Inequality and Mobility

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May, 2003
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May 2003
Barcelona Economics WP n° # 23

Abstract

This paper presents a theoretical model that can aid the understanding of how wage inequality and mobility are jointly determined. The model shows that the correlation between inequality and mobility can be used to identify the cause of changes in inequality. Changes in the production sector lead to a positive correlation between the two variables, while changes in the education sector lead to a negative correlation. A better understanding of the causes for changes in inequality over time or across countries can therefore be achieved by observing changes in mobility as well. The model is applied to some empirical issues, like the recent rise in the skill premium, and international differences in inequality and mobility. The paper also shows that skilled parents might prefer more public education than unskilled parents, if the educational barriers for children of unskilled are high.

JEL Classification: E6, J24, J62.

Keywords: Income inequality, Income Distribution, Intergenerational Mobility, Education.

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Inequality and Mobility

1. Introduction
This paper presents a simple macroeconomic model of wage inequality and intergenerational mobility. The inequality we focus on is between wages of skilled and unskilled workers. This inequality has recently become a topic of intense research due to its recent rise. The paper shows how wage inequality and intergenerational mobility are endogenously determined in equilibrium, and how they change together as a result of exogenous changes. The model also enables us to examine some important policy issues.

There are three main reasons why we strongly believe that wage inequality and mobility should be studied together. The first reason is that they strongly affect one another. The skill premium affects the incentive to acquire education and through it upward and downward mobility. But the skill premium itself is affected by mobility, through the distribution of skill. The second reason is related to welfare. Mobility affects the chances of children to climb up the social ladder and thus affects altruistic parents’ welfare. The third and main motivation of the paper is empirical. Many economic factors can account for changes in inequality. For example, both skill-biased-technical-change and a reduction in public education increase wage inequality. But our paper shows that the former increases mobility while the latter reduces it. Thus, observing differences in mobility in addition to inequality can be informative in identifying the causes of changes in inequality.

We next describe the main ingredients of the model. Workers can be either skilled or unskilled. Since it is costly to become educated, equilibrium income of the
skilled is higher than income of the unskilled and we use the ratio between the two levels of income as a measure of wage inequality. We further assume that the income of the unskilled depends on their number, motivated, for example, by the existence of a fixed factor like some natural resources. Hence, inequality is endogenous and increases with the number of unskilled workers.

Skill is acquired through education, which is provided by teachers, who are skilled and therefore demand the same income as skilled workers. We assume that parents cannot borrow against the future income of their children and hence have to finance the education of their children from current income. The amount of education needed to become skilled differs both by the child’s innate ability and by the parent’s education. Abilities are stochastic, for mobility to be non-trivial.¹ We also assume that skilled parents have an advantage over unskilled parents in helping their children to acquire education. Skilled parents have better knowledge of what books to buy or which tutors to hire, etc. As a result, success in education, given innate ability, depends not only on the parents’ income, but on their education as well. This effect of skilled parents has not been sufficiently studied in the literature.²

We assume that parents know the educational ability of their children.³ They allocate income between their own consumption and their child’s education. That creates ability thresholds, above which children become skilled. These thresholds define the probabilities of becoming skilled for children of skilled and unskilled. These probabilities

¹ Differential abilities can be modeled in two possible ways: ability in education, or ability in production after education. We tried both ways and got similar results. We use this specification for simplicity only.
² Some recent studies supply empirical evidence to this effect. See Rubinstein and Tsiddon (1998).
³ In appendix 3 we analyze the case of missing information on ability.
measure intergenerational mobility. We focus mainly on the probability to become skilled of a child of unskilled. This rate of upward mobility is our main measure of mobility.

The equilibrium is analyzed by looking at two relationships between inequality and mobility. The first describes how inequality affects investment in education and through it the rate of upward mobility. We identify two opposite effects here. On the one hand, higher inequality increases the gains from education, which increases upward mobility. We call this the incentive effect. On the other hand, higher inequality reduces the ability of unskilled parents to pay for education, which is supplied by skilled workers and its cost is indexed to their wage. We call this the distance effect. We find that the distance effect dominates only if inequality is very high, so that this relationship between inequality and mobility is mainly positive. The second relationship reflects the effect of mobility on inequality through the labor market. Higher mobility increases the net flow from unskilled to skilled and thus reduces the number of unskilled in the long run. This raises their wage and reduces inequality, creating a negative relationship between mobility and inequality. The interaction between two relationships between inequality and mobility determines the equilibrium.

We then examine the effects of various exogenous changes, divided into changes in the production sector and changes in the education sector. The first changes tend to shift inequality and mobility in the same direction. Intuitively, such changes affect the returns to factors of production and thus affect inequality. Increased inequality raises mobility through the incentive effect and hence the correlation is positive, unless initial inequality is very large. Changes in the education sector lead to a negative correlation between inequality and mobility. Such changes increase access to education and thus
increase mobility. This reduces the number of unskilled, reducing inequality. Thus we get a negative correlation in this case.

The paper then more specifically studies the effect of public education. Similar to other improvements in education, public education reduces inequality and increases upward mobility. Interestingly, skilled and unskilled parents differ on how much public education they want. Skilled parents pay more for it, as taxes used to finance education are proportional to income, but they also make better use of it, due to their higher productivity in using education. We show that if the educational barrier faced by unskilled parents is large enough, they may even prefer to have less publicly financed education than skilled parents.\footnote{This paper is related to the growing literature that brings inequality and mobility into macroeconomics. The modern theory of inequality and mobility began with the pioneering work of Becker and Tomes (1979) and Loury (1981). The macroeconomic implications of inequality and of mobility have been explored in Galor and Zeira (1993), Banerjee and Newman (1993), Durlauf (1996), Owen and Weil (1998), Maoz and Moav (1999), Hassler and Mora (2000), Benabou (2001), and others. Empirical support to the macroeconomic importance of inequality appears in Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996) and Barro (2000). The relation between public education and inequality and mobility has been studied in Glomm and Ravikumar (1992), Fernandez and Rogerson (1995, 1998), and Benabou (2002). The main theoretical contribution of this paper is to add to the analysis of mobility a flexible production technology, so that the wage ratio between skilled and unskilled workers becomes an}
endogenous variable, which depends on their relative supplies. This creates mutual
dependence between wage inequality and mobility, as described above. In contrast to
several papers mentioned above, we also assume that parents care directly about the
welfare of their children, facilitating normative analysis.

Recent years have also seen growing empirical research on intergenerational
mobility. Many papers have measured mobility in the US, like Cooper, Durlauf and
Johnson (1993) and other papers surveyed in Solon (1999) and in Graw and Mulligan
(2002). Recently new data enable mobility comparisons across countries. Thus, Checchi,
Ichino and Rustichini (1999) find that Italy is more equal but less mobile than the US.
Bjorklund et. al (2001) find that Nordic countries are more equal and also more mobile
than the US. Dahan and Gaviria (2001) find that Latin American countries are both less
equal and less mobile than the US. Solon (2002) surveys these and other international
studies. One of the main goals of this paper is to gain a better understanding to these
international empirical findings.5

The paper is organized as follows. Section 2 presents the model, and Section 3
describes the steady state equilibrium. Section 4 analyzes changes in the production
sector, while Section 5 analyzes the effects of changes in the education sector. Section 6
examines the effects of public education and its desirability and Section 7 summarizes.
The appendix contains some proofs and an analysis of the case when parents have no
information on their children’s ability when deciding on their education.

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4 That public education might benefit high-income students more than low-income students has been
acknowledged in many studies, such as Fernandez and Rogerson (1995). Our contribution is to relate it to
the non-pecuniary advantage in education of skilled parents.
5 It is important to note that the measures used in these studies for inequality and mobility are not identical
to our definitions of these two variables. Our measures are dictated, of course, by the simplified model we
use. Since our results are qualitative and not quantitative, these differences are not critical.
2. The Model

Consider an economy with a single physical good, which is produced by two alternative technologies. In one technology only skilled labor is used and each skilled worker produces $a$. In the second technology the good is produced by use of unskilled labor and natural resources, as in agriculture, mining and similar sectors. Each worker produces

$$y_n = a_n x^\alpha,$$

where $x$ is the amount of natural resources used by the worker, $a_n$ is productivity and $\alpha \in (0,1)$. We further assume for simplicity that natural resources are distributed equally to all unskilled workers for use in each period.\(^6\) A second good produced in the economy is education, namely teaching skills to the young. Only skilled workers can be teachers.\(^7\) Each skilled worker produces $h$ units of education, where $h \leq 1$.

The economy consists of overlapping generations with no population growth. Each person has one child, and each generation is a continuum of size $P$. Individuals live two periods each. In the first period of life they go to school, or not. In the second period of life they work, as skilled or unskilled, consume, and invest in education of their child. They derive utility both from own consumption and from the utility of their offspring:

$$V = \ln c + \beta E V_{off},$$

where $c$ is own consumption, $V_{off}$ is utility of offspring, $\beta \in (0,1)$ is an intergenerational discount factor, and $E$ is the expectation operator.

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\(^6\) Without being critical for the main results, this assumption simplifies the analysis by simultaneously allowing a fixed skilled wage, a unskilled wage that decreases in the number of unskilled and no profits or other factor returns.

\(^7\) This is a very realistic assumption, as workers in the education sector are teachers, namely skilled workers. Adding unskilled workers to the education sector has no effect on the main results of the paper.
We next describe education acquisition. Children differ in the amount of education they need to become skilled. This difference is due both to different endowments of innate ability for learning and to differences in parental education. Specifically, we assume that educated parents help their children in getting educated in non-pecuniary ways in addition to paying for education. In other words, children of unskilled parents face some educational barrier that children of skilled parents do not face. Our (inverse) measure of innate ability is “inaptitude.” It is the amount of education a child needs to become skilled, if born to a skilled parent. We denote inaptitude by $e$ and assumed that it is random, independent across families and over time, and distributed uniformly on $[0, 1]$. A child of an unskilled parent with inaptitude $e$ needs $be$ units of education to become skilled. We assume $b \geq 1$ so that $b-1 \geq 0$ measures the additional barrier to education faced by such children of unskilled. This barrier reflects many factors, like social norms, cultural barriers, and even technology. It is further discussed in Sections 5 and 6.

Capital markets are imperfect. We assume that neither the parent nor the child can borrow to finance education, committing the offspring to return the loan later in life. As a result, parents pay for education out of their income. We assume first that education is a private good, and introduce public education in Section 6. Regarding the information structure of the model, we assume that the education decision is made after the child’s inaptitude is revealed. Hence, a parent observes her child’s inaptitude and then decides whether or not to invest the required amount of education to make the child skilled. In Appendix 3 we consider the case when the educational decision is done before any information on inaptitude is revealed, an assumption that creates an interesting bargaining
situation between the child and the parent, allowing an analysis of the effect of intra-
family organization on inequality and mobility.

3. Steady State Equilibrium

We analyze the equilibrium steady state only. This is both due to tractability and because
the main focus of the paper is on direction of changes, rather than their exact magnitudes.
It can be shown that the convergence of the dynamic system is monotonic, so that the
steady state analysis gives us the right qualitative predictions.

3.1. Income Distribution

The income (wage) earned by skilled workers in the industrial sector is \( y_s = a_s \). This is
also the income of education suppliers, due to free entry to both occupations by educated
workers. The income (wage) of an unskilled worker is:

\[
y_u = a_x x^\alpha = a_s \left( \frac{X}{N} \right)^\alpha,
\]

where \( X \) is the aggregate amount of natural resources, \( N \) is the total number of unskilled
workers, \( a_s \) is technology parameter and \( \alpha \in (0,1) \) determines the curvature of the
unskilled production function. Since \( \alpha \) is positive, income of unskilled workers falls if
their number increases.

Denoting \( a \equiv a_s / a_n \), we define inequality as the ratio between incomes of
skilled and of unskilled:

\[
I = \frac{y_s}{y_u} = a \left( \frac{N}{X} \right)^\alpha.
\]
Hence, labor market equilibrium implies that inequality rises with the number of unskilled $N$, declines with natural resources $X$, and rises with the relative productivity of skilled vs. unskilled $a$.

Equation (4) describes how income inequality is related to the distribution of skill, namely to the number of unskilled $N$ and the number of skilled $S = P - N$. Specifically, given exogenous variables, the distribution of skills as well as the distribution of income can be written as a function of one endogenous variable $I$. Similarly, the Gini coefficient too, for example, depends solely on income inequality $I$. While (4) describes the equilibrium in the labor market taking the supplies of the two types of labor as given, we next describe how these supplies are determined through educational decisions.

3.2. Investment in Human Capital

The decision to invest in a child’s education depends on the cost of education for the parent on the one hand, and on the expected gains from education for the child, on the other hand. Parents consider the expected utility of her child, which depends only on whether the child becomes skilled or not. We denote the expected utility of a skilled person by $V_s$ and of an unskilled by $V_n$, evaluated before the inaptitude of her child is revealed. A parent who earns income $y$ invests in education of her child as long as education costs $i$ satisfy:

$$
\ln(y - i) + \beta V_s \geq \ln y + \beta V_n.
$$

The Gini coefficient is not monotonically related to $I$. When $I=1$ income is equal for all and the Gini is zero. As $I$ increases the Gini increases as well up to a point where it begins to decline. As $I$ reaches its maximum at $I^* = a(P/X)^*$, when all workers are unskilled, the Gini is zero again.
Hence, individuals invest the required amount in education of child, as long as it does not exceed a share $m$ of their income, defined by:

$$-\ln(1 - m) = \beta(V_s - V_a).$$

As we see, the maximum share of income spent on education, $m$, is equal for both skilled and unskilled, depending positively on the expected gains from education, the RHS of (6).

The cost of one unit of education is $y_s/h$ and the necessary amount of education for a child of inaptitude $e$ is $e$ if she is born to skilled and be otherwise. Knowing the maximum share of income spent on education $m$, we can calculate the threshold levels of investing in education for children of skilled and unskilled. The threshold level for children of skilled workers is:

$$e_s = hm.$$  

It is also equal to the probability of getting educated for a child of a skilled parent. Since $1-e_s$ is the probability of such a child to become unskilled, $1-e_s$ measures downward mobility. The education threshold for a child of an unskilled worker is:

$$e_u = \frac{hm}{bI}.$$  

This is the probability that a child of an unskilled becomes skilled and is therefore a measure of upward mobility.

According to equation (6), investment in education depends on the expected gains from education, $V_s - V_a$. We next calculate these gains, using the above threshold levels. The expected utility of skilled, before their child’s inaptitude becomes known, is
The last term in (9) represents the expected incremental utility a skilled person gets from the possibility that her child will become skilled. Using (6) and (7), this term can be written

\[ e_s \beta (V_s - V_n) = hm \ln(1-m). \]

Evaluating the integral, we then get

\[ V_s = \ln y_s + e_s V_s + \frac{h}{\beta} \left( \frac{h}{b} \right) \ln(1-m) - m. \]

Similarly, the expected utility of unskilled, before inaptitude of their child is known, is

\[ V_n = \ln y_n + e_n V_n + \frac{h}{\beta} \left( \frac{h}{b} \right) \ln(1-m) - m. \]

From these expected utilities we derive the gains from education:

\[ V_s - V_n = \ln I + \frac{h}{\beta} \left( 1 - \frac{1}{bI} \right) \ln(1-m) - m. \]

The gains from education depend positively on income inequality, both directly through the income effect and indirectly, by increasing the child’s ability to acquire education, if the parent is skilled. Furthermore, the gains from education depend positively on the equilibrium maximum share of income invested in education \( m \). Higher \( m \) increases the difference in expected educational spending between skilled and non-skilled. However, this is more than compensated for two other effects. First, the fact that \( m \) is chosen optimally in the future, reflecting larger future differences \( V_s - V_n \), higher \( m \) increases the educational gains. Second, higher \( m \) increases the difference in the probabilities

\[ e_s - e_n = mh \left( 1 - 1/bI \right), \]

which also increases the educational gains.

[Insert Figure 1 here]
Equations (6) and (11) describe two relationships between the gains to education and $m$. Equation (6) describes how the choice of $m$ depends positively on the gains from education and is drawn in Figure 1 as the *incentive curve*. Equation (11) describes how the gains from education of a child depend positively on the maximum share of income current and future generations will invest on education. This relationship is drawn in Figure 1 as the *intergenerational curve*. It can be shown that the incentive curve is necessarily steeper than the intergenerational curve, as the feedback effect from $m$ to the gains from education is smaller than the direct incentive effect. Hence, a unique equilibrium exists, as shown in Figure 1, and the intersection of the two curves determines the steady state level of maximum investment in education $m$. This level is defined by the following equation, which is derived from (6) and (11):

$$
-\frac{1}{B} \ln(1-m) = \ln I + h\left[1-\frac{1}{bI}\right] - \ln(1-m) - m.
$$

### 3.3. Steady State Equilibrium

The equilibrium maximum spending on education $m$, as described in Figure 1 and equation (12), depends on income inequality $I$, through the gains from education. Higher inequality shifts the intergenerational curve upward, and increases the maximum share of income spent on education $m$. This relationship, which reflects the incentive effect of inequality, is described by a function $m = M(I)$ and by the curve $M$ in Figure 2.

Income inequality and $m$ are related through the labor market as well. If $m$ is higher, more young people become skilled, of both skilled and unskilled parents. This increases the steady state number of skilled workers, and reduces the number of unskilled workers. Hence, income of unskilled rises and inequality falls. This determines a negative
relation between $m$ and $I$. To derive it formally, note that in the steady state the upward flow of children of unskilled, who get education, must be equal to the downward flow of children of skilled, who do not get education. Hence, the steady state must satisfy:

$$e_s N = (1 - e_s) S.$$  

Let us denote the ratio of unskilled to skilled labor by $n \equiv N/S = N/(P-N)$.

From equation (4) we:

$$n = \frac{X}{P} a^{-\frac{1}{a} I^a} \left(1 - \frac{X}{P} a^{-\frac{1}{a} I^a}\right)^{-1} = n(I, a, \frac{X}{P}).$$  

Hence, $n$ depends positively on inequality $I$, negatively on the relative productivity of skilled to unskilled $a$, and positively on the amount of natural resources per capita: $X/P$.

Using $n$ and substituting the thresholds of upward and downward mobility from (7) and (8) into (13), we get the following equilibrium condition:

$$m = \frac{1 - bI}{bI + n}.$$  

This equation defines a negative relation between the maximum income share $m$ and inequality $I$, which we denote the function $m = L(I)$, and depict as the curve $L$ in Figure 2. The $L$ curve describes the labor market steady state equilibrium condition.

[Insert Figure 2 here]
education. This in turn determines the degree of intergenerational mobility, to be analyzed in the next subsection. Note, that we do not derive a closed form solution to the model for two reasons. First, there is no clean analytical solution to equations (12) and (15). Second, the diagrammatic analysis is sufficient to derive all the result in this paper. Extensions of the model can use numerical methods to derive further results.

3.4. Upward Mobility

Figure 2 shows how the equilibrium is determined in terms of inequality and investment in education. Next, we describe the equilibrium in terms of the main variables at focus, namely inequality and mobility. We use the probability of upward mobility $e_u$ as our main measure of mobility.

The level of upward mobility is equal, according to (8), to:

$$e_u = \frac{h M(I)}{b I}.$$  

Hence, the decision to invest in education by the unskilled is affected by inequality in two opposite ways. One effect, through $M(I)$, is positive, and we call it the *incentive effect*. The other effect, through the denominator, is negative, and it reflects the difficulty of an unskilled parent to pay skilled teachers for education out of her lower income. We call it the *distance effect*. It is easy to see that the incentive effect dominates for low levels of inequality while at high levels the distance effect dominates. The mathematical reason for this is that $m$ is bounded by 1 and the result reflects the fact that the marginal utility of consumption increases towards infinity as $m$ approach unity. Hence, the incentive effect is diminishing as inequality rises. Equation (16) is drawn in Figure 3 as the $MM$ curve, where the additional $M$ stands for mobility.
Next we consider the relationship between mobility and inequality through the labor market. We use the equation of upward mobility (8) to rewrite (15):

\[ e_n = \frac{1}{n + bI}. \]

This relation between the upward mobility and inequality is described by the \textit{ML} curve in Figure 3, which is downward sloping. The equilibrium described by the intersection of the two curves in Figure 3 is fully identical to that in Figure 2, only it shows the level of upward mobility instead of the share of income invested in education. It is therefore only a different presentation of the same steady state equilibrium.

The steady state equilibrium in Figure 3 is an intersection of two relations. One is the \textit{MM} curve, which describes upward mobility as a result of optimal education decisions of parents, who consider the level of inequality to be given. The second is the \textit{ML} curve, which describes how mobility affects the distribution of skill, and through it the level of income inequality. In the rest of the paper we examine how these two relations are affected by various exogenous changes and how inequality and mobility change as a result.

3.5. Welfare Considerations

From equations (10) the expected utility of unskilled follows immediately as

\[ V_u = \ln y_s - \ln I + \frac{h}{bl} \left[ -\ln(1-m) - m \right], \]

and the expected utility of skilled is derived by adding the gains from education using (6) (6):
Expected utilities are related negatively to inequality $I$, and positively to $m$. Intuitively, inequality, which is related inversely to income of unskilled, reduces utility of unskilled directly and utility of skilled indirectly, as their children have some chance of becoming unskilled. Maximum investment in education $m$ is positively related to utility of both, since it increases the chances of children becoming skilled. But equilibrium $m$ and $I$ are related to one another from the labor market equilibrium. Using equilibrium condition (12) to eliminate $I$ from (18) and then using (19), we can express the expected utility of skilled workers as:

$$V_s = V_n + \frac{-\ln(1-m)}{\beta}.$$  

Similarly the expected utility of unskilled workers is:

$$V_n = \frac{1}{1-\beta} \left[ \ln y_s + (1-h) \ln(1-m) - hm \right].$$

These expressions are important for welfare evaluations, since given the wage of skilled, and holding educational productivity $h$ constant, $m$ is an indicator of welfare. For both types of individuals, $m$ is negatively related to welfare. The reason is that along the $L$ curve, $m$ rises with inequality. Hence, despite the greater chance of being skilled, utility falls as the income of unskilled declines.9

### 4. Changes in the Production Sector

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9 Of course, measuring aggregate welfare runs into the standard problems of how to weigh the different types of individuals and how to take into account dynamic transitions to new steady states.
In this Section we begin the analysis of the steady state equilibrium, which is derived in Section 3. This is a comparative static analysis, which can be interpreted in two possible ways: as explaining cross-country differences and as explaining changes within the same country over time.

We first focus on exogenous changes in the production sector. More specifically, we analyze the case of skill-biased-technical-change, having been a topic of much research recently. Within our model such a change amounts to an increase in the productivity of skilled workers only, namely a rise in \( a_s \), and therefore in \( a \). Such a change does not affect the relation between the gains from education and parental willingness to invest in education. Therefore, it does not have any effect on the \( M \) or \( MM \) curves in Figures 2 and 3. However, an increase in \( a \) does affect the \( L \) and \( ML \) curves, shifting them up and to the right, as follows immediately from equations (14), (15) and (17). These shifts lead to an increase in inequality \( I \), and an increase in the maximum share of income invested in education \( m \). Note that the higher the level of inequality is, the smaller is the effect on \( m \), and the larger is the effect on \( I \). Hence, skill-biased technical change raises inequality more in economies which are already unequal. The intuition behind these results is as follows. Skill-biased technical change increases the market skill premium, which raises the gains from education, so that parents invest more in education. Since \( m \) is bounded, the effect on it becomes smaller as inequality \( I \) increases.

As for the effect of skill-biased technical change on mobility, we learn from Figure 3 that upward mobility \( e_n \) rises if the incentive effect is dominant, while it declines when the distance effect dominates. Namely, the positive effect of skill-biased technical
change on mobility falls with inequality. Intuitively, higher inequality provides both greater incentives for education, which increases mobility, but also increases the cost of education to unskilled parents, which reduces mobility. We believe that for most developed countries the incentive effect dominates, and skill-biased-technical-change raises upward mobility. In all countries, though, it reduces downward mobility, since \( m \) is higher.

The above analysis can be very helpful in understanding the rise in wage inequality in Western countries in recent decades, which has been attributed by many economists to skill-biased-technical-change.\(^{10}\) Our analysis offers a way to assess this explanation, which is to examine whether it has been accompanied by an increase in intergenerational mobility and in the share of income devoted to education. Such a positive correlation can lend support to the skill-biased-technical-change hypothesis. If, on the contrary, mobility and investment in education have declined in recent decades, we may have to look for an alternative explanation to the rise of inequality.

The above analysis applies to other changes in the production sector as well, such as globalization. Many claim that cheap imports from poor countries have reduced the income of unskilled workers in the developed countries. This claim can be formalized in our model as a reduction of the productivity coefficient of unskilled labor \( a_n \). This also increases in the parameter \( a \), and hence has the same effect as skill-biased-technical-change, which is discussed above. Another variable, which is important to growth and development, is the amount of natural resources in a country.\(^{11}\) In our model it is measured by natural resources per capita \( X/P \). Increasing it has an exactly opposite effect.

\(^{10}\) See Katz and Murphy (1992), Berman, Bound and Griliches (1994) and others for this hypothesis.

\(^{11}\) See Sachs and Warner (2001) and others, who claim that natural resources impede economic growth.
to skill-biased-technical-change. It shifts the $L$ and $ML$ curves downward and leaves the $M$ and $MM$ curves unchanged. Hence, inequality and mobility are reduced and investment in education is reduced as well. Hence, countries with more natural resources have lower inequality, since their unskilled workers earn more, but they also invest less in education. That reduces social mobility in these countries and it might also have a significant adverse effect on economic growth.$^{12}$

We can now relate the results of this section to the findings on international differences in inequality and mobility. As mentioned in the introduction those findings are that the US is less equal and more mobile than Italy, more equal and more mobile than Latin American countries, and less equal and less mobile than the Nordic countries. The only finding that conforms to differences in the production sector is that of Italy and the US in Checchi, Ichino and Rustichini (1999). As it is hard to believe that Italy and the US differ much in technology, a possible explanation could be wage compression in Italy. Such wage compression can be modeled in our framework as an administrative reduction of $y_s$ below $a_s$, which reduces income inequality and mobility.

5. Changes in the Educational Sector

In this section we turn from the production sector to the education sector and examine how changes in this sector affect the steady state equilibrium levels of inequality and mobility. In other words, how inequality and mobility are correlated, when the education sector is going through changes. More formally, we examine the effects of changes in the parameters $h$ and in $b$ on the steady state equilibrium, where $h$ describes the overall

$^{12}$ Despite these adverse effects on mobility and education, welfare increases according to (20) and (21), since $m$ is reduced. This reflects the positive effect of natural resources on the income of unskilled.
productivity of education, while $b$ describes the socio-cultural barriers to education faced by children of unskilled parents.

Intuitively, improvements in the productivity of education should lead to higher mobility and to lower inequality. Better education enables children of unskilled to acquire more education for their money so that more of them become skilled. Hence, upward mobility increases. As better education increases the number of skilled and reduces the number of unskilled, inequality falls. However, this argument does not take into account general equilibrium effects arising from the fact that educational choices are endogenous. We therefore turn to a formal analysis of changes in $h$ and $b$.

Consider first an increase in overall productivity of education, namely an increase in $h$. While the response of parents to the gains from education remains unchanged, as indicated by equation (6), the gains from education rise, since investment in education becomes more productive. As a result, the probability of the grandchild to become educated increases, which raises the present gains from education. In terms of Figure 1, the incentive curve remains unchanged while the intergenerational curve shifts upward, which increases maximum investment in education $m$. Hence, the $M$ curve shifts up as well. Upward mobility increases too, as parents invest more money and as this money is more productive. Hence, as shown in (16), the $MM$ curve shifts upward as well.

We next turn to the labor market. If education is more productive, the $ML$ curve remains unchanged, as shown by equation (17). The intuition is simple: upward mobility and downward mobility change together when education becomes more efficient. Thus, if upward mobility is unchanged, so is downward mobility, and the distribution of skill remains unchanged. Hence, the economy moves along the $ML$ curve
in Figure 3 to the left: inequality falls and upward mobility rises. Note, that the effect on investment in education $m$ is ambiguous. The intuition: lower inequality reduces the gains from education, which is opposite to the direct positive effect described above.\footnote{This can be shown diagrammatically as well. While an increase in $h$ shifts the $M$ curve up, it shifts the $L$ curve down in Figure 2. Hence, while $I$ is clearly reduced, the effect on $m$ is ambiguous.}

We next turn to examine the effect of a reduction of $b$, namely a decline in the educational barrier faced by children of unskilled. As shown by equation (13), a reduction in $b$ reduces the amount of investment in education $m$. Intuitively, it increases the chance of the grandchild to get education, even if the child is unskilled, and hence it reduces the gains from education of the child. This reduces spending on education and shifts the $M$ curve down. As for the $MM$ curve in Figure 3, the reduction of $b$ has two opposite effects: on one hand, unskilled parents pay less for education, but on the other hand, their investment is more productive. Appendix 1 shows that the elasticity of $m$ with respect to $b$ is smaller than 1, so the productivity effect dominates and $MM$ shifts up.

In the labor market, a reduction in $b$ shifts the $ML$ curve up and to the right, as seen from equation (17). The intuitive explanation is as follows. If upward mobility is kept unchanged while the educational barrier to children of unskilled is reduced, it means parents invest less in education. Hence downward mobility increases, there are more unskilled workers in the steady state, inequality rises, and the $ML$ curve shifts to the right. Since both curves in Figure 3 shift up, mobility rises, while the effect on inequality is less clear. For a better understanding of this ambiguity, we should look at Figure 2. The $M$ curve shifts down, as shown above, and the $L$ curve shifts down as well, as shown by equation (15). Hence, a lower $b$ reduces maximum investment in education $m$, which increases downward mobility, as skilled parents invest less in education. As shown
above, upward mobility increases as well. Hence, the effect on the steady state number of unskilled is ambiguous and so is the effect on inequality. Appendix 1 carries a more careful analysis, which resolves this ambiguity and shows that reducing education barriers for children of unskilled always reduces inequality.

As mentioned above, $b$ measures the barrier to education for children of unskilled. It reflects social norms, cultural aspects, and sometimes even the state of the technology. Periods of rapid technical progress increase the relative ability of educated parents to help their children in school. Thus such periods lead to changes in $b$ and in mobility and inequality. It is also interesting to see what is the impact of reducing $b$ on welfare. As shown above, lowering $b$ reduces $m$, and as equations (20) and (21) show, that increases utility to both skilled and unskilled. Clearly, utility of unskilled rises by more, since the gains from education fall.

In summary, both changes in the education sector have similar qualitative effects: mobility rises and inequality declines. Hence, changes in the education sector lead to a negative correlation between inequality and mobility, unlike changes in the production sector, which lead to a positive correlation between them. This can be helpful in explaining changes in inequality over time or across countries. If mobility moves with inequality, the underlying reason probably lies in the production sector, while if mobility is negatively related to inequality, the underlying reason is in the education sector. In the next section we show that the effects of changes in public education are similar to changes in the education sector discussed above.
6. Public Education

In this section we introduce public support to education, allowing parents to add to it by purchasing more education privately. Thus, unlike Glomm and Ravikumar (1992), public education does not rule out private expenditures on education. In this model, public education helps parents increase the probability of their children becoming skilled. We assume that public education is tax financed, and the tax is proportional to income.

Intuitively, an increase in public education should lead to higher mobility and lower inequality. Increased public education enables more children of unskilled to go to school, so upward mobility rises. It also increases the amount of skilled workers, which reduces inequality. But some forces might work in opposite directions. For example, public education reduces private investment in education, both due to crowding out, and also because it reduces the gains for education, as the ability of the child to finance education of the grandchild becomes less important. Another effect is that of taxation. Increased public education raises taxes, which reduce post-tax income differentials, which reduces the gains from education. To assess these mixed effects, we turn to a formal analysis of the overall effect of public education on inequality and mobility.

The government supplies each student with $p$ units of education. A relevant issue is what happens to students who need less education to graduate. We assume that their parents do not reveal information on their low inaptitude, and thus they are subsidized by the same amount as the others. This assumption is made for simplification, but it is quite reasonable. In our model, parents would not mind the government to pay for excess education to their children. Adding a small amount of uncertainty about innate educational aptitude would make parents (and children) strictly prefer more education as
long as it is free. The subsidy is financed by a tax of rate $T \in [0, 1]$ on income. For simplicity we assume that the productivity of education is fixed and equal to 1, namely: $h = 1$.

First note that the maximum share of investment in education out of income is determined in the same way as in the benchmark case in Section 3 and hence equation (6) holds here as well. Given the maximum share of income invested in education $m$, the threshold of education for children of skilled is

$$e_s = p + m(1 - T),$$

and the threshold of education for children of unskilled is

$$e_u = \frac{1}{b} \left[ p + \frac{m}{t}(1 - T) \right].$$

Note that public education seems to benefit skilled parents more than unskilled, since $b=1$, namely, they use education better. This is an important point, which has recently received some empirical support. Dynarsky (2000) finds that increased public support to college education raises participation of middle class students more than poor students.

Note though, that the overall effect of public education on skilled and unskilled is ambiguous, once we take into consideration the effect of taxation on income and on the ability to finance education, as skilled parents pay more taxes than unskilled parents. We return to this issue below, when we analyze the welfare aspects of public education.

The expected utility of skilled and unskilled in the case of tax and subsidy is calculated in a similar way to the benchmark model of private education:
Calculation yields the following expected utility for skilled:

\begin{equation}
V_s = \ln y_s + \ln(1-T) + \beta V_a - \left[ \frac{p + 1-T}{b} \right] \ln(1-m) - \frac{1-T}{bI}m.
\end{equation}

Similarly, the expected utility of unskilled is

\begin{equation}
V_a = \ln y_a + \ln(1-T) + \beta V_a - \left[ \frac{1-T}{bI} \right] \ln(1-m) - \frac{1-T}{bI}m.
\end{equation}

From equations (25) and (26) we derive the gains from education and together with equation (6) we get a condition that defines the maximum investment in education:

\begin{equation}
\frac{1}{\beta} \ln(1-m) = \ln I + (1-T) \left( 1 - \frac{1}{bI} \right) \left[ -\ln(1-m) - m \right] - p \left( 1 - \frac{1}{b} \right) \ln(1-m).
\end{equation}

This condition describes how the maximum share of investment \( m \) is determined. Note, that it depends positively on public education \( p \) if and only if \( b \) is strictly larger than unity. This is because when \( b>1 \), public education can be more efficiently used by skilled parents creating an additional incentive for education. Clearly, given \( p \) and \( I \), educational investments fall in the tax rate \( T \), since it reduces after tax wage inequality.

Now, consider the relation between the subsidy and the tax rate due to the public budget constraint, which is

\begin{equation}
pS = \frac{T[y_sN + y_sS]}{y_s}.
\end{equation}

Hence the tax rate depends on the amount of public education in the following way:

\begin{equation}
T = p \frac{I}{n+I}.
\end{equation}
Substituting the budget constraint (29) in the equilibrium condition (27) we get:

\[
-\frac{1}{\beta} \ln(1-m) = \ln I + \left(1 - p - \frac{I}{n+I}\right) \left(1 - \frac{1}{bI}\right) \ln(1-m) - \left(1 - \frac{1}{b}\right) \ln(1-m). 
\]

This condition describes a modified \( M \) curve (modified to the budget constraint), which outlines how the maximum private investment in education \( m \) is determined. It is clear from (30) that \( m \) depends positively on inequality \( I \), hence the modified \( M \) curve is increasing. The modified \( MM \) curve is described by the following condition where \( m \) is derived from (30):

\[
e_n = \frac{1}{b} \left( p + \frac{m}{I} - \frac{pm}{n+I} \right).
\]

The \( ML \) curve is derived from the labor market steady state condition (13), which in this case is

\[
e_n = \frac{1 + p(I - 1)}{n + bI}.
\]

The \( ML \) curve is downward sloping in the inequality-mobility plane. Conditions (31) and (32) jointly determine the unique steady state equilibrium and the levels of inequality and mobility, at the intersection of the modified \( MM \) and \( ML \) curves.

We next turn to analyze the effects of changes in public education on the equilibrium. According to (30) the direct effect of an increase in public education \( p \) on the maximum spending on education \( m \) is ambiguous due to the opposite effects of \( p \) and of the tax rate \( T \). Clearly, if \( b = 1 \), public education is equally beneficial for skilled and unskilled parents. Hence the negative effect of the tax rate dominates, the gains from education decline with public education and the modified \( M \) curve shifts down. Since also the modified \( L \) curve, given by
shifts down, \( m \) unambiguously falls in this case. As the barrier to children of unskilled \( b \) increases, the gains from public education to skilled increase, and from some point on \( m \) rises with \( p \). As shown by equation (31), public education raises upward mobility even if \( m \) declines, due to its direct effect. Hence, the modified \( MM \) curve shifts up. The labor market curve \( ML \) shifts up as well.\(^ {14} \) Hence, upward mobility rises with public education.

As for inequality, we show in Appendix 2 that unless \( \beta \) and \( b \) are both too close to unity, the shift in \( MM \) is larger than the shift in \( ML \), so inequality falls. Hence, increasing public education should typically raise mobility and reduces inequality, and the opposite effect of taxation on private spending on education is secondary in size. Thus, changes in public education create a negative correlation between inequality and mobility.

We next turn to examine the effect of public education on welfare. In a similar way to Section 3 we can show that expected utility of the unskilled is:

\[
V_u = \frac{1}{1-\beta} \left[ \ln y_s + \ln(1-T) - \left( 1-T + p - \frac{1}{\beta} \right) \ln(1-m) - (1-T)m \right].
\]

Expected utility of skilled, \( V_s \), equals (34) plus the gains from education: \(- \ln(1-m)/\beta \). Public education has two main effects on expected utilities. The positive effect is due to reduction of private cost of education. This effect diminishes with public education, since lower inequality reduces the gains from education. Hence the marginal benefit of public education is diminishing. The negative effect is the tax cost of public education. This cost is increasing since marginal utility of consumption increases with tax. This heuristically
explains why expected utility is concave in public education. Hence, there is a level of public education that maximizes expected utility, where the tax costs outweigh the benefits of public education. This level differs for skilled and unskilled.

To analyze preference differences over public education between skilled and unskilled, we note the existence of two opposing effects. On one hand the marginal cost of the tax is higher for skilled parents since they pay more taxes. On the other hand, skilled parents use public education relatively more effectively if \( b > 1 \), so their marginal benefit from public education is higher and in addition, they are more likely to send their children to school. This latter effect clearly increases in \( b \), implying that if unskilled prefer more support to education when \( b \) is low, this may reverse as \( b \) increases.

This is illustrated in Figure 4, which presents a numerical example of how public education \( p \) affects expected utilities of skilled and of unskilled, \( V_s \) and \( V_n \). These are calculated for parameter values: \( \alpha = .3, \beta = .75, a = 1, X/L = .2, \) and for \( b = 1.1 \) and 5. Figure 4 shows that optimal public education is unique for each type. It further shows that at low \( b \) unskilled workers prefer more public education than skilled workers, but at high \( b \) they might prefer less. This is since skilled parents make better use of public education than unskilled parents. This result might have consequences for the political economy of public education. A country with high barriers to children of unskilled has high inequality and low mobility, as shown in Section 5. This is further amplified if

\[ \text{See equation (30). Intuitively, increasing } p \text{ raises upward mobility and reduces downward mobility. To keep inequality and } n \text{ intact, the ratio between upward and downward mobility must be restored, so } m \text{ falls. This has a larger effect on downward mobility, and hence upward mobility ends up higher.} \]

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unskilled parents are a majority in the population. They vote for less public education, which further reduces mobility and increases inequality.\textsuperscript{15}

We next return to some empirical implications of our results on public education, focusing on two issues. The first is the rise in wage inequality between skilled and unskilled workers in the US in recent decades, which is also discussed in Section 4. While many economists attribute it to skill-biased-technical-change, some have recently explored the possibility that it might be caused by some reduction in public education. Goldin and Katz (1999) claim that the wage gap began to increase already in 1950, much earlier than the IT revolution. They also find that the wage gap was reduced significantly in the first half of the 20\textsuperscript{th} Century and attribute it to expansion of public education. Hence, their paper hints at the possibility that the rise in inequality in the second half of the 20\textsuperscript{th} Century could be attributed, among other things, to ending the expansion of public education. More explicitly Card and Lemieux (2001) claim that the rise in inequality is a result of a slowdown in the supply of skill in the US. Our paper shows that one way this issue can be clarified, is by observing what happened to intergenerational mobility in the US in recent decades. If the skill-biased-technical-change is the right explanation we should observe a rise in mobility, while if it is due to a slow down in public education, we should observe a reduction in mobility.

The second empirical issue has to do with comparisons across countries. For example, Checchi, Ichino and Rustichini (1999) find that while Italy is more equal than the US, it is much less mobile. They attribute these differences to higher public education in Italy. Our model shows, that if this were the case, Italy should have been more mobile.

\textsuperscript{15} Interestingly, Alesina and Ferrara (2001) find that perceptions on barriers to mobility have an effect on preferences for redistribution in the US.
Hence, our model points at differences in the production sector between Italy and the US, such as wage compression, as a better explanation for these findings. Our model can also shed light on other international differences in inequality and mobility. Dahan and Gaviria (2001) find that Latin American countries are both less equal and less mobile than the US. Furthermore, they find that within Latin American countries inequality and mobility are negatively correlated. This points at differences in public education. A similar conclusion applies to the findings of Bjorklund et. al. (2002) that the Nordic countries are both more equal and more mobile than the US.\textsuperscript{16}

7. Conclusions
This paper presents a simple theoretical model that analyzes the joint determination of income inequality, skill distribution and intertemporal mobility. Its main focus is on the expected correlation between inequality and mobility. Empirical studies have shown that this correlation across countries can be either positive or negative. Our paper gives a simple explanation to these findings. We show that the correlation between inequality and mobility is positive if the underlying changes are in the production sector, while the correlation is negative if the underlying changes are in the education sector. Our model therefore helps in understanding observed differences in inequality across countries. It can also help in understanding differences in inequality in a single country over time, if we can find data on how mobility changes over time.

The model also shows that the non-pecuniary advantage of skilled parents in getting education for their children plays an important role in the economy. It affects

\textsuperscript{16} We should be cautious here as empirical measures of mobility and inequality differ somewhat from our definition, as noted in footnote 4.
private decisions on education and as we also show it affects the political decisions on public education as well. Hence, the barrier to education, which is faced by unskilled parents, plays an important role in the development of the economy and should be considered when economic policy is discussed.

Finally, our model can be used to examine additional economic, social and cultural variables. For example, the analysis of the missing information in the appendix shows, that countries, in which parents are more dominant within the family, tend to have less education, less mobility and higher inequality. We believe that the framework presented here can be further extended in many other interesting directions.
Appendix

1. Effects of Reducing $b$ on Inequality and Mobility

We first show that the $MM$ curve shifts up when $b$ is reduced despite the fall in $m$. We therefore calculate the elasticity of $m$ with respect to $b$ from (12) and show that it is smaller than 1. This elasticity is:

$$\left. \frac{b}{m} \frac{\partial m}{\partial b} \right|_{(12)} = \frac{1-m}{m} \left[ -\ln(1-m) - m \right] \frac{h}{bI} < 1.$$  

(A1)

Hence, a reduction of $b$ shifts the $MM$ curve up, despite the reduction in $m$. Since the $ML$ curve shifts up as well, it means that upward mobility always rises when $b$ is reduced.

We next show that a reduction of $b$ lowers income inequality. From equation (15) we derive:

$$bI = \frac{hmn}{1-hm}.$$  

Substituting in equation (12) we get:

$$\ln I + (h + hn)[-\ln(1-m) - m] + \frac{\ln(1-m)}{m} + 1 + \frac{1}{\beta} \ln(1-m) = 0.$$  

(A2)

Equation (A2) describes the loci in the $(I, m)$ plane of the various equilibria as $b$ changes. Note, that (A2) itself is not affected by changes in $b$. The LHS of (A2) is decreasing in $m$, from $\ln I$ at $m=0$ to $-\infty$ at $m=1$. To see this note, that if it increases with $m$, when $n$ is high, (A2) does not hold and there is no equilibrium. The LHS of (A2) is increasing with inequality $I$. Hence, $I$ and $m$ are positively related as $b$ changes. We therefore conclude that a reduction in $b$, which lowers $m$, reduces inequality $I$ as well. Hence, smaller
barriers to education of children of unskilled lead to greater mobility and to lower inequality.

2. Effect of Public Education on Inequality

In this appendix we prove that the modified $MM$ curve shifts upward by more than the $ML$ curve, if the intergenerational preference parameter $\beta$ is not too high. The shift in the $MM$ curve is given by:

$$\frac{d e_u}{d p}_{(29)} = \frac{1}{b} \left[ 1 - \frac{m}{n + I} + \left( \frac{1}{l} - \frac{p}{n + I} \right) \frac{\partial m}{\partial p}_{(28)} \right].$$

The upward shift in the $ML$ curve is given by:

$$\frac{d e_u}{d p}_{(30)} = \frac{l - 1}{n + bI}.$$

A simple calculation shows that the $MM$ shifts by more than the $ML$ curve if:

$$\phi = n + bI + b - m \frac{n + bI}{n + I} + \frac{n + bI}{l} \frac{n + I - pI}{n + I} \frac{\partial m}{\partial p}_{(28)} > 0.$$

From (30) we get that the shift in the modified $M$ curve is:

$$\frac{\partial m}{\partial p}_{(28)} = \left( 1 - \frac{1}{b} \right) (1 - m) \ln(1 - m) + \frac{bl - 1}{bn + bl} [\ln(1 - m) + m](1 - m).$$

As can be seen from (A6) and as discussed in Section 6, when $b$ is low, the shift in $M$ is negative, but when $b$ is higher the shift is positive. If this is the case and (A6) is positive, then (A5) holds, since:

$$b - m \frac{n + bI}{n + I} > b - m \frac{bn + bI}{n + I} = b - mb > 0.$$
Hence, for a sufficiently large $b$ the $MM$ curve always shifts up by more than the $ML$ curve does. We next turn to the case that $b$ is low and (A6) is negative. Even then we can find a lower bound and show that:

$$\frac{\partial m}{\partial p}_{(28)} > -\frac{bl - 1}{bn + bl} \frac{\beta}{1 - \beta} \exp(-1).$$

Substituting in (A5) and using (A7) we get:

$$\phi > n + bl - \frac{n + bi}{n + I} - \frac{pI - bi}{n + I} - \frac{bl}{1 - \beta} \frac{\beta \exp(-1)}{(n + I)(1 - \beta)}. $$

Hence, if $\beta$ is not too close to 1, this expression is positive and (A5) holds.

3. Unknown Inaptitude

In the paper we assume that inaptitude of child is already known to when education decision is made. In this appendix we explore the possibility that inaptitude is unknown at the time of this decision. We find that the main results of the paper still hold under this specification. However, one important new issue appears here. Under missing information parents and children bargain over the amount of investment in education, and it therefore depends on their relative bargaining strengths. This opens the discussion to the effect of differences in social and cultural norms on inequality and mobility.

We assume that the amount of education is decided before inaptitude is known. When education begins, the child immediately observes whether she can finish school or not, namely whether $e$ is below or above the amount of education purchased. If she can, she remains in school, and if not she leaves school and the parents get their money back. Note that under these informational assumptions, the offspring can bargain with parent on the size of investment in education, threatening not to go to school altogether if the
amount spent is not high enough. This bargaining is of course impossible when inaptitude is known. The underlying conflict here reflects the different interests of parents and children. While the latter want always to have more education, to ensure success as much as possible, parents share this desire, but they also care about own consumption. This conflict of interests is resolved in bargaining. We assume that parent and offspring use a simple form of asymmetric Nash bargaining, which we describe next.

The expected utility of parents at time of bargaining is:

\[
\ln y_j + \beta V_n + e_j [\ln(1 - m_j) + \beta (V_s - V_n)],
\]

where \(j=s\) if parent is skilled and \(j=n\) if parent is unskilled. The threshold levels are: \(e_s = m_s\) and \(e_n = m_n/bI\), as we assume for simplicity that \(h = 1\). The expected utility of children to parents of type \(j\) is:

\[
V_n + e_j (V_s - V_n).
\]

The threat points are \(\ln y_j + \beta V_n\) and \(V_n\) for parent and child respectively, and if the relative bargaining power of parents is denoted by \(q\), the logarithm of the Nash-product is

\[
\ln e_j + q \ln[\ln(1 - m_j) + \beta (V_s - V_n)] + (1 - q) \ln(V_s - V_n).
\]

Substituting the threshold levels and maximizing yields the same level of education out of income for skilled and unskilled, \(m\), which is determined by:

\[
\beta (V_s - V_n) = q \frac{m}{1-m} - \ln(1 - m).
\]

Calculating and adding the optimal expected utilities yield the equilibrium condition, which determines the \(M\) curve:

\[
\frac{1}{\beta} \frac{qm}{1-m} - \frac{1}{\beta} \ln(1-m) - \left(1 - \frac{1}{bl}\right) \frac{qm^2}{1-m} = \ln I.
\]
It is easy to verify that this defines an increasing $M$ curve. The derivation of the rest of the curves is the same as in the main model.

It is clear that the main results of the paper remain intact. The novelty of this version is the ability to examine the effect of a change in the bargaining power of parents, $q$. Since the left hand side of (A12) increases with $q$, it leads to a downward shift of both the $M$ and $MM$ curves, while the $L$ and $ML$ curves remain unchanged. Intuitively, when parents have more bargaining power over their children, they consume more and pay less for their education. Hence, downward mobility decreases and inequality increases. Thus, another way to reduce inequality and increase mobility in a country is to reduce social power of parents and give more power and importance to the young generation.
References


Checchi, Daniele; Ichino, Andrea and Rustichini, Aldo. “More Equal but Less Mobile?


The incentive curve

The intergenerational curve

Figure 1
Figure 2
Figure 3