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# A New Model for Designing a Product Line Using TURF Analysis 

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# A new model for designing a product line using TURF analysis ${ }^{1}$ 

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#### Abstract

This paper introduces the approach of using TURF analysis to design a product line through a binary linear programming model. This improves the efficiency of the search for the solution to the problem compared to the algorithms that have been used to date. Furthermore, the proposed technique enables the model to be improved in order to overcome the main drawbacks presented by TURF analysis in practice.


Key words: TURF analysis, Binary programming, product design
JEL: C61, M31

[^0]
## 1. INTRODUCTION

TURF (Total Unduplicated Reach and Frequency) analysis is a technique used in the world of marketing to optimize product lines. Specifically, it involves selecting the combination of product variants that will ensure that the overall product reaches its maximum penetration. Its applications are extensive, both in the world of mass commodities, and in durable goods and even in services (Conklin \& Lipovetsky, 2000, Cohen 1993). The technique is applicable when we have a product that we want to launch on the market as a range in which only one attribute changes, for example, the choice of different flavours for an ice-cream, colours for an MP3 player or fragrances for an airfreshener.

In 1990 Miaoulis, Free and Parsons presented the technique. It involves an adaptation of tools from the world of advertising where the aim is to design a communication plan that will reach the highest number of potential customers. Transferred to the world of marketing, the technique concerns choosing out of all the possible combinations the product line that, in a similar way, will attract the highest number of potential consumers. The problem prioritizes new consumers over those who duplicate consumption, what it tries to maximize is the penetration of the product line as a whole, not of each of the variants that make it up.

An example will help to clarify the concepts. Suppose we have three variants of a product that are candidates for forming a product line. In our case, we will assume that we are limited to putting 2 varieties out of the possible 3 onto the market. We have to find the optimal combination that will achieve the greatest penetration of the product line. The following table shows the situation considered and the data used for the example.

|  | A | B | C |
| :---: | :---: | :---: | :---: |
| Interviewee 1 | 1 | 1 | 0 |
| Interviewee 2 | 1 | 1 | 0 |
| Interviewee 3 | 1 | 0 | 0 |
| Interviewee 4 | 0 | 0 | 1 |
| Total | $\mathbf{3}$ | $\mathbf{2}$ | $\mathbf{1}$ |

If we were seeking to maximize the penetration of each of the variants separately, then we should opt for marketing the combination $\mathrm{A}+\mathrm{B}$, as each of them has 3 and 2 potential consumers, respectively. However, our goal is to maximize the penetration of the product
line as a whole, and since the two consumers who choose B have already been reached by variant A , including B in the combination does not bring any increase in penetration, with the penetration for the complete range being 3 individuals. If, on the other hand, we choose the combination $\mathrm{A}+\mathrm{C}$ we are increasing the overall product penetration by 1 individual, because C has been chosen by a consumer that did not choose A. Our overall penetration will be 4 , the maximum that we could reach.

The calculation method proposed by the authors is that of exhaustive enumeration, calculating the overall penetration of all the possible combinations. This guarantees the optimal combination. However, as is acknowledged in the same article, as the alternatives being considered in the problem increase, the computation time required increases exponentially, and therefore, as the authors propose, a more efficient search algorithm should be found, using mathematical programming methods.

In 2000, Kreiger and Green proposed an alternative method to exhaustive enumeration: the greedy algorithm. This method consists of choosing, firstly, the alternative that attracts the highest number of individuals, including it in the final solution. Then the alternative that achieves the greatest penetration taking into account only the individuals who were not attracted by the first alternative is chosen, in other words, the unduplicated penetration is calculated, conditioned to the first alternative chosen. The process continues until all the individuals are covered or until none of the remaining alternatives manages to increase the overall penetration. As the authors themselves show, this process does not guarantee the optimal solution (Mullet, 2001). Markowitz $(2004,2005)$ extends the TURF model to different business marketing situations and compares the TURF model with others.

In the following section we present the TURF model as an integer linear programming problem, the method which we establish as the framework for the analysis in order to apply an optimal search algorithm. The results obtained through our exact algorithm and through the greedy algorithm are compared below, in order to calculate to what extent the latter does not give optimal solutions. Furthermore, the new method is applied to a real case, extended in such a way that some of the main drawbacks of TURF analysis are overcome. Finally, a series of adaptations of the model to multiple situations that may arise in real life are proposed.

## 2. Application of a binary linear programming model to TURF

## analysis

The problem we are going to study below comes under the group of binary linear programming models, linear programming because all the functions of the model (both the target function and all the constraints) are linear and binary functions because the variables we are going to introduce will be variables that can only have values of zero or one.

Once the model has been established, a series of mathematical algorithms can be applied that let us obtain the solution to the problem. In this regard, there are exact algorithms, which guarantee the optimal solution, and heuristic algorithms, which do not always give optimal solutions but do provide a good sub-optimal solution. In this case, we will use an exact algorithm, thus assuring that we will obtain the optimal solution.

As we have indicated above, the problem consists of finding the product line that, overall, attracts the highest possible number of customers. The person responsible for the product could present it as follows:

What is the minimum number of varieties I have to put on the market in order to attract the maximum possible number of buyers?

## What varieties should I market?

To answer these questions, the first thing we have to do is compile the data. As required by TURF analysis, data observation is carried out through surveys in which the interviewee is asked to rate each of the proposals based on how attractive it is. This analysis does not need the use of a certain scale or volumetric measure (Miaoulis, Free, \& Parsons, 1990), and it is common practice to use an intention to buy scale (Ortega 1998). The data we have to input in the model are binary data (buy / not buy) and therefore a criterion has to be established to distinguish between the two. Normally the highest score ("top box") or highest two scores ("top two boxes") of the scale are used as buy and the rest as not buy.

With the data matrix obtained, we will have for each individual consulted the alternatives he or she is willing to buy and those he/she is not willing to buy, in other
words, we can calculate the penetration of each alternative separately and the penetration of each product line as a whole. ${ }^{3}$

The following phase consists of translating our real problem into a mathematical model. Let us suppose a problem in which we have $n$ alternative flavours for an ice-cream. We carry out a survey of $m$ individuals as to what flavours they would buy and what flavours they would not. We have a data matrix in which for each individual we know the set of flavours he or she would be willing to buy. We can define the following set:

$$
N_{i}=\{j / \text { consumer i chooses variety } j\}
$$

In other words, for every individual it is the set of flavours he or she indicates that he or she would be willing to buy. In this way we introduce the data in the model.

We also need to know what varieties we will put on the market, and what others we will not. These will be the binary variables of our model:

$$
x_{j}=\left\{\begin{array}{l}
1, \text { we put variety } j \text { on the market } \\
0, \text { we don't }
\end{array}\right.
$$

The problem, as we have presented it, consists of finding the minimum combination of varieties that means that all the individuals can buy at least one of the varieties they have chosen. Therefore, our target function will consist of keeping to a minimum the number of product varieties we will put on the market, in other words:

$$
\operatorname{Min} Z=\sum_{j=1}^{n} x_{j}
$$

However, bearing in mind that all the interviewees have to be able to buy at least one of the varieties they have chosen, that is:

$$
\sum_{j \in N_{i}} x_{j} \geq 1 \quad i=1 \ldots m
$$

[^1]We force at least one of the varieties chosen by each individual to be put on the market. ${ }^{4}$

Thus, the final formulation of the TURF model will be:

$$
\begin{aligned}
& \operatorname{Min} Z=\sum_{j=1}^{n} x_{j} \\
& \text { s.a. } \sum_{j \in N_{i}} x_{j} \geq 1 \quad i=1 \ldots m \\
& x_{j}=(0,1) \quad j=1 \ldots n
\end{aligned}
$$

The use of this model fulfils the purpose of the TURF analysis, because when an individual has already been "reached" by a variety, he or she is no longer considered for the choice of the rest of varieties. Thus the problem takes into account new buyers and does not consider at any time to what extent individuals who are already buyers duplicate the purchase of the product with more than one variety.

Presenting the analysis in this way lets us benefit from a series of advantages compared to the algorithms used previously. The main advantages are:

- Compared to exhaustive enumeration: it represents a considerable saving in computation time. The algorithm that solves the problem no longer has to calculate each and every one of the possible combinations. This fact becomes relevant when the problems presented have a great quantity of variables and/or records. The main software on the market that enables TURF to be obtained does so using the exhaustive enumeration method and therefore presents limitations as to the number of varieties to be introduced in the problem. The implementation of TURF with the proposed model would let this limitation be extended. Likewise, it would also improve the efficiency of such software.

[^2]- Compared to the greedy algorithm: it guarantees the optimal solution. In the following section we compare the two methods. The conclusions of this comparison show the advisability of using an exact algorithm.
- Furthermore, the presentation of the problem through binary linear programming means that the adaptations of the model to real situations can be done easily since the model contributes great flexibility to the problem. In section no. 4, we will present some of the situations that habitually arise in real life and the adaptations of the model in order to obtain the goal.
- Calculating the solution for the basic analysis no longer needs specific software. The most commonly used spreadsheets normally include add ins that let mathematical programming problems be solved, such as the Solver add--in that comes with Excel.


### 2.1. Binary programming $v$. greedy algorithm

As we have discussed, the use of the greedy algorithm does not guarantee that the optimal solution to the problem will be obtained. In order to quantify to what extent this deviation of results occurs, we have compared this method with the method proposed in this study, both in randomly generated samples and in real samples.

In the test with random samples, a total of 100 samples were generated, considering 500 records (individuals) for 15 product options. On the other hand, we applied the same comparison in 68 real samples collected in several studies on different consumer goods. As we show below, the results of the analysis differ according to the type of sample analysed, although we can reach the same conclusion: the advisability of using the exact algorithm.

The test carried out on random samples shows alarming results, as only 6 of every 10 solutions obtained through the greedy algorithm provided the optimal solution.

In the case of the comparison of methods with real samples, the results are less shocking, due to the greater concentration of replies in the more attractive varieties. The analysis showed that of 68 samples analysed, in 9 of them the optimum result was not obtained when the greedy algorithm was used, in other words, $13 \%$ of the solutions recommended by that algorithm were erroneous. The error is much smaller compared to that obtained with random samples, however it is still significant because not reaching the
optimum result means the erroneous recommendation to market at least one variety more than those that are strictly necessary.

## 3. IMPROVING TURF

### 3.1. Main criticism of classic TURF analysis

Although in theory TURF analysis appears to be a very useful tool in decision-making in this type of problems, it has received important criticisms, especially due to the fact that in practice the results are not as clear as would be expected. Below we detail some of the main drawbacks of the analysis:

- The most important problem of this tool is that it usually provides very similar (if not equal) results for different product lines, making it difficult to decide on the choice of a single combination.
- Furthermore, it usually includes in the solution varieties that only attract a small number of consumers and which are not very attractive to a broader public. This leads, in practice, to the analysis recommending that "strange" varieties be marketed that contribute very little added penetration and which obtain results that are not very favourable in other indicators.
- On the other hand, marketing personnel do not have total control over the product line marketed. It may occur that, due to lack of shelf space or lack of stocks, the product line is not fully at the point of sale at the time the consumer makes his or her choice. These cases are not contemplated in the analysis because it only calculates the result assuming that all the varieties are available.


### 3.2. Possible improvements of the analysis

Bearing in mind the criticisms of TURF analysis, we present a procedure that in light of the practical results presented below would appear to be an adequate system for overcoming the main problems of this analysis, always taking into account the information that TURF provides us with in order to gain a better understanding of the behaviour of the product options we handle.

The data we are going to use to illustrate the procedure come from a real sample obtained for a study in which a total of 14 product options were presented to a sample of 150 individuals. The individual penetration of each of the different varieties is shown in graph 1 .

Graph 1: individual penetration of each of the different varieties


Application of the basic TURF model shows us that with four varieties we reach the maximum possible penetration, which is 127 individuals, in other words, with the optimal combination of varieties we can reach $85 \%$ of the potential consumers. The analysis also shows us what varieties we have to choose for our product line (varieties shown as shaded areas in the above graph). As can be seen, there are varieties included in the final solution with an individual penetration notably lower than others that are not included.

It seems advisable to check whether there are other possible optimal combinations for our product line. A quick way of exploring possible alternative optimal solutions is to force each of the varieties not included in the first final combination to have a value of one, and in each case we recalculate the TURF. If the problem does not find an optimal solution or increases the minimum number of varieties necessary to obtain maximum penetration, the variety in question will never become part of an optimal combination. On the contrary, in
other words, if by forcing a variety to be put on the market we obtain a combination of four varieties that reaches maximum penetration, this variety forms part of an optimal solution.

In our case, on carrying out the process described above, we reached the conclusion that there are indeed several optimal solutions. So, what is the best solution out of all the optimal solutions?

The answer to this question depends fundamentally on the specific goals of each problem. However, a good generic solution can be obtained by weighing each of the varieties according to its individual penetration. In other words, if we have two varieties that could form part of an optimum combination, we will opt for the variety which individually obtains greater penetration. As we have already discussed in the introductory section, the analysis prioritizes new buyers over buyers who duplicate varieties, however, under equal conditions we will always opt for the varieties that are contributing more penetration. In this way we also allow the consumer to have the greatest possible range of products letting him or her vary consumption in the case of goods with a high buying frequency or simply to have more options when making his or her choice.

The adaptation of the model is very simple, all we have to do is weigh the binary variables of the target function on the basis of the individual penetration of each of them so that if a variety attracts a higher number of buyers, it has priority for forming part of the optimal solution over other less attractive varieties.

Since in the basis problem we seek to minimize our target function, we have to weigh using a lower coefficient the greater the penetration of the variety it is associated to, for example the inverse of the individual penetration of the variety:
$\operatorname{Min} Z=\sum_{j=1}^{n} \frac{1}{p_{j}} x_{j}$
s.a. $\sum_{j \in N_{i}} x_{j} \geq 1 \quad i=1 \ldots m$
$x_{j}=(0,1) \quad j=1 \ldots n$
Where $p_{j}$ is the individual penetration of each variety.
Applying this system to our example gives the following combination of varieties:
Graph 2: individual penetration of each of the different varieties


In other words, we change variety number 4 of our initial solution (the fourth-worst individual penetration) and replace it with number 7 (third in the ranking of attractiveness to buying), without affecting our overall penetration.

The use of this adaptation of the model, appears, in light of the results, to be fairly recommendable. In addition, it also lets us overcome the main drawbacks of the analysis we mentioned above:

- It lets us highlight one out of the possible optimal combinations, and therefore facilitates decision-making considerably.
- Out of all the optimal combinations, we choose those with the best possible results in individual penetration therefore reducing the possibility of finding varieties that only attract very minority publics.
- Furthermore, if the product line is not fully available at the point of sale, we know that at least individually each of the varieties available to the public at that time will be a sufficiently attractive variety, and therefore problems of lack of stocks or lack of shelf space will be less critical.


### 3.3. Comparison of solutions

On the other hand, we must not forget that all the information we handle is not drawn directly from the population that is the object of the study but from a sample. This sample will be more or less representative but in no case will it reflect perfectly the reality of the population. If we handle different product lines that are candidates for going on the market, the comparison between them should be carried out through comparing hypotheses in order to check whether the differences observed are significant at statistical level.

One of the checks that should be carried out by default is the comparison of the overall penetration of the best varieties according to TURF with the combination of varieties with the greatest individual penetration. If the overall penetration of the two combinations is statistically equal, we may be interested in opting for the second, even though the TURF suggests a different combination of varieties.

In our example, the combination of the four varieties with the greatest penetration (14, 5,6 and 7) obtains an overall penetration of 125 individuals out of 150 , two individuals fewer than the optimal combination. By applying a signification test for dependent proportions ${ }^{5}$ we find that the two penetrations we are handling are statistically equal and therefore it may be of interest to sacrifice two individuals of overall penetration to market the four varieties that obtain the greatest individual penetration. Likewise, we can compare any combination that is of special interest for the goals of the personnel responsible for the product.

In the event that the two proportions were significantly different, we would have to opt for one strategy or the other, always bearing in mind that the TURF analysis aims at maximizing the penetration of the line as a whole, sacrificing buyers who duplicate in favour of new buyers.

[^3]
## 4. ADAPTATIONS OF THE BASIC MODEL

As we have already seen, the fact of having found a theoretical model in which to place our problem makes our tool much more flexible when it comes to adapting to the different situations that may arise in real life. In this section, we present some of the most common problems that can be treated with TURF analysis.

### 4.1. Limited number of varieties

It is very common to have a limit to the number of varieties we wish to put on the market. In this case, the formulation of the problem would be:

What is the maximum penetration we can reach if we want to put a certain number of varieties on the market? Which varieties should I market?

In this case we have two groups of variables, not only the binary variables that determine what varieties will be marketed, but also variables that will let us know the number of buyers we will reach with the final combination. In other words, for each individual considered the solution will tell us whether he or she buys or not. We add these variables to the problem:
$y_{i}=\left\{\begin{array}{l}1, \text { customer } i \text { will purchase } \\ 0, \text { customer } i \text { will not purchase }\end{array}\right.$

Also in this case the number of varieties is pre-determined so we have to introduce a new constraint to the problem:
$\sum_{j=1}^{n} x_{j}=V$
Where V is the number of varieties we want to put on the market.
In this case, not necessarily all the individuals considered will end up consuming, so we have to reformulate the constraint as follows:

$$
\sum_{i \in N_{i}} x_{j} \geq y_{i} \quad i=1 \ldots m
$$

Finally, the problem no longer tries to minimize the number of varieties but to maximize the number of consumers, bearing in mind our limitation of varieties. Therefore, our new target function and our new model will be:

$$
\begin{aligned}
& \operatorname{Max} Z=\sum_{i=1}^{m} y_{i} \\
& \text { s.a. } \sum_{j=1}^{n} x_{j}=V \\
& \sum_{j \in N_{i}} x_{j} \geq y_{i} \quad i=1 \ldots m
\end{aligned}
$$

$$
y_{i}=(0,1) \quad i=1 \ldots m, x_{j}=(0,1) \quad j=1 \ldots n
$$

In other words, the target maximizes the number of buyers subject to a limitation on the number of varieties. When the problem decides that a variety goes on the market $\left(x_{j}=1\right)$ all the individuals who include it in their preferences will be considered buyers (their $y_{i}$ will become 1) because the problems seeks to make the number of buyers maximum. On the contrary, when the problem decides that none of the varieties chosen by an individual goes on the market (all the $x_{j}$ chosen by the individual are zero) then that individual will not be considered a buyer (we force his $y_{i}=0$ )

In this case, we consider that all the individuals are equal. However, it may be interest to us to differentiate between individuals on the basis of certain criteria. An extension of this version of TURF is presented in point 4.4, taking into account the frequency with which each individual buys the product.

### 4.2. Extension of a product line already being marketed

On a considerable number of occasions we are not dealing with a totally new product line, but one that already exists, in other words, we already have some varieties marketed that we do not want to stop selling. In this case, the adaptation of the model is very simple, all we have to do is force the corresponding variables to have a value of one, in other words, for each of the varieties we want to include in the final solution we introduce a new constraint:
$x_{j}=1$
Where $j$ is the variety we want to continue marketing.

### 4.3. Maximizing the volume

Furthermore, just as we have previously weighed on the basis of the individual penetration of each of the varieties, it may be of interest to us to consider the volume we can obtain with each of the varieties. In goods with a high buying frequency it may be more worthwhile to market a variety even though it is not as attractive in comparison with another if it obtains a more frequent statement of intention to buy.

In this case, we weigh the varieties by assigning greater weight to the varieties with more frequent purchases and less weight to the most sporadic. To do so, we weigh by the inverse of the buying frequency obtained, so that the varieties with a more frequent consumption will have more possibilities of being put on the market.

The procedure is as follows. Each of the data in the frequency matrix is multiplied by the intention to buy the variety for each individual, so that if the individual has no intention to buy the specific variety, the declared frequency does not count for this variety. Once this has been done, we add up the columns of frequencies for each variety $\left(f_{j}\right)$. All that remains for us to do is obtain the inverse of each of the $f_{j}$. These data will be the weightings used to weigh the binary variables of our problem so that the varieties showing the greatest frequency will be weighed by a smaller factor which will have more weight on making our target function minimal.
$\operatorname{Min} Z=\sum_{j=1}^{n} \frac{1}{f_{j}} x_{j}$
$s_{1} a_{i} \sum_{j \in N_{i}} x_{j} \geq 1 \quad i=1 \ldots m$
$x_{j}=(0,1) \quad j=1 \ldots n$

### 4.4. Taking into account the different characteristics of the buyers

If our number of varieties is limited and is lower than the number of varieties offered by the optimal solution to the basic problem, in other words, if we do not manage to attract all the individuals considered, we can discriminate between buyers based on the criterion we are most interested in, for example, the frequency with which they consumer usually buys the product as a whole (if we are designing a new line of ice-creams, we could consider the frequency with which each individual usually buys ice-cream).

We use this information to weigh the interviewees' preferences so that if an individual buys very often, his or her preferences should have more weight than individuals who purchase more sporadically.

Since our number of varieties is limited, this version of TURF is an extension of the adaptation presented in section 4.1., in which we had a variable for each individual considered in the problem, a variable which we will weigh on the basis of the frequency of buying declared by each individual:

$$
\begin{aligned}
& \operatorname{Max} Z=\sum_{i=1}^{m} f_{i} y_{i} \\
& \text { s.an } \sum_{j=1}^{n} x_{j}=V \\
& \sum_{i \in N_{i}} x_{j} \geq y_{i} \quad i=1 \ldots m \\
& \qquad y_{i}=(0,1) \quad i=1 \ldots m, x_{j}=(0,1) \quad j=1 \ldots n
\end{aligned}
$$

Where $f_{i}$ is the number of times individual $i$ buys the product in a given period ${ }^{6}$.

### 4.5. More than one variety per consumer

On other occasions, it can be considered that attracting the consumer with just one variety limits the consumer's capacity to alternate between varieties, very common behaviour especially in food products. Accordingly, what interests us is that each consumer considered feel attracted by at least $2,3, \ldots$ varieties. The problem is also usually applied to products that include different varieties in the same products, for example, an air-freshener that combines three different fragrances, a box with a selection of biscuits, etc. In these cases, we can consider that the consumer will only buy the product provided he or she feels attracted by at least a determined number of varieties (greater than one).

The adaptation in this case is also very simple, and consists of changing the constraints relating to consumption as follows:
$\operatorname{Min} Z=\sum_{j=1}^{n} x_{j}$
s.a. $\sum_{j \in N_{i}} x_{j} \geq P \quad i=1 \ldots m$
$x_{j}=(0,1) j=1 \ldots n$
We consider that each individual has to feel attracted by at least a number of varieties greater than $1(P)^{7}$.

### 4.6. Combined with other variables

The TURF analysis calculated in this way permits the introduction in the model of other variables not included in the study and that have more to do with the

[^4]production/organization of the business, such as the production cost of each of the varieties, their net profit, or the time necessary to produce them.

## 5. CONCLUSIONS

This study represents a revision of the popular TURF analysis used in the world of marketing for the definition of product lines. The aim of this analysis is to maximize the penetration of a product line taken as a whole, sacrificing buyers attracted by different product alternatives in favour of new buyers.

Our contribution to the analysis involves considering the problem as a Linear Programming problem. This has enabled us to develop a model that fits the objective of the TURF analysis perfectly and allows us, through mathematical programming and the search algorithms used by it, to reach the solution to the problem in a more efficient and exact manner than with the previously proposed calculation algorithms.

Furthermore, the theoretical framework found lets the problem be adapted simply to multiple situations that may arise in practice. These extensions of the model have shown themselves to be very useful in overcoming the main drawbacks of the analysis.

The proposed working method shows that we must not forget the individual penetration of each of the proposed varieties, despite the fact that the objective of the TURF analysis does not consider this information at any time. As we have seen, it has been shown to be a very useful criterion when it comes to discriminating between different optimal solutions, making the objective of the analysis compatible with other objectives the person responsible for the product may have, such as choosing the varieties that are most attractive individually.

Finally, we should never make the final decision considering only the results obtained through the TURF analysis, particularly if we take into account that these are decisions based on information provided by market studies that do not reflect perfectly the reality of the population. However, and although ultimately the decision does not coincide with the recommendation of the analysis, the fact of carrying out the entire process provides us with very valuable information on the behaviour of the different alternatives we are handling, the optimum size of our
product line and the number of individuals we manage to attract with each of the combinations being considered.

## REFERENCES

Agresti, A. (2002). Categorical Data Analysis. Florida: Wiley Interscience.
Cohen, E. (1993). TURF analysis. Quirk's Marketing Research Review.
Conklin, M., \& Lipovetsky, S. (2000). A Winning Tool for CPG. Marketing Research.
Kreiger, A. M., \& Green, P. E. (2000). Turf Revisited: Enhancements to Total Unduplicated Reach and Frequency Analysis. Marketing Research.

Markowitz, L. (2004). Appropiate Applications of TURF and Shapley Value for Product Line Optimization. Ipsos Insight.

Markowitz, L. (2005). Going Beyond Turf to Complement and Extend Existing Product Lines. Ipsos Insight, November 2005.

Miaoulis, G., Free, V., \& Parsons, H. (1990). Turf: A New Planning Approach for Product Line Extensions. Marketing Research.

Mullet, G. M. (2001). TURF redux. Quirk's Marketing Research Review.


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[^1]:    ${ }^{3}$ We should point out that in the original database it is necessary to screen out those individuals who do not show an interest in any of the varieties presented. We will not be able to attract them to any combination and therefore they should not be included in the problem. The model presented below works provided that this screening of the data is carried out. Otherwise, an easily adaptable alternative model has to be used.

[^2]:    ${ }^{4}$ If we work with a database that has not been screened, the constraint $\sum_{j \in N_{i}} x_{j} \geq 1$ will not let us solve the problem because not all the individuals will be buyers. In this case, the constraint has to be reformulated as follows:
    $\sum_{j \in N_{i}} x_{j} \geq c_{j}$
    With $c_{j}=1$ if the individual shows interest in buying at least one variety, $c_{j}=0$ if the individual does not show interest in any variety.
    In any case, we reiterate the advisability of working with a screened database.

[^3]:    ${ }^{5}$ The text applied was McNemar's test used to compare proportions in dependent samples. The level of signification obtained for the Chi-squared statistic was 0.5 and therefore the null hypothesis of equality between proportions is clearly accepted.

[^4]:    ${ }^{6}$ Since, in this case, we want to maximize the target function, we can multiply directly by the frequency with which each individual declares that he/she buys the product.
    ${ }^{7}$ In this case we have to exclude all those who do not feel attracted by at least $P$ varieties because we assume that they will never consider buying my product and therefore we have to screen our database, retaining only those who have declared an intention to buy at least $P$ product alternatives.

