain-loss dichotomy, or the probabilities.

[^0]Following Kahneman and Tversky's path, the study of the gain-loss dichotomy has been largely confined to "reflected" choices where all the money amounts of a positive prospect are multiplied by minus one. We submit that such a reflection has two components: a translation (or change of origin) of the probability distributions of the money outcomes, which naturally captures the gain-loss asymmetry, and a switch of probabilities between the good and the bad outcomes. (The bad outcome is "no gain" in choices between certain and uncertain gains, and is a "loss" when choosing between certain and uncertain losses: see Section 2.1 below for precise definitions.)

Our treatments $G$ and $L$ differ only by a translation from gains to losses, and so do treatments $G^{\prime}$ and $L^{\prime}$. Treatments $G$ and $G^{\prime}$, on the contrary, differ only in the switch of the probabilities of the favorable and unfavorable outcomes, and so do treatments $L$ and $L^{\prime}$. Taken together, they explore the effects of the two components "translation" (gain vs. loss) and "probability switch" ( 0.2 vs. 0.8 ), on risk attraction, so permitting a better understanding of any gain-loss asymmetry for each amount of money at stake. In a nutshell, we find:

* Translating gains into losses increases the frequency of choices that display risk attraction, but only if the probability of the bad outcome is low (0.2); if, on the contrary, the probability of the bad outcome is high (0.8), then translating gains into losses has no effect on risk attitudes. We call this increase in risk attraction a translation effect and, therefore, claim that a translation effect occurs for low but not for high probabilities of the bad outcome.
* Switching the probability of the bad outcome (from 0.2 to 0.8 ) increases the frequency of risk attraction when choices involve gains. But the effect is substartially weaker when these choices involve losses. We call this increase in risk attraction a switch effect and, therefore, claim that there is a strong switch effect for gains, but a weak one for losses.

Accordingly, we confirm the previously observed reflection effect for high probabilities of gains and losses, i.e., risk attraction increases when moving from a high probability of gain (which entails a low probability of the bad outcome "no gain") to a high probability of loss (i.e., high probability of the bad outcome). But risk attraction slightly decreases when moving from a low probability of gain (which entails a high probability of the bad outcome) to a low probability of loss (i.e., low probability of the bad outcome).

The design of Treatments $L$ and $L^{\prime}$ (see Section 2.2 .2 below) addresses a basic difficulty in real-money experiments with losses and gains, namely the need for the experimenter and the
subjects to agree on their perceptions of what is a loss and what is a gain. Because subjects cannot legally lose relative to their pre-experiment wealth, all real-money experiments with losses require that subjects previously receive, or earn, from the experimenter the money that they may eventually lose. Therefore, in the experiments with losses, either the losses are hypothetical or subjects play with "house money," which seems to increase their willingness to accept risk. In order to mitigate these difficulties, our subjects make their choices between certain and uncertain losses several months after ear ning some income by taking a quiz. This delay made most subjects feel that they had fully spent the earned cash by the time they made their decisions.

Section 4 below compares our results with those reported in the literature, while Appendix 3 comments on their theoretical implications, arguing that, while both the translation effect and the switch effect contradict the expected utility hypothesis, the translation effect implies a deeper violation of standard preference theory, because it negates the existence of consistent, referenceindependent preferences, a precondition for non-paternalistic normative analysis. If the person lacks consistent preferences, then the evaluation of any policy that affects her reference point must appeal to an external notion of welfare. ${ }^{2}$

## 2. Gains vs. Losses

### 2.1. What is the "corresponding choice involving losses"?

As noted in the introduction, Kahneman and Tversky replaced gains by losses through a "reflection," i.e., the multiplication by minus one of all money amounts. But it is important to observe that, as long as the probabilities are not 50-50, this reflection involves two distinct operations: a switch of the probability masses of the good and bad money outcomes, and a translation by which a gain becomes the absence of a loss. The distinction can be illustrated as follows.

Consider a choice between the positive or negative amount of money $z$ with probability $p$ (and zero with probability $1-p$ ) and the certain positive or negative amount of money $p z$. Denote such a choice as ? $z, p$ ?. Let $z>0$, and consider first the following choice.

Choice $\boldsymbol{G}$ ? ? z, 0.8?. A person with given actual wealth whas to choose between a certain gain of $\$ 0.8 z$ vs. an uncertain gain of $\$ z$ with probability 0.8 .

[^1]If she chooses the certain gain, he r ex post money balance is $x=w+0.8 z$, whereas if she chooses the uncertain gain, then her ex post money balance is $x=w+z$ with probability 0.8 , and $x=$ $w$ with probability 0.2 . Thus, she is choosing between the two discrete probability density functions of the NW cell in Figure 1. The certain choice is the degenerate pdf, depicted as a solid green bar, whereas the two hollow red bars depict the uncertain choice.

Now assume that she is not making choice $G$, but the following one.

Choice $\boldsymbol{G}^{\prime}$ ? ? z, 0.2?. A person with given actual wealth whas to choose between a certain gain of $\$ 0.2 z$ vs. an uncertain gain of $\$ z$ with probability 0.2

Choice $G^{\prime}$ is depicted in the SW cell of Figure 1. Note that Choice $G^{\prime}$ is obtained from Choice $G$ by switching the probabilities of the good and bad outcomes, while preserving actuarial fairness. More generally, consider the probability switch operator $s$ defined by $s(? z, p ?)=? z, 1-$ $p$ ?. Of course, there is no reason why the person could not choose the certain gain in $G$ and the uncertain gain in $G^{\prime}$.

Assume next that she does choose the certain gain in $G$, thus displaying risk aversion in that choice. Would she actually choose the uncertain loss, and hence display risk attraction, in the "corresponding choice involving losses"? Both Kahneman and Tversky's reflection and our translation create choices involving losses that in some way correspond to $G$ : translation gives choice $L$ depicted in the NE cell of Figure 1, while reflection gives $L^{\prime}$ in the SE cell.

Choice L? ? $-z, 0.2$ ?. A person with given actual wealth whas to choose between a certain loss of $\$ 0.2 z$ vs. an uncertain loss of $\$ z$ with probability 0.2.

Note that choice $L$, involving losses, can be derived from choice $G$, involving gains, by a leftward translation of the discrete probability density functions along the money axis, so that the good outcome is now $w$ (no loss) instead of $w+z$ (a gain), whereas the bad outcome is now $w-z$ (a loss), instead of $w$ (no gain). As a result, a translation keeps unchanged the probabilities of the good and bad outcomes.

More generally, define the translation operator $t$ by $t(? z, p ?)=?-z, 1-p ?$.

Choice $L^{\prime}$ ? ? $-z, 0.8$ ?. A person with given actual wealth $w$ has to choose between a certain loss of $\$ 0.8 z$ vs. an uncertain loss of $\$ z$ with probability 0.8

Note that choice $L^{\prime}$ can be derived from choice $G$ by applying a switch and a translation, in any order. More generally, define the reflection operator $r$ as the composite transformation:

$$
r(? z, p ?) ? s(t(? z, p ?))=t(s(? z, p ?))=?-z, p ?
$$

i.e., Reflection = Translation + Switch.

Which one of the two loss choices, $L$ or $L^{\prime}$ is the more natural counterpart to the gain choice $G$ ? Choice $L$ involves a minimal transformation of $G$. After all, what makes an ex post money balance the result of a gain or a loss is the starting point: an ex post balance of 900 is due to a gain if you start at 800 , but it is due to a loss if you start at 1000 . Choice $L$ is a choice involving losses obtained by simply moving the reference point in $G$, from the outcome $w$ meaning "unluckily, I did not gain" to meaning "luckily, I did not lose," without altering the probability of being unlucky. Accordingly, we view $L$ as the more appropriate counterpart to $G$.

Our experiment targets the effect of both translations and probability switches on risk attitudes. The treatments are named after the type of choices that the subjects are asked to make.

The comparison of the risk attitudes displayed in type $G$ vs. $L$ choices, or in type $G$ vs. $L^{\prime}$ choices, enables us to test for asymmetries when gains are translated into losses, and nothing else is changed. The comparison of $G$ vs. $G^{\prime}$ choices, or $L$ vs. $L^{\prime}$ choices throws light on the effect of switching the probabilities between the good and the bad outcome while maintaining the sign of the prospects and keeping choices fair. Last, the comparison of $G$ vs. $L^{\prime}$ choices or $G^{\prime}$ vs. $L$ choices tests for reflection effects.

## Choice $G$



Figure 1
Reflection $=$ Translation + Switch
$G$ ? $L$ : translation
$G^{\prime}$ ? $L^{\prime}$ : translation
$G$ ? $G^{\prime}$ : switch
$L$ ? $L^{\prime}$ : switch
$G ? \quad L^{\prime}$ : reflection (main diagonal)
$G^{\prime}$ ? $L$ : reflection (skew diagonal)
$z>0$, probabilities are measured vertically

### 2.2. The experiment

### 2.2.1. Treatment $G$ : gains at low probability of the bad outcome

Our four treatments share a basic design. As with the rest of them, we performed Treatment $G$ in a single session (no preliminary pilot sessions) with students from Universitat Pompeu Fabra who volunteered. We only selected students who had not taken courses in economics or business and tried to maintain an equal proportion of sexes. In Treatment $G$ we used twenty-one subjects, but we ended up with a ratio of males to females of $6 / 15$. Subjects were told that they would be randomly assigned, without replacement, to one of seven classes corresponding to the seven money amounts, in pesetas, 500, 1000, 2000, 5000, 7500, 10000 and 15000 (i.e., approximately, US\$ 3, 7, 13, 33, 50, 67 and 100). A subject was asked to choose, for each of the seven classes and before knowing to which class she would eventually belong, between the certain gain of 0.8 times the money amount of the class and the uncertain prospect giving the money amount of the class with probability 0.8 and nothing with probability 0.2 . In what follows we say that a subject displays risk attraction (resp. risk aversion) in a particular choice if she chooses the uncertain (resp. certain) alternative. ${ }^{3}$

Subjects were given a 7-page folder to record their decisions, one page for each class. Every page had five boxes arranged vertically. The certain gain was printed in the first box, and the amount of money of the uncertain prospect in the second one, with the statement that the probability of winning was 0.8 . The third box contained two check cells, one for choosing the certain gain, and another one for choosing the uncertain prospect. ${ }^{4}$ Below a separating horizontal line, two more boxes were later used to record the random outcome and the take-home amount. In order to facilitate decisions, a matrix on the back of the page showed all the amounts of money involved. The information was given to the subjects as written instructions (available on request), which were read aloud by the experimenter. The treatment began after all questions were privately answered.

Once all subjects had registered their seven decisions (under no time constraint: nobody used more than 15 minutes), their pages were collected. Subjects were then called one by one to an office with an urn that initially contained twenty-one pieces of paper: each piece indicated one class, and each of the seven classes occurred three times. A piece of paper was randomly drawn (without replacement): the experimenter and the subject then checked the subject's choice for that particular class. If her choice was the certain gain, she would take home 0.8 times the amount of money of her

[^2]class. If, on the contrary, she chose the uncertain prospect, then a number from one to five was randomly drawn from another urn. If the number one was drawn, then the subject would take nothing home. Otherwise, she would take home the amount of money of her class. The subject was then paid and dismissed, and the next subject was escorted into the office.

The experimental data are presented in Table A1 of Appendix 1.

### 2.2.2. Treatment $L$ : losses at low probability of the bad outcome

Since experimenters should not earn money from their subjects, any experiment with losses must involve either hypothetical losses or the provision of sufficient initial cash. Doubts have been raised about the reliability of the results from experiments with hypothetical losses (see Charles Holt and Susan Laury, 2002). But providing money to the participants has its pitfalls too. First, if participants do not earn the cash, then the cash provision will easily be interpreted as a windfall gain. Second, even if subjects earn the necessary cash through their own skills and effort, they will still be playing with "house money." There are grounds for suspecting that playing with windfall gains or house money increases risk attraction. ${ }^{5}$

Our Treatment $L$ implements a design that we believe avoids to a large measure both the windfall-gains effect and the house-money effect. In order to counteract the windfall-gains effect, participants first took a quiz on general education and earned cash according to the number of correct answers: 15000 pesetas to the participants ranked one to six, 10000 pesetas to the next six, 7500 pesetas to the third group of six, while the last group receive d 5000 pesetas.

To alleviate the house-money effect, we temporally separated the just described cash-earning quiz from the actual decisions involving losses by four months and a semester break. Because we guaranteed that the losses would never exceed the cash received in the quiz, the subjects could admittedly feel that they were playing with house money. But we hoped to reduce this effect by the temporal separation of the treatment's two parts. It is hard to know to what extent we managed this, but some evidence indicates that we mostly succeeded. ${ }^{6}$ The answers to some questions asked to a different group of people also indicated the importance of the time lag for the perception of loss. ${ }^{7}$

[^3]In the treatment, subjects were told that they would be randomly assigned to one of seven classes corresponding to the seven money amounts to lose (500, 1000, 2000, 5000, 7500, 10000 and 15000 pesetas). In any event, a subject could not be assigned to a class with an amount of money exceeding the amount earned in the quiz. Now, a subject was asked to choose, for each of the possible classes and before knowing to which class she would eventually belong, between the certain loss of 0.2 times the money amount of the class and the uncertain prospect of losing the money amount of the class with probability 0.2 and nothing with probability 0.8 . To record their decisions, subjects were given a 7-page folder that contained one page for each class. In each page, they were required to register, under no time constraint, their choice between the certain loss and the uncertain prospect. Subjects were then called one by one to an office where the subject's class was randomly drawn. Next, the experimenter and the subject checked the subject's choice for that particular class. If her choice was the certain loss, she would pay 0.2 times the amount of money of her class. If, on the contrary, she chose the uncertain prospect, then a number from one to five was randomly drawn from an urn. If the number one was drawn, then the subject would pay the amount of money of her class. Otherwise, she would pay nothing.

Twenty-one subjects took part in the second part of the treatment. ${ }^{8}$ The male/female ratio ended up being 10/11. The experimental data are presented in Table A2 of Appendix 1.

### 2.2.3. Treat ment $G^{\prime}$ : gains at high probability of the bad outcome

The treatment was identical to Treatment $G$, except that the probability of the uncertain gain was now 0.2 , instead of 0.8 . Twenty-four subjects participated, with a male/female ratio of $9 / 15$. The experimental data are displayed in Table A3 of Appendix 1.

[^4]
### 2.2.4. Treatment $L^{\prime}$ : losses at high probability of the bad outcome

Treatment $L^{\prime}$ was performed with thirty-four students, with a male/female ratio of $18 / 16 .{ }^{9}$ The treatment had exactly the same format as Treatment $L$, except that the probability of the uncertain loss was now 0.8 , instead of 0.2 . The experimental data are displayed in Table A4 of Appendix 1.

## 3. Statistical analysis and results

### 3.1. Statistical analysis

We first estimate the following logit regression model with random intercept

$$
\ln \frac{p_{i j}}{1 ? p_{i j}}=? ?+u_{i}+?_{1} d_{1}+?_{2} d_{2}+?_{3} d_{1} d_{2}+b z_{j}
$$

$i$ ? $\{1, \ldots, I\}$, where $I$ is the number of subjects.
$j ?\{1, \ldots, 7\}$, the seven levels of money, $z_{j} ?\{0.5,1,2,5,7.5,10,15\}$,
$d_{1}=0$ if gains (treatments $G, G^{\prime}$ ),
$d_{1}=1$ if losses (treatments $\left.L, L^{\prime}\right)$,
$d_{2}=0$ if the probability of the bad outcome is 0.2 (treatments $G, L$ ),
$d_{2}=1$ if the probability of the bad outcome is 0.8 (treatments $G^{\prime}, L^{\prime}$ ),
i.e., a change of the value of $d_{1}$ indicates a translation, whereas one of $d_{2}$ indicates a probability switch. ${ }^{10}$

The variable $p_{i j}$ is the probability that subject $i$ chooses the certain alternative (and thus displays risk aversion) when the amount of money at stake is $z_{j}$ (or $-z_{j}$ ) in thousands of pesetas (so as to avoid too many decimals in the estimates of the regression coefficients) for the four regimes described by the values of the dummy variables. ${ }^{11}$ The individual effect $u_{i}$ allows for heterogeneous individual tastes, assumed to be normally distributed with mean zero and standard deviation ${ }^{n}$ ? ? o that?? ? $u_{i}$ is the random intercept.

[^5]The estimation results appear in Table 1.

| var | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | . 5128323 | . 058686 | 8.739 | 0.000 | . 3978098 | . 6278548 |
| $d_{1}$ | -2.318966 | 1.108522 | -2.092 | 0.036 | -4.491629 | -. 1463026 |
| $d_{2}$ | -3.276675 | . 7781808 | -4.211 | 0.000 | -4.801882 | -1.751469 |
| $d_{3}$ | 2.24134 | 1.413743 | 1.585 | 0.113 | -. 5295455 | 5.012226 |
| _cons | -. 0454774 | . 5373737 | -0.085 | 0.933 | -1.098711 | 1.007756 |
| $\ln ?^{2}{ }^{\text {u }}$ | 1.897352 | . 4738794 | 4.004 | 0.000 | . 968565 | 2.826138 |
| ?u | 2.582288 | . 6118466 |  |  | 1.62301 | 4.108546 |
| ? | . 8695915 | . 0537389 |  |  | . 7248334 | . 9440721 |
| kelihood | ratio test | of ? = 0: | $?^{2}(1)=102.81$ |  | Prob $>?^{2}=0.0000$ |  |

Table 1: ML estimation with interaction regressor

A first observation is that the hypothesis of no individual effect $(?=0)$ is rejected by the $?^{2}$ test. More interestingly, the estimates for $b$ and the coefficient of the switch dummy $d_{2}$ are highly significant. On the other hand, the estimate for the coefficient of the translation dummy $d_{1}$ is significant at the $5 \%$ level, although not at the $1 \%$. Finally, that of the interaction term is not significant, and, hence, we cannot reject the hypothesis that there is no interaction. Accordingly, we estimate the more parsimonious model obtained by dropping the interaction term, i.e.,

$$
\ln \frac{p_{i j}}{1 ? p_{i j}}=? ?+u_{i}+?_{1} d_{1}+?_{2} d_{2}+b z_{j} \text {, }
$$

with results displayed in Table 2.

| var | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | . 5064414 | . 0550028 | 9.208 | 0.000 | . 3986379 | . 614245 |
| $d_{1}$ | -. 9521154 | . 5836578 | -1.631 | 0.103 | -2.096064 | . 1918328 |
| $d_{2}$ | - 2.453579 | . 6715575 | -3.654 | 0.000 | -3.769808 | -1.13735 |
| _cons | -. 4860929 | . 5446004 | -0.893 | 0.372 | -1.55349 | . 5813041 |
| $\ln ?^{2}$ | 1.825409 | . 3008473 | 6.068 | 0.000 | 1.235759 | 2.415058 |
| $?^{2}$ | 2.49105 | . 3747128 |  |  | 1.85499 | 3.345209 |
| ? | . 8612139 | . 0359586 |  |  | . 7748249 | . 9179684 |
| Likelihood ratio test of ? = 0: |  |  | $?^{2}(1)=$ | 106.38 | Prob > ${ }^{2}=0.0000$ |  |

Table 2: ML estimation without interaction regressor

The estimate for the coefficient $b$ of the amount of money continues to be positive and highly significant, proving that risk aversion increases with the amount of money at stake.

The estimate of the coefficient $?_{2}$ of the switch dummy also continues to be highly significant, but now we cannot reject the hypothesis that the coefficient $?_{1}$ of the translation dummy is zero. Prima facie, this suggests that the observed increase in risk attraction under a translation from gains to losses could be due to chance variation, whereas a probability switch does impact the degree of risk attraction. Nevertheless, the lack of significance of the translation variable is conditional to specifying no interaction, motivated by the lack of statistical significance of the estimate of $?_{3}$ in the previous regression. This lack of significance may be genuine, but could also be caused by the sample size or even by an inexact specification of the structural model. All things considered, it seems safe to conclude that a translation from gains to losses could be influencing risk attraction, but that, even if it did, a switch in the probabilities would have a stronger effect.

### 3.2. The fundamental role of the amount of money at stake

Figure 2 below displays, in the manner of Figure 1, the raw data of our four treatments, as detailed in Appendix 1. Each row in each of the four panels corresponds to a subject, whereas the columns correspond to the amounts of money at risk. A letter $c$ indicates choosing the certain gain or loss (thus displaying risk aversion, color green), while the letters un indicate choosing the uncertain gain or loss (thus displaying risk attraction, color red). In each panel subjects have been ordered to help reading the table: the bottom ones violate the "standard pattern" (see Result 2 below), whereas the other subjects are ordered by increasing risk attraction. The statistical analysis above, the visual inspection of the panels of Figure 2, and the percentages calculated in Table 3, allow us to state the following results.

Result 1. Diversity. The majority of subjects display risk attraction for choices involving some amounts of money, and risk aversion for some others, the number of safe choices varying across individuals.

Result 2. Standard pattern. Most individuals (85\%) follow the standard pattern, defined as follows: whenever risk attraction is displayed in a choice involving a given money amount, risk attraction is also displayed for any smaller (in absolute value) amount of money. ${ }^{12}$

Result 3. Increasing frequency of risk aversion with the amount of money at stake. The proportion of subjects who display risk aversion in a particular choice increases with the amount of money at stake.

Result 4. Risk aversion by the majority for large amounts of money at stake. For both gains and losses, and for low and high probabilities, a majority of our subjects display risk attraction for low amounts of money at stake (the red area in Table 3), but risk aversion for large amounts (the green area in Table 3).

Result 4, which states that, when examining the risk attitudes of the majority, what matters is the amount of money at stake, and not whether the choices are on gains or on losses, or whether the probabilities are high or small (for our nonextreme values of 0.2 and 0.8 ), is justified by Table 3 . The key to this observation, the most surprising of the present paper, was designing an experiment that involved relatively substantial losses.

Result 5. Translation effect for low probability of the bad outcome. For all amounts of money at stake, if gains and losses are related by a translation and the probability of the bad outcome is 0.2 , then subjects are more likely to display risk attraction with losses than with gains. In other words, risk attraction becomes more frequent as we move to the right along the top row of figures 1 and 2 .

[^6]

Figure 2. Raw data of the four treatments, $G, L, G^{\prime}$ and $L^{\prime}$. The color green indicates risk aversion and the color red risk attraction. Each row corresponds to the decisions of a single subject for each of the seven money amounts.

Amount of Money (in pesetas and in US\$ rounded off to the nearest dollar)

|  | $\begin{aligned} & \mathbf{5 0 0} \\ & \$ 3 \end{aligned}$ | $\begin{gathered} 1000 \\ \$ 7 \end{gathered}$ | $\begin{gathered} 2000 \\ \$ 13 \end{gathered}$ | $\begin{gathered} 5000 \\ \$ 33 \end{gathered}$ | $\begin{array}{r} 7500 \\ \$ 50 \end{array}$ | $\begin{aligned} & 10000 \\ & \$ 67 \end{aligned}$ | $\begin{gathered} 15000 \\ \$ 100 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Treatment } \boldsymbol{G} \\ & \text { (gains with prob }=0.8 \text { ) } \\ & \text { (i.e., prob. of bad outcome }=0.2 \text { ) } \end{aligned}$ | 0.57 | 0.57 | 0.29 | 0.05 | 0.10 | 0.10 | 0.05 |
| $\begin{aligned} & \hline \text { Treatment } L \\ & \text { (losses with prob. }=0.2 \text { ) } \end{aligned}$ | 0.86 | 0.71 | 0.62 | 0.29 | $\begin{gathered} 0.24 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.17) \end{gathered}$ | $\begin{gathered} 0.33 \\ (0.17) \end{gathered}$ |
| $\begin{aligned} & \text { Treatment } \boldsymbol{G}^{\prime} \\ & (\text { gains with prob. }=0.2 \text { ) } \\ & \text { (i.e., prob. of bad out. }=0.8 \text { ) } \end{aligned}$ | 0.92 | 0.92 | 0.79 | 0.46 | 0.50 | 0.17 | 0.17 |
| $\begin{aligned} & \text { Treatment } L^{\prime} \\ & \text { (losses with prob }=0.8 \text { ) } \end{aligned}$ | 0.91 | 0.97 | 0.71 | 0.47 | $\begin{gathered} 0.50 \\ (0.43) \end{gathered}$ | $\begin{gathered} 0.35 \\ (0.29) \end{gathered}$ | $\begin{gathered} 0.37 \\ (0.24) \end{gathered}$ |

Table 3. Fraction of subjects in treatments $G, L, G^{\prime}$ and $L^{\prime}$ who display risk attraction (by choosing the uncertain alternative) for the various amounts of money a stake. The color red highlights a majority of subjects displaying risk attraction. The color green a majority displaying risk aversion.

Because of the small number of answers for the amounts 7500, 10000 and 15000 in Treatments $L$ and $L^{\prime}$ (as described in Section 2), two fractions are reported in the corresponding cells. The fraction in parentheses is extrapolated by assuming that a subject with the standard pattern of behavior who has chosen the certain alternative for some amount of money would continue to display the standard pattern, and hence would choose the certain alternative for larger a mounts. This procedure is inspired by the pervasiveness of the standard pattern, see Result 2. (If either the subject has failed to display the standard pattern, or if she has always chosen the uncertain alternative, then we assume that her probability of choosing the uncertain alternative is $50 \%$.) The other figure is the actual number of subjects who choose the uncertain alternative divided by the number of respondents.

### 3.3. Translations, switches and reflections

Again, the visual inspection of the panels of Figure 2, together with the percentages collected in Table 3, suggest the following statements, which are consistent with our statistical analysis.

Result 6. Negligible translation effect for high probability of the bad outcome. Translating gains into losses when the probability of the bad outcome is 0.8 leaves risk attraction unchanged, with the possible exception of the two largest amounts of money at risk, where we have fewer data.

Result 7. Large switch effect for gains. Subjects are far more likely to display risk attraction when the probabilities of the uncertain gain are low than when they are high. In fact, at a low probability of gains, a substantial majority of individuals show risk attraction, even though all choices involve gains.

Result 8. Moderate switch effect for losses. Subjects are more likely to display risk attraction when the probabilities of the uncertain loss are high than when they are low. The effect shows along the whole range of amounts of money at risk, but it is substantially weaker than the corresponding effect when the probability switch affects gains.

The remaining results refer to the "reflection effect" as defined by Kahneman and Tversky (1979, quotation in the first paragraph of this paper). Recall that this reflection effect occurs if risk attraction increases when all the money amounts of a positive prospect are multiplied by minus one, i.e., when gains are translated into losses, and the probabilities of the bad and good outcomes are switched.

Result 9. Large reflection effect for high probability of gains and losses. The frequency of risky choices substantially increases when moving from a prospect with a high probability of gain (low probability of the bad outcome) to a prospect to a high probability of loss, i.e., along the main diagonal of figures 1 and 2 . Interestingly, this effect is of the same order of magnitude as the switch effect for gains referred to in Result 7.

Result 10. Weak reverse reflection effect for low probability of gains and losses. The frequency of risky choices moderately increases when moving from a prospect with a low probability of loss to a prospect to a low probability of gain, i.e., along the skew diagonal of figures 1 and 2. We call it
"reverse reflection effect" because now risk attraction decreases when moving from gains to losses.

Results 5 to 10 can be summarized in Figure 3, arranged as figures 1 and 2. The move from $G$ to $L^{\prime}$, along the main diagonal, entails a large increase in risk attraction, i.e., a strong, direct reflection effect (Result 9). While this confirms the oft-observed reflection effect for high probabilities of gains and losses, our decomposition of reflection into translation and switch yields novel perspectives.

First, the lack of a clear translation effect for a 0.8 probability of the bad outcome is noteworthy, particularly when contrasted with the evident translation effect when the probability of the bad outcome is 0.2 .

Second, this reflection effect can be decomposed as the sum of a strong switch effect for gains (Result 7) and a negligible translation effect for high probabilities of the bad outcome (Result 6). This decomposition reduces this reflection effect to the consequence of a probability switch in the gain domain. ${ }^{13}$

## 4. Relation to the literature

A summary of the features of our approach will help the comparison with other experiments reported in the literature. First, we deal with real money, and our subjects' losses are designed to be more real than in previous experiments. Second, the elicitation method consists of choices between uncertain prospects and their expected value. ${ }^{14}$ Third, we consider seven amounts of money at stake, with substantial quantities at the higher end. Fourth, we choose the probabilities 0.2 and 0.8 away from the zero and one extremes. Last, we study asymmetries in risk attitudes when the choices faced by the agents are transformed by a reflection, but also when they are separately transformed by the two components of a reflection, namely a translation and a probability switch.

[^7]

Figure 3
Informal Summary of Results

Kahneman and Tversky (1979) inspired what may be called the gain-loss asymmetry view, according to which people tend to display risk aversion for gains and risk attraction for losses. But they later posited the more complex view of a fourfold pattern of risk attitudes (Tversky and Kahneman, 1992), here summarized in Table 4, where, one should note, only the entries of the main diagonal are justified by the assumptions of prospect theory (see Appendix 2 below): Tversky and Kahneman base the skew-diagonal entries of their proposed pattern on "empirical regularities" instead. ${ }^{15}$

| Risk aversion for high-probability gains <br> (low probability of the bad outcome) | Risk aversion for low-probability losses |
| :--- | :--- |
| Risk attraction for low -probability gains <br> (high probability of the bad outcome) | Risk attraction for high-probability losses |

Table 4. Tversky-Kahneman (1992) fourfold pattern of risk attitudes

In a sense, our experiment contradicts both the gain-loss asymmetry view and the TverskyKahneman fourfold pattern, because, as stated in Result 4 above, the risk attitude of the majority of our subjects is essentially risk attraction for low amounts of money at risk, and risk aversion for large amounts, irrespective of whether they face gains or losses, and whether the probability is 0.2 or 0.8. ${ }^{16}$

But both the asymmetry view and the four-fold pattern imply effects that admit some comparison with our findings. They both predict a reflection effect along the main diagonal of Figures 1-3 and Table 4: in either case, risk attraction increases when moving from highprobability gains to high-probability losses, in agreement with our findings as well as those of the

[^8]early experimental literature. ${ }^{17}$ But, contrary to the gain-loss asymmetry view, the TverskyKahneman fourfold pattern also implies a switch effect for gains, which agrees with our findings, as well as a switch effect for losses, which is not contradicted by our Result 8 above. Moreover, the gain-loss asymmetry view implies a conventional reflection effect along the skew diagonal of Figures 1-3 and Table 4, whereas the four-fold pattern implies a reverse reflection effect there (i.e., it implies that risk attraction decreases when moving from low-probability gains to low-probability losses), the latter being in line with our Result 10 . Hence, we disagree with the gain-loss asymmetry view in additional respects. ${ }^{18,19}$

How do our results compare with the newer experimental literature that uses real money? Robin Hogarth and Hillel Einhorn (1990) run one experiment with real gains and losses (together with two with hypothetical ones). It covers three probability values, namely $0.1,0.5$ and 0.9 (vs. our 0.2 and 0.8 ) and two amounts of money, namely a very low $\$ 0.1$ and a low $\$ 10$ (versus our seven amounts, ranging from $\$ 3$ to $\$ 100$ ). Their $\$ 10$ treatment lies between our $\$ 7$ and $\$ 13$ treatments.. Some of their results agree with ours. In particular, the increases in the percentages of risk-attracted choices induced by an increase in the probability of the bad outcome are similar to ours. ${ }^{20}$ But for a large ( 0.9 ) probability of the bad outcome, the fraction of their subjects that display risk attraction in gains is substantially lower than in losses ( 0.57 vs . 0.86 , see their Table 4 , page 795), in sharp contrast with our Result 6 for these money amounts (see Table 1 above).

Because they do not deal with money amounts higher than $\$ 10$, it is not possible to compare their work with our high-money results.

Laury and Holt (2000) investigate the reflection effect under both hypothetical and real gains and losses: indeed one of their objectives is to unearth discrepancies in hypothetical vs. real money, which is the main theme of Holt and Laury (2002). The methodology of these two papers is quite different from ours. Instead of actuarially fair choices, their subjects face pairs of

[^9]sequences, one for losses and one for gains, of ten binary choices, each choice involving an " $S$ " (for safe) and an " $R$ " (for risky) alternative. Loss choices are reflected from gain choices by multiplying all money amounts by minus one. The difference "expected $\$ S$ minus expected $\$ R$ " increases along the ten-choice loss sequence, and a risk-neutral decision maker would choose the pattern $R R R R R / S S S S S$ for losses. It follows that the difference in expected money amounts decreases along the gain sequence, and that risk neutrality requires the pattern $\operatorname{SSSSS} / R R R R R$ for gains. Thus, a subject choosing $R R R R R / R R S S S$ in the loss sequence displays risk attraction in losses, and, if she chooses SSSSS/SSRRR in the gain sequence, then she displays risk aversion in gains, a combination that exhibits a reflection effect. The main result in Laury and Holt (2000) is that the conventional reflection effect is widespread in the experiments with hypothetical gains and losses, but becomes substantially less frequent with real money. In Holt and Laury (2002) subjects tend to display more risk aversion for higher than for lower amounts. None of these conclusions contradicts our finding that subjects become risk averse for high amounts of money at stake.

Harbaugh et al. (2002a) conduct a series of experiments with subjects from age 5 to 64, using a range of payoffs from $\$ 10$ to $\$ 90$ for adults and $\$ 1$ to $\$ 9$ for children, and varying the probabilities from 0.02 to 0.98 . Choices of adults seem consistent with the use of objective probabilities, especially when evaluating a gamble over a gain, and are, on average, risk neutral. Unfortunately Harbaugh et al. (2002a) do not provide a disaggregation of choice data in terms of payoffs. ${ }^{21}$ Without it, we can only speculate that the stated risk neutrality may be an average of more risk attraction for low payoffs and more risk aversion for high payoffs, in agreement with our observations; and that their results may be, in any case, biased towards a more risky behavior as a result of their subjects gambling with house money.

Harbaugh et al. (2002b) use real payoffs and vary the probabilities, while keeping the amount of money at stake a constant $\$ 20$. One of their objectives is to discover discrepancies in risk attitudes due to the elicitation method: the certainty equivalent of a lottery (as in Tversky and Kahneman, 1992) vs. the choice between a lottery and its expected value (as we do). We limit the comparison between our results and the ones in Harbaugh et al. (2002b) for the latter elicitation method. Their results seem to contradict Tversky and Kahneman fourfold pattern: notably, they show a reverse reflection effect along the main diagonal of Figures 1-3 and Table 4. Contrary to

[^10]the findings of the present paper, they report a reverse switch effect. However, none of these observations is statistically different from risk neutrality: their only statistically significant result is a translation effect along the top row of Figures 1-3 and Table 4, in agreement with our Result 5.

## 5. Conclusions

Historically, there are three main views on money risk attitudes. The oldest one (Daniel Bernoulli, 1738) sees most people as risk averse most of the time. By positing a fundamental gain-loss asymmetry, Kahneman and Tversky subverted in 1979 this then dominant wisdom, advancing the now popular view that risk attraction is the norm for losses, whereas risk aversion is the norm for gains. Later developments focused on the role of probabilities, leading Tversky and Kahneman in 1992 to propose a four-fold pattern, where the risk attitude depends on both the probabilities and the gain-loss dichotomy.

Our paper experimentally tests these views while including an important third factor: the amount of money of risk. Methodologically, we address a basic difficulty in real-money experiments with losses and design a treatment to alleviate the "house money" effect that pervades real-money experiments with losses.

A main result is that, contrary to the recent views, and as Bernoulli thought, risk aversion is the majority attitude when substantial amounts of money are at stake, both for gains and for losses. Much of the previous experimental evidence of risk taking in the face of losses is confined either to hypothetical losses or the loss of small amounts, limitations that our experimental design avoids. To paraphrase Kahneman and Tversky, the relevant distinction appears to be, not the domain of gains vs. domain of losses, as they claim, but between the domains of large vs. small money amounts. ${ }^{22}$

The claim should not be construed as negating the presence of risk-attitude patterns based on the gain-loss dichotomy or the probabilities involved. On the contrary, the body of this paper is also devoted to the analysis of such patterns.

Kahneman and Tversky's formulated the gain-loss asymmetry in terms of a reflection. We propose the decomposition "reflection = translation + switch," which throws new light on the risk-

[^11]attitude patterns, and we submit that translations, rather than reflections, capture the gain-loss dichotomy in a natural and parsimonious manner. Our experimental findings on risk-attitude patterns can be cast in these terms as follows.
(a) A translation effect for low probabilities of the bad outcome, but not for high ones;
(b) A strong switch effect for gains, but not for losses;
(c) A strong reflection effect for high probabilities of gains, but not for low ones. The strong reflection effect is similar in magnitude to the switch effect for gains described in (b) above, and, on the basis of our decomposition, it can simply be interpreted as the result of the probability switch involved.

Finally, the distinction between the translation and the switch effect helps clarify the implications of the observed behavior on preference theory. We argue that, while both the translation and the switch effect contradict the expected utility hypothesis, the translation effect implies a deeper departure from standard preference theory because, contrary to the switch effect, it negates the existence of consistent, endowment-independent preferences, and, accordingly, has more serious implications for normative analysis, favoring the new developments in paternalistic welfare economics.

With our 2?2? 7 experimental design, we have attempted systematically to approach a central issue in the analysis of risk attitudes. But our work is subject to the limitations inherent to the experimental method in the social sciences, first and foremost the non-randomness and small size of the samples. Accordingly, our results have the character of initial discoveries that require the scientific test of replication. While experimenting with substantial money losses can be a difficult endeavor, the study of risk attitudes is a keystone in the understanding of choice, and demands additional efforts.

## Appendix 1. Experimental data

|  |  |  |  |  |  |  | mount |  | ney | etas |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 500 |  | 1000 |  | 2000 |  | 5000 |  | 7500 |  | 10000 |  |  |
| Subject AG |  | c |  | C |  | C |  | c |  | C |  | c | c |
| Subject BG |  | C |  | C |  | C |  | C |  | C |  | c | c |
| Subject CG |  | c |  | c |  | c |  | c |  | c |  | c | c |
| Subject DG |  | c |  | c |  | c |  | c |  | C |  | C | c |
| Subject EG |  | c |  | c |  | c |  | c |  | C |  | C | c |
| Subject FG |  | c |  | C |  | c |  | c |  | C |  | c | c |
| Subject GG |  | C |  | c |  | C |  | C |  | c |  | c | c |
| Subject HG |  | C |  | C |  | c |  | c |  | C |  | C | c |
| Subject JG |  | un |  | c |  | c |  | C |  | c |  | C | c |
| Subject KG |  | un |  | un |  | c |  | c |  | C |  | c | c |
| Subject LG |  | un |  | un |  | c |  | C |  | c |  | C | c |
| Subject MG |  | un |  | un |  | c |  | c |  | C |  | c | c |
| Subject NG |  | un |  | un |  | c |  | C |  | C |  | C | c |
| Subject OG |  | un |  | un |  | c |  | c |  | c |  | c | c |
| Subject PG |  | un |  | un |  | un |  | c |  | c |  | C | c |
| Subject QG |  | un |  | un |  | un |  | C |  | C |  | C | c |
| Subject RG |  | un |  | un |  | un |  | c |  | C |  | c | c |
| Subject IG |  | c |  | un |  | un |  | c |  | c |  | c | c |
| Subject SG |  | un |  | un |  | un |  | c |  | C |  | un | c |
| Subject TG |  | un |  | un |  | un |  | ur |  | un |  | c | C |
| Subject UG |  | un |  | un |  | C |  | c |  | un |  | un | un |

Table A1. Treatment $G$. A letter $c$ (green cell) indicates choosing the certain gain (thus displaying risk aversion), while the letters un (red cell) indicate choosing the uncertain gain (thus displaying risk attraction). In this table, as in similar tables below, subjects have been ordered to help reading the table.


Table A2. Treatment $L$. A letter $c$ (green cell) indicates choosing the certain loss (thus displaying risk aversion), while the letters un (red cell) indicate choosing the uncertain loss (thus displaying risk attraction). The dashes indicate that the subject was not asked to make the corresponding choice.


Table A3. Treatment $G^{\prime}$. A letter $c$ (green cell) indicates choosing the certain gain (thus displaying risk aversion), while the letters un (red cell) indicate choosing the uncertain gain (thus displaying risk attraction).


Table A4. Treatment $L^{\prime}$. A letter $c$ (green cell) indicates choosing the certain gain (thus displaying risk aversion), while the letters un (red cell) indicate choosing the uncertain gain (thus displaying risk attraction).

## Appendix 2. Prospect theory and the fourfold pattern

Prospect Theory (Kahneman and Tversky, 1979, Tversky and Kahneman, 1992) postulates that, when deciding between the uncertain gain or loss $z$ with probability $p$ (and zero with probability $1-p$ ) and the certain gain or loss $p z$, the decision maker chooses the uncertain (resp. the certain) gain or loss, thus displaying risk attraction (resp. aversion) if and only if

$$
?(p ? \operatorname{sign} z) v(z)-v(p z)>0(\text { resp. }<0),
$$

where $v: \mathbf{R}$ ? $\mathbf{R}$ is her "value function," which can be normalized so that $v(0)=0$, and ? is a "weighting function" that transform probabilities into decision weights. ${ }^{23}$ The following assumptions are standard.
A. 1 (Convexity-concavity). The function $v$ is increasing on $\mathbf{R}$, strictly concave on $\mathbf{R}_{+}$and strictly convex on $\mathbf{R}_{-}$.
A. 2 (Weak distortion of probabilities). If $p$ is small, then $?(p ?) \geq$.$p ; if p$ is large, then $?(p ?) \leq$.$p .$

Lemma. Under A. 1 and A.2, if $p$ is large, then the individual displays: (i) risk aversion for all $z>0$, and (ii) risk attraction for all $z<0$.

The proof is immediate. By strict concavity, $v(p z)>p v(z)$, and hence ? $(p$ ? .) $v(z)-v(p z)$ $<?(p ?) v.(z)-p v(z) \leq 0$, by A.2, proving (i). A similar argument proves (ii).

But the model has no theoretical implications on risk attitudes when the probabilities are small. Indeed, A. 2 implies that $?(p ?) \geq$.$p , whereas, by A. 1, p v(z) / v(p z)<1$, and the sign of

$$
\begin{equation*}
\text { ?(p? .) } v(z)-v(p z)=v(p z) \stackrel{?}{?} \frac{(p \mid \cdot)}{p} \frac{p v(z)}{v(p z)} ? 1 \stackrel{?}{?} \tag{1}
\end{equation*}
$$

depends on the functional forms of ? and $v$.
Tversky and Kahneman (1992, section 2.3) propose the following form for the value function, which in particular implies A.1.

[^12]
## A.3.2. Reference-dependent preferences under certainty

Consider the basic model of consumer choice under certainty. There is a list of $N$ economic variables, or goods, that affect the consumer's welfare: the underlying space of economic goods can thus be modeled as $\mathbf{R}^{N}$, and we focus on a subset $X$ of it, called the consumption set, that specifies possible physical constraints, e.g., $X=\mathbf{R}^{N}{ }_{+}$.

Economic activity involves acquiring or relinquishing various amounts of these goods, as for instance in the process of buying commodities, selling labor, or saving. The consumer has an initial position or endowment?? $\mathbf{R}^{N}$, that corresponds to autarkic consumption, i.e., when the decision maker "does nothing." Society offers a set $Z$ ? $\mathbf{R}^{N}$ of opportunities to acquire and relinquish goods: let us call it a trading set. The endowment ? and the trading set $Z$ define the attainable consumption set $(Z+\{?\})$ ? $X$.

The traditional interpretation of ? as the autarkic or "do nothing" consumption point is not essential: we could as well think of ?, à la Kahneman and Tversky, as a subjective "reference" consumption point, perhaps determined by the status quo or by expectations and aspirations. One only needs to reinterpret the fact that $\left(z_{1}, \ldots, z_{N}\right)$ belongs to $Z$ as the ability to consume $?+\left(z_{1}, \ldots\right.$, $\mathrm{z}_{N}$ ). In order to accommodate both interpretations, we may refer to ? as the "reference point."

The theory has the positive aim of understanding and predicting the choice of a consumer with reference or endowment point ? facing the set $Z$, as well as the normative aim of judging consumer welfare, e. g., whether or not the consumer is better off at $\left(?^{0}, Z^{0}\right)$ than at $\left(?^{1}, Z^{1}\right)$, and thus evaluating economic policies that affect? or $Z$.

Standard economic theory postulates a well-defined preference relation ? on $X$. Given ? and $Z$, the consumer chooses $z ?\left(z_{1}, \ldots, z_{N}\right) ? Z$ in order to maximize ? on the attainable set $(Z+$ $\{?\}) ? X$. This induces an indirect preference relation ? * on (?, Z) pairs expressing whether the consumer is better off at $\left(?^{0}, Z^{0}\right)$ than at $\left(?^{1}, Z^{1}\right)$, or not, for all possible such pairs.

It follows that, under the standard assumptions, the consumer's choice of $z$ at $\left(?^{0}, Z\right)$ will typically differ from that at ( $?^{1}, Z$ ) in this sense, her choice depends on ?.$^{26}$ But, obviously, whenever $\left(?^{0}, Z^{0}\right)$ and $\left(?^{1}, Z^{1}\right)$ satisfy $Z^{0}+\left\{?^{0}\right\}=Z^{1}+\left\{?^{1}\right\}$, both her consumption points $?^{0}+z^{0}$ and $?^{1}+z^{1}$, and her welfare levels will be the same at $\left(?^{0}, Z^{0}\right)$ and at $\left(?^{1}, Z^{1}\right)$.

[^13]However, the experimental literature on the endowment effect and loss aversion documents the choice of different consumption points $?^{0}+z^{0}$ and $?^{1}+z^{1}$ in cases where $Z^{0}+\left\{?^{0}\right\}=Z^{1}+$ $\left\{?^{1}\right\} .{ }^{27}$ Understanding these choices by preference maximization requires a family $\{?: ? ? ?\}$ of preference relations on $X$, where ? is an index set of possible endowment or reference points, instead of a single preference relation ?..$^{28}$ We then say that preferences are reference dependent if they vary with ?, i.e., if ? ? ? ? ? for some ?, ?'? ? . By this definition, reference dependence requires the possibility of changes in ? : no reference dependence could arise if ? never varied, i.e., if? were the singleton $\{?\} .{ }^{29}$

So let ? have many elements. Prediction then requires the knowledge of the family of preference relations \{? ? :? ? ? \}. But, even if available, this knowledge is not sufficient to evaluate consumer welfare, because one cannot derive from it an indirect preference relation ? * on $(?, Z)$ pairs. In other words, we cannot tell from observing the consumer's behavior whether the consumer is better off at $\left(?^{0}, z^{0}\right)$ than at $\left(?^{1}, z^{1}\right)$, because the consumer never has a chance to choose between $\left(?^{0}, z^{0}\right)$ and $\left(?^{1}, z^{1}\right) .{ }^{30}$

Perhaps the consumer does have a preference relation on $(?, z)$ pairs. We may be unable to infer it from her choices, but perhaps we can ask her. Suppose that she asserts to be better off at $\left(?^{0}\right.$, $z^{0}$ ) than at $\left(?^{1}, z^{1}\right)$, where $z^{0}=0$ and $?^{0}=?^{1}+z^{1}$, i.e., she asserts that she would be better off if her endowment or reference point were $?^{0}$ and she stayed at it than if it were $?^{1}$ and she moved to ? ${ }^{0}$ : this is usually referred to as an "endowment effect." But suppose also that $\left(?^{0}, 0\right)$ is socially more costly than $\left(?^{1}, ?^{0}-?^{1}\right)$. (Perhaps the implementation of $\left(?^{0}, 0\right)$ requires more bureaucracy). It is not clear that in this case her preferences should be respected. ${ }^{31}$

[^14]At the crux of the matter is the question, why does she prefer $\left(?^{0}, 0\right)$ to $\left(?^{1}, z^{1}\right)$ ? Is ? a true "do nothing" consumption vector, or a subjective reference point? If endowments can change, what distinguishes a change in endowments from a trade? The normative relevance of an expressed preference of $\left(?^{0}, 0\right)$ over $\left(?^{1}, ?^{0}-?^{1}\right)$ has to be justified by appealing to basic principles.

## A.3.3. Translation-dependent risk attitude in the space of contingent money balances

The model of Section A.3.2 can be extended to decisions under uncertainty, with the interpretation that preferences are ex ante, before the uncertainty is resolved. The space of ex ante economic variables is now either that of vectors of contingent money balances or the space of lotteries over fixed prizes. These are still finite-dimensional Euclidean spaces for many applications. ${ }^{32}$

We consider the space of contingent money balances first. There are $S$ states: the number $x_{s}$ denotes an amount of money available to the consumer in the contingency that state $s$ occurs. A vector $x=\left(x_{1}, \ldots, x_{S}\right) ? \mathbf{R}_{+}^{S}$ is interpreted as a point of contingent money balances. The consumer has an initial position or reference point? ? $\mathbf{R}^{S}{ }_{+}$, and society offers her a trading set $Z$ ? $\mathbf{R}^{S}$ of opportunities to change contingent money balances. As in Section A.3.2,? and $Z$ define the attainable set $(Z+\{?\}) ? \mathbf{R}^{S}$.

Let $S=2$, with the interpretation that the first state is the bad one. Recalling the definitions in Section 2.1 above, for $z>0$, choice $? z, p$ ? is the choice between the uncertain point of contingent money balances $(?, ?+z)$ and the certain point $(?+p z, ?+p z)$, whereas, for $z<0$, the choice is between $(?+z, ?)$ and $(?+p z, ?+p z)$.

A decision-maker's risk attitude is translation dependent if she displays risk aversion for choice ? $z, p$ ? (as in the NW cell of Figure 1 above), but risk attraction for choice $?-z, 1-p$ ? (NE cell), for a wide range of initial wealth values $w$. It is easy to see that translation-dependent risk attitudes imply reference-dependent preferences. ${ }^{33}$ Let $z=100$ and $p=0.8$. Assume that, both for initial wealth 1000 and 1100 , she displays risk aversion for the choice ? 100, 0.8 ? (NW cell of Figure 1), but risk attraction for the choice ? $-100,0.2$ ? (NE). In the graph of contingent money

[^15]balances of Figure A1, this means that she prefers $S$ to $C$ when her endowment or reference point is $C^{\prime}$, but $C$ to $S$ if it is $C^{\prime \prime}{ }^{34}$ Thus, no single set of indifference curves on the space of contingent money balances can rationalize her behavior.

## A.3.4. Translation-dependent risk attitude in lottery space

There is a finite number $M$ of possible final money balances $\left\{x_{1}, \ldots, x_{M}\right\}$. The space of economic variables is now the ( $M-1$ )-standard simplex: we call a point $\left(p_{1}, \ldots, p_{M}\right)$ in the simplex a lottery, interpreted as the vector of the probabilities of ending up with the various possible money balances.

The decision maker's endowment or reference point is a lottery ( $p^{?}{ }_{1}, \ldots, p^{?}{ }_{M}$ ). Perhaps she is initially endowed with wealth $x_{m}$, in which case we say that her endowment is the (degenerate) lottery with $p^{?}{ }_{m}=1$ and $p^{?}{ }_{m^{\prime}}=0$ for $m^{\prime} ? m$.

Translation-dependent attitudes cannot be rationalized by a set of indifference curves in the simplex. Consider the previous example: when the consumer's wealth is 1000 , she prefers a sure gain of 80 to a gain of 100 with probability 0.8 , whereas when her wealth is 1100 she prefers a loss of 100 with probability 0.2 to a certain loss of 20 . Let $M=3$, and $\left\{x_{1}, x_{2}, x_{3}\right\}=\{1000,1080,1100\}$. A wealth of 1000 is then the lottery $(1,0,0)$ as endowment, represented as point $C^{\prime \prime}$ in the Marschak-Machina triangle of Figure A2, whereas a wealth of 1100 is the lottery $(0,0,1)$, represented as point $C^{\prime} .{ }^{35}$ Points $C$ and $S$ have an expected money value of $1080 .{ }^{36}$ Risk aversion for gains means that, when the endowment or reference point is $C^{\prime \prime}$, the consumer prefers point $C$ (a certain gain of 80 added to 1000 ) to point $S$ (the gain of 100 with probability 0.8 ). But risk attraction for losses means that, when the endowment or reference point is $C^{\prime}$, the consumer prefers uncertain point $S$ to the certain point $C$. Thus, no single set of indifference curves can rationalize her behavior.

## A.3.5. Switch-dependent risk attitude

A decision-maker's risk attitude is switch dependent if she displays risk aversion for choice $? z, p$ ? (as in the NW cell of Figure 1 above), but risk attraction for choice ? $z, 1-p$ ? (SW), for a wide

[^16]range of initial wealth values $w$ and of gain values $z$. It is intuitively clear that, typically, switchdependent risk attitudes are incompatible with the expected utility hypothesis. ${ }^{37}$

But switch-dependent risk attitudes are compatible with consistent preferences. For instance, as in rank-dependent utility theory (John Quiggin, 1982, 1993), we order the components of ( $x_{1}, \ldots$, $x_{M}$ ) so that $x_{1} \leq \ldots \leq x_{M}$ and let, in obvious notation,

$$
U\left(x_{1}, \ldots, x_{M} ; p_{1}, \ldots, p_{M}\right)=?{ }_{i ? 1}^{M} x_{i}^{\prime \prime} q^{\prime \prime} ?_{{ }_{j 21}}^{i} p_{j}{ }^{\prime} ? ? q^{\prime} ?{ }_{j 21}^{i ? 1} p_{j}{ }^{\prime \prime \prime}
$$

for an appropriately chosen increasing function $q:[0,1] ?[0,1]$ that overweighs low probabilities.
For instance, let $q(0.2)>0.2, q(0.8)=1-q(0.2)$, and $q(1)=1$. Then the decision-maker, at any $w$ :

* prefers the uncertain gain of $z$ with probability 0.2 to the certain gain of $0.2 z$; because

$$
\begin{gathered}
U(w, w+z ; 0.8,0.2)=w q(0.8)+(w+z) q(0.2)=w[q(0.8)+q(0.2)]+z q(0.2) \\
=w+z q(0.2)>w+0.2 z=U(w+0.2 z ; 1),
\end{gathered}
$$

* yet prefers the certain gain of $0.8 z$ to the uncertain gain of $z$ with probability 0.8 , because

$$
\begin{gathered}
U(w, w+z ; 0.2,0.8)=w q(0.2)+(w+z) q(0.8)=w q(0.2)+w q(0.8)+z q(0.8) \\
=w+z q(0.8)<w+0.8 z=U(w+0.8 z ; 1) .
\end{gathered}
$$

Therefore, even if switch-dependent attitudes violate the expected utility hypothesis, they are indeed compatible with consistent, reference-independent preferences.

[^17]

Figure A1. Translation-dependent attitudes do not allow for consistent preferences on the space of contingent money balances.


Figure A2. Translation-dependent attitudes do not allow for consistent preferences on the space of lotteries

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[^1]:    ${ }^{2}$ See Colin Camerer et al (2003) and Richard Thaler and Cass Sunstein (2003) for new developments in paternalistic welfare economics.

[^2]:    ${ }^{3}$ A risk-neutral subject could choose either the certain or the uncertain prospect, his or her choice being at random. But the likelihood that the results of the experiment consist of random variation is statistically indistinguishable from zero. ${ }^{4}$ Note that there is no default, i.e., "doing nothing" is not an option.

[^3]:    ${ }^{5}$ See, e.g., Martin Weber and Heiko Zuchel (2001), Scott Boylan and Geoffrey Sprinkle (2001), Kevin Keasy and Philip Moon (1996) and Thaler and Eric Johnson (1990). According to Thaler and Johnson (1990), p. 657, "... after a gain, subsequent losses that are smaller than the original gain can be integrated with the prior gain, mitigating the influence of loss-aversion and facilitating risk-seeking." But see Jeremy Clark (2002) for small sums of money.
    ${ }^{6}$ After registering their choices, subjects answered a questionnaire about the prospective pain of losing money in the experiment. The majority (59\%) agreed that it would be very painful lo lose money because "the money was theirs,"

[^4]:    $9 \%$ accepted that they would feel some pain since it was "as if the money was theirs," $21 \%$ claimed to no pain since the money "was not actually theirs," and $11 \%$ gave other answers. Postexperiment personal intervie ws showed similar results.
    ${ }^{7}$ The questions concerned hypothetical gains and losses, as well as different time lags. In one situation, a Mr. A and a Mr. B gained and lost the same amounts of money, but while Mr. A experienced the gain and the loss on the same day, Mr. B's loss occurred several weeks later. Respondents were asked to compare the happiness of Mr. A. and Mr. B after losing the money. Of the 107 respondents, 71 considered Mr. A the happier one, whereas 27 thought it was Mr. B and 9 said that both were equally happy. In another situation, we asked to compare Mr. C, who won and lost some money on the same day, and Mr. D, who earned an amount equal to the difference between Mr. C's gain and loss. The response was overwhelmingly in favor of Mr. D being happier (87), with 4 favoring Mr. C and 15 voting for indifference. Taken at face value, the answers to this questionnaire indicate that individuals feel more pain if a loss does not coincide in time with a gain, suggesting that their capability of integrating the loss with a previous gain is limited (even when the loss and the gain are presented together as in the examples described), when time separates the moment of the gain from the moment of the loss.
    ${ }^{8}$ Twenty-four subjects had taken the quiz, but three did not show up for the second part of the $L$ experiment.

[^5]:    ${ }^{9}$ Thirty-six subjects had taken the quiz, but two did not show up for the second part of the $L^{\prime}$ ' experiment.
    ${ }^{10}$ Recall that our Experiments $L$ and $L^{\prime}$ were designed in such a way that low earners in the quiz did not have to decide on the higher money losses. Our estimation data allows for the missing data to be missing at random (MAR). It does not require the stronger assumption of missing completely at random (MCAR), which would imply the patterns of missing data to be uncorrelated with the observable variable $z$.
    ${ }^{11}$ Note that we assume a common slope $b$ in the four different regimes: we had previously estimated the overparametrized model with RHS ? $?_{i} u_{i}+?_{1} d_{1}+?_{2} d_{2}+?_{3} d_{1} d_{2}+\left(b+?_{4} d_{1}+?_{5} d_{2}\right) z_{j}$, and found that the estimates for $?_{4}$ and $?_{5}$ were statistically insignificant.

[^6]:    ${ }^{12}$ This was also observed and is reported in Bosch-Domènech and Silvestre $(1999,2002)$.

[^7]:    ${ }^{13}$ Alternatively, this reflection effect is also the sum of a moderate translation effect for low probabilities of the bad outcome (Result 5) and a small switch effect for losses (Result 8). A similar pair of decompositions applies to the move along the skew diagonal, as described in Result 10.
    ${ }^{14}$ See Harbaugh et al., 2002b, for some results on different elicitation methods.

[^8]:    ${ }^{15}$ The (perhaps unconventional) arrangement of the fourfold pattern in Table 4 agrees with our figures 1-3.
    ${ }^{16}$ Of course, the appropriateness of our discussion to the Tversky-Kahneman fourfold pattern is in any event contingent on interpreting the probabilities that we deal with ( 0.8 and 0.2 ,) as sufficiently "high" and "low" for their pattern. Because of the related probability weighting (see Appendix 2 below), Figures 3.3 in Tversky and Kahneman (1992) do suggest that 0.2 is "low" and 0.8 is "high," whereas the earlier Figure 2.4 in Kahneman and Tversky (1979) suggests that 0.2 is not low enough.

[^9]:    ${ }^{17}$ Kahneman and Tversky (1979, Table 1, Problem 3) provide one instance of "reflection" corresponding to the main diagonal of our Figure 1 above. Table 3.3 in Tversky and Kahneman (1992) has more instances (all the experiment pairs in the $6^{\text {th }}$ column, where the probability of gain or loss is 0.75 , and in the $7^{\text {th }}$ column, where the probability is 0.90). John Hershey and Paul Shoemaker (1980) critically examine the reflection effects submitted by Kahneman and Tversky's (1979), yet their only experiment with a probability close to our 0.80 yields a strong and significant reflection effect (Table 3, Experiment 11). Gains and losses are hypothetical in all these experiments.
    ${ }^{18}$ Most of the examples in Tversky and Kahneman (1992) Table 3.3 fail to evidence a translation effect. An exception is provided by rows three and four for the probabilities 0.25 and $0.75\left(4^{\text {th }}\right.$ and $6^{\text {th }}$ columns). There they report risk aversion for a 0.75 probability of a gain of $\$ 100$, but risk attraction for a 0.25 probability of a loss of $\$ 100$.
    ${ }^{19}$ The data in Tversky and Kahneman (1992), Table 3.3, systematically show a switch effect.
    ${ }^{20}$ Although not the levels: the fractions of their subjects displaying risk attraction are systematically smaller than ours.

[^10]:    ${ }^{21}$ While they disaggregate choices by age, by probability, by gains and losses, they do not provide data showing how choices evolve as payoffs change.

[^11]:    ${ }^{22}$ In particular, when studying decisions in the face of uncertainty, behavior observed for small money amounts cannot be extrapolated to large amounts. The surprising analysis of Matthew Rabin (2000) points in all likelihood towards a related phenomenon.

[^12]:    ${ }^{23}$ The dependence of ? on the sign of $z$ means that the weighting may be different for gains and for losses, see Tversky and Kahneman (1992). Gains and losses are defined relative to a reference point, which can be current assets or some subjective aspiration level. Kahneman and Tversky's (1979) notation does not cover changes in the reference point. In fact, they believe that any such changes can be ignored: in their words (1979, p. 277) "the preference order of prospects is not greatly altered by small or even moderate variations in asset positions." Bosch-Domènech and Silvestre (2002) suggest, on the contrary, an important role for wealth.

[^13]:    ${ }^{26}$ Kahneman and Tversky (1979) downplay this dependence in the context of choices between uncertain prospects.

[^14]:    ${ }^{27}$ See Thaler (1992) for an informal survey of endowment effects in both certain and uncertain choices.
    ${ }^{28}$ More generally, the preference relation could be indexed by both? and $Z$, capturing conceivable "dependence on irrelevant alternatives."
    ${ }^{29}$ Trivially, if $?=\{?\}$, then endowment dependence by any definition is irrelevant.
    ${ }^{30}$ At least unless some a priori restrictions on the overall preferences are imposed. The problem displays formal similarities with the estimation of preferences for nonmarketed goods, such as quality or the environment, for which ingenious positive results con be obtained by a priori postulating particular forms of complementarity or substitutability between marketed and nonmarketed goods (see, e.g., Robert Willig, 1978, and Douglas Larson, 1992). Some of these methods could conceivably be adapted to the present context.
    ${ }^{31}$ If, on the contrary, changing endowments were exactly as costly as trading, then Pareto efficiency in a society where everybody had this type of preferences would require the redistribution of endowments and no trade.

[^15]:    ${ }^{32}$ But not for all, in particular, not for a continuum of possible lottery prizes.
    ${ }^{33}$ Note that the attitude reversal is assumed to occur for a range of initial wealth values. There would be no problem if it only occurred for a single $w$, in which case the expected utility hypothesis could be maintained, with a vNM utility function convex in the interval $(w-z, w)$ and concave in $(w, w+z)$.

[^16]:    ${ }^{34}$ The line of slope - $1 / 4$ through point $C$ in Figure A1 is a fair-odds (or iso-expected-money) line.
    ${ }^{35}$ The Marschak-Machina triangle is a 2-dimensional Cartesian representation of the standard 2-simplex $\left\{\left(p_{1}, p_{2}, p_{3}\right) ? \mathbf{R}_{?}^{3}: p_{1} ? p_{2} ? p_{3} ? 1\right\}$ where we read $p_{1}$ as the abscissa, $p_{3}$ as the ordinate, and $p_{2}$ as $1-p_{1}-p_{3}$.
    ${ }^{36}$ The expected money balances are 1080 along any points of the line of slope 4 through $C$ in Figure A2.

[^17]:    ${ }^{37}$ Again, there would be no problem if the attitude change only took place for a single $w$ and $z$, in which case the expected utility hypothesis could be maintained, with a $v N M$ utility function $u$ convex in the interval ( $w, w+0.5 z$ ) and concave in $(w+0.5 z, w+z)$. But assume that, for any $w$ ? [1000, 1100] and any $z$ ? [0, 100], (a) the consumer prefers the uncertain gain of $z$ with probability 0.2 to the certain gain of $0.2 z$; but (b) she prefers the certain gain of $0.8 z$ to the uncertain gain of $z$ with probability 0.8 . Under the expected utility hypothesis we can $\operatorname{set} u(1000)=0$, and $u(1100)=100$. Then (a) implies that $u(1020)<20$, and (b) that $u(1080)>80$, which, as long as $u$ is continuous, imply that? $z^{\prime}$ ? (20, 80) and $z^{\prime \prime}$ ? ( 80,100$]$ such that (i) $u\left(1000+z^{\prime}\right)=z^{\prime}$, (ii) $u\left(1000+z^{\prime \prime}\right)=z^{\prime \prime}$, and (iii) $u(1000+z)>z, ? z$ ? ( $\left.z^{\prime}, z^{\prime \prime}\right)$. Consider $w$ ? $1000+z^{\prime}$ and $z=z^{\prime \prime}-z^{\prime}$. By (a), the consumer prefers the uncertain gain of $z$ with probability 0.2 to the certain gain of $0.2 z$, i.e., $0.8 u\left(1000+z^{\prime}\right)+0.2 u\left(1000+z^{\prime}+z^{\prime \prime}-z^{\prime}\right)>u\left(1000+z^{\prime}+0.2\left(z^{\prime \prime}-z^{\prime}\right)\right)$, or, using (i)-(ii), 0.8 $z^{\prime}+0.2 z^{\prime \prime}>u\left(1000+0.8 z^{\prime}+0.2 z^{\prime \prime}\right)$, contradicting (iii), because $0.8 z^{\prime}+0.2 z^{\prime \prime} ?\left(z^{\prime}, z^{\prime \prime}\right)$. Thus, (a) and (b) are incompatible with the expected utility hypothesis.

