Preference Shocks that Destroy Party Systems

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Preference shocks that destroy party systems*

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Abstract

We propose a two party electoral competition model to analyze the effects of an exogenous shock over voters' preferences on the strategic policy choices of the parties. We find that if the shock affects voters' ideology regarding an issue that is already salient, then both parties strategically adapt their already moderated policy choices in the direction of the new median voter. However, if the shock changes the relative issue salience, then both parties strategically shift their policy choices from their ideal points towards the ideal point of the median voter of the newly salient issue. The asymmetry of the distribution of the voters preferences, that is possibly intensified by the shock, produces a disadvantage for one of the parties, which is forced to implement a large policy shift. We argue that a large policy shift may break a party internal balance among its different factions, which in turn may produce important disruptions in the party system. We illustrate our arguments with an analysis of recent events in Catalonia and the UK.

Keywords: preference shock, relative salience, party consistency.

JEL Classification: D72

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"*Not all issues flow with predictability from past decisions. The violent civil disorders of the 1960s or the OPEC oil boycott blow up like summer thunderstorms and burst upon the country in magnified form via the mass media. A great national crisis suddenly is proclaimed by commentators and citizens alike and Senators are expected to take immediate action. 'Pressing problems' of this kind force their way onto the Senate's agenda, whether or not feasible solutions are in sight. Action of some kind, even if it is merely symbolic, must be taken as quickly as possible. Problems of this type have arisen frequently in the past few decades, catching most Senators by surprise. They occur almost randomly and when they arise displace all other agenda items that can be delayed or pushed aside.*"

(Walker 1977, p. 426)

1 Introduction

Unexpected events like terrorist attacks, natural disasters, political scandals or even sports results have impact on voters' moods and policy preferences and thus unravel important political consequences. This paper examines how, in a context of electoral party competition, the proposals of political parties adjust to respond to such sudden changes in voters preferences.

When an exogenous shock affects the policy preferences of voters, the equilibrium that competing political parties had attained prior to the shock is upset. Changes in voters preferences change voting decisions and parties must realign their policy positions in order to react optimally. The magnitude of the policy shifts needed by each party depend on the magnitude of the shift produced on the voters' preferences, and thus, on the intensity of the shock, but also on the optimal reaction to opponents' re-adjustments. Changes in voter's preferences favour some parties and damage others. Hence, the optimal policy revisions that follow after a preference shock are asymmetric across parties.

An exogenous shock can affect the voters' policy preferences and their voting behavior in two different ways. On the one hand, a shock might shift the voters ideal points on a given issue. For example, in the case of a terrorist attack it is plausible to think that all voters shift their preference towards an increase in national security. Thus, in the policy dimension affected by the shock, the ideal points of all voters change and they all change in the same direction. On the other hand, a shock may change the balance between the different policy dimensions. Voters are likely to shift their attention to the policy dimension affected by the shock, and regard it as more important and relevant. Thus the salience of a previously unimportant policy dimension may increase, turning it into the dominating issue after the shock.

A preference shock benefits some parties and hurts others. When a shock drives the ideal policy of all voters closer to the ideal policy of a given party, then this party will enjoy an electoral advantage because earning each vote becomes cheaper in ideological cost: to attract votes after the shock this party must make policy proposals that are closer to his ideal points that the policies required prior to the shock. At the same time, for a party with opposed views, the shock causes a disadvantage because the shock drives the voters' ideal points away from the ideal policy of the party. Each single vote becomes more expensive in ideological cost after the shock: to attract votes after the shock requires policy choices further away from his ideal points than those proposed before the shock. Consequently, the parties policy adjustment as a reaction to a voters' preference shift are asymmetric.

A main focus of our interest are the consequences of a preference shock that changes the relative salience of issues; that is, situations where voters attention shifts unexpectedly and an issue that was previously disregarded suddenly becomes the relevant dimension of political competition. When voters turn their attention to a new issue their decisions are based on their evaluation of the parties' policies on this new salient issue, and parties must strategically choose policy positions on the new relevant issue. If the two parties have different ideal points in this new salient issue, then they will move to compete in the new dimension, aiming to attract the voters with ideal points located between their opposed ideal points. Party competition in the new issue may face a symmetric distribution of voters, in which case both parties will optimally moderate a bit their positions. But, the new salient issue may be one where voters views are closer to one of the two parties; thus one party is favored by the proximity of the median voter to its ideal point and the other party is damaged by the larger distance between the median voter and its ideal point. In this case, the optimal policy choices of parties would imply asymmetric moves from their ideal points. This is the scenario where the shock delivers the most interesting consequences while maintaining the electoral competition among the initial parties. There is one last possible scenario: If both parties hold similar positions regarding the new issue, i.e. both parties share the same ideal point in the new most salient issue, then new parties may find it profitable to enter the electoral competition with proposals that confront the one shared by the two old parties, so that the change on issue salience might break up the stability of the party system. We focus our analysis on the case with asymmetric voters' preferences. In particular, we consider the effects of a biased preference distribution and those of a polarized one.

To analyse these phenomena we construct a model of two party electoral competition with policy motivated parties and sincere voters. We assume that there is a decisive voter whose ideal point is unknown to the parties and parties have beliefs about it that are common and common knowledge. In this model we introduce two assumptions that relate closely to the two effects described above: an exogenous shock on a given policy dimension produces a shift of the ideal points of all voters on that dimension, and it also produces an increase of the salience of the policy dimension affected by the shock.

We characterize the equilibrium policy choices of two parties that compete in an election both before and after the shock takes place, and compare the different outcomes that arise.

Our analysis disentangles the distinct effects of the policy preference shift and the changes in issue salience. We first analyze a one dimensional policy space model and consider only the effect of the shift of the voters' ideal points on the policy proposals of the parties in equilibrium. Then we analyze a two dimensional policy space model in which we consider both effects.

In the one dimensional model, in equilibrium, parties that are policy motivated choose policies that are moderate compared to their ideal points. If there is no shock these policies are symmetric with respect to the expected location of the ideal point of the decisive voter (the median voter). These results are in line with the ones described in Calvert (1985) and Wittman (1977 and 1983). If the voters' preferences are affected by a shock that shifts their ideal points the policy choices of both parties change moving in the same direction, that is, following the direction of the change in voters' preferences. However, the resulting policy shifts by the parties are asymmetric because the magnitude of the adjustment is different for each party. In fact, if all the voters' ideal points move in the same direction then the ideal point of the decisive voter moves closer to the ideal point of one of the two parties, and away from the ideal point of the other party. This implies that one party benefits from the shock while the other party suffers damage. The party that is damaged by the shock needs a greater adjustment in its policy proposal in order to remain competitive in the electoral contest, while the party favored by the shock needs a smaller change, only to react to his opponent change of strategy. Thus, compared to the policies chosen prior to the shock, the policy choice of the party disfavored by the shock is more moderate, while the policy of the party favored by the shock can be less moderate (closer to the party's ideal point).

In order to examine the effect of shocks over voter's preferences in multi dimensional policy spaces we consider a simple extension of the model where the two parties take policy positions over two dimensions. The equilibrium policies depend on the relative salience that the voters assign to each of the issues. The more salient an issue the more moderate are the parties policy choices on that issue. In particular, if only one issue is salient, that is, under the assumption that voters base their voting decision only on the policy proposals regarding the issue they consider most important then the equilibrium policies on the salient issue coincide with the outcomes described in the one-dimensional model, while the equilibrium policies on the non salient issue coincide with the parties ideal points. Indeed, there is no reason for the parties to compromise their policy positions on an issue if voters are not paying attention to it.

Now consider the effects of a shock that boosts the importance of a given policy dimension but does not change the ideal policies of voters in each issue. Here we will consider two possibilities. On the one hand, it is possible that the policy dimension affected by the shock was already the salient issue. In this case, the equilibrium strategies of the parties remain un-changed. On the other hand, it is possible that the shock affects an issue previously disregarded by the voters, and this issue becomes the salient issue as a consequence of the shock. In this case, the comparison between the policies chosen by the parties before and after the shock is a bit more complex. Before the shock, the newly salient issue was not salient, and thus both parties could implement their ideal points on that dimension. After the shock parties must react by moderating their policies on this issue. The equilibrium choices of the parties in this case are explained with the analogous argument of the one dimensional model. As in the one dimensional model, the party that is damaged by the shock reacts by moderating its policy, while the party favored by the shock does not need to moderate its policy so much. However, now the outcomes after the shock have to be compared to the parties' ideal points on that dimension, since this was their optimal choice prior to the shock, when the affected issue was not salient. Consequently, when a shock changes the issue that voters consider salient, both parties react by a substantial moderation of their choices, moving their policies towards the ideal point of the expected median voter, in the newly relevant dimension, while they turn radical, towards their ideal points in the dimension that is no longer salient.

Combining the previous analysis we can turn to consider the effects of a shock that produces both a shift in the voters' ideal points and a sizeable increase in the importance of the policy dimension affected by the shock. Once more we will consider two possibilities. On the one hand, it is possible that the shock affects the policy dimension that was already the salient issue. In that case, the effects of the shock are only driven by the voters' preference shift, and the analysis and results coincide with the ones obtained in the one dimension model. On the other hand, if the dimension affected by the shock was not salient prior to the shock, and it becomes salient as a consequence of the shock, the results follow by a combination of the previous analysis. On the issue that becomes salient due to the effects of the shock both parties react by moderating their choices, that is, moving their policies towards the ideal point of the expected median voter. In addition, both parties are moderating their policies in an asymmetric way: in equilibrium the party damaged by the shock chooses a policy far away from his ideal point while the party favored by the shock can choose a policy relatively close to his ideal point.

Finally, we also analyze the parties' equilibrium strategies when the shock induces a polarization of the voters' preferences. We consider scenarios that initially deliver equilibrium conditions where parties' policy choices are moderate relative to their ideal points. We show that in these cases, when the shock produces a very polarized electorate, then parties may react by choosing divergent policies.

The asymmetry in policy reactions that follows whenever a shock affects the voters' ideal policies, is consistent with the empirical evidence provided by Plümper and Epifanio (2015). They analyze the changes in antiterrorist policies implemented by incumbents of different countries after a terrorist attack.

They find that the intensification of antiterrorist policy is greater for leftist incumbents than for rightist incumbents. The formal arguments developed here offer a theoretical explanation for the empirical observation provided by Plümper and Epifanio.

Our results also point out that shocks that induce changes in voters preferences affect the electoral balance between the two competing parties. By favoring one party over the other, the shock imposes different electoral costs to the parties: it makes electoral competition much lighter for one party and much more costly for the other. Implementing a large policy shift may be very costly for a party, because compromising ideological principles may cause internal frictions among the different factions within the party and may end up destroying the party's internal equilibrium. At the same time, when a shock brings to salience a new policy issue, if the positions of established parties on the new issue are too similar, the shock might induce the entry of new parties and reconfiguring the political landscape.

We illustrate this argument with a discussion of the recent political upheaval in Catalonia and the United Kingdom. In 2010 high court decisions in Spain boosted the salience of the independence issue among Catalan voters'. As a consequence the two main political parties faced demands for drastic changes in their policy proposals and suffered major crisis. The Catalan party system of more than thirty years has been knocked down. The Brexit referendum is another shock that has turned on the salience of a previously disregarded issue and brought about a major crisis in the political party system.

The rest of the paper is organized as follows. In the next section we argue that unexpected events that change voters preferences are a relevant fact and review empirical evidence that supports this claim. Section 3 turns to the theoretical model and analyzes the effect of preference shifts on parties political choices in a one dimensional model. In section 4 we examine the two dimensional model and we show the effects of an increase of issue salience on the parties equilibrium policy choices. The effects of shocks that impact salience and cause a preference shift simultaneously are discussed in section 5. In section 6 we analyze the effect of a shock that produces polarized voters' preferences. In section 7 we illustrate how changes in issue salience that require a large policy shift cause instability on party's internal consistency and may break up party systems. Section 8 concludes with some final remarks.

2 Do unexpected events change voter's preferences?

It is commonly argued that the policy preferences of the voters are conditioned and even determined by the policy stands of the parties during campaigns, and by the policies implemented by incumbents. Indeed, it is in the interest of the parties to try to manipulate voters' opinions through the media, and electoral campaigns. This priming phenomenon is discussed in numerous experimental and empirical studies in psychology, political psychology and political science including Bartels 2006; Iyengar 1990; Iyengar and Kinder 1987; Iyengar, Kinder, and Peters 1982; Kahneman and Tversky 1979, 1981, 1984; Krosnick and Kinder 1990; Sheafer and Weimann 2005. For a critique, see also Lenz 2009. Similarly, the literature on policy and policy preference responsiveness (Barbera *et al.* 2019, Clinton 2006; Kastellec *et al.* 2015; Shapiro *et al.*1999)) shows that policy agendas are mainly driven by the political parties, and finds weak empirical support for the claim that politicians are responsive to the general public.

However, public opinion and voter's preferences are also shaped by dramatic, unexpected events that are exogenous to parties' actions. This has long been recognized in the literature on agenda-setting, that identifies "focusing events" as a leading cause for changes in public opinion and shifts in the political debate (Bangartner and Jones, Birkland, 1998; 1993; Kingdon, 1995; Walker, 1977). According to Birkland 2017, p. 74: "*Focusing events are sudden, relatively rare events that spark intense media and public attention because of their sheer magnitude or, sometimes, because of the harm they reveal. Focusing events thus attract attention to issues that may have been relatively dormant. Examples of focusing events include terrorist attacks (September 11, 2001 was, certainly, a focusing event), airplane accidents, industrial accidents such as factory fires or oil spills, large protest rallies or marches, scandals in government, and everyday events that gain attention because of some special feature of the event. Two examples of the latter are the alleged beating of motorist Rodney King by the Los Angeles Police Department in the early 1990s and O. J. Simpson's murder trial in 1995; the Rodney King incident was noteworthy because, unlike most such incidents, the event was caught on videotape, while the Simpson trial was noteworthy because of the fame of the defendant. Focusing events can lead groups, government leaders, policy entrepreneurs, the news media, or members of the public to pay attention to new problems or pay greater attention to existing but dormant (in terms of their standing on the agenda) problems, and, potentially, can lead to a search for solutions in the wake of perceived policy failure.*"

There is a substantial empirical literature documenting the effects of dramatic, extraordinary events on public opinion and election outcomes. Studies examine the consequences of nuclear power accidents (van der Brug, 2001), natural disaster and accidents involving loss of life (Birkland 1998, Slovic, Lichtenstein, & Fischhoff 1984), the assassinations of important social or political figures such as Martin Luther King (Hofstetter, 1969) or terrorism attacks. Balcells and Torrats-Espinosa 2018 examine eight violent attacks perpetrated between 1989 and 1997 by the a Basque terrorist organization Euskadi Ta Askatasuna (ETA). They measure the effect of attacks on public opinion surveys that were being fielded when the attacks occurred to estimate the causal effect of terrorist violence on individuals' intent to participate in elections as well as on professed support for the incumbent party. They find that terrorist attacks significantly increase individuals' intent to participate in a future election (with a greater impact in attacks against civilians) but find no evidence that the attacks change support for the incumbent party. In general, identifying the casual effect of exogenous shocks on voters' preferences is not a simple exercise. Muñoz *et al.* 2019 review studies that exploit the role of exogenous shocks during the fieldwork of public opinions surveys taking them as natural experiments to explore causal estimates, and discuss the general strengths and limitations of this identification strategy.

The terrorist attacks of March 2004 in Madrid and their impact in the subsequent election to the Spanish Congress is an interesting case. On March 11, only three days before the election, a major Islamist terrorist attack killed 193 people and injured around 2,000. Polls taken before the bombings showed a clear advantage of the incumbent conservative party over the socialist party. However the actual vote delivered a clear victory to the socialist party. The important difference between the results of the polls taken before the attack and the election results seemed to indicate that the voters might have changed their vote intention as a result of the bombings. Indeed, the conservative party, incumbent at that time, might have been considered by many as responsible of the attack, because of the government decision to support the US and UK in Iraq sending Spanish military forces just a few months prior to the election. Instead, in his electoral platform, the socialist leader had promised to bring the troops back home in case of victory. Papers that analyze the relationship between the attacks and the election results on the Spanish congress using post election surveys deliver inconclusive results. While the results of Bali 2007 seem to imply that the bombings were decisive, Lago and Montero 2005 find that they had no effect. Montalvo 2011, 2012 takes a different approach; instead of survey responses he examines actual voting results. He compares the outcome of the vote on election day with a control group of individuals that voted before the terrorist attacks: residents abroad had cast their vote before March 7 at a Spanish consulate or by post, thus they voted before the bombing took place. It turns out that the results of the vote for this control group showed an advantage to the conservative party, as predicted by the polls, and their turnout participation level was consistent with the normal trend of previous elections. Instead, the votes cast on election day, after the bombings, reversed the results predicted by the polls and showed a significant advantage for the socialist party over the conservative party and, in addition, a significant increase in turnout compared to previous congressional elections. Hence, this natural experiment approach provides evidence that the bombings changed the voters's political views and induced voters to turnout in larger numbers. The change of political views in this case was a demand to reconsider international alliances, and the higher turnout arised from the enhanced political engagement by citizens when the relevance of the policy dimension affected by the shock increased.

Overall, the literature provides strong arguments and substantial empirical evidence to support the claim that, shocks that induce changes in voters preferences and the relative salience of issues do occur, and that they are exogenous to the actions of political parties. And therefore, they have major political consequences in re-aligning policy proposals and party systems.

3 Voters' preference shift

This section describes the effects of a shock that shifts a voters' preference on a one dimensional model of two party electoral competition. The case analyzed here corresponds to a shock that affects the issue that is salient for the voters before the shock is realized. Thus, voters base their voting decision on the policies proposed on this single issue before and after the shock. Hence a one dimensional policy space is sufficient to analize this case.

3.1 The one dimensional model

There are two parties, A and B, that compete in an election. The policy space X is one dimensional and represented by the unit interval $X = [0, 1]$. Let $x \in X$ denote a policy position on the unique issue. Before the election, parties simultaneously choose policy platforms x_A and x_B respectively from the policy space X. Without loss of generality, we assume that there is a unique decisive voter who has preferences over policies represented by a utility function $u_m(x) = -(x - x_m)^2$ where $x_m \in X$ represents the voter's ideal policy.

We assume that parties do not know the exact location of x_m and they have beliefs about it that are common knowledge and are represented by a probability distribution $F : X \to [0, 1]$ with support over X and with density function $f : X \to [0, 1]$. If parties knew the exact value of x_m, they could anticipate the probability with which the decisive voter votes for each party and they could use it to decide their optimal strategy. Since we assume that parties have beliefs about the value of x_m, they can only anticipate the expected probability with which the decisive voter votes for each party.

Parties have policy preferences, just like voters. We assume that the ideal point of party A is 0 and the ideal point of party B is 1. The parties ideal points are common knowledge. Parties are policy motivated and their utility for policy x is represented by $u_A(x) = -x^2$ and $u_B(x) = -(1 - x)^2$ respectively. Each party maximizes his expected utility that is given by:

$$\begin{aligned} U_A(x_A, x_B) &= \pi_A \left[-(x_A)^2 \right] + (1 - \pi_A) \left[-(x_B)^2 \right] \\ U_B(x_A, x_B) &= \pi_A \left[-(1 - x_A)^2 \right] + (1 - \pi_A) \left[-(1 - x_B)^2 \right] \end{aligned}$$

where $\pi_A = \pi_A(x_A, x_B)$ denotes the probability of winning for party A and $1 - \pi_A$ denotes the probability of wining for party B as a function of the parties policy choices (x_A, x_B).

The game takes place in two stages. In the first stage, parties simultaneously choose positions in X. As in the standard Downsian model, we assume that parties implement their announced positions if they win the election. In the second stage, the decisive voter votes for the party whose election would give him the higher utility, if elected. In case of indifference, the voter is assumed to vote for each party with probability equal to 1/2. Since the behavior of the voter is unambiguous in this model, we define an equilibrium of the game only in terms of the policy strategies of the two parties in the first round.

Let $(x_A, x_B) \in X^2$ denote a pair of pure strategies for parties A and B respectively. we solve for the Nash equilibria of this game, that is, those pairs of pure strategies $(x_A^*, x_B^*) \in X^2$ such that $U_A(x_A^*, x_B^*) \geq U_A(x_A, x_B^*)$ for all $x_A \in X$ and $U_B(x_A^*, x_B^*) \geq U_B(x_A^*, x_B)$ for all $x_B \in X$.

3.2 Shock on the one dimensional policy space

Since we are interested in the effects of a shock on the voters' preferences that produce biases on the policy preferences of the decisive voter we assume that the beliefs of the parties about the location of the ideal point of the decisive voter are represented by $F(x) = F(x, a)$ where $F(x, a)$ denotes a uniform distribution over a support equal to $[a, 1]$ with $a \in [0, 1)$. Thus we represent the effect of the shock on the policy preferences of the voters with the parameter $a \in [0, 1)$. Before the shock occurs we assume that $a = 0$ that is, parties believe that the expected ideal point of the decisive voter is distributed uniformly over the unit interval. After the shock, parties believe that the expected ideal point of the decisive voter is distributed uniformly over the interval $[a, 1]$ with $a > 0$. This implies that voters' ideal points have shifted towards the right hand side of the policy space. Thus the parties' beliefs are represented by the following density function $f(x, a) = \begin{cases} 0 & if\ x \in [0, a) \\ \frac{1}{1-a} & if\ x \in [a, 1] \end{cases}$.

For every pair of policy choices $(x_A, x_B) \in X^2$ we have that the probability of winning for party A is given by $\pi_A(x_A, x_B) = \begin{cases} 0 & if\ \frac{x_A+x_B}{2} \in [0, a] \\ \frac{x_A+x_B-2a}{2(1-a)} & if\ \frac{x_A+x_B}{2} \in (a, 1] \end{cases}$ and the probability of winning for party B is given by $\pi_B(x_A, x_B) = 1 - \pi_A(x_A, x_B) = \begin{cases} 1 & if\ \frac{x_A+x_B}{2} \in [0, a] \\ \frac{2-x_A-x_B}{2(1-a)} & if\ \frac{x_A+x_B}{2} \in (a, 1] \end{cases}$.

See figures 1 and 2. Now we will compare the equilibrium policy choices of the parties when there is no shock, $a = 0$, and when there is a preference shock, $a > 0$. First, we describe the equilibrium results in the absence of any shock.

Proposition 1: *If* $a = 0$ *then the parties equilibrium strategies are* $x_A^*(0) = \frac{1}{4}$ *and* $x_B^*(0) = \frac{3}{4}$.

All proofs are relegated to the appendix.

This case is a direct application of the model of two party competition with policy motivated parties described in Calvert (1985) and Wittman (1977 and 1983). Both parties moderate their policies in a symmetric way, by choosing platforms halfway between their respective ideal points and the expected median voter ideal point, and they both win with equal probability. This result describes the parties optimal policy choices before the shock takes place. Next, we analyze the effect of the shock on the parties' policy choices in equilibrium.

Proposition 2: *If $a > 0$ then the parties equilibrium strategies are* $x_A^*(a) = \frac{9a-2+\sqrt{(2-9a)^2+32}}{16}$ *and* $x_B^*(a) = \frac{2+\frac{9a-2+\sqrt{(2-9a)^2+32}}{16}}{3}$.

This result shows that after the shock the parties' policy choices in equilibrium are conditioned on the value of the parameter that represents the intensity of the shock, a. Figure 3 represents graphically the parties' policy choices on the one dimensional policy space. After the shock is produced the expected ideal point of the decisive voter has shifted to the right side of the policy space (from $\frac{1}{2}$ to $\frac{1+a}{2} > \frac{1}{2}$). This produces an important change in the optimal strategy of party A, since now electoral competition has become much more costly in terms of policy compromise. Instead, party B has to face a much lighter electoral competition, since his ideal point has become more popular from the voters' point of view. Indeed, when analyzing the best response functions of both parties one observes that the best response function of party B does not change much after the shock. His final policy choice will change only because party A's policy choice is conditioned by the magnitude of the shock, and thus party B reacts to a different strategy of his opponent. Since his opponent is forced to choose more moderate policies, party B can choose a policy closer to his ideal point without compromising his electoral success.

Notice that the policies chosen in equilibrium by the two parties are located on both sides of the expected ideal point of the decisive voter. Even though party A produces a larger move towards the expected ideal point of the decisive voter than party B, in equilibrium party B's policy choice ends up being much closer to the expected ideal point of the decisive voter than party A's. For larger values of the shock magnitude both parties' policy choices move towards the right side of the policy space, closer to each other, and converge to 1 when the value of a approaches 1.

The probability of winning for party A is always smaller than the probability of winning for party B, and it decreases with the size of the shock. Similarly, the expected payoffs in equilibrium for party A are smaller than those of party B and they are decreasing with the size of the shock.

Finally, when we compare the equilibrium policy choices before and after the shock, that is, the results obtained in propositions 1 and 2, we find that the policy shift produced by party A is much larger than the shift produced by party B. Given that these two shifts are produced in the same direction on the policy space, it implies that the shock produces a double disadvantage on party A with respect to party B: party A has to produce a larger shift from his ideal point than party B's shift and more important, party A has to shift his policy away from his ideal point, while party B can move his policy closer to his ideal point.

4 Issue salience increase

This section describes the effects of a shock that produces an increase of the salience of a policy dimension on a two dimensional model of two party electoral competition. The case analyzed here corresponds to a shock that affects an issue that was not considered as the most salient for the voters even before the shock was produced, and that becomes the most salient issue after the shock. Since the voting decision depends mostly on the parties policy choices on the most salient issue, in this case the voters' decision is based on different issues before and after the shock. Therefore, we extend the previous model in order to consider a two dimensional policy space, in which each dimension represents a different issue.

4.1 The two dimensional model

There are two parties, A and B, that compete in an election. The policy space $\mathbf{X}$ is two dimensional and represented by the unit square $\mathbf{X} = [0,1] \times [0,1]$. Let $(x, y) \in X$ denote a policy position on each one of the issues. Before the election, parties simultaneously choose policy platforms (x_A, y_A) and (x_B, y_B) respectively from the policy space $\mathbf{X}$. Without loss of generality, we assume that there is a unique decisive voter who has preferences over policies represented by a utility function $u_m(x, y) = -\sigma (x - x_m)^2 - (1 - \sigma)(y - y_m)^2$ where $(x_m, y_m) \in X$ represents the voter's ideal policy and $\sigma \in [0, 1]$ denotes the relative salience of issue x over issue y.

We assume that parties do not know the exact location of (x_m, y_m) and they have beliefs about it that are common knowledge and are represented by a probability distribution $F : \mathbf{X} \rightarrow [0, 1]$ with support over $\mathbf{X}$ and with probability mass function $f : \mathbf{X} \rightarrow [0, 1]$.

Parties have policy preferences, just like voters. We assume that the ideal point of party A is $(0, 0)$ and the ideal point of party B is $(1, 1)$. The parties ideal points are common knowledge. Parties are policy motivated and, their utility for policy platform (x, y) is represented by $u_A(x, y) = -x^2 - y^2$ and $u_B(x, y) = -(1 - x)^2 - (1 - y)^2$ respectively. Each party maximizes his expected utility that is given by:

$$\begin{aligned} U_A((x_A, y_A), (x_B, y_B)) &= \pi_A \left[-(x_A)^2 - (y_A)^2 \right] + (1 - \pi_A) \left[-(x_B)^2 - (y_B)^2 \right] \\ U_B((x_A, y_A), (x_B, y_B)) &= \pi_A \left[-(1 - x_A)^2 - (1 - y_A)^2 \right] + (1 - \pi_A) \left[-(1 - x_B)^2 - (1 - y_B)^2 \right] \end{aligned}$$

where $\pi_A = \pi_A((x_A, y_A), (x_B, y_B))$ denotes the probability of winning for party A and $1 - \pi_A$ denotes the probability of wining for party B as a function of the parties policy choices $((x_A, y_A), (x_B, y_B))$.

As before, the game takes place in two stages. In the first stage, parties simultaneously choose positions in $\mathbf{X}$. As in the standard Downsian model, we assume that parties implement their announced positions if they win the election. In the second stage, the decisive voter votes for the party whose election would give him the higher utility, if elected. In case of indifference, the voter is assumed to vote for each party with probability equal to 1/2. Since the behavior of the voter is unambiguous in this model, we define an equilibrium of the game only in terms of the location strategies of the two parties in the first round.

Let $((x_A, y_A), (x_B, y_B)) \in \mathbf{X}^2$ denote a pair of pure strategies for parties A and B respectively. We solve for the Nash equilibria of this game, that is, those pairs of pure strategies $((x_A^*, y_A^*), (x_B^*, y_B^*)) \in \mathbf{X}^2$ such that

$U_A((x_A^*, y_A^*), (x_B^*, y_B^*)) \geq U_A((x_A, y_A), (x_B^*, y_B^*))$ for all $(x_A, y_A) \in \mathbf{X}$ and
$U_B((x_A^*, y_A^*), (x_B^*, y_B^*)) \geq U_B((x_A^*, y_A^*), (x_B, y_B))$ for all $(x_B, y_B) \in \mathbf{X}$.

4.2 Shock on the two dimensional policy space

In this case the parameter that determines the parties' equilibrium policy choices is the relative issue salience (σ). We assume that the value of σ before the shock is 0. This implies that before the shock the most salient issue is y, and thus this is the only issue that voters take into account when deciding their vote before the shock. We assume that the value of σ after the shock is 1. This implies that after the shock the most salient issue is x, and thus this is the only issue that voters take into account when deciding their vote before the shock.

First, we describe the equilibrium results in the absence of any shock.

Proposition 3: *If* $\sigma = 0$ *the parties equilibrium strategies are* $(x_A^*(0), y_A^*(0)) = \left(0, \frac{1}{4}\right)$ *and* $(x_B^*(0), y_B^*(0)) = \left(1, \frac{3}{4}\right)$.

Before the shock, issue y is infinitely more salient than issue x, thus the voting decision is only affected by the parties' positions on issue x. In this case, since parties are policy motivated, their optimal choices on the non salient issue coincide with their ideal points. On the salient issue they are going to moderate their policy choices, thus in equilibrium we obtain the solution predicted for the one dimensional case (proposition 1): both parties moderate their policies in a symmetric way, by choosing platforms halfway between their respective ideal points and the expected median voter ideal point.

Now we introduce a preference shock that affects the relative issue salience. Notice that as the value of the parameter that represents the relative issue salience increases, issue x becomes more salient, and thus the decision of the voting decision is conditioned by the policy choices of the parties on both issues. As the value of σ increases the voting decision shifts weights between issues: the utility that voters derive from policies implemented on issue x becomes more important, and thus more relevant to decide to whom to give their vote, and this forces parties to moderate their positions also on that issue.

Thus if the shock affects the voters relative issue salience, after the shock parties will choose a moderate policy position on the issue that has become salient due to the shock. The next result describes the equilibrium results after the shock is produced.

Proposition 4: *If* $\sigma = 1$ *then the parties equilibrium strategies are* $(x_A^*(0), y_A^*(0)) = \left(\frac{1}{4}, 0\right)$ *and* $(x_B^*(0), y_B^*(0)) = \left(\frac{3}{4}, 1\right)$.

Since x has become the most salient issue after the shock, voters will base their decision only on the parties policy choices on issue x. Thus parties have to moderate their positions on issue x exactly as if they were competing on a one dimensional model, and they can propose their ideal points on the non-salient issue.

Finally, in order to compare the equilibrium policy choices before and after the shock we have to compare the equilibrium policy choices corresponding to no shock $(\sigma = 0)$, with those corresponding to $(\sigma = 1)$. Since issue y becomes irrelevant in terms of the voting decision, parties are going to reverse their symmetric and moderate policy choices to their ideal points on that issue, and they will abandon their ideal points on the new salient issue and choose instead moderate policies.

5 Voters' preference shift and issue salience increase

This section describes the effects of a shock that produces a voters' preference shift on one dimension and an increase of the salience of this same dimension, that is, we analyze the combination of the effects described in the previous section. The case analyzed here corresponds to a shock that affects an issue that was not considered as the most salient for the voters before the shock was produced, and that becomes the most salient issue after the shock and at the same time affects the voter's preferences on this issue. As before, voters' decision is going to be based on different issues before and after the shock. Thus, here we are going to consider the two dimensional policy space described before.

Notice that if the shock that affects the voters preference on a given issue does not make this issue the most salient, then the voting decision remains unaffected: after the shock voters only consider the parties' positions on issue y to decide their vote, as they did before the shock. In this case, we have that voters preferences are only changing with respect to an issue that they consider irrelevant for their voting decision. Since the voters' preferences on the salient issue are not affected by the shock, then the parties' optimality conditions are also unaffected, and the equilibrium obtained coincides with the one obtained in the absence of a shock.

We assume, as in the previous case, that the effect of the shock on the policy preferences of the voters is represented by the parameter $a \in [0, 1)$. In this case we assume that the beliefs of the parties about the location of the ideal point of the decisive voter are represented by $F(x, y) = F(x, y; a)$ where $F(x, y; a)$ denotes a uniform distribution over a support equal to $[a, 1] \times [0, 1]$ with $a \in [0, 1)$ and therefore the density function is given by

$$f(x, y, a) = \begin{cases} 0 & if\ x \in [0, a) \\ \frac{1}{1-a} & if\ x \in [a, 1] \end{cases} .$$

See figure 4. As in the previous case, before the shock occurs we assume that $a = 0$ that is, parties believe that the expected ideal point of the decisive voter is distributed uniformly over the unit square. After the shock, when $a > 0$, parties believe that the ideal points of voters have shifted towards the right side of the policy space on the dimension affected by the shock (x) and they are remain uniformly distributed over the unit interval on the dimension that is not have been affected by the shock (y).

In this case we have two parameters that are going to determine the parties' equilibrium policy choices. On the one hand, there is the relative issue salience (σ). we assume that the value of σ before the shock is 0. This implies that before the shock the most salient issue is y, and thus this is the only issue that voters take into account when deciding their vote before the shock. And we assume that the value of σ after the shock is 1. This implies that after the shock the most salient issue is x, and thus this is the only issue that voters take into account when deciding their vote before the shock. On the other hand, there is the effect of the shock on the voters' policy preferences represented by the parameter a. Thus in this case we will compare the equilibrium policy choices of the parties when there is no shock, $a = 0$ and $\sigma = 0$ to the equilibrium policy choices when there is a preference shock $a > 0$ and $\sigma = 1$.

In this case we will have that since the shock affects the voters relative issue salience, after the shock parties will choose moderate policy position on the issue that has become salient due to the shock. In addition, after the shock voters' preferences have shifted to the right side of the policy space on that issue. Therefore, the parties policy choices have to take into account both effects. Notice that as before party A suffers a disadvantage from the voters' preference policy shift, because voters' ideal points move away from party A's ideal point, while party B obtains a strategic advantage since voters' ideal points move closer to party B's ideal point. The next result describes the equilibrium results after the shock is produced.

Proposition 5: *If $\sigma = 1$ then for $a > 0$ the parties equilibrium strategies are*

$$(x_A^*(a), y_A^*(a)) = \left(\frac{9a - 2 + \sqrt{(2-9a)^2 + 32}}{16}, 0 \right)$$

and

$$(x_B^*(a), y_B^*(a)) = \left(\frac{2 + \frac{9a-2+\sqrt{(2-9a)^2+32}}{16}}{3}, 1 \right)$$

This result shows once more that after the shock the parties' policy choices in equilibrium are conditioned on the value of the parameter that represents the intensity of the shock, a. Figure 5 represents graphically the parties' policy choices on the two dimensional policy space. After the shock is produced the expected ideal point of the decisive voter has shifted to the right side of the policy space on the x dimension and has remained unchanged on the y dimension. Since x has become the most salient issue after the shock, voters will base their decision only on the parties policy choices on issue x. Thus parties have to moderate their positions on issue x exactly as if they were competing on a one dimensional model, and they can propose their ideal points on the non-salient issue.

Finally, in order to compare the equilibrium policy choices before and after the shock we have to compare the equilibrium policy choices corresponding to no shock $(a = 0, \sigma = 0)$, with those corresponding to $(a > 0, \sigma = 1)$, that is a change in the voters policy preferences for issue x and an increase of the salience of issue x.Since issue y becomes irrelevant in terms of the voting decision, parties are going to reverse their symmetric and moderate policy choices to their ideal points on that issue, and they will abandon their ideal points on the new salient issue and choose instead moderate policies in an asymmetric way: party A is forced to more moderate policies because of its electoral disadvantage.

In order to determine the effect of a preference shock on a given issue on the parties' policy choices on that issue, we have to compare the equilibrium strategies before and after the shock on the issue that suffers the shock. The following corollary offers these results.

Corollary: *If a shock produces a change from* $(a = 0, \sigma = 0)$ *to* $(a > 0, \sigma = 1)$ *then the parties' equilibrium strategies on the dimension affected by the shock change from* $(x_A^*, x_B^*) = (0, 1)$ *to*

$$(x_A^*, x_B^*) = \left(\frac{9a - 2 + \sqrt{(2-9a)^2 + 32}}{16}, \frac{2 + \frac{9a-2+\sqrt{(2-9a)^2+32}}{16}}{3} \right).$$

This implies that on the issue affected by the shock both parties move their policy choices from their ideal points toward the ideal point of the expected decisive voter, thus both parties moderate their final policy choices on that issue. As before, the policy shift produced by party A is much larger than the shift produced by party B. However, in this case these two shifts are produced in opposite directions on the x dimension. This implies that both parties are suffering a disadvantage (of different intensities) from the shock, since both of them are forced to moderate their positions.

6 Polarized voters' preferences

This section replicates the previous analysis for the case in which the change of preferences produced by a shock polarizes the distribution of the voters ideal points on the issue affected by the shock. First, we analyze the equilibrium parties' platforms in a one dimensional model of two party electoral competition when the voters' preferences are polarized. Then we study the effect of a relative salience change due to the shock.

In order to represent the polarized voters' preferences we assume that the beliefs of the parties about the location of the ideal point of the decisive voter are represented by $F(x) = F(x;\alpha,\beta)$ where $F(x;\alpha,\beta)$ denotes a uniform distribution over a support equal to $[0, \beta - \alpha] \cup [\alpha + \beta, 1]$ with $0 \leq \alpha \leq \beta \leq \frac{1}{2}$. Thus the parties' beliefs are represented by the following density function

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{1-2\alpha} & if \quad 0 \leq x \leq \beta - \alpha \; or \; \alpha + \beta \leq x \leq 1 \\ 0 & if \qquad \beta - \alpha < x < \alpha + \beta \end{cases}$$

This function assigns positive probability of having a rather rightist voter, and positive probability of having a rather leftist voters, while it assigns zero probability to relative moderate positions. See figure 6. Parameter β represents the magnitude of the bias of the distribution: when β approaches 1/2 the distribution becomes symmetric, and for small values of β it becomes more likely that the voter's ideal point is on the right hand side of the policy space. Parameter α represents the magnitude of the polarization: when α approaches zero polarization disappears and the distribution becomes uniform over the whole policy space, and for larger values of α larger probabilities accumulate on the extremes of the policy space. Notice that when α approaches β the distribution becomes completely biased to the right hand side of the policy space: it becomes a uniform distribution over the interval $[2\beta, 1]$ and coincides with the analysis proposed in the previous section with $a = 2\alpha = 2\beta$. Without loss of generality we assume that this bias always favors the most rightist policies by considering $\beta \leq \frac{1}{2}$. Finally, as both α and β approach$\frac{1}{2}$ it becomes a degenerate distribution that assigns probability $\frac{1}{2}$ to each extreme of the policy space.

For every pair of policy choices $(x_A, x_B) \in X^2$ we have that the probability of winning for party A is given by $\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = F\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = \begin{cases} \frac{x_A+x_B}{2(1-2\alpha)} & if \quad 0 \leq \frac{x_A+x_B}{2} \leq \beta - \alpha \\ \frac{\beta-\alpha}{1-2\alpha} & if \quad \beta - \alpha < \frac{x_A+x_B}{2} < \alpha + \beta \\ \frac{x_A+x_B-4\alpha}{2(1-2\alpha)} & if \quad \alpha + \beta \leq \frac{x_A+x_B}{2} \leq 1 \end{cases}$

and the probability of winning for party B is given by $\pi_B(x_A, x_B; \alpha, \beta) = 1 - \pi_A(x_A, x_B; \alpha, \beta)$

The following proposition describes the pure strategies Nash equilibrium of the one dimensional model with polarized voters' preferences.

Proposition 6: *The equilibrium pure strategies for* $\beta \geq \overline{\beta}(\alpha)$ *are:* $x_A^* = 0$ *and* $x_B^* = 1$ *for* $\beta \geq \overline{\beta}(\alpha)$ *and the equilibrium pure strategies for* $\beta \leq \overline{\overline{\beta}}(\alpha)$ *are*

$$x_A^* = 3\frac{5 + 3\alpha + \sqrt{9\alpha^2 - 2\alpha + 1}}{8} - 2$$

and

$$x_B^* = \frac{5 + 3\alpha + \sqrt{9\alpha^2 - 2\alpha + 1}}{8}$$

This proposition characterizes two different types of equilibrium strategies. One in which the two parties diverge completely by implementing their ideal points, and another one in which the two parties partially converge. The magnitude of the bias of the distribution of the voters' preferences, represented by the parameter β, determines which equilibrium holds. The two sets of parameter values for which each one of these equilibria exist do not intersect, therefore we have uniqueness of equilibrium in each one of these sets. See figure 7.

The equilibrium with extreme strategies exists whenever the values of both α and β are relatively large, that is, whenever the distribution of the voters' ideal points is very polarized but not too biased. This case can be illustrated by a population consisting of two large groups of extremist, that induces parties to stay extreme because the possible policy deviations that may increase their expected vote shares are so far away from their ideal points that are not profitable.

When the distribution of the voters' ideal points is more biassed, that is, for smaller values of β, the extreme strategies are no longer an equilibrium. In this case, one of the parties, party B in our case, enjoys an advantage because the proportion of rightist voters is much larger than the proportion of leftist voters. In this case, the disadvantaged party, party A, has a strong incentive to moderate his position in order to increase his vote share beyond the group of his close supporters. This induces party B to moderate his position in order to defend his close supporters. Therefore in this case we obtain an interior equilibrium in which both parties are partially converging.

Notice that the strategies of both parties in this interior equilibrium depend only on the value of the parameter α. Thus the degree of convergence of the parties strategies is determined solely by the degree of polarization of the voters' preferences, not by the bias of the distribution.

Observe that in this interior equilibrium the strategies of both parties become more rightist whenever α increases, because for a given distribution bias, when polarization increases party A has to choose a more aggressive strategy and thus party B can optimally relax his response. If instead we consider the case of $\alpha = 0$ we find a direct application of the model of two party competition with policy motivated parties described in Calvert (1985) and Wittman (1977 and 1983) in which $x_A^* = \frac{1}{4}$ and $x_B^* = \frac{3}{4}$, that is, both parties moderate their policies in a symmetric way, by choosing platforms halfway between their respective ideal points and the expected median voter ideal point, and they both win with equal probability.

Notice that the policies chosen in equilibrium by the two parties are located on both sides of the expected ideal point of the decisive voter. Even though party A produces a larger move towards the expected ideal point of the decisive voter than party B, in equilibrium party B's policy choice ends up being much closer to the expected ideal point of the decisive voter than party A's. For larger values of α both parties' policy choices move towards the right side of the policy space, closer to each other, and converge to 1 when the value of α approaches 1/2.

In equilibrium the probability of winning for party A is always smaller than the probability of winning for party B, and it decreases with the polarization. The policy shift produced by party A relative to his ideal point is much larger than the shift produced by party B. Thus, the expected payoffs in equilibrium for party A are smaller than those of party B and they are both decreasing with the polarization of the voters' distribution.

Whenever neither of the conditions described in the proposition holds we do not existence of pure strategy equilibrium. This is the case when large values of β coexist with small values of α, that is, when the distribution is rather symmetric and not very polarized. Both parties choosing their ideal points cannot be an equilibrium, because the disadvantaged party would have an incentive to moderate his position and increase his probability of winning. The internal equilibrium does not hold because given that party B is choosing a moderate position and polarization is low party A is better off securing his group of supports with a leftist policy. Thus for large values of β and small values of α only mixed strategy equilibrium exists, and its characterization is beyond the scope of this paper.

7 Party consequences of a large policy shift

Our analysis shows that a preference shock forces one of the parties to a large change on his policy position away from its ideal point. Since parties' internal consistency is based on a balance between the different ideological positions of the different factions, it is plausible to think that the need for such sudden policy change might affect the equilibrium of forces inside a party, and might break up the stability of party systems.

In order to illustrate this phenomenon we review recent events in Catalonia and the UK where where a sudden boost in the relative salience of an issue has imposed the need to make major adjustments in the policy proposals of the dominant parties and this has affected the stability of the party system.

Catalonia has recently experienced drastic changes in a party system that was stable for over 30 years. The Catalan political debate has always been spread over two policy dimensions: the ideological or economic dimension and the sovereignty or decentralization dimension. Until the first decade of the twenty first century political preferences of the Catalan society on the economic dimension covered most of its range: from extreme right to extreme left. However on the sovereignty dimension political preferences were rather moderate. There were claims for different moderate degrees of decentralization but on the extremes these claims were very weak: demands for policies close to full centralization or to full independence were supported by a very small part of the population. Accordingly, the policy positions of the political parties over these two issues were moderate on the sovereignty issue and covered the full range of policies on the economic issue.

From 1980 up until 2010 the Catalan party system appeared as a stable system that contained five political parties. Convergència i Unió (CIU), a Catalan center-right coalition that stood for increasing decentralization had the largest electoral support. Partit Socialista de Catalunya (PSC), a Catalan center-left party that supported different decentralization claims over time was the second largest party. Two other parties had much smaller electoral supports, but they played significant roles in the governing coalitions that formed during the period: a Spanish rightist and centralist party, Partido Popular (PP) and a Catalan leftist and independentist party, Esquerra Republicana de Catalunya (ERC). Finally, Iniciativa per Catalunya-Verds (ICV) a leftist green party that mildly stood for decentralization had the smallest electoral support.

The possibilities of majoritarian coalitions involved all kinds of cross ideological agreements among parties on both issues[1]. However, we only observe two kinds of governments: those supported by rightist parties (CIU and PP) and those supported by leftist parties (PSC, ERC, and ICV). There was never a government formed or supported by parties that shared the same political views on the sovereignty issue. This observation leads us to conclude that it was more costly for political parties to compromise their positions on the economic issue than on the sovereignty issue. The implication of this observation is that the salience of the sovereignty issue was clearly dominated by the salience of the economic issue during this period.

Things changed after 2010, when the Spanish Constitutional Court ruled to dismiss the reform of

[1] For a detailed analysis of the governments formed during this period in Catalonia see Aragones (2007).

the Catalan self-government charter (which had been approved with a very strong consensus in 2005). This ruling of the Spanish high court has played a the role of a focusing event that has greatly switched the attention of the Catalan voters to the sovereignty dimension, and has induced a notable change in the political preferences of the Catalan electorate. While Catalan independence was supported only by a minority of about 10-15 percent from 1980 until 2009, this support experienced a very sharp increase to values right after the after the Constitutional Court ruling, and has since then consistently exhibited supports close to 50 percent in polls and electoral results[2]. This implies that on the one hand, the relevant policy space that parties must cover has been enlarged: it includes extreme decentralization positions that are supported by increasing number of voters. And on the other hand, the relative salience of the two issues has changed dramatically with the sovereignty issue becoming much more salient than the economic issue.

At the same time three new parties entered the political area. Ciudadanos (C) claims no position on the economic issue but a strong position on the sovereignty issue advocating for extreme centralization. Candidatura d'Unitat Popular (CUP) has a strong independentist and leftist position. And finally Podemos (P) holds a strong leftist position and a moderate position on the sovereignty issue.

Parties policy positions had to adapt to the new political environment. It is interesting to notice how different parties have used very different strategies to deal with the new preferences of the constituency[3]. On the one hand, we observe that a few parties have adapted in a very easy and natural way: some of them by not moving from their initial positions (C, PP, CUP, P, ICV) and others by reverting their compromised moderate policies to their original ideal points (ERC, originally defined as independentist). However, the two largest parties have had a harder time to adapt to the new environment. PSC suffered severe internal party tensions that drove it to break into several small factions with leftist-independentist positions and a somewhat larger faction holding a leftist-centralist position. Similarly, CIU also broke into several small factions holding different positions on the sovereignity issue and a larger faction with a rightist-independentist position. Thus, the effect of a voters' preference shock was mostly suffered by the two largest parties, which were parties that had to support a more complicated and perhaps fragile internal equilibrium of forces. The breakdown of the two major parties into different factions produced a drastic change in the existing Catalan party system.

In the UK, the Brexit referendum brought about another shock causing a major upheaval in voters political preferences and the political party system. While ambivalence regarding membership in the EU has been a constant in British politics, this was not a major subject in the electoral competition until

[2] For a detailed analysis of the evolution of the political preferences in Catalonia during this period see Guinjoan and Rodon (2016).

[3] For a detailed analysis of the evolution of the Catalan party system during this period see Aragones and Ponsati (2016).

David Cameron prior to June 2016. After the referendum delivered its unexpected majority for Brexit, membership in the EU has become the salient issue in the political debate. Both the Conservative and Labour parties, having campaigned on both sides of the question, faced the imperative need to clarify their proposals deeply divided. The minor Brexit party and the LibDems, with clear views in favor and against Brexit, have become a major threat to the two big parties under the new scenario. Tories have adjusted to extreme Brexit positions, while Labor remains diffident.

In Scotland, where a large majority opposed Brexit, the shock has boosted the salience of the independence-union debate, greatly benefiting the agenda of the Scottish National Party. In Wales opposition to Brexit has also increased support to Plaid Cymru. Reunification may gain support in Northern Ireland if Brexit re-established the border breaking the good Friday agreements. What the British political landscape will look like after the present crisis, and weather the UK will survive it, are still very open questions.

8 Concluding remarks

We have argued that exogenous shocks change voters' policy preferences and demand policy adjustments by the political parties. When these shocks occur they have important implications on the survival of political parties and on the stability of party systems. The shift of the voters' political preferences caused by the shock induces parties to react in an asymmetric way, because while one party gains from the shock the other party suffers a disadvantage. Shocks also produce changes in the relative salience of issues and therefore political competition induces parties to moderate their policies on an issue in which they could have otherwise implemented their ideal policies (because as in Ansolabehere and Puy 2018 parties are not interested in competing on issues that are not relevant for the voters). The overall effect of a shock forces one of the parties, the disadvantaged one, to produce a large policy shift on the issue hit by the shock. We have illustrated how such a drastic policy change may destroy the internal consistency of the party.

The analysis presented relies on several assumptions. In particular, we assume that parties ideal points are not affected by the shock. This is a point that can hardly be proved nor disproved by empirical evidence. If they were assumed to be affected by the shock in the same way than voters' preferences are, then none of the results presented would hold. However, if they were assumed to be less affected by the shock than voters' preferences are, then the results would be qualitatively the same.

We have assumed that parties utility functions are quadratic. If they were assumed to be linear the results would not be as interesting, since in this case we have that some equilibrium policies deliver corner solutions, and thus the comparative statics analysis of the shock would not be as rich as the one provided by concave utility functions.

We have also assumed that parties are policy motivated. If parties were only office motivated, as in a standard model a la Downs 1957 then in equilibrium they would both always converge to the ideal point of the current expected median of the dimension that is most salient. In this case, the parties' policy choices would change from ((0,1/2),(1,1/2)) before the shock to ((1+a/2,0),(1+a/2,1)) after the shock. Thus, the qualitative results of moderate policies and asymmetric reactions would still hold in this case. If instead parties were assumed to be both policy and office motivated, then the results presented here would be slightly modified by the fact that in this case both parties will have stronger incentives to moderate their policy choices.

Finally, we have assumed that parties assign the same weight to the utility loses produced on all issues, that is, all issues are considered equally salient and their relative issue salience is not affected by the shock. We think that these are reasonable assumptions since on the one hand a party represents an aggregated pool of different sensitivities and ideological views, and thus it is plausible to assume that in general it would suffer a similar utility cost from any policy loss on any issue. And, on the other hand, voters' sensitivity is probably more likely to be affected by a shock, and thus voters are going to react more intensively to a preference shock. However, it would be interesting to see how results might change if these assumptions were relaxed.

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9 Appendix

Proof of Proposition 1:

If $a = 0$ then $f(x,a) = 1$ for $x \in [0,1]$. Thus, for every pair of policy choices $(x_A, x_B) \in X^2$ we have that the probability of winning for party A is given by $\pi_A(x_A, x_B) = \frac{x_A + x_B}{2}$ and the probability of winning for party B is given by $\pi_B(x_A, x_B) = 1 - \pi_A(x_A, x_B) = 1 - \frac{x_A + x_B}{2}$.

parties' payoffs functions can be written as

$$\begin{aligned} U_A(x_A, x_B) &= -\frac{x_A + x_B}{2} (x_A)^2 - \left[1 - \frac{x_A + x_B}{2}\right] (x_B)^2 \\ &= \frac{x_A + x_B}{2} \left[-(x_A)^2 + (x_B)^2\right] - (x_B)^2 \\ U_B(x_A, x_B) &= -\frac{x_A + x_B}{2} (1 - x_A)^2 - \left[1 - \frac{x_A + x_B}{2}\right] (1 - x_B)^2 \\ &= \frac{x_A + x_B}{2} \left[(1 - x_B)^2 - (1 - x_A)^2\right] - (1 - x_B)^2 \end{aligned}$$

The first order conditions are:

$\frac{\partial U_A(x_A, x_B)}{\partial x_A} = \frac{-(x_A)^2 + (x_B)^2}{2} - \frac{x_A + x_B}{2} 2x_A = 0$

$\frac{\partial U_B(x_A, x_B)}{\partial x_B} = \frac{(1-x_B)^2 - (1-x_A)^2}{2} - \frac{x_A + x_B}{2} 2(1 - x_B) + 2(1 - x_B) = 0$

which imply that their reaction functions are:

$x_A(x_B) = \frac{x_B}{3}$

$x_B(x_A) = \frac{2 + x_A}{3}$

The second order conditions are:

$\frac{\partial^2 U_A(x_A, x_B)}{\partial (x_A)^2} = -3x_A - x_B < 0$

$\frac{\partial^2 U_B(x_A, x_B)}{\partial (x_B)^2} = x_A + 3x_B - 4 < 0$

Thus in equilibrium we must have: $x_A = \frac{1}{4}$ and $x_B = \frac{3}{4}$

with $x_A < x_B, x_A + x_B = 1, x_A - x_B = 1/2$,

and $U_A(x_A, x_B) = U_B(x_A, x_B) = -\frac{5}{16}$.♦

Proof of Proposition 2:

If $a > 0$ then $f(x,a) = \begin{cases} 0 & if\ x \in [0,a) \\ \frac{1}{1-a} & if\ x \in [a,1] \end{cases}$. Thus, for every pair of policy choices $(x_A, x_B) \in X^2$ we have that the probability of winning for party A is given by

$\pi_A(x_A, x_B) = \begin{cases} 0 & if\ \frac{x_A + x_B}{2} \in [0,a] \\ \frac{x_A + x_B - 2a}{2(1-a)} & if\ \frac{x_A + x_B}{2} \in (a,1] \end{cases}$

and the probability of winning for party B is given by

$\pi_B(x_A, x_B) = 1 - \pi_A(x_A, x_B) = \begin{cases} 1 & if\ \frac{x_A+x_B}{2} \in [0,a] \\ \frac{2-x_A-x_B}{2(1-a)} & if\ \frac{x_A+x_B}{2} \in (a,1] \end{cases}$

In order to compute the probability of winning of the two parties we have to consider two cases:

a) those (x_A, x_B) such that $\frac{x_A+x_B}{2} \in (a,1]$, that is, $2a < x_A + x_B \leq 2$; and

b) those (x_A, x_B) such that $\frac{x_A+x_B}{2} \in [0,a]$, that is, $0 \leq x_A + x_B \leq 2a$.

First suppose that we are in case a) and (x_A, x_B) are such that $\frac{x_A+x_B}{2} \in (a,1]$. In this case, parties' payoffs functions are given by

$$\begin{aligned} U_A(x_A, x_B) &= \frac{x_A + x_B - 2a}{2(1-a)} \left[-(x_A)^2 + (x_B)^2 \right] - (x_B)^2 \\ U_B(x_A, x_B) &= \frac{x_A + x_B - 2a}{2(1-a)} \left[(1-x_B)^2 - (1-x_A)^2 \right] - (1-x_B)^2 \end{aligned}$$

and their first order conditions are:

$\frac{\partial U_A(x_A,x_B)}{\partial x_A} = \frac{1}{2(1-a)} \left[-(x_A)^2 + (x_B)^2 \right] - \frac{x_A+x_B-2a}{2(1-a)} 2x_A = 0$

$\frac{\partial U_A(x_A,x_B)}{\partial x_B} = \frac{1}{2(1-a)} \left[(1-x_B)^2 - (1-x_A)^2 \right] - \frac{x_A+x_B-2a}{2(1-a)} 2(1-x_B) + 2(1-x_B) = 0$

The second order conditions are:

$\frac{\partial^2 U_A(x_A,x_B)}{\partial (x_A)^2} = -\frac{1}{(1-a)}(3x_A + x_B - 2a) < 0$ for $3x_A + x_B > 2a$ which is satisfied since $\frac{x_A+x_B}{2} \in (a,1]$

$\frac{\partial^2 U_A(x_A,x_B)}{\partial (x_B)^2} = \frac{x_A+3x_B-4}{(1-a)} < 0$

Their reaction functions are as follows. For party A

$x_A(x_B) = \frac{(2a-x_B)+\sqrt{4\left[(x_B-a)^2+ax_B\right]}}{3}$

with

$x_A(x_B) \geq 0$

$x_A(x_B) \leq x_B$ if $x_R \geq a$

$\frac{\partial x_A(x_B)}{\partial x_B} \geq 0$ iff $x_R > a$

$\frac{\partial x_A(x_B)}{\partial a} \geq 0$

For party B

$x_B(x_A) = \frac{2+x_A}{3}$

with

$x_B(x_A) \leq 1$

$x_B(x_A) \geq x_A$

$\frac{\partial x_B(x_A)}{\partial x_A} \geq 0$

$\frac{\partial x_B(x_A)}{\partial a} = 0$

Next suppose that (x_A, x_B) are such that $\frac{x_A+x_B}{2} \in [0,a]$, that is, $0 \leq x_A + x_B \leq 2a$.

Notice that if $x_B \leq a$ we have that $BR_A(x_B \leq a) = [0, x_B]$ because $U_A(x_A, x_B) = -(x_B)^2 > -(\hat{x}_L)^2 = U_A(\hat{x}_A, x_b)$ for all $x_A \in [0, x_R]$ and for all $\hat{x}_A \in (x_B, 1]$. Thus, parties payoff functions are given by

$$\begin{aligned} U_A(x_A, x_B) &= -(x_B)^2 \\ U_B(x_A, x_B) &= -(1 - x_B)^2 \end{aligned}$$

If $x_B > a$ we have that the only best response of party A is

$BR_A(x_B > a) = x_A(x_B) = \frac{(2a - x_B) + \sqrt{4\left[(x_B - a)^2 + a x_B\right]}}{3}$

and notice that $x_A(x_B) + x_B \geq 2a$, because in this case the policy outcome is a convex combination of x_B and $x_A(x_B) < x_B$ which leads to better payoffs for party A than any x_A such that $0 \leq x_A + x_B \leq 2a$ that produces $U_A(x_A, x_B) = -(x_B)^2$. Therefore, party A reaction function is given by:

$$BR_A(x_B) = \begin{cases} \frac{(2a - x_B) + \sqrt{4\left[(x_B - a)^2 + a x_B\right]}}{3} & if\ x_B > a \\ [0, x_R] & if\ x_B \leq a \end{cases}$$

Regarding B we have that if $x_A \leq a$ then $BR_B(x_A) = \max\left\{2a - x_A, \frac{2+x_A}{3}\right\}$. Otherwise, if $x_A > a$ then $BR_B(x_A) = \frac{2+x_A}{3}$. Therefore, party B reaction function is given by:

$$BR_B(x_A) = \begin{cases} \frac{2+x_A}{3} & if\ x_A > \frac{3a-1}{2} \\ 2a - x_A & if\ 2a - 1 < x_A < \frac{3a-1}{2} \\ 1 & if\ 2a - 1 > x_A \end{cases}$$

Notice that for $a < \frac{1}{3}$ we have that $BR_B(x_A) = \frac{2+x_A}{3}$, and for $a < \frac{1}{2}$ we always have $BR_B(x_A) < 1$.

Thus, in equilibrium we will have that $a < \frac{x_A^* + x_B^*}{2}$ and :

$x_A^* = \frac{9a - 2 + \sqrt{(2-9a)^2 + 32}}{16}$ and $x_B^* = \frac{2 + \frac{9a - 2 + \sqrt{(2-9a)^2+32}}{16}}{3}$.

with

$\frac{\partial x_A^*}{\partial a} > 0$

$\frac{\partial x_B^*}{\partial a} > 0$

$\frac{\partial x_A^*}{\partial a} = 3\frac{\partial x_B^*}{\partial a} > \frac{\partial x_B^*}{\partial a}$

$x_A^* \leq x_B^*$

$x_A^* \leq \frac{1+a}{2} \leq x_B^*$

$\lim_{a \to 1} x_A^* = 1; \lim_{a \to 1} x_B^* = 1$

$2a < x_A^* + x_B^* \leq 2$

$\frac{\partial\left(x_A^*+x_B^*\right)}{\partial a} = \frac{4}{3}\frac{\partial x_A^*}{\partial a} > 0$

$x_A^* + x_B^* > 1$

$\frac{1+a}{2} - x_A^* > x_B^* - \frac{1+a}{2}$

$\frac{\partial\left(x_B^*-x_A^*\right)}{\partial a} = -\frac{2}{3}\frac{\partial x_{LA}}{\partial a} < 0$

$\pi_A = \frac{1+2x_A-3a}{3(1-a)} = \frac{6-15a+\sqrt{(2-9a)^2+32}}{24(1-a)} < \frac{1}{2}$

$1 - \pi_A = 1 - \frac{1+2x_{LA}-3a}{3(1-a)} = \frac{2(1-x_A)}{3(1-a)} = \frac{18-9a+\sqrt{(2-9a)^2+32}}{24(1-a)}$

$\frac{\partial \pi_A}{\partial a} < 0$

$\frac{\partial U_A(x_A,x_B)}{\partial a} < 0$

$\frac{\partial U_B(x_A,x_B)}{\partial a} > 0.$ ♦

Proof of proposition 3:

If $\sigma = 0$ and $a = 0$ then the decisive voter's utility function is given by:

$u_m(x,y) = -\sigma(x-x_m)^2 - (1-\sigma)(y-y_m)^2 = -(y-y_m)^2$

and the probability of winning for party A is given by

$\pi_A((x_A,y_A),(x_B,y_B)) = \frac{y_A+y_B}{2}$.

Thus, the decisive voter's problem is identical to the one analyzed in proposition 1. The parties' optimization problem contains no restrictions on the x dimension, therefore they can implement their ideal points on this dimension with no electoral cost. The parties' optimization problem with respect to the y dimension are identical to the ones analyzed in proposition 1. Therefore, the parties equilibrium strategies are given by $(x_A^*(a), y_A^*(a)) = \left(0, \frac{1}{4}\right)$ and $(x_B^*(a), y_B^*(a)) = \left(1, \frac{3}{4}\right)$.♦

Proof of proposition 4:

If $\sigma = 1$ and $a = 0$ then the decisive voter's utility function is given by:

$u_m(x,y) = -\sigma(x-x_m)^2 - (1-\sigma)(y-y_m)^2 = -(x-x_m)^2$

and the probability of winning for party A is given by

$\pi_A((x_A,y_A),(x_B,y_B)) = \frac{x_A+x_B}{2}$.

Once more the problem is unidimensional. The parties' optimization problem contains no restrictions on the y dimension, therefore they can implement their ideal points on this dimension with no electoral cost. The parties' optimization problem with respect to the x dimension are identical to the ones analyzed in proposition 1. Therefore, the parties equilibrium strategies are given by $(x_A^*(0), y_A^*(0)) = \left(\frac{1}{4}, 0\right)$ and $(x_B^*(0), y_B^*(0)) = \left(\frac{3}{4}, 1\right)$.♦

Proof of proposition 5:

If $\sigma = 1$ and $a > 0$ then the decisive voter's utility function is given by:

$u_m(x,y) = -\sigma(x-x_m)^2 - (1-\sigma)(y-y_m)^2 = -(x-x_m)^2$

and the probability of winning for party A is given by

$$\pi_L((x_A,y_A),(x_B,y_B)) = \begin{cases} 0 & if\ \frac{x_L+x_R}{2} \in [0,a] \\ \frac{x_L+x_R-2a}{2(1-a)} & if\ \frac{x_L+x_R}{2} \in (a,1] \end{cases}$$

In this case the decisive voter's problem is identical to the one analyzed in proposition 2. The parties' optimization problem contains no restrictions on the y dimension, therefore they can implement their ideal points on this dimension with no electoral cost. The parties' optimization problem with respect to the x dimension are identical to the ones analyzed in proposition 2. Therefore, the parties equilibrium strategies are given by: $(x_A^*(a), y_A^*(a)) = \left(\frac{9a-2+\sqrt{(2-9a)^2+32}}{16}, 0\right)$

and

$(x_B^*(a), y_B^*(a)) = \left(\frac{2+\frac{9a-2+\sqrt{(2-9a)^2+32}}{16}}{3}, 1\right)$. ♦

Proof of Proposition 6:

Observe that for $\alpha = 0$ we have the biased distribution function analyzed in the previous sections. Here we assume that $\alpha > 0$.

If $\alpha > 0$ then $f(x;\alpha,\beta) = \begin{cases} \frac{1}{1-2\alpha} & if \quad 0 \le x \le \beta-\alpha\ or\ \alpha+\beta \le x \le 1 \\ 0 & if \qquad \beta-\alpha \le x \le \alpha+\beta \end{cases}$. Thus, for every pair of policy choices $(x_A, x_B) \in X^2$ with $x_A \le x_B$ we have that the probability of winning for party A is given by $\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = F\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = \begin{cases} \frac{x_A+x_B}{2(1-2\alpha)} & if \quad 0 \le \frac{x_A+x_B}{2} \le \beta-\alpha \\ \frac{\beta-\alpha}{1-2\alpha} & if \quad \beta-\alpha \le \frac{x_A+x_B}{2} \le \alpha+\beta \\ \frac{x_A+x_B-4\alpha}{2(1-2\alpha)} & if \quad \alpha+\beta \le \frac{x_A+x_B}{2} \le 1 \end{cases}$

with

$$\frac{\partial \pi_A}{\partial x_A}\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = \begin{cases} 0 & if \quad \beta-\alpha \le \frac{x_A+x_B}{2} \le \alpha+\beta \\ \frac{1}{2(1-2\alpha)} & otherwise \end{cases}$$

and $\frac{\partial^2 \pi_A}{\partial (x_A)^2}\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = 0$

And the probability of winning for party B is given by $\pi_B(x_A,x_B;\alpha,\beta) = 1 - \pi_A(x_A,x_B;\alpha,\beta)$.

The parties' payoffs functions and their first and second order conditions are given by:

$$U_A(x_A,x_B;\alpha,\beta) = \pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right)\left[(x_B)^2-(x_A)^2\right]-(x_B)^2$$
$$\frac{\partial U_A}{\partial x_A}(x_A,x_B;\alpha,\beta) = \frac{\partial \pi_A}{\partial x_A}\left[(x_B)^2-(x_A)^2\right]-2x_A\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right)=0$$
$$\frac{\partial^2 U_A}{\partial (x_A)^2}(x_A,x_B;\alpha,\beta) = -4x_A\frac{\partial \pi_A}{\partial x_A}-2\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right)\le 0$$
$$U_B(x_A,x_B;\alpha,\beta) = \pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right)\left[(1-x_B)^2-(1-x_A)^2\right]-(1-x_B)^2$$
$$\frac{\partial U_B}{\partial x_B}(x_A,x_B;\alpha,\beta) = \frac{\partial \pi_A}{\partial x_B}\left[(1-x_B)^2-(1-x_A)^2\right]+2(1-x_B)\left[1-\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right)\right]=0$$
$$\frac{\partial^2 U_B}{\partial (x_B)^2}(x_A,x_B;\alpha,\beta) = -4(1-x_B)\frac{\partial \pi_A}{\partial x_B}-2\left[1-\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right)\right]\le 0$$

Notice that since we have $\frac{\partial \pi_A}{\partial x_A}, \frac{\partial \pi_A}{\partial x_B} \ge 0$, the second order conditions are always satisfied. In order to compute the equilibrium policy choices we have to consider three cases:

a) those (x_A, x_B) such that $\frac{x_A+x_B}{2} \in [0, \beta-\alpha]$,

b) those (x_A, x_B) such that $\frac{x_A+x_B}{2} \in [\beta-\alpha, \beta+\alpha]$,

c) those (x_A, x_B) such that $\frac{x_A+x_B}{2} \in [\beta+\alpha, 1]$,

First suppose that we are in case a) thus we have that (x_A, x_B) are such that $\frac{x_A+x_B}{2} \in [0, \beta-\alpha]$. In this case, the probability of winning for party A is given by $\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = \frac{x_A+x_B}{2(1-2\alpha)}$

and the first order condition for party A is $\left[(x_B)^2 - (x_A)^2\right] - 2x_A(x_A+x_B) = 0$

which holds if and only if $[x_B - 3x_A](x_A+x_B) = 0$ thus the best response of party A in this case is given by $BR_A(x_B) = \frac{x_B}{3}$. Notice that $BR_A(x_B) \geq 0, BR_A(x_B) \leq x_B$. The first order condition for party B is $(1-x_A)^2 - (1-x_B)^2 = 2(1-x_B)[2(1-2\alpha) - (x_A+x_B)]$

which holds if and only if

$x_B = \frac{4(1-\alpha)-x_A \pm \sqrt{[4(1-\alpha)-x_A]^2+3\left((x_A)^2-4(1-2\alpha)\right)}}{3}$

Notice that $\frac{4(1-\alpha)-x_A + \sqrt{[4(1-\alpha)-x_A]^2+3\left((x_A)^2-4(1-2\alpha)\right)}}{3} > 1$

thus $BR_B(x_A) = \frac{4(1-\alpha)-x_A - \sqrt{[4(1-\alpha)-x_A]^2+3\left((x_A)^2-4(1-2\alpha)\right)}}{3}$

Notice that $BR_B(x_A) \leq 1$ always, and $BR_B(x_A) \geq x_A$ if and only if $x_A \leq 1-2\alpha$.

Thus for $\frac{x_A+x_B}{2} \in [0, \beta-\alpha]$ the equilibrium candidates have to satisfy

$x_A = \frac{x_B}{3}$ and $x_B = \frac{4(1-\alpha)-x_A - \sqrt{[4(1-\alpha)-x_A]^2+3\left((x_A)^2-4(1-2\alpha)\right)}}{3}$,which implies that

$x_A = \frac{3(1-\alpha)\pm\sqrt{9\alpha^2-2\alpha+1}}{8}$.

Notice that in this case we need to satisfy $\frac{x_A+x_B}{2} = 2x_A < \beta-\alpha$ and this holds if and only if $\beta > \frac{3(1-\alpha)\pm\sqrt{9\alpha^2-2\alpha+1}}{4} + \alpha$. Since $\frac{3(1-\alpha)\pm\sqrt{9\alpha^2-2\alpha+1}}{4} + \alpha > \frac{1}{2}$ for all $\alpha < \frac{1}{2}$ and we have assumed that $\beta < \frac{1}{2}$, we cannot have an equilibrium in this case.

Next suppose that we are in case b) thus we have that (x_A, x_B) are such that $\frac{x_A+x_B}{2} \in [\beta-\alpha, \beta+\alpha]$. In this case, the probability of winning for party A is given by $\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = \frac{\beta-\alpha}{1-2\alpha}$ and we have that $\frac{\partial U_A}{\partial x_A} < 0$. Thus the best response for party A in this case is: $BR_A(x_B) = \min\left\{x_A : \beta-\alpha \leq \frac{x_A+x_B}{2}\right\} =$
$$\begin{cases} 0 & if \quad 2(\beta-\alpha) \leq x_B \\ 2(\beta-\alpha) - x_B & if \quad 2(\beta-\alpha) \geq x_B \end{cases}$$
Notice that $BR_A(x_B) \geq 0$, and $BR_A(x_B) \leq x_B$ iff $\beta - \alpha \leq x_B$.

Similarly we have that $\frac{\partial U_B}{\partial x_B} > 0$. Thus the best response for party B in this case is:

$$BR_B(x_A) = \max\left\{x_B : \frac{x_A+x_B}{2} \leq \alpha+\beta\right\} = \begin{cases} 2(\beta+\alpha) - x_A & if \quad 2(\beta+\alpha) - 1 \leq x_A \\ 1 & if \quad 2(\beta+\alpha) - 1 \geq x_A \end{cases}$$

Notice that $BR_B(x_A) \leq 1$ and $BR_B(x_A) \geq x_A$ iff $\beta+\alpha \geq x_A$.

Thus for $\frac{x_A+x_B}{2} \in [\beta-\alpha, \beta+\alpha]$ the equilibrium candidates have to satisfy

$$x_A = \begin{cases} 0 & if \quad 2(\beta-\alpha) \leq x_B \\ 2(\beta-\alpha) - x_B & if \quad 2(\beta-\alpha) \geq x_B \end{cases} \text{ and } x_B = \begin{cases} 2(\beta+\alpha) - x_A & if \quad 2(\beta+\alpha) - 1 \leq x_A \\ 1 & if \quad 2(\beta+\alpha) - 1 \geq x_A \end{cases}$$

which implies that

for $\beta + \alpha \geq \frac{1}{2}$ we have $x_A = 0$ and $x_B = 1$, and for $\beta + \alpha \leq \frac{1}{2}$ we have $x_A = 0$ and $x_B = 2(\beta + \alpha)$. Notice that these two equilibrium candidates satisfy all the required conditions: $\beta + \alpha \geq x_A, \beta - \alpha \leq x_B$, and $\beta - \alpha \leq \frac{x_A + x_B}{2} \leq \beta + \alpha$.

Finally, suppose that we are in case c) thus we have that (x_A, x_B) are such that $\frac{x_A+x_B}{2} \in [\beta + \alpha, 1]$. In this case, the probability of winning for party A is given by $\pi_A\left(\frac{x_A+x_B}{2}; \alpha, \beta\right) = \frac{x_A+x_B-4\alpha}{2(1-2\alpha)}$ and first order condition for party A is $\left[(x_B)^2 - (x_A)^2\right] - 2x_A(x_A + x_B - 4\alpha) = 0$

which holds if and only if $x_A = \frac{4\alpha - x_B \pm \sqrt{(x_B-4\alpha)^2 + 3(x_B)^2}}{3}$. Since $\frac{4\alpha - x_B - \sqrt{(x_B-4\alpha)^2+3(x_B)^2}}{3} < 0$ the best response for party A is $BR_A(x_B) = \frac{4\alpha - x_B + \sqrt{(x_B-4\alpha)^2+3(x_B)^2}}{3}$. Notice that $BR_A(x_B) \geq 0, BR_A(x_B) \leq x_B$ for $2\alpha \leq x_B$. The first order condition for party B is $(1 - x_B)^2 - (1 - x_A)^2 + 2(1 - x_B)[2(1 - 2\alpha) - (x_A + x_B - 4\alpha)] = 0$

which holds if and only if $x_B = \frac{2+x_A}{3}$. Thus the best response for party B is $BR_B(x_A) = \frac{2+x_A}{3}$. Notice that $BR_B(x_A) \leq 1$ and $BR_B(x_A) \geq x_A$.

Thus for $\frac{x_A+x_B}{2} \in [\beta + \alpha, 1]$ the equilibrium candidates have to satisfy

$x_A = \frac{4\alpha - x_B + \sqrt{(x_B-4\alpha)^2+3(x_B)^2}}{3}$ and $x_B = \frac{2+x_A}{3}$ which implies that $x_B = \frac{5+3\alpha \pm \sqrt{9\alpha^2 - 2\alpha + 1}}{8}$.

First consider $x_B = \frac{5+3\alpha - \sqrt{9\alpha^2-2\alpha+1}}{8}$ and notice that $\frac{x_A+x_B}{2} = 2x_B - 1 \geq \beta + \alpha$ iff

$\beta \leq \frac{5+3\alpha-\sqrt{9\alpha^2-2\alpha+1}}{4} - 1 - \alpha$. Since $\frac{5+3\alpha-\sqrt{9\alpha^2-2\alpha+1}}{4} - 1 - \alpha < 0$ and we have assumed $\beta > 0$ this cannot be an equilibrium candidate.

Next consider $x_B = \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}$ and notice that $\frac{x_A+x_B}{2} = 2x_B - 1 \geq \beta + \alpha$ iff

$\beta \leq \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{4} - 1 - \alpha$. Let $\beta \leq \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{4} - 1 - \alpha = \widetilde{\beta}(\alpha)$ be denoted as condition A. Notice that condition A implies $\beta + \alpha \leq \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{4} - 1$ and we have that $\frac{1}{2} \leq \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{4} - 1 \leq 1$. Observe that we also have $x_B = \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8} \geq 2\alpha$. Thus in this case we have that whenever condition A holds the equilibrium candidate is

$x_A^* = 3\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8} - 2$ and $x_B^* = \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}$.

Observe that $\frac{\partial\left(\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}\right)}{\partial\alpha} > 0$ for all $\alpha < \frac{1}{2}; \frac{3}{4} \leq x_B^* = \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8} \leq 1$ for all $\alpha < \frac{1}{2}; \frac{1}{4} \leq x_A^* = 3\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8} - 2 \leq 1$ for all $\alpha < \frac{1}{2}$.

Next we have to check possible profitable deviations for the equilibrium candidates found in cases b) and c).

In case b) we have $x_A = 0$ and $x_B = 1$ for $\beta + \alpha \geq \frac{1}{2}$ and $x_A = 0$ and $x_B = 2(\beta + \alpha)$ for $\beta + \alpha \leq \frac{1}{2}$. Notice that in this last case we have that $\frac{x_A+x_B}{2} = \beta + \alpha$ thus the analysis of case c) applies and generically the best response of party A to $x_B = 2(\beta + \alpha)$ is different that $x_A = 0$, and the best response of party B to $x_A = 0$ is different that $x_B = 2(\beta + \alpha)$. Thus, we can conclude that whenever $\beta + \alpha \leq \frac{1}{2}$ the equilibrium candidate is the one found in case c).

For $\beta+\alpha \geq \frac{1}{2}$, the equilibrium candidate in case b) is $x_A=0$ and $x_B=1$. Clearly party B does not have a profitable deviation unless party A has one, thus we have to check whether in this case party A has a profitable deviation given $x_B=1$, that is, whether $U_A(BR_A(1),1)>U_A(0,1)$ where $BR_A(1)=\frac{4\alpha-1+\sqrt{(1-4\alpha)^2+3}}{3}$ whenever $\frac{BR_A(1)+1}{2} \geq \alpha+\beta$, that is, $\frac{2(1-\alpha)+\sqrt{(1-4\alpha)^2+3}}{6} \geq \beta$.

We have that $U_A(BR_A(1),1)>U_A(0,1)$ if and only if
$\frac{(BR_A(1)+1-4\alpha)(1-(BR_A(1))^2)}{2}+\alpha>\beta$, which can be written as $\left[\left(1+\frac{(1-4\alpha)^2}{3}\right)\left(4\alpha-1+2\sqrt{4\alpha^2-2\alpha+1}\right)+4-7\right.$
β

Let $\left[\left(1+\frac{(1-4\alpha)^2}{3}\right)\left(4\alpha-1+2\sqrt{4\alpha^2-2\alpha+1}\right)+4-7\alpha\right]\frac{1}{9}=\overline{\beta}\left(\alpha\right)>\beta$ be denoted as condition B.

Thus we have that $x_A=0$ and $x_B=1$ is the unique pure strategy equilibrium for high enough values of β, that is, whenever condition B does not hold.

Finally, we have to check whether there are profitable deviations from the equilibrium candidate found in case c): $x_A^*=3\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}-2$ and $x_B^*=\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}$ which hold under condition A. Notice that a deviation to case a) is not possible because $BR_A(x_B^*)=\frac{x_B^*}{3}$, and this means that $\frac{BR_A(x_B^*)+x_B^*}{2}=\frac{2}{3}x_B^* \geq \frac{1}{2}>\beta-\alpha$ since $x_B^* \geq \frac{3}{4}$. Thus we only have to consider deviations to case b), that is, $x_A'=2\left(\beta-\alpha\right)-x_B^*$.

Notice that $x_A=0$ is a deviation that leads to case b) whenever $2\left(\beta-\alpha\right) \leq x_B^*$ or $\beta \leq \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{16}+\alpha$. Let $\widehat{\widehat{\beta}}\left(\alpha\right)=\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{16}+\alpha$. In this case, party A can guarantee a probability of winning of $\frac{\beta-\alpha}{1-2\alpha}$ by choosing $x_A=0$.

We have that $U_A(0,x_B^*)<U_A(x_A^*,x_B^*)$ if and only if
$\frac{\beta-\alpha}{1-2\alpha}\left(x_B^*\right)^2<\frac{2x_B^*-1-2\alpha}{1-2\alpha}\left[\left(x_B^*\right)^2-\left(3x_B^*-2\right)^2\right]$
which holds if and only if $\beta<\left(2x_B^*-1-2\alpha\right)\left[1-\frac{\left(3x_B^*-2\right)^2}{\left(x_B^*\right)^2}\right]+\alpha$.

Let $\beta<\widehat{\beta}\left(\alpha\right)=\left(2x_B^*-1-2\alpha\right)\left[1-\frac{\left(3x_B^*-2\right)^2}{\left(x_B^*\right)^2}\right]+\alpha$ be denoted as condition C. Notice that
$\widehat{\beta}\left(\alpha\right)<x_B^*-1-\alpha$ for all $\alpha<\frac{1}{2}$, thus condition C is below condition A, that is $\widehat{\beta}\left(\alpha\right)<\widetilde{\beta}\left(\alpha\right)$; and the intersection of condition C with $\beta \leq \widehat{\widehat{\beta}}\left(\alpha\right)$ defines the set of parameter values for which the candidate of case c) is an equilibrium, that is $\beta \leq \overline{\overline{\beta}}\left(\alpha\right)=\min\left\{\widehat{\beta}\left(\alpha\right),\widehat{\widehat{\beta}}\left(\alpha\right)\right\}$

Condition B:
$\beta<\frac{(BR_A(1)+1-4\alpha)(1-(BR_A(1))^2)}{2}+\alpha=\overline{\beta}(\alpha)$ with $BR_A(1)=\frac{4\alpha-1+\sqrt{(1-4\alpha)^2+3}}{3}$ implies that
$\overline{\beta}(0)=\frac{16}{27}>\frac{1}{2}$
$\overline{\beta}(\frac{1}{4})=\frac{17}{36}<\frac{1}{2}$
$\overline{\beta}(\frac{1}{2})\frac{1}{2}$
$\frac{\partial\overline{\beta}(\alpha)}{\partial\alpha}=\left[4\left(1-4\alpha\right)^2-3-\frac{2}{3}\left(1-4\alpha\right)\left[8\sqrt{4\alpha^2-2\alpha+1}+\frac{3+(1-4\alpha)^2}{\sqrt{4\alpha^2-2\alpha+1}}\right]\right]\frac{1}{9}$

$\frac{\partial \overline{\beta}(0)}{\partial \alpha} = \frac{1}{18} > 0$
$\frac{\partial \overline{\beta}(1/4)}{\partial \alpha} = -\frac{1}{3} < 0$
$\frac{\partial \overline{\beta}(1/2)}{\partial \alpha} = 1 > 0$

Condition C:

$\beta \leq (2x_B^* - 1 - 2\alpha)\left[1 - \frac{(3x_B^*-2)^2}{(x_B^*)^2}\right] + \alpha$ with $x_B^* = \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}$ implies that

$\beta \leq \left(\frac{1-5\alpha+\sqrt{9\alpha^2-2\alpha+1}}{4}\right)\left[1 - \frac{\left(3\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}-2\right)^2}{\left(\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}\right)^2}\right] + \alpha = \widehat{\beta}(\alpha)$

$\widehat{\beta}(0) = \frac{4}{9} < \frac{1}{2}$

$\widehat{\beta}(\frac{1}{4}) = \left(\frac{-1+\sqrt{17}}{16}\right)\left[\frac{504+34\sqrt{17}}{(23+\sqrt{17})^2}\right] + \frac{1}{4} = 0,41 > \frac{1}{4}$

$\widehat{\beta}(\frac{1}{2}) = \frac{1}{2}$

$\frac{\partial \widehat{\beta}(\alpha)}{\partial \alpha} = \left(\frac{-5+\frac{9\alpha-1}{\sqrt{9\alpha^2-2\alpha+1}}}{4}\right)\left[1 - \frac{\left(3\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}-2\right)^2}{\left(\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}\right)^2}\right] - \left(\frac{1-5\alpha+\sqrt{9\alpha^2-2\alpha+1}}{4}\right)\frac{\left(3\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}-2\right)3}{\left(\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}\right)} + 1$

$\frac{\partial \widehat{\beta}(0)}{\partial \alpha} = -\frac{4}{3} - \frac{1}{2} + 1 = -\frac{5}{6}$

$\frac{\partial \widehat{\beta}(1/4)}{\partial \alpha} = \left(\frac{-5+\frac{5}{\sqrt{17}}}{4}\right)\left[1 - \frac{(5+3\sqrt{17})^2}{(23+\sqrt{17})^2}\right] - \left(\frac{-1+\sqrt{17}}{16}\right)\frac{15+9\sqrt{17}}{23+\sqrt{17}} + 1 = 0,09$

$\frac{\partial \widehat{\beta}(1/2)}{\partial \alpha} = 1$

Comparing conditions B and C we have that condition B lies above condition C, that is $\overline{\beta}(\alpha) > \widehat{\beta}(\alpha)$.

See figure 8, where the light green line represents condition A, the red line represents condition B, the dark green line represents condition C, and the purple line represents $\widehat{\widehat{\beta}}(\alpha)$.

Therefore we have that when condition B does not hold, that is, for $\overline{\beta}(\alpha) \leq \beta$, there is a unique equilibrium in pure strategies with $x_A^* = 0$ and $x_B^* = 1$.

And for $\beta \leq \overline{\overline{\beta}}(\alpha) = \min\left\{\widehat{\beta}(\alpha), \widehat{\widehat{\beta}}(\alpha)\right\}$ there is a unique equilibrium in pure strategies with $x_A^* = 3\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8} - 2$ and $x_B^* = \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}$.

Notice that for $x_A^* = 0$ and $x_B^* = 1$ we have $\pi_A\left(\frac{x_A+x_B}{2};\alpha,\beta\right) = \frac{\beta-\alpha}{1-2\alpha} < \frac{1}{2}$ iff $\beta < \frac{1}{2}$

and $U_A(0,1) = \frac{\beta-\alpha}{1-2\alpha} - 1 < U_B(0,1) = -\frac{\beta-\alpha}{1-2\alpha}$

And similarly for $x_A^* = 3\frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8} - 2$ and $x_B^* = \frac{5+3\alpha+\sqrt{9\alpha^2-2\alpha+1}}{8}$ we have

$\frac{x_A^*+x_B^*}{2} = \frac{2+6\alpha+2\sqrt{9\alpha^2-2\alpha+1}}{8}$ and

$\pi_A\left(\frac{x_A^*+x_B^*}{2};\alpha,\beta\right) = \frac{2-10\alpha+2\sqrt{9\alpha^2-2\alpha+1}}{8(1-2\alpha)} < \frac{1}{2}$ iff $\alpha < \frac{1}{2}$

and since $x_A^* > 1 - x_B^*$ we also must have $U_A(x_A^*, x_B^*) < U_B(x_A^*, x_B^*)$.♦

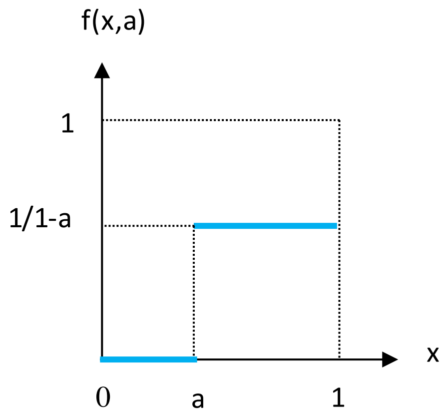


Figure 1: density function

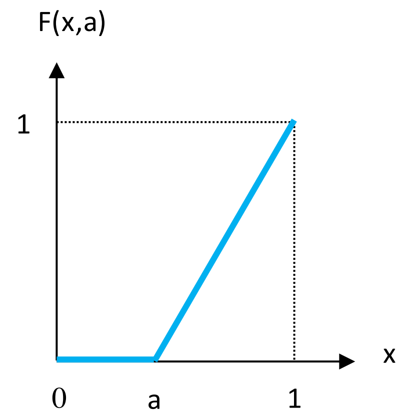


Figure 2: cdf

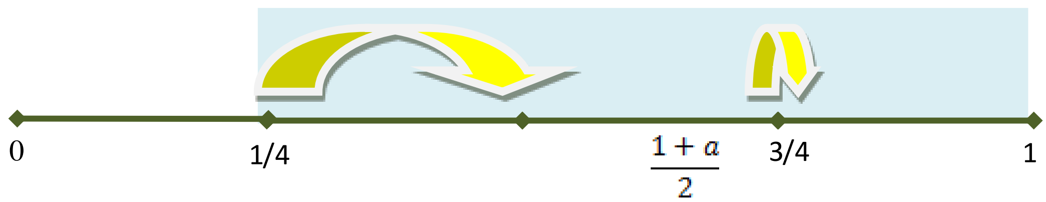


Figure 3: Equilibrium after the shock

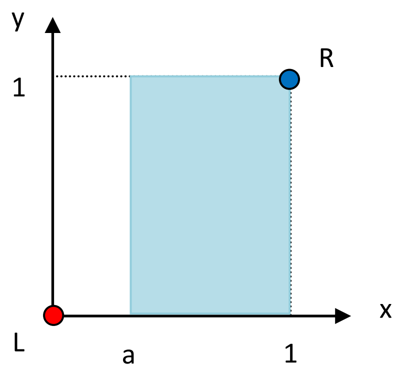
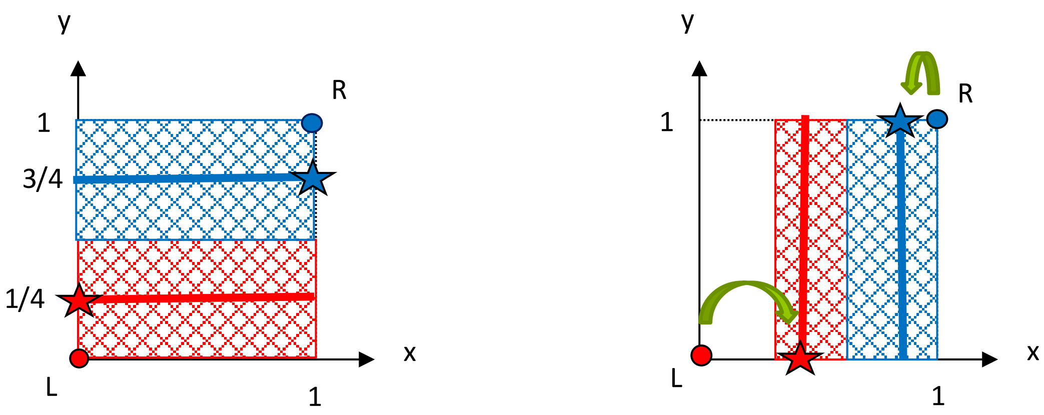


Figure 4: density function



$a = 0$
$\sigma = 0$

$a > 0$
$\sigma = 1$

Figure 5: overall effect of the shock

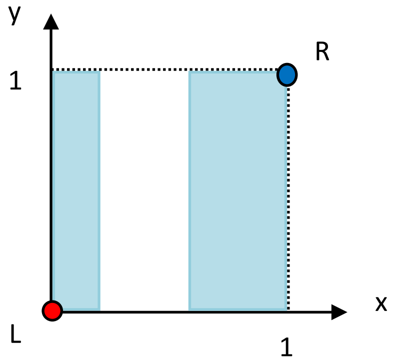


Figure 6: Polarized preferences

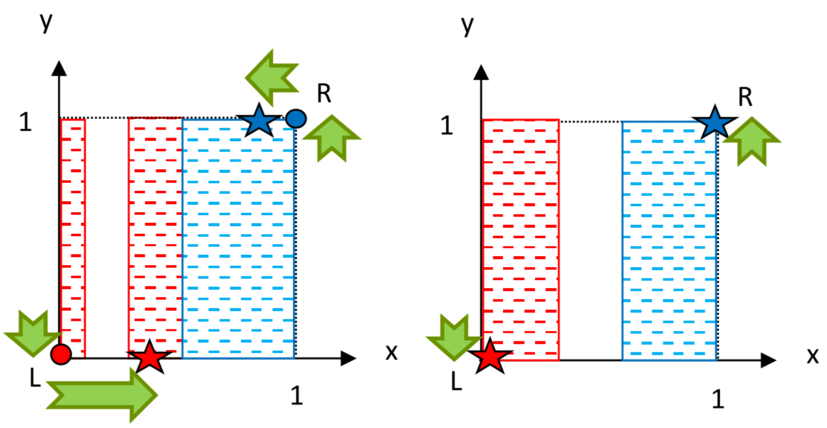


Figure 7: overall effect of the shock for polarized preferences

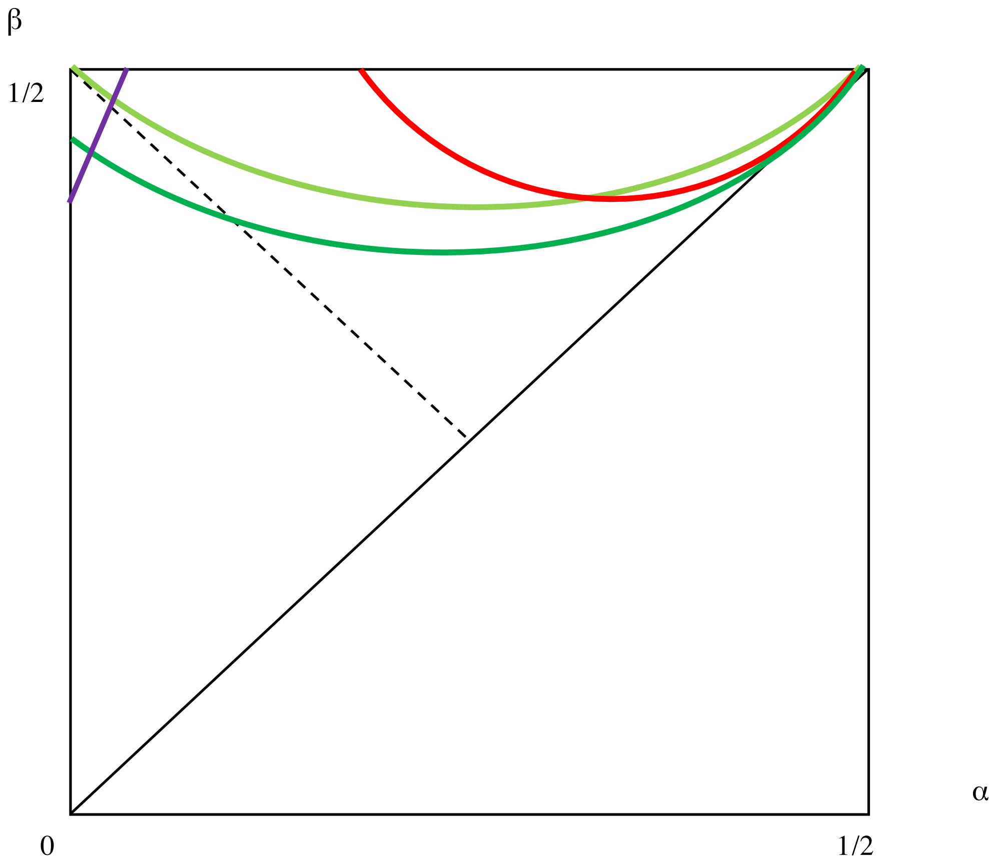


Figure 8: Above the red curve, extreme equilibrium strategies. Below the purple and dark green curves, interior equilibrium strategies.