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**Financing Constraints, Irreversibility,
and Investment Dynamics**

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Abstract

We develop a model of an industry with many heterogeneous firms that face both financing constraints and irreversibility constraints. The financing constraint implies that firms cannot borrow unless the debt is secured by collateral; the irreversibility constraint that they can only sell their fixed capital by selling their business. We use this model to examine the cyclical behavior of aggregate fixed investment, variable capital investment, and output in the presence of persistent idiosyncratic and aggregate shocks. Our model yields three main results. First, the effect of the irreversibility constraint on fixed capital investment is reinforced by the financing constraint. Second, the effect of the financing constraint on variable capital investment is reinforced by the irreversibility constraint. Finally, the interaction between the two constraints is key for explaining why input inventories and material deliveries of US manufacturing firms are so volatile and procyclical, and also why they are highly asymmetrical over the business cycle.

JEL classification: D21, E22, E32, G31
Keywords: Financing Constraints, Irreversibility, Investment

1   Introduction

A body of theoretical literature has shown that asymmetric information and contract incompleteness may limit firms access to external finance and thus make them unable to fund profitable investment opportunities.[1] Over the past 20 years several authors have investigated if financing constraints are an important determinant of the investment at firm level and of the cyclical fluctuations of aggregate investment and output. During the same period other authors have studied the effect of the irreversibility of investment on capital accumulation and aggregate investment dynamics.[2]

The motivation of this paper is that these constraints have always been analyzed in isolation, with very few exceptions (Scaramozzino, 1997 and Holt, 2003). Therefore little is known about the interactions between irreversibility and financing constraints. In this paper we develop a model of firm investment with both irreversibility of fixed capital and borrowing constraints. We show that these constraints interact and reinforce each other, that these interactions amplify the effect of these constraints on firm level and aggregate investment, and that they are important to understand the dynamics of investment over the business cycle.

Our model has three distinctive features. First, output is produced by a risk neutral profit maximizing firm which operates a concave risky technology using two complementary factors of production, fixed and variable capital. Second, new investment in fixed capital takes one period to produce output, while new investment in variable capital is immediately productive. Moreover, the fixed capital stock cannot be disinvested unless the whole business is sold, while variable capital is fully reversible. Third, the firm's only source of external finance is debt secured by collateral assets.

We show that the irreversibility and the financing constraints interact and their effects amplify each other. This amplification is both static and dynamic. The static component is intuitive. If a firm is financially constrained then it can only invest if it has available internal funds. Therefore a negative exogenous shock that reduces the financial wealth of a firm also reduces its investment capacity. If fixed capital is irreversible, then this reduction is absorbed by a reduction in variable capital investment. The dynamic component is less intuitive but not less important. It implies that a binding irreversibility constraint greatly increases the intensity of future financing constraints. As an example, consider a firm which faces a persistent negative productivity shock and expects negative profits for some periods. Fixed capital is inefficiently high because of the irreversibility constraint, and this reduces profits, the return on capital and the market value of the firm. Suppose now that the firm also faces capital market imperfections. Negative profits reduce the financial wealth of the firm and increase the probability of facing a binding financing constraint in the future. If the negative shock is very persistent, financial wealth may be reduced

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[1]Stiglitz and Weiss (1981), Besanko and Thakor (1986), Milde and Riley (1988), Hart and Moore (1998), Albuquerque and Hopenhayn (2000).

[2]See Caballero (1997) for a review of the literature about investment with real constraints and Hubbard (1998) for a review of the literature about investment with financing constraints.

up to the point where both constraints are simultaneously binding. In this case fixed capital is inefficiently high but also variable capital is inefficiently low. The unbalanced use of the two factors of production increases losses and further reduces financial wealth and variable capital investment. When the bad period ends and productivity starts to increase, the firm, if it managed to avoid default, is very cautious about investing in fixed capital. This is because it fears the consequences of future irreversibility and financing constraints. Therefore the interaction between these constraints increases the volatility of variable capital during downturns and reduces the volatility of fixed capital during upturns.

We use this model with interacting financing and irreversibility constraints to explain why inventories are very volatile and why they respond asymmetrically over the business cycle. The high volatility of inventories has been documented by Ramey (1989), Blinder and Maccini (1991) and Ramey and West (1999). Moreover, the same authors show that inventory investment is procyclical, and that the drop in inventories accounts for a large part of the GDP decline in recessions. This indicates that inventory dynamics are important to understand business cycle fluctuations. However it remains an open question if there is more commovement between inventories and output during recessionary than during expansionary periods. In the empirical section of the paper we provide new evidence on this issue. We analyze the dynamics of output, material deliveries and input inventories of several two digits durable goods manufacturing sectors, using quarterly data from 1962 to 1996. We find that both input inventories and deliveries are significantly more procyclical around recessionary than expansionary periods. In almost all the sectors considered, the procyclicality of inventories completely disappears if we only analyze periods in which output is above its trend.

In order to explain this empirical evidence we calibrate our model to simulate the behaviour of an industry with many heterogeneous firms. We model material deliveries and input inventories as the gross flow and the end of the period stock of variable capital respectively. We simulate several artificial industries, over many periods, with and without financing imperfections and irreversibility of fixed capital. The simulated data show that the interactions between the financing and the irreversibility constraint are essential to generate the high volatility of inventories and deliveries relative to fixed capital, as well as their asymmetric behavior in the different phases of the business cycle.

Four recent papers adopt a similar approach to our paper, and analyze an economy with heterogenous firms where financing constraints are binding for a fraction of them in equilibrium: Cooley and Quadrini (2001), Cooley, Marimon and Quadrini (2003), Gomes (2001) and Jermann and Quadrini (2003). In particular Cooley and Quadrini (2001) show that financing imperfections in a model of industry dynamics explain a stylized fact regarding growth dynamics of firms which is not explained by models based only on technological shocks. Moreover our paper is also related to the irreversibility literature, and in particular to Bertola and Caballero (1994), Abel and Eberly (1999) and Veracierto (2002). Finally, our paper is related to Holt (2003), who analyses the effects of the

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interactions between the financing and the irreversibility constraints on the dividend policy along the life cycle pattern of firms.

Yet, our paper is substantially different from all those above. We focus on the interactions between financing and irreversibility constraints as well as on business cycle dynamics rather than growth dynamics of firms. We model theoretically and quantify with simulations a precautionary effect of future expected financing constraints on risky investment. We model theoretically an amplification effect between the irreversibility and the financing constraint and show that such effect is essential for explaining the cyclical fluctuations of variable capital in the US manufacturing sector.

The paper is organized as follows: section 2 illustrates the empirical evidence regarding the dynamics of input inventories and deliveries in the US manufacturing sector; section 3 illustrates the theoretical model; section 4 compares the empirical evidence with the simulation results; section 5 presents the conclusions.

2 Summary of the empirical evidence

The aim of this section is twofold. First, we will analyze the dynamics of input inventories in the durable manufacturing sector. Ramey and West (1999) show that inventory investment is procyclical, and that even though the stock of inventories at the economywide level is small relative to GDP, the reduction in inventories account for a large part of the decline in GDP during recessions. One explanation of this evidence is that inventories generally exhibit very high short term volatility (Hornstein, 1998), and there is nothing specific about the behavior of inventories during recessions with respect to the other phases of the business cycle. If this is true, then we should observe that inventory investment increases as sharply (relative to sales) during upturns as it decreases during downturns. In this section we will show that this is not the case for the input inventories in the US manufacturing sector, which are significantly more procyclical during downturns than upturns periods. This happens both for the whole of the manufacturing sector and for single two digits durable manufacturing industries. We focus on input (materials and work in progress) rather than output inventories, because Ramey (1989), Blinder and Maccini (1991) and Humpreys, Maccini and Schuh (2001) show that input inventories are larger and much more volatile than finished goods inventories, especially in the durable goods sectors.

Second, we will show that the procyclicality and asymmetric behavior of inventory investment is always mirrored by a similar procyclicality and asymmetric behavior of material deliveries. In order to explain this empirical evidence, in the next sections we will model variable capital as a factor of production. We will interpret the gross investment in variable capital as the flow of deliveries, and the net change in the end of the period stock of variable capital as the investment in input inventories. Following Humpreys, Maccini and Schuh (2001) we define deliveries as follows:

$$deliveries_{y,q} = usage\ of\ materials_{y,q} + \Delta input\ inventories_{y,q}$$

where y denotes the year (from 1962 to 1996) and q denotes the quarter. Input inventories are materials plus work in progress. Since quarterly data about the usage of materials are not available, we estimate them as follows:

$$usage\ of\ materials_{y,q} = \frac{usage\ of\ materials_y}{output_y} * output_{y,q}$$

This calculation should be sufficiently accurate since the ratio between usage of materials and output is fairly stable in the short term. In tables 1-5 we illustrate the volatility, procyclicality, and asymmetric behavior of material deliveries and input inventories. Table 1 shows the main statistics for the two digits durable manufacturing sectors for which the Bureau of the Census provides detailed historical data.[3] Industry statistics confirm the empirical evidence, documented by Humpreys, Maccini and Schuh (2001) for the total manufacturing sector, that input inventories are more volatile than finished goods inventories. Moreover deliveries are more volatile than sales in all sectors. The last two columns show that both investment in input inventories and deliveries are procyclical.[4] The fact that the high volatility and procyclicality of input inventories has a direct counterpart in the volatility and procyclicality of the flow of material deliveries is already noted by Humpreys, Maccini and Schuh (2001). Using annual data about the durable goods manufacturing sector, they show that input inventories are very volatile and procyclical mainly because deliveries drop more than usage of materials during downturns and increase more during upturns.

The next three tables answer the following question: is such procyclicality uniform in the different phases of the business cycle? Tables 2, 3 and 4 provide a negative answer to this question. Table 2 shows the correlation between $\frac{\Delta inventories_t}{sales_t}$ and $sales_t$ separately for different sub-periods. This correlation is significantly positive in the periods in which sales are below their trend, while is not significantly different from zero in the periods in which sales are above their trend. This is true for the total of the manufacturing sector and for 5 out of 7 two digits sectors. The second part of the table shows that this correlation is larger when detrended sales are decreasing than when they are increasing. Table 3 provides a similar picture regarding the procyclicality of deliveries.

These statistics are consistent with the fact that the drop in inventories accounts for a large fraction of the drop of GDP during recessions. They also show that the asymmetric behavior of input inventories is mirrored by the one of material deliveries. In the next section we will show that the interaction between the financing and the irreversibility constraint generates firm dynamics consistent with this empirical evidence. The explanation provided by our theoretical model is based on the following intuition. At the beginning of a downturn the negative aggregate productivity shock implies that some

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[3]Source: M3 survey (http://www.census.gov/indicator/www/m3/hist/m3bendoc.htm)

[4]The correlation coefficients regarding inventory investment are smaller in absolute values than the similar coefficients estimated by Ramey and West (1999). This is due to some differences in the way we compute the statistics. We consider durable manufacturing sectors and input inventories while Ramey and West consider the whole of domestic sales and total inventories. We apply the HP filter while Ramey and West apply a linear trend.

Table 1: volatility and procyclicality of deliveries and inventories

Sector	Standard Deviations					Correlations	
	Sales	Deliveries	Material inv.	W.i.p. inv.	Final goods i.	$\left(\frac{\Delta inv_t}{s_t}, s_t\right)$	$\left(\frac{del_t}{s_t}, s_t\right)$
SIC 32	0.043	0.056	0.034	0.039	0.032	0.038** (0.009)	0.006 (0.065)
SIC 33	0.086	0.094	0.050	0.044	0.059	0.038** (0.012)	0.068 (0.023)
SIC 34	0.043	0.059	0.056	0.033	0.034	0.076** (0.026)	0.180** (0.060)
SIC 35	0.061	0.080	0.061	0.058	0.051	0.078** (0.021)	0.162** (0.053)
SIC 36	0.057	0.075	0.075	0.061	0.058	0.118** (0.019)	0.155** (0.059)
SIC 37	0.066	0.083	0.058	0.066	0.053	0.040** (0.016)	0.191** (0.035)
SIC 38	0.039	0.073	0.068	0.069	0.059	0.027 (0.031)	0.312 (0.099)**

Quarterly data. Standard deviations are calculated on the deviations from the trend computed using the Hodrick-Prescott filter (smoothing parameter λ=1600). Standard errors are in parenthesis.

Table 2: correlation $\left(\frac{\Delta inventories_t}{sales_t}, sales_t \right)$

	all observations	below trend	above trend	decreasing sales	increasing sales
Total	0.08**	0.109**	0.038	0.076**	0.084**
Manufacturing	(0.014)	(0.025)	(0.033)	(0.021)	(0.020)
SIC 32	0.038**	0.037*	0.040*	0.041**	0.036*
	(0.009)	(0.017)	(0.020)	(0.013)	(0.015)
SIC 33	0.038**	0.070**	0.006	0.037*	0.042*
	(0.012)	(0.023)	(0.023)	(0.017)	(0.019)
SIC34	0.076**	0.101**	0.040	0.128**	0.022
	(0.026)	(0.044)	(0.058)	(0.036)	(0.037)
SIC35	0.078**	0.174**	-0.036	0.104**	0.072*
	(0.021)	(0.042)	(0.048)	(0.032)	(0.029)
SIC36	0.118**	0.063	0.175**	0.139**	0.102**
	(0.019)	(0.037)	(0.038)	(0.029)	(0.027)
SIC37	0.041**	0.071**	0.002	0.0486*	0.047*
	(0.016)	(0.027)	(0.031)	(0.022)	(0.026)
SIC38	0.027	0.113*	-0.072	0.089*	-0.007
	(0.031)	(0.052)	(0.057)	(0.046)	(0.043)

Quarterly data. Both series $\frac{\Delta inventories_t}{sales_t}$ and $sales_t$ are percentage deviations from their trend computed using the Hodrick-Prescott filter (smoothing parameter λ=1600). Standard errors are in parenthesis. **significant at the 99% confidence level; *significant at the 95% confidence level.

Table 3: correlation $\left(\frac{deliveries_t}{sales_t}, sales_t\right)$

	all observations	below trend	above trend	decreasing sales	increasing sales
Total	0.089**	0.145**	0.091*	0.083**	0.096**
Manufacturing	(0.017)	(0.029)	(0.039)	(0.024)	(0.024)
SIC 32	0.006	0.053**	-0.029	-0.006	0.021*
	(0.065)	(0.136)	(0.114)	(0.088)	(0.099)
SIC 33	0.068**	0.108*	0.028	0.099**	0.044
	(0.023)	(0.044)	(0.045)	(0.036)	(0.032)
SIC 34	0.180**	0.280**	0.030	0.230*	0.120
	(0.060)	(0.110)	(0.130)	(0.090)	(0.090)
SIC 35	0.162**	0.411**	-0.133	0.213**	0.155*
	(0.053)	(0.103)	(0.118)	(0.078)	(0.074)
SIC 36	0.155**	0.003	0.317**	0.108	0.199*
	(0.059)	(0.112)	(0.117)	(0.088)	(0.081)
SIC 37	0.190**	0.239**	0.129	0.186**	0.225**
	(0.035)	(0.057)	(0.067)	(0.046)	(0.055)
SIC 38	0.312**	0.569**	0.017	0.533**	0.169
	(0.099)	(0.168)	(0.185)	(0.146)	(0.138)

Quarterly data. Both series $\frac{deliveries_t}{sales_t}$ and $sales_t$ are percentage deviations from their trend computed using the Hodrick-Prescott filter (smoothing parameter λ=1600). Standard errors are in parenthesis. **significant at the 99% confidence level; *significant at the 95% confidence level.

Table 4: elasticity of deliveries to sales : $\frac{\Delta deliveries_t}{\Delta sales_t}\frac{sales_t}{deliveries_t}$

	Contractions in Sales			Expansions in Sales		
	1st quarter	2nd quarter	3rd quarter	1st quarter	2nd quarter	3rd quarter
Total Manufacturing	1.23** (0.15)	1.27** (0.12)	1.20** (0.37)	0.93** (0.18)	1.21** (0.19)	1.17** (0.28)
SIC 32	0.70* (0.33)	0.78* (0.41)	0.99* (0.51)	0.82** (0.31)	1.35** (0.34)	1.22 (0.72)
SIC 33	0.79** (0.07)	0.75** (0.11)	1.18** (0.12)	1.27** (0.09)	1.17** (0.11)	1.18** (0.17)
SIC 34	1.15** (0.36)	1.66** (0.29)	1.98* (0.83)	1.14* (0.49)	1.18** (0.31)	0.91** (0.31)
SIC 35	1.41** (0.24)	1.77** (0.45)	2.06** (0.29)	0.67* (0.34)	0.53 (0.38)	1.00** (0.34)
SIC 36	1.37** (0.31)	0.99** (0.34)	1.30** (0.40)	0.20 (0.46)	1.28** (0.42)	1.34** (0.34)
SIC 37	1.09** (0.12)	1.21** (0.08)	0.76** (0.17)	0.96** (0.07)	1.00** (0.14)	0.58 (0.48)
SIC 38	1.19* (0.46)	1.66* (0.77)	1.77 (1.22)	1.98** (0.58)	0.59 (0.61)	1.79* (0.83)

The estimated elasticities conditional on more than three consecutive periods of contraction or expansion in detrended real sales are not reported because too few observations do not allow us to estimate significant coefficients. Standard errors are in parenthesis. **significant at the 99% confidence level; *significant at the 95% confidence level.

firms would like to downsize their fixed assets, but they are prevented to do so by the presence of the irreversibility constraint. As the downturn continues and productivity and revenues worsen, some of these firms may also have a binding financing constraint, and hence they may be forced to reduce their investment in variable capital. If this explanation is correct, we expect that the elasticity of deliveries to sales increases conditional on a sequence of negative shocks. Table 4 confirms this. It shows the estimated elasticity of deliveries to sales conditional on the number of periods of subsequent reduction or increase in detrended real sales. During periods of contraction in sales we observe that in 5 out of 7 sectors the elasticity tends to increase conditional on the contraction lasting longer. This is especially true for sectors 34, 35, and 38. These sectors are also those with the highest degree of asymmetry in the procyclicality of input inventories and deliveries (see tables 2 and 3). The same is not true during periods of expansion in sales. Here the elasticity of deliveries does not have any clear increasing or decreasing pattern.

Finally, our theory predicts that financing constraints are an essential factor to gen-

Table 5: summary statistics from the 1992 survey of manufacturers

	SIC code	N. firms	Total payroll	% of total payroll from firms smaller than 50 employees
Stone, Clay, Glass Products	32	16254	13113	30.8
Primary Metal	33	6501	22202	7.2
Fabricated metal products	34	36429	38962	40.8
Industrial machinery	35	53956	57231	24
Electronic and other electric equip.	36	16922	8978	4.7
Transportation Equipment	37	11287	62733	3.5
Instruments and related products	38	11354	33067	8.5

erate the asymmetry in the dynamics of input inventories and deliveries. If this is true, we should observe that such asymmetry is stronger for smaller rather than larger firms. Unfortunately we do not have the availability of data disaggregated in the size dimension. Nonetheless in table 5 we show the size distribution of the two digits sectors considered before. Apart from sector SIC 32, which does not show any significant procyclicality of deliveries in the first place, we observe that the three sectors that showed the highest degree of asymmetry in the procyclicality of deliveries and input inventories (SIC 34, SIC 35 and SIC 38) are also those with the highest density of small firms. Conversely the only sector in which the asymmetry goes in the opposite direction (SIC 36), is also one of the sectors with lower density of small firms.

3 The model

In this section we develop a model that studies the interactions between financing and irreversibility constraints. We consider an industry populated by many firms. Each firm chooses investment in fixed and variable capital in order to maximize the expected discounted sum of future dividends. As all firms have access to the same risky technology, we will describe and solve the optimal investment problem of one generic firm. k_t and l_t are respectively the stock of fixed and variable capital. Variable capital represents variable inputs such as raw materials and work in progress, while fixed capital represents fixed inputs such as plant and equipment. For simplicity, labour is not considered in the analysis. However, the inclusion of an additional factor of production would not affect the results. Output y_t is produced according to a Cobb-Douglas production function:

$$y_t = e^{\theta_t} k_t^\alpha l_t^\beta \text{ with } \alpha > 0; \beta > 0; \alpha + \beta < 1 \qquad (1)$$

θ_t is a stationary autoregressive stochastic process representing the productivity shock. Variable capital l_t is immediately productive, and is reversible. Fixed capital purchased at time t takes one period to become productive and once installed it cannot be liquidated unless the whole of the assets is sold. This means that, if the firm continues the activity in period t, fixed capital k_{t+1} is subject to the following irreversibility constraint:

$$k_{t+1} \geq (1 - \delta_k) k_t \tag{2}$$

δ_k is the depreciation rate of fixed capital. Constraint (2) is justified by the fact that in many industries plant and equipment do not have a secondary market because they cannot be easily converted to other productions. It is also consistent with the empirical evidence from a large sample of US manufacturing plants analyzed by Caballero, Engel and Haltiwanger (1995). Moreover, constraint (2) implies that fixed capital is fully irreversible. Another possibility would be to assume partial irreversibility (fixed capital has a resell price lower than its value in the firm). As long as the wedge between the selling price and the internal value is not negligible, both full and partial irreversibility would generate similar qualitative results.

We introduce financial markets imperfections by assuming that equity finance and risky debt are not available. At time t the firm can borrow (and lend) one period debt, with face value b_t, at the market riskless rate r. A positive (negative) b_t indicates that the firm is a net borrower (lender). We assume that, due to an enforceability problem, lenders only lend secured debt, and the only collateral they accept is the next period residual value of physical capital. Therefore at the time t the amount of borrowing is limited by the following constraint:[5]

$$b_t \leq \tau_k k_t + \tau_l l_t \tag{3}$$

τ_k and τ_l are the shares of fixed and variable capital that can be used as collateral:

$$\tau_k \leq (1 - \delta_k)^2 \tag{4}$$

$$\tau_l \leq 1 - \delta_l \tag{5}$$

δ_l is the depreciation rate of variable capital. Fixed capital is accepted as collateral because constraint (2) does not apply when the whole of the assets are liquidated. Constraints (4) and (5) holding with equality imply that the end of period t residual capital

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[5]The rationale for this collateral constraint is that the firm can hide the revenues from the production. Being unable to observe such revenues the lenders can only claim, as repayment of the debt, the value of the firm's physical assets (Hart and Moore, 1998). Therefore the firm can only lend or borrow one period secured debt at the market interest rate r offered by the lenders. We also implicitly assume that in case of default and renegotiation of the debt with the lender, the firm has all the bargaining power. Otherwise the lender could use the threat of liquidation of fixed capital to enforce the repayment of uncollateralised debt.

is fully collateralisable.[6] In the case of constraint (4) the term $1 - \delta_k$ is squared because at the beginning of period t capital k_t has already depreciated at the rate δ_k (see equation 8), and therefore at the end of period t the residual value of one unit of k_t is equal to $(1 - \delta_k)^2$.

The timing of the model is the following: the firm inherits from time $t - 1$ the fixed capital k_t and the net worth w_t. At the beginning of period t the shock θ_t is realized, and the firm can either liquidate or continue activity. If the firm is liquidated its assets are sold and its net worth w_t is distributed as dividends. If the firm continues activity it pays the fixed cost $F > 0$ and borrows one period debt with face value b_t, receiving the discounted value b_t/R. The net worth w_t plus the new borrowing b_t/R are allocated between dividends, variable capital and fixed capital:

$$d_t + k_{t+1} + l_t = w_t + \frac{b_t}{R} - F \tag{6}$$

$$d_t \geq 0 \tag{7}$$

d_t are the dividends distributed at time t. After producing the firm repays the debt b_t at the end of the period. Therefore residual net worth at the end of period t is:[7]

$$w_{t+1} = y_t - b_t + (1 - \delta_k) k_{t+1} + (1 - \delta_l) l_t \tag{8}$$

The fixed cost F ensures that on average a significant fraction of firms earn negative profits. In addition to generating realistic dynamics of revenues and profits, this assumption ensures that a positive fraction of firms have both the financing and the irreversibility constraint binding. In the next section we will show that the interaction between the two constraints is essential for the results obtained in the simulations of the model. Furthermore, the timing assumptions in equations (6) and (8) imply that fixed capital takes two period to generate revenues. This is because fixed capital purchased at time $t - 1$ generates output in period t, but y_t cannot be used to finance period t investment, because it enters the law of motion of wealth at time $t + 1$ (see equation 8). This timing assumption is necessary because if we allow y_t to enter the law of motion of wealth at time t, then equation (6) would imply that variable capital investment could be paid with its own revenues. In this case financing constraints would be binding only for a negligible share of firms in the simulated industries.[8]

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[6]Constraints (4) and (5) holding with disequality are equivalent to assume that the firm can 'steal' a $(1 - \tau)$ fraction of the residual value of capital. To assume that l_t is collateralisable is equivalent to assume that the firm is given short term trade credit by its suppliers, who then discount such credit at a bank, which is willing to assume the liability because this is secured by the value of variable capital. Regarding fixed capital, the results of the model are not sensitive to whether we assume k_t or k_{t+1} to be available as collateral in period t. Therefore we chose the more realistic assumption. k_t is phisical capital existing at the beginning of time t, and can be observed by the lenders at the time they lend the funds. Conversely k_{t+1} is fixed capital that will be in place only at the end of period t.

[7]Equations (1) and (8) imply that one unit of installed fixed capital k_{t+1} is fully productive in period $t + 1$, while it's market value is reduced by the depreciation rate δ_k.

[8]This is because variable capital investment, $l_t - (1 - \delta_l) l_{t-1}$, is on average much larger than fixed capital investment, $k_{t+1} - (1 - \delta_k) k_{t-1}$. In the calibration of the model with quarterly data, l_t/k_{t+1} is on average around 1, δ_k around 3% and δ_l around 50%.

Liquidation at the beginning of period t can happen for two different reasons: i) exogenous liquidation, with probability $1 - \gamma$. This ensures that the distribution of firms does not degenerate to the point where all firms are very wealthy and no one is financially constrained. ii) Voluntary liquidation. After observing θ_t, the firm decides to liquidate before paying the fixed cost F if the net present value of continuing activity is lower than the liquidation value of the assets of the firm. In theory the contemporaneous presence of the financing and the irreversibility constraint can also cause an endogenous "inefficient" liquidation where a firm is forced to sell the business in order to repay the debt, even though the business has a positive net present value (see appendix B for details). In practice, however, inefficient liquidation is never part of the set of optimal choices conditional on the set of parameters we use for the simulations.

We formulate now the intertemporal maximization problem. We denote the expected value of the firm at time t, after θ_t is realized, and conditional on not liquidating in period t, by $V_t^{stay}(w_t, \theta_t, k_t)$, where w_t, θ_t and k_t are the three state variables of the problem:

$$V_t^{stay}(w_t, \theta_t, k_t) = \max_{l_t, k_{t+1}, b_t} d_t + \frac{1}{R} E_t \left[V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1}) \right] \tag{9}$$

$V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1})$ is defined as follows:

$$V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1}) = \gamma S_{t+1} V_{t+1}^{stay}(w_{t+1}, \theta_{t+1}, k_{t+1}) + (1 - \gamma S_{t+1}) w_{t+1} \tag{10}$$

S_t is the following binary variable:

$$S_t = \left\{ \begin{array}{l} 1 \text{ if } V_t^{stay}(w_t, \theta_t, k_t) > w_t \text{ and } F \leq w_{t+1} - \left[(1 - \delta_k) - \frac{\tau_k}{R} \right] k_{t+1} \\ 0 \text{ otherwise} \end{array} \right. \tag{11}$$

The condition $\left[F \leq w_{t+1} - \left[(1 - \delta_k) - \frac{\tau_k}{R} \right] k_{t+1} \right]$ rules out inefficient liquidation, as explained in appendix B. The firm's problem is defined by equation (9) subject to constraints (2), (3), (6) and (7). The solution to the problem is obtained using a numerical method, and is illustrated in detail in appendix A. A unique solution to this problem may not exist in general, because equation (10) implies that V_{t+1} may not be concave. However we can show that, for the sets of parameter values chosen for the simulations presented in the next sections, the function $E_t \left[V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1}) \right]$ is concave in (w_t, θ_t, k_t), and therefore also the function $V_t^{stay}(w_t, \theta_t, k_t)$ is concave, and the solution found is thus unique.

In the remaining parts of this section we provide a description of the first order conditions of the problem in the special case when endogenous exit never happens in equilibrium ($S_t = 1$ for any t). This simple case is useful to illustrate the effects of the interactions between the financing and the irreversibility constraint on investment decisions. We substitute $S_t = S_{t+1} = 1$ and $V_t = V_t^{stay}$ in equations (9) and (10), obtaining:

$$V_t(w_t, \theta_t, k_t) = \max_{l_t, k_{t+1}, b_t} d_t + \frac{1}{R} E_t \left[\gamma V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1}) + (1 - \gamma) w_{t+1} \right] \tag{12}$$

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Let μ_t λ_t and ϕ_t be the Lagrangian multipliers associated to the constraints $(2), (3)$ and (7). Also let π_{t+1} be the value at the beginning of period $t+1$ of the gross economic profits realized by the firm during period t:

$$\pi_{t+1} = y_t - \Lambda_{t+1}\delta_k k_t - \delta_l l_t - RF \tag{13}$$

$$\Lambda_{t+1} = \frac{R}{\gamma}\frac{1+\gamma\phi_t}{1+\gamma\phi_{t+1}}$$

The cost of fixed capital depreciation, $\delta_k k_t$, is multiplied by the term Λ_{t+1} which takes into account that the cost of capital is affected by the shadow value of money in period $t+1$ relative to the shadow value in period t. Λ_{t+1} is inversely related to γ because with probability $1-\gamma$ the firm liquidates at the beginning of period t before the installed capital k_t can be used for production. Substituting d_t in equation (12) using equation (6) and taking the first order conditions of equation (12) with respect to b_t, l_t and k_{t+1}, it is possible to show that the solution of the problem is given by the optimal sequence of $\{k_{t+1}, l_t, b_t, d_t, \lambda_t, \mu_t, \phi_t \mid k_t, w_t, \theta_t, \Theta\}_{t=0}^{\infty}$ which satisfies equations (2), (14), (15), (16) and (17), plus the standard complementary slackness conditions on λ_t, μ_t and ϕ_t:

$$\phi_t = R\lambda_t + \gamma E_t\left(\phi_{t+1}\right) \tag{14}$$

$$\left[1 + \gamma E_t\left(\phi_{t+1}\right)\right]\left(\pi_{t+1}^l - r\right) = (R - \tau_l)R\lambda_t \tag{15}$$

$$\frac{\gamma}{R}E_t\left[\left(1 + \gamma\phi_{t+2}\right)\pi_{t+2}^k\right] - (1 + \phi_t)r = R\lambda_t - \gamma\tau_k E_t\left(\lambda_{t+1}\right) - R\mu_t + \gamma\left(1 - \delta_k\right)E_t\left(\mu_{t+1}\right) \tag{16}$$

$$d_t + \left(1 - \frac{\tau_l}{R}\right)l_t + k_{t+1} \leq w_t + \frac{\tau_k}{R}k_t \tag{17}$$

$\pi_{t+2}^k = \frac{\partial\pi_{t+2}}{\partial k_{t+1}}$ and $\pi_{t+1}^l = \frac{\partial\pi_{t+1}}{\partial l_t}$ are the marginal gross profits of fixed and variable capital respectively.[9] Equation (17) combines the budget constraint (6) and the collateral constraint (3) and implies that the downpayment necessary to finance k_{t+1}, l_t and d_t must be lower than the net wealth of the firm plus the additional funds borrowed using k_t as collateral. $\left(1 - \frac{\tau_l}{R}\right)$ is the downpayment necessary to buy one unit of variable capital. λ_t is positive when the financing constraint is binding in period t (equation 17 is holding with equality), and is equal to zero otherwise. ϕ_t is positive when the financing constraint is binding in period t or has some probability to bind in the future. This can be easily seen by iterating forward equation (14):

$$\phi_t = R\sum_{j=0}^{\infty}\gamma^j E_t\left(\lambda_{t+j}\right) \tag{18}$$

Equation (18) implies that a positive ϕ_t represents the additional value of financial wealth for the firm in terms of its ability to reduce financing constraints. As long as

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[9] π_{t+1}^l is known at time t because it does not depend on the term Λ_{t+1}.

$\phi_t > 0$ then the gross return on money invested in the firm is higher than the gross market return R, and the firm does not distribute dividends ($d_t = 0$). Equations (15) and (16) are the first order conditions for variable capital and fixed capital respectively. In equation (16) the terms $(1 + \phi_t) r$ and $\frac{\gamma}{R} E_t \left[\left(1 + \gamma \phi_{t+2} \right) \pi_{t+1}^k \right]$ are the marginal cost and the marginal gross profits of fixed capital respectively. Capital k_{t+1} is purchased at time t, generates output during period $t + 1$ but the corresponding profits become available at the beginning of period $t + 2$, and therefore they are evaluated at their expected shadow value ϕ_{t+2}. μ_t is positive when the irreversibility constraint (2) is binding, and is equal to zero otherwise. The term $(1 - \delta) \beta E_t \left(\mu_{t+1} \right)$ is the cost of future expected irreversibility constraints. In the next subsections we will describe the main qualitative features of the model. We will first analyze the solution without the financing constraint, then we will analyze the solution without the irreversibility constraint, and finally we will explain how the two constraints interact with each other.

3.1   Solution with the irreversibility constraint only

In this subsection we rule out current and future expected financing constraints by assuming that w_0, the initial wealth of the firm, is high enough so that the borrowing constraint (3) is never binding. This means that $\lambda_t = 0$ for any t, and the first order conditions (14), (15) and (16) can be simplified as follows:

$$\phi_t = 0 \tag{19}$$

$$\pi_{t+1}^l - r = 0 \tag{20}$$

$$\frac{\gamma}{R} E_t \left(\pi_{t+2}^k \right) - r = \gamma \left(1 - \delta_k \right) E_t \left(\mu_{t+1} \right) - R \mu_t \tag{21}$$

Equations (2), (20) and (21) jointly determine μ_t, l_t and k_{t+1}. They describe the solution to a version of a well known irreversible investment problem (e.g. see Bertola and Caballero, 1994). Since we allow for a multifactor production technology the consequence is that l_t, the reversible factor, is more volatile than k_t, the irreversible factor, both after a positive and a negative shock. This follows from the comparison of equations (20) and (21). Equation (20) implies that variable capital always reacts to both positive and negative productivity shocks. Constraint (2) instead implies that, after a negative productivity shock at time t, k_{t+1} cannot be reduced below $(1 - \delta_k) k_t$, and as a consequence $\frac{\gamma}{R} E_t \left(\pi_{t+2}^k \right)$ decreases and is compensated by an increase in μ_t on the right hand side of equation (21). Instead after a positive productivity shock $\mu_t = 0$ but $E_t \left(\mu_{t+1} \right) > 0$ because constraint (2) can be binding at time $t+1$ conditional on a future negative shock. The positive value of $E_t \left(\mu_{t+1} \right)$ in equation (21) is compensated by a reduction in k_{t+1} that increases $\frac{\gamma}{R} E_t \left(\pi_{t+2}^k \right)$.

3.2   Solution with the financing constraint only

In this section we rule out current and future expected irreversibility constraints by assuming that both variable capital and fixed capital are reversible, and the irreversibility

constraint (2) no longer applies. Substituting $\mu_t = E_t\left(\mu_{t+1}\right) = 0$ in (16) we obtain the following:

$$\frac{\gamma}{R}E_t\left[\left(1 + \gamma\phi_{t+2}\right)\pi_{t+2}^k\right] - (1 + \phi_t)\,r = R\lambda_t - \gamma\tau_k E_t\left(\lambda_{t+1}\right) \tag{22}$$

In this case equations (14), (15), (17) and (22) jointly determine ϕ_t, λ_t, l_t and k_{t+1}. If the firm does not have enough resources to invest optimally, then constraint (17) is binding with equality and both λ_t and ϕ_t are positive. Equations (15) and (22) imply that a positive λ_t increases both π_{t+1}^l and π_{t+2}^k, and this reduces both l_t and k_{t+1}. Suppose instead that the constraint (17) is not binding, but there is a positive probability to face future financing constraints. In this case $\lambda_t = 0$ but $E_t\left(\lambda_{t+1}\right)$ and ϕ_t are positive. Equation (15) simplifies to equation (20), because future expected financing constraints do not directly affect l_t, the variable capital investment decisions. Regarding fixed capital, equation (22) implies that financing constraints expected at time $t + 1$ have two counteracting effects on k_{t+1}. First, higher expected financing constraints increase $\gamma\tau_k E_t\left(\lambda_{t+1}\right)$, the expected collateral value of fixed capital in period $t + 1$. This reduces $\frac{\gamma}{R}E_t\left[\left(1 + \gamma\phi_{t+2}\right)\pi_{t+2}^k\right]$, the required gross return on capital, and increases k_{t+1}. Second, equation (15) implies that $E_t\left(l_{t+1}\right)$ decreases in $E_t\left(\lambda_{t+1}\right)$. This means that the more the firm is expected to be financially constrained in the next period, the lower is the expected level of variable capital investment l_{t+1}. Since k_{t+1} and l_{t+1} are complementary, it follows that the lower is $E_t(l_{t+1})$, the lower is the expected profitability of fixed capital, and the lower is k_{t+1}. Simulation results show that this negative effect always dominates on the positive effect of the term $\gamma\tau_k E_t\left(\lambda_{t+1}\right)$. The implication is that two firms with identical technology and identical profitability, and both currently not financially constrained, may choose very different fixed capital investment levels depending on their level of wealth, because the latter affects future expected financing constraints. In section 4 we will show that this "precautionary" effect on investment is quantitatively important, and reduces the volatility of fixed capital in a way that is similar to the effect of convex adjustment costs.

3.3 Solution with the financing and the irreversibility constraints

We now consider the solution of the problem with both constraints. Instead of describing in detail such solution, we focus only on the most interesting feature, namely the fact that the irreversibility and the financing constraint interact and reinforce each other. Not only do financing constraints increase the cost of future irreversibility constraints, but also the irreversibility of fixed capital amplifies the effects of financing constraints on variable capital investment. In order to see this we evaluate equation (17) for $d_t = 0$ and $k_{t+1} = (1 - \delta)k_t$. This allows us to define an upper limit on new investment in variable capital:

$$\left(1 - \frac{\tau_l}{R}\right)l_t \le w_t - \left[(1 - \delta_k) - \frac{\tau_k}{R}\right]k_t \tag{23}$$

The left hand side of equation (23) is the downpayment necessary to buy the variable capital l_t. $\left[(1 - \delta_k) - \frac{\tau_k}{R}\right]k_t$ is the minimum financing needed to sustain the current level of fixed capital. Suppose now that the firm is hit by a persistent and negative productivity

shock, so that both the financing and the irreversibility constraints are binding at times t and $t+1$. By evaluating equation (23) with equality both at times t and $t+1$ it follows that:

$$\Delta l_{t+1} = \frac{R}{R - \tau_l} \left\{ \Delta w_{t+1} + \delta_k \left[(1 - \delta_k) - \frac{\tau_k}{R} \right] k_t \right\} \tag{24}$$

Because δ_k, the quarterly depreciation rate of fixed capital, is typically very small, equation (24) implies that the reduction in wealth caused by the negative shock ($\Delta w_{t+1} < 0$) is mainly absorbed by a negative change in variable capital ($\Delta l_{t+1} < 0$). This reduces the ratio $\frac{l_{t+1}}{k_{t+1}}$ below the optimal level. The unbalanced use of the factors of production reduces the productivity of fixed capital and the firm's output. This means that if a similar negative shock also hits the firm in period $t+2$ it will cause a bigger reduction in w_{t+2} and l_{t+2}, and will further reduce the productivity of fixed capital and output, and so on.

We now illustrate the opposite amplification effect: future expected financing constraints increase the expected cost of a binding irreversibility constraint and reduce the incentive to invest in fixed capital. First we notice that, when both constraints are binding, μ_t is determined by equation (16). By substituting recursively we obtain:

$$\mu_t = \frac{1}{R} \sum \left[\frac{\gamma (1 - \delta_k)}{R} \right]^j E_t \left[\left(1 + \phi_{t+j} \right) r - \frac{\gamma}{R} \left(1 + \gamma \phi_{t+2+j} \right) \pi^k_{t+1+j} + R \lambda_{t+j} - \gamma \tau_k \lambda_{t+1+j} \right] \tag{25}$$

In order to interpret equation (25) it is useful to evaluate it for the case of no financing constraints ($\lambda_t = \phi_t = 0$ for any t):

$$\mu_t = \frac{1}{R} \sum \left[\frac{\gamma (1 - \delta_k)}{R} \right]^j \left[r - \frac{\gamma}{R} E_t \left[\pi^k_{t+1+j} \right] \right] \tag{26}$$

The term $r - \frac{\gamma}{R} E_t \left[\pi^k_{t+1+j} \right]$ is positive if the irreversibility constraint is binding in period $t+j$. Therefore equation (26) shows that μ_t is the expected discounted sum of the marginal loss in revenues caused by the fact that the stock of fixed capital is inefficiently high when the irreversibility constraint is binding. Now compare equation (26) with equation (25). In the latter case, where the firm also faces financing imperfections, μ_t increases in $E_t \left(R \lambda_{t+j} - \gamma \tau_k \lambda_{t+1+j} \right)$. This term is positive, and represents the net cost of future expected financing constraints. The intuition is that the lower return on capital caused by the irreversibility constraint is going to be a bigger problem when the firm is financially constrained, and must rely on internal funds to finance investment. Therefore financing constraints increase the cost of irreversibility μ_t. But this implies that they also increase the value of $E_t \left(\mu_{t+1} \right)$ when the irreversibility constraint is not currently binding, and so they make the firm more cautious about investing in fixed capital.

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4 Numerical Solution and simulations

4.1 The solution of the investment problem

We solve the intertemporal maximization problem using a numerical method (see appendix C for details). Adding the subscript i to indicate the generic $i-th$ firm, the solution consists of the optimal policy functions $k_{i,t+1}(w_{i,t}, \theta_{i,t}, k_{i,t})$ and $l_{i,t}(w_{i,t}, \theta_{i,t}, k_{i,t})$, the associated Lagrange multipliers $\lambda_{i,t}(w_{i,t}, \theta_{i,t}, k_{i,t})$, $\mu_{i,t}(w_{i,t}, \theta_{i,t}, k_{i,t})$ and $\phi_{i,t}(w_{i,t}, \theta_{i,t}, k_{i,t})$, the value function $V_{i,t}^{stay}(w_{i,t}, \theta_{i,t}, k_{i,t})$ and the liquidation rule $S_t(w_{i,t}, \theta_{i,t}, k_{i,t})$. $\theta_{i,t}$ is defined as follows:

$$\theta_{i,t} = \theta_{i,t}^f \varepsilon_t \tag{27}$$

$\theta_{i,t}^f$ is the idiosyncratic productivity shock, and ε_t is the industry-wide shock common to all firms. ε_t is introduced in order to study the implications of the model for the dynamics of investment at the industry level. The model is otherwise partial equilibrium, with constant interest rate and constant relative prices normalized to 1. Both $\theta_{i,t}^f$ and ε_t are first order autoregressive stochastic processes:

$$\begin{aligned} \theta_{i,t}^f &= \rho_\theta \theta_{i,t-1}^f + \zeta_{i,t}^\theta \\ \zeta_{i,t}^f &\sim iid\ N\left(\overline{\theta}, \sigma_\theta^2\right) \end{aligned} \tag{28}$$

$$\begin{aligned} \varepsilon_t &= \rho_\varepsilon \varepsilon_{t-1} + \zeta_t^\varepsilon \\ \zeta_t^\varepsilon &\sim iid\ N\left(0, \sigma_\varepsilon^2\right) \end{aligned} \tag{29}$$

In all the following simulations $\theta_{i,t}^f$ and ε_t are discretised respectively as two states and eight states symmetric Markov processes:

$$\theta_{i,t}^f \{\theta_L, \theta_H\} \text{ where } \theta_L < \theta_H$$

$$\varepsilon_{i,t} \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_8\} \text{ where } \varepsilon_1 < \varepsilon_2 < ... < \varepsilon_8$$

The benchmark parameters are illustrated in table 6. We calibrate the model to

match the long run averages of output and capital stock for the "Fabricated Metals Sector" in the US from 1962 to 1995 (source: NBER-CES manufacturing industry database, SIC code 34). Therefore the ability of the model to explain the cyclical fluctuations of investment will be measured by comparing the statistics from the simulated industry with the corresponding empirical data for the SIC 34 sector. This sector has been chosen as a generic representative one, but the theoretical model can be applied to any other sector where productive units use a combination of reversible and irreversible factors of production and can be subject to borrowing constraints. In fact in section 2 we showed that the stylized facts that this model aims at explaining are common to the whole of the US manufacturing sector as well as to almost all the 2 digits durable goods manufacturing sectors. The main advantage of using the SIC 34 sector, rather than the whole manufacturing industry, is that this sector has an high density of small firms. As table 5 shows, the SIC 34 sector is populated by many firms, 36429 according to the statistics for year

	Base calibration	Empirical restriction	Empirical data	Simulated data
α	0.072	k_t/l_t	1.06 (SIC 34)	1.02
β	0.898	returns to scale $(\alpha + \beta)$	0.97 (SIC 34)	0.97
δ_l	0.5	Materials usage / Input invent.	$\approx$1 (SIC 34)	1
ρ^a	0.9	$corr\,(y_t, y_{t+1})$	0.77 (SIC 34)	0.70
σ^a	0.0097	st.dev.$(sales_t)$	0.043 (SIC 34)	0.043
$\overline{\theta}$	0.82578	average size of fixed assets	58 851 (SIC 34)	
γ	0.964	% firms $\leq$ 90% avg. size	71% (SIC 34)	70%
F^1	0.012	quarterly turnover	3.67% (SIC 34)	3.75%
w_0^1	0.33	fraction of negative profits	37% (Worldscope)	42%
δ_k	0.03	$\frac{i_t}{k_t}$	0.145 (Compustat)	0.140
σ_θ	0.0182	$\text{std}\left(\frac{i_t}{k_t}\right)$	0.139 (Compustat)	0.095
ρ_θ	0.7	$\text{corr}\left(\frac{i_t}{k_t}, \frac{i_{t-1}}{k_{t-1}}\right)$	0.239 (Compustat)	0.139
τ_l	0	(imperfect enforceability)		
τ_k	0.679	debt/assets ratio	0.3 (corporate sector)	0.32
r	0.01	quarterly real interest rate	1%	1%

1: both F and w_0 are expressed as fractions of the average size of fixed capital of a financially unconstrained firm.

1992, the majority of which were small firms: 40.8% of the total payroll was generated from firms with less than 50 employees. Therefore it is reasonable to assume that a large share of these firms is affected by financing imperfections, like the firms in the simulated industry.

Table 7 summarizes the empirical evidence about the volatility of deliveries and inventories for the SIC 34 sector. The standard deviations are computed on the deviations from the Hodrick-Prescott nonlinear trend. We choose λ=1600 for quarterly series and λ=100 for yearly series. Table 7 confirms the empirical evidence, illustrated in section 2,

that input inventories are more volatile than output inventories and that deliveries are more volatile than materials usage, sales and output. Moreover deliveries drop relatively more than sales during recessions, while at the same time input inventories drop relatively more than finished goods inventories.

In calibrating the model the set of technological parameters $\alpha, \beta, \delta_l, \overline{\theta}, \rho_a$ and σ_a are chosen to match the aggregate statistics about the SIC 34 sector. δ_l is set in order to match the average life of materials, with the following procedure: in the model $(1-\delta_l)l_t$ is the residual value of variable capital at the end of period t, after production takes place. Therefore we interpret $(1-\delta_l)l_t$ as the stock of input inventories, and the gross investment in variable capital can be interpreted as deliveries:

$$\text{deliveries}_t = l_t - (1-\delta_l)l_{t-1} \tag{30}$$

Total variable capital available for production in period t is l_t, and $\delta_l l_t$ can be interpreted as the usage of materials in period t. Therefore δ_l must satisfy the following

Table 7: summary statistics, Fabricated Metals Sector (SIC 34)

	Yearly average[1]	St.deviation[2]		Avg. % drop in recessions relative to the drop in Sales
	Y	Y	Q	Q
Sales	129.512	0.0564	0.0429	100
Output	129.611	0.0568	0.0440	109
Mat. deliveries	65975	0.0654	0.0594	143
Mat. inventories	5689	0.0734	0.0556	58
W.i.p.. inventories	4977	0.0527	0.0325	44
Finished g. inv.	4276	0.0350	0.0343	18
Materials usage	65892	0.0584		
Employment (n. workers)	1439	0.0515		
Employment (hours)	2208	0.0544		
Capital stock (equipment)	34087	0.0234		
Capital stock (plant)	24764	0.0110		

Y=yearly; Q=quarterly; 1.Values are in real terms. 2.Standard deviations of percentage deviations from the trend calculated with the HP filter.

equation:

$$\frac{(1 - \delta_l)l_t}{\delta_l l_t} = \frac{\text{input inventories at the end of year } t}{\text{usage of materials in year } t} \tag{31}$$

Solving for δ_l we obtain:

$$\delta_l = \frac{1}{\frac{\text{input inventories at the end of year } t}{\text{usage of materials in year } t} + 1} \tag{32}$$

In the empirical data the ratio $\frac{\text{input inventories at the end of year } t}{\text{usage of materials in year } t}$ is approximately equal to one during the sample period. Thus δ_l is calibrated to be equal to 0.5. $\bar{\theta}$ is calibrated to match the average size of the firm.[10] α and β match a chosen level of return to scale of 0.97, which is consistent with empirical micro studies (Burnside, 1996), and the average ratio of fixed over variable capital. ρ_ε and σ_ε are chosen in order to match the volatility and autocorrelation of aggregate output of the SIC 34 sector.[11]

The parameters F, w_0 and γ affect the distribution and the turnover of firms in the simulated industry. They are jointly set to match the following statistics: i) the average fraction of firms that report negative net income in the US. Since this information is not available for the SIC 34 sector, we calculate it from a sample of more than 1300 US industrial companies drawn from the Worldscope Database and analyzed in Caggese (2004). ii) The turnover rate. In the model a firm that exits production is a firm that liquidates all its assets and is replaced by a newborn firm. Therefore this turnover can be interpreted

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[10]Since the technology has decreasing returns to scale, then for a given value of $\alpha + \beta$ the average size of the firm in the steady state is monotonously increasing in θ.

[11]Since the simulated data are stationary, we detrend the empirical data using the Hodrick-Prescott filter.

as either firms destruction or as property change (the new firm acquires the assets of the liquidating firm). The annual average plant destruction rate in the SIC 34 sector has been 6.4% for the 1995-2000 period.[12] The rate of ownership change for the plants in the manufacturing sector can be inferred from "The Manufacturing Plant Ownership Change Database" (Nguyen, 1998). By dividing the number of ownership changes with the total number of plants surveyed, we calculate an annual rate of approximately 7.5%. In the model this corresponds to a quarterly turnover rate of 3.67%.[13] iii) The size distribution of firms.

δ_k, σ^f and ρ^f are calibrated to jointly match the average, the standard deviation and the autocorrelation of the investment rate in fixed capital. Since this information is not available for the SIC 34 sector, we match the values for the Compustat sample provided by Gomes (2001). Table 6 shows that the matching of the volatility and the autocorrelation of firm level investment is not perfect. One reason is that the mapping between the parameters and the moments matched is very nonlinear, and the volatility and persistency of the idiosyncratic shock affects contemporaneously many of the other moments, both at the firm and at the industry level. In particular it is difficult to match the autocorrelation coefficient of the fixed capital investment rate because of the absence of convex adjustment costs in the model. Despite this, the model with both constraints is still able to generate a positive autocorrelation of the fixed capital investment rate, because current and future expected financing constraints dampen the reaction of fixed capital to the productivity shocks, as illustrated in section 3.2 and in figure 2.

Regarding the financial parameters we set r, the quarterly real interest rate, equal to 1%. τ_l, the fraction of variable capital that can be used as collateral, is set equal to zero, while τ_k matches the debt/assets ratio observed on average in the US corporate sector. This choice of allocating all the collateral capacity to the fixed capital is not necessarily consistent with the empirical evidence. Therefore in the next section we analyze the sensitivity of the results to different assumptions regarding τ_k and τ_l. The last two columns of table 6 report the empirical moments matched in the calibration and the corresponding moments in the benchmark simulated industry.

In the next sections we compare the simulations of the industry with the financing and irreversibility constraints with three alternative industries, one with only the financing constraint, one with only the irreversibility constraint, and one with no constraints. The benchmark parameters for these industries are illustrated in table 8.[14]

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[12]Both the exit rate statistics and the ownership change statistics actually refer to plants rather than to firms. But most of the firms in the SIC34 sector are small single plant firms, and therefore these statistics are a good approximation of their rate of turnover.

[13]This value is calculated by considering that the annual destruction rate and the property change rate are calculated on firms existing at the beginning of the year, and by assuming that the destruction probability is independent from property change probability. Since we set $\gamma = 0.965$, the exit rate of 3.75% in the simulated industry corresponds to 3.35% quarterly rate of exogenous liquidations and 0.4% of voluntary liquidations.

[14]Table 8 illustrates the calibrated parameters for the four simulated industries. The main difference between the parameter sets of each industry is the difference in the volatility and persistency of the productivity shock. In the industry with only the irreversibility constraint it is necessary a very high persistency of the idiosyncratic shock to generate some autocorrelation in the fixed capital investment

Table 8: calibrated parameters in the four industries

	Both constraints	only fin. const.	only irr. const.	no constraints
α	0.072	0.088	0.075	0.08
β	0.898	0.882	0.895	0.89
δ_l	0.5	0.5	0.5	0.5
ρ^a	0.9	0.95	0.9	0.9
σ^a	0.0085	0.0075	0.002	0.0015
$\overline{\theta}$	0.82578	0.82672	0.835596	0.835387
γ	0.964	0.98	0.968	0.97
F^1	0.012	0.010	0.013	0.013
w_0^1	0.33	0.19	n.a.	n.a.
δ_k	0.03	0.035	0.035	0.035
σ_θ	0.0182	0.01	0.002	0.0015
ρ_θ	0.7	0.7	0.98	0.98
τ_l	0	0	n.a	n.a
τ_k	0.679	0.679	n.a	n.a
r	0.01	0.01	0.01	0.01

1: both F and w_0 are expressed as fractions of the average size of fixed capital of a financially unconstrained firm.

Figures 1(I) and 1(II) show the value of the multipliers $\lambda_t (w_t, \theta_t, k_t)$ and $\phi_t (w_t, \theta_t, k_t)$ and the policy functions $l_t (w_t, \theta_t, k_t)$ and $k_{t+1} (w_t, \theta_t, k_t)$ in the space of w_t/k^{avg} and for selected values of k_t and θ_t. k^{avg} is the average optimal fixed capital for an unconstrained firm. θ_t is chosen so that figures 1(I) and 1(II) represent the investment decision of a firm with a negative idiosyncratic productivity shock ($\theta^f = \theta_L$ and $\varepsilon = \varepsilon_7$). Figure 1(II) illustrates the policy function of a firm that is only subject to the financing constraint, and hence is free to adjust fixed capital, while figure 1(I) illustrates the policy functions of a firm which is also subject to the irreversibility constraint. For both figures k_t is chosen at an intermediate level so that, in the case of figure 1(I), the irreversibility constraint is not binding unless wealth is very low. The most interesting comparison is for values of wealth lower than W^*, when the financing constraint is binding and λ_t is positive. In this case equation (17) holds with equality and $d_t = 0$. Therefore a decrease in w_t is compensated by a one to one decrease in $\left(1 - \frac{\tau_l}{R}\right) l_t + k_{t+1}$. In the case of figure 1(II), the drop in wealth is mostly absorbed by a drop in fixed capital. This is because the financing constraint is binding and hence the value of internally generated money is higher than the market price of it. ϕ_t (the "fi" line), the extra return of the funds invested in the firm, can be as high as 7% for very low levels of financial wealth. This means that it is very profitable to generate more output today, and the firm prefers to use its limited financial resources to invest in variable capital, which is immediately productive, and to reduce the

rate. ρ_θ is equal to 0.98 against $\rho_\theta = 0.7$ in the industry with both constraints. In the industries with only the financing constraint and with no constraints it is not possible to match the autocorrelation of the fixed capital investment rate, no matter how close to 1 ρ_θ is. Therefore we choose to set $\rho_\theta = 0.7$ in the industry with only the financing constraint (the same as for the industry with both constraints). and $\rho_\theta = 0.98$ for the industry with no constraints (the same as for the industry with only the irreversibility constraint).

Figure 1: Policy functions conditional on $\theta^f = \theta_L$, $\varepsilon = \varepsilon_7$ and $k_t = 87576$
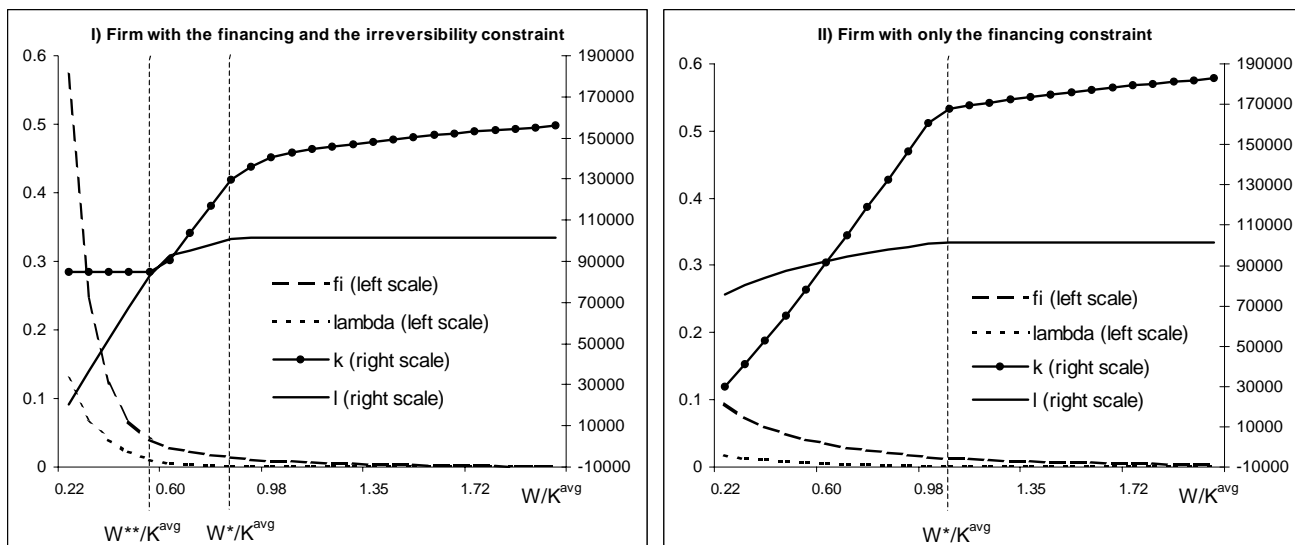
investment in fixed capital.

In figure 1(I) both constraints are binding for levels of wealth below W^{**}. In this case a drop in financial wealth must be absorbed by a drop in variable capital. Lower variable capital means that the firm produces less output today, generates less cash flow, and is expected to face even worse financial conditions in the future. The magnitude of this amplification effect is summarized by the fact that when w_t is very low, λ_t is up to ten times higher in figure 1(I) than in figure 1(II). The consequence is that ϕ_t can be as high as 58% in figure 1(II). This extreme value of ϕ_t has a very low probability to happen in equilibrium, while values of ϕ_t between 10% and 15% are more frequent among the simulated firms with both constraints binding. Such values imply that the firms would be willing to pay between 44% and 64% annual interest rate to obtain additional finance. This may still seem an unrealistically high value, but actually the annualized interest rate that firms implicitly pay on trade credit is often found to be above 40% (Ng et Al, 1999).

One interesting feature of the policy functions, present in both figures 1(I) and 1(II), is the fact that fixed capital is sensitive to financial wealth when the financing constraint is not currently binding. As we mentioned in the previous section, this happens because fixed capital investment is negatively affected by future expected financing constraints. This effect is shown more clearly in figure 2, which considers a firm that has a very low stock of capital at the beginning of period t. The black line represents the optimal investment in fixed capital k_{t+1}. For very high levels of wealth, the firm is never constrained (ϕ_t is equal to zero), and k_{t+1} reaches the optimal unconstrained level, which we call k^*_{t+1}. This maximum investment level takes into account expected irreversibility constraints. That is, the firm would invest even more if fixed capital was reversible. The gray line is the maximum feasible investment in fixed capital if the firm borrows up to the limit and

Figure 2: Effect of future expected financing constraints on fixed capital investment. Policy functions conditional on $\theta^f = \theta_L$, $\varepsilon = \varepsilon_7$ and $k_t = 15924$
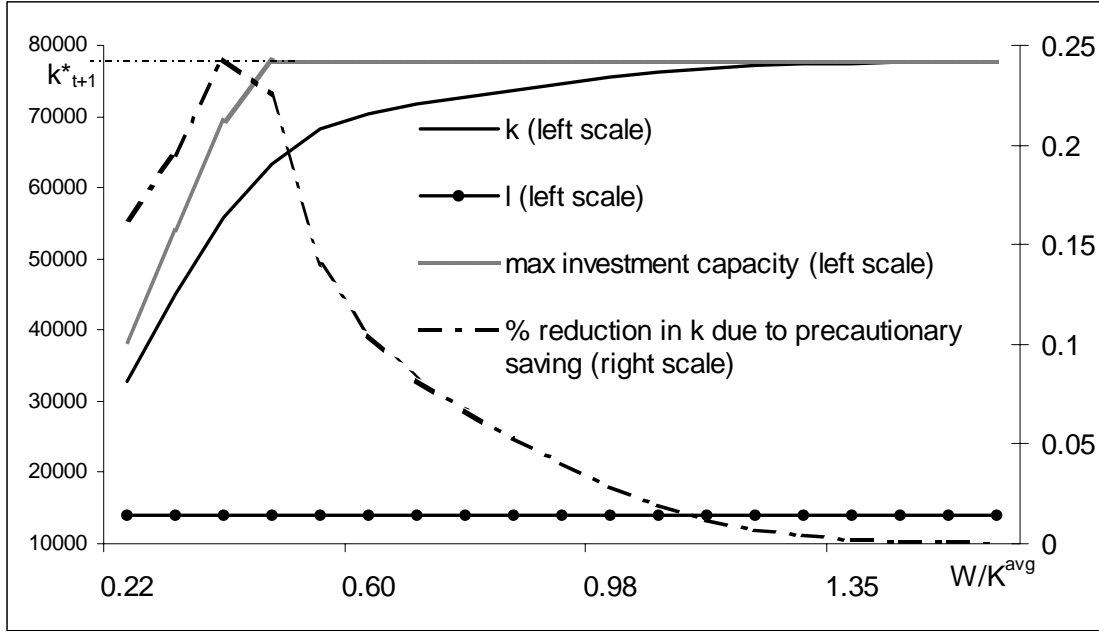
invests all available wealth. When the black line is below the grey line it means that the borrowing constraint is not binding, but still the firm chooses a level of k_{t+1} lower than k_{t+1}^*. The distance between the gray line and the black line is the reduction in capital due to future expected financing constraints. In figure 2 this reduction can amount to 24% of the value of k_{t+1}^*. This is because the difference between current capital ($k_t = 15924$) and optimal desired capital ($k_{t+1}^* = 77649$) is very large. This means that future expected financing constraints act in a similar way as convex adjustment costs. The bigger is the gap between current and desired capital, the more these constraints reduce optimal investment.

4.2   Dynamics of aggregate output and investment

In this section we will use the solution of the model to simulate the investment and production path of many heterogeneous firms. We will show that the combination of the irreversibility and the financing constraint generates cyclical fluctuations of investment and output consistent with the empirical evidence illustrated in tables 1-4. In the simulated industry all firms are identical ex ante, but each of them is subject to a different realization of the idiosyncratic productivity shock $\theta_{i,t}^f$, which is uncorrelated across firms and serially correlated for each firm. The distribution of $\{w_{i,t}, k_{i,t}\}$ across firms depends on the set of exogenous parameters, on the initial distribution of $\{w_{i,0}, k_{i,0}\}$ and on the history of aggregate shocks $\{\varepsilon_j\}_{j=0}^{t}$. In this section we compare the empirical data from the SIC 34 Sector with the data of several artificial industries. For each industry we

simulate 50000 firms for 10000 periods. In each period a fraction $1 - \gamma$ of firms is liquidated. We assume that an identical number of new firms enters production, so that the total number remains constant. Each newborn firm draws the initial value of θ^f from a uniform distribution, has an endowment of w_0 and a fixed capital level of k_0. w_0 is a key parameter to determine the aggregate distribution of wealth and the intensity of financing constraints. If w_0 is too small then no firm ever manages to expand enough to become unconstrained, and all firm are liquidated after few periods of life. If w_0 is too large then all firms can expand to the level at which they are never financially constrained. It is possible to show that for the chosen value of w_0 a stochastic steady state exists such that a fraction of firms is on average financially constrained, and among those a significant fraction is contemporaneously financially and irreversibility constrained. Regarding k_0, we assume that the initial level of fixed capital of a firm that enters production at time t is ex ante optimal, conditional on the information set at time $t - 1$.

Tables 9-12 compare output and investment dynamics of the firms in the SIC 34 sector (first column) with the dynamics of the firms in four different artificial industries. These are simulated using the parameters shown in table 8, and correspond to the four versions of the model described in the previous section: without any constraint, with one of the two constraints only, and with both constraints. For the empirical data we compute fixed capital considering only the stock of equipment. This is because land and buildings are more likely to be subject to other type of adjustment costs besides the irreversibility constraint, which is the only type of real constraint considered in the model. For the simulated data, since we do not model finished goods inventories, sales and output coincide. The objective of this simulation exercise is to verify the ability of the model to replicate the following empirical evidence: i) the relative volatility of deliveries with respect to fixed capital; ii) the procyclicality and asymmetric behaviour of deliveries.

The last column in table 9 reports the simulated statistics for the industry without the irreversibility and the financing constraint. In this case the fixed capital stock is approximately as volatile as sales are. Deliveries are more volatile than fixed capital because they are measured as the gross investment in variable capital (see equation 30). This is, by construction, more volatile than the stock of variable capital l_t, which in the frictionless case is as volatile as fixed capital k_t. The correlation between the deliveries/sales ratio and the level of sales is positive and much larger than zero. This is due to the time to build assumption about fixed capital. After a positive shock firms can immediately increase variable capital input, while investment in fixed capital takes one period to become productive. Therefore deliveries increase proportionally more than sales after a positive shock. This can be interpreted as an increase in capacity utilization of the existing fixed capital stock. The time to build assumption alone can account for the procyclicality of deliveries, but it cannot account for its asymmetry: the procyclicality of deliveries is symmetrical across expansion and contraction phases.

The next column illustrates the statistics for the industry with the irreversibility constraint only, and shows a reduction in the volatility of fixed capital relative to the volatil-

<div align="center">Table 9: simulated statistics</div>

	Empirical data	Simulated data[3]			
		Both Const.	Only f.c.	Only Irr.	No Const.
st.dev. Sales[2]	0.0429	0.0434	0.0449	0.0482	0.0424
$\frac{st.dev.(deliveries)}{st.dev.(sales)}$	1.38 - 1.16[1]	1.26	1.40	1.26	1.36
$\frac{st.dev.(fixedcap.)}{st.dev.(sales)}$	0.41[1]	0.37	0.72	0.93	1.05
$\frac{st.dev.(deliv.)}{st.dev(fixedcap.)}$	2.79[1]	3.42	1.94	1.35	1.29
$\text{corr}(deliveries_t/sales_t, sales_t)$					
All periods	0.18**	0.24	0.33	0.28	0.33
Below trend	0.28**	0.28	0.24	0.22	0.29
Above trend	0.03	0.13	0.31	0.26	0.29
Decrease in sales	0.23*	0.30	0.21	0.16	0.25
Increase in sales	0.12	0.04	0.27	0.24	0.23
average % of binding fin. const.	N/A	39%	43%	0%	0%
average % of binding irrev. const.	N/A	29%	0%	36%	0%
average % of both constraints binding	N/A	9%	0%	0%	0%

1.Based on yearly data. 2. Standard deviations of percentage deviations from the trend. 3. Despite simulate data are stationary, we apply the same Hodrick Prescott filter used on the empirical data, in order to ensure that similar frequencies are filtered. **significant at the 99% confidence level; *significant at the 95% confidence level.

ity of deliveries and sales. This is because in every period the irreversibility constraint is binding for a fraction of firms. These firms cannot change fixed capital in response to a negative productivity shock. Despite the irreversibility constraint is on average binding for as much as 32% of the firms, the increase in the volatility of variable capital relative to fixed capital is small. This is because the two factors of production are complementary. Therefore after a negative shock if a firm is unable to reduce fixed capital it has also less incentive to reduce variable capital deliveries. This implies that the lower volatility of fixed capital also causes a lower volatility of deliveries.

Regarding the asymmetric behaviour of deliveries in the business cycle, deliveries are more procyclical during upturns than during downturns. Therefore the introduction of an element of asymmetry in the model, the irreversibility constraint, generates an asymmetry in deliveries that is opposite to the one observed in the empirical data. Also this finding is explained by the complementarity of the two factors of production. In the simulated industry a downturn begins with a negative aggregate productivity shock that reduces output. As long as the low productivity persists, aggregate fixed capital is gradually reduced towards the new optimal level. The fact that irreversibility is binding for some firms implies that aggregate fixed capital is inefficiently high, and hence also variable capital deliveries are higher than otherwise. The inefficiently large capital implies that output drops more than variable capital during a downturn, and hence the deliveries/sales ratio is less procyclical in this phase than during an upturn.

The next column shows the simulated data for an industry with the financing constraint only. The relative volatility of deliveries is higher than in the two previous industries. The reason is that in this industry on average 43% of the firms have a binding financing constraint. For these firms the level of variable capital is inefficiently low, and hence its marginal productivity is high, leading to an higher sensitivity of variable capital to the productivity shock. Interestingly, financing constraints reduce the volatility of fixed capital more than the irreversibility constraint. This is due to the dampening effect of future expected financing constraints on current fixed capital investment decisions. Regarding the cyclical behaviour of variable capital, also this simulated industry is not consistent with the asymmetry of deliveries over the business cycle. Financing constraints increase the procyclicality of deliveries during expansion phases, as it happened in the industry with only the irreversibility constraint.

The next column shows the results of the simulation of an industry with both the financing and the irreversibility constraint. In this industry the ratio of the volatility of deliveries with respect to the volatility of fixed capital increases substantially, and is closer the one observed in the empirical data. Moreover table 10, on the second row, shows that in this industry the relative volatility of input inventories is almost identical to the one in the data. The ratio of the volatility of input inventories over fixed capital is 3.14 in the data and 3.36 in the industry with the financing and the irreversibility constraint. Surprisingly, no additional convex or concave adjustment cost is needed to generate this result. This is due to the interactions between the two constraints. As explained in

Table 10: Simulated statistics, input inventories

	Empirical data	Simulated data			
		Both. Const.	Only F.C.	Only Irr.	No constr.
$\frac{st.dev.(inventories)}{st.dev.(sales)}$	1.29-1.30[1]	1.24	1.54	1.29	1.38
$\frac{st.dev.(inventories)}{st.dev.(fixedcapital)}$	3.14[1]	3.36	1.71	1.37	1.30
corr($\Delta inventories_t/sales_t, sales_t$)					
All firms					
All periods	0.076**	0.23	0.37	0.30	0.33
Below trend	0.101*	0.26	0.25	0.23	0.30
Above trend	0.04	0.12	0.38	0.29	0.27
Decrease in sales	0.128**	0.30	0.20	0.18	0.29
Increase in sales	0.02	0.03	0.36	0.25	0.17
Smaller firms					
Below trend	N.A.	0.22	0.21	0.19	0.30
Above trend	N.A.	0.04	0.29	0.21	0.27
Larger firms					
Below trend	N.A.	0.32	0.33	0.23	0.26
Above trend	N.A.	0.27	0.35	0.30	0.25

1. Based on yearly data. **significant at the 99% confidence level; *significant at the 95% confidence level.

section 3.3, on the one hand the financing constraint amplifies the cost of future expected irreversibility constraints and discourages fixed capital investment. On the other hand the irreversibility of fixed capital amplifies the effect of financing constraints on deliveries when both constraints are binding. One way to quantify this amplification effect is to notice that if we add the irreversibility constraint to the industry with no frictions, the ratio between the volatility of deliveries relative to the volatility of fixed capital increases only by 5%. Instead, if we add the irreversibility constraint on top of the financing constraint, this ratio increases by 75%.

More importantly, the model with both constraints is consistent with the asymmetric behaviour of deliveries in expansion and contraction phases. Deliveries are more volatile than sales during periods in which sales are below trend, as it happens in the empirical data. The intuition for this result is that in this industry at any point in time a fraction of firms has both constraints currently binding. These firms, during a downturn, are forced to reduce deliveries in response to the reduction in profits and wealth. This effect more than counterbalances the fact that the more wealthy firms in the sample only have the irreversibility constraint binding and hence do not reduce deliveries with the same intensity. Less intuitive is the reason why deliveries are not procyclical during upturns. In figure 1 and in section 4.1 we have shown that when both constraints are binding the shadow cost of money for the firm reaches very high levels. Therefore also the marginal productivity of variable capital is very high, and when a positive aggregate shock hits and positive profits are realized, a small increase in deliveries generates large increases in output, and hence deliveries increase less than output for these firms. The ability of the

Table 11: Dynamics of deliveries, small and large firms

	Both const.	only f.c.	only irr.	no const.
Smaller firms[1]				
$\text{corr}(\text{del}_{t+1}/\text{sales}_t,\text{sales}_t)$ (all periods)	0.18	0.33	0.19	0.30
$\text{corr}(\text{del}_{t+1}/\text{sales}_t,\text{sales}_t)$ (below trend)	0.24	0.24	0.16	0.26
$\text{corr}(\text{del}_{t+1}/\text{sales}_t,\text{sales}_t)$ (above trend)	0.04	0.32	0.17	0.26
Larger firms				
$\text{corr}(\text{del}_{t+1}/\text{sales}_t,\text{sales}_t)$ (all periods)	0.35	0.30	0.29	0.33
$\text{corr}(\text{del}_{t+1}/\text{sales}_t,\text{sales}_t)$ (below trend)	0.33	0.25	0.22	0.28
$\text{corr}(\text{del}_{t+1}/\text{sales}_t,\text{sales}_t)$ (above trend)	0.27	0.27	0.28	0.28

1.Every period we select as smaller firms the 50% of firms with smaller output.

model to match the observed behaviour of deliveries is important, because the asymmetric behaviour of deliveries is empirically closely related to the asymmetric dynamics of input inventories, as shown in tables 2 and 3. Table 10 reports the procyclicality of inventory investment for the empirical data and for the simulated industries. Since we interpret the stock of variable capital at the end of period t, $(1 - \delta_l)l_t$, as input inventories, $(1 - \delta_l)(l_t - l_{t-1})$ can be interpreted as inventory investment. Also in this case the industry with both constraints is the only one that reproduces the asymmetry in inventory dynamics found in the empirical data. Our model therefore provides a theoretical justification of the stylized fact that input inventories are very procyclical especially during recessionary periods. Furthermore, it is consistent with the finding of Gertler and Gilchrist (1994) and Bernanke, Gertler and Gilchrist (1996) who observe that at the beginning of a recession inventories decline much more in small rather than in large firms. Tables 10 and 11 show that the asymmetric behaviour of input inventories and deliveries in the industry with both constraints is entirely driven by the smaller firms.

The asymmetric behaviour of deliveries in the industry with both constraints is also reflected in the elasticity of deliveries to sales. Table 12 shows the estimated elasticity of deliveries to sales conditional on the number of consecutive periods of decreasing and increasing sales. For the simulated data the elasticity of deliveries to sales is very large in the first period of both expansion and contraction phases because fixed capital does not initially adjust to the new productivity shock. Therefore the sensitivity of output to the shock is less strong than the sensitivity of variable capital. In order to abstract from this effect we compare, in the bottom part of table 12, the ratio between the elasticity of deliveries to sales in contraction and expansion periods. In the industry with both constraints this ratio is higher in the second and third period with respect to the first period, as it is in the data, while it is generally lower in the other simulated industries.

Table 12: sales elasticity of deliveries

	Empirical Data	Simulated Data			
Increase in sales		Both Constraints	Only F.C.	Only Irr.	No Const
$\frac{\partial del_{t+1}}{\partial s_t}\frac{s_t}{del_{t+1}}$ (1st qt.)	1.14	2.26	2.63	2.94	2.86
$\frac{\partial del_{t+1}}{\partial s_t}\frac{s_t}{del_{t+1}}$ (2nd qt.)	1.18	0.43	1.14	1.47	1.25
$\frac{\partial del_{t+1}}{\partial s_t}\frac{s_t}{del_{t+1}}$ (3rd qt.)	0.91	0.64	0.73	0.60	0.58
Decrease in sales					
$\frac{\partial del_{t+1}}{\partial s_t}\frac{s_t}{del_{t+1}}$ (1st qt.)	1.15	2.17	2.64	2.98	3.01
$\frac{\partial del_{t+1}}{\partial s_t}\frac{s_t}{del_{t+1}}$ (2nd qt.)	1.66	0.84	0.76	1.05	1.23
$\frac{\partial del_{t+1}}{\partial s_t}\frac{s_t}{del_{t+1}}$ (3rd qt.)	1.98	0.95	0.86	0.81	0.56
Decrease/increase					
1st quarter	1.01	0.96	1.00	1.01	1.05
2nd q.	1.41	1.98	0.67	0.72	0.98
3rd q.	2.17	1.48	1.17	1.36	0.98

4.3   Sensitivity analysis

We now present a sensitivity analysis of the main results obtained in table 9. In figures 3, 4 and 5 we analyze the effect of a gradual increase in w_0, and hence of a reduction in the intensity of financing constraints, on the simulated statistics. In the previous section we claimed that the interactions between the financing and the irreversibility constraint are necessary to generate the high volatility of deliveries relative to fixed capital and their asymmetric dynamics. Here we confirm the claim by showing that these features of the simulated industry gradually disappear when w_0 increases.

Figures 3 and 4 illustrate the sensitivity analysis for the industry with both constraints. Figure 5 for the industry with only the financing constraint. On the x axis it is reported the ratio w_0/k^{avg}. The smallest value on the x axis corresponds to the value of w_0 chosen in the simulations in the previous section. In figure 3(I) we illustrate the relationship between w_0 and the fraction of firms with a binding constraint. As w_0 increases, the fraction of firms with a binding borrowing constraint decreases. The fraction of firms with both constraints binding is initially relatively large, but it rapidly decreases towards zero. Therefore only firms with low wealth may experience both constraints contemporaneously binding. On the contrary the fraction of firms with the irreversibility constraint binding increases with w_0. This is because the higher is wealth, the more firms accumulate fixed capital, and the more the irreversibility constraint is likely to bind conditional on a negative idiosyncratic shock.[15]

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[15]The fraction of firms with a binding irreversibility constraint is as high as 75% when no firm is ever financially constrained. This value is higher than the 32% obtained from the simulations that use the calibrated parameters for the industry with only the irreversibility constraint (table 9, column 4) because in figure 3 we use the calibrated parameters for the industry with both constraints, which imply an higher volatility of the idiosyncratic shock. For the same reason the ratio between the volatilities of deliveries and fixed capital in figure 3(II) decreases from a maximum of 3.4 to 1.9, which is less than the difference, shown in table 9, between the calibrated economies with both constraints and with the irreversibility constraint only.

Figure 3: economy with the financing and the irreversibility constraint
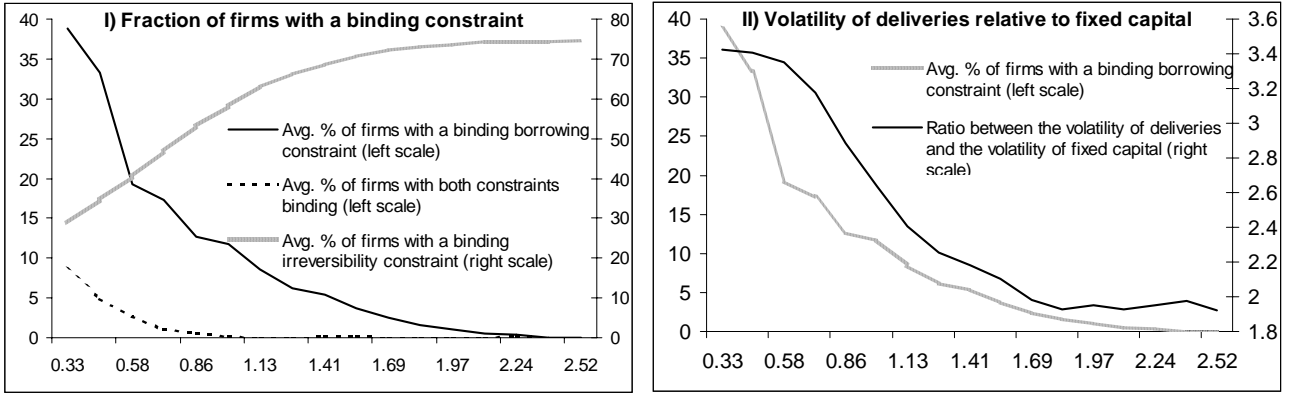

Figure 3(II) shows that the volatility of deliveries relative to the volatility of fixed capital is very high for levels of w_0/k^{avg} between 0.3 and 0.6. This happens despite in the same range of values of w_0 the fraction of financially constrained firms drops from 39% to 20%. The reason is that these firms become unconstrained but most of them still expect future financing problems. From our analysis in section 3.2 we know that future financing constraints increase the cost of future irreversibility constraints and dampen the volatility of fixed capital. Therefore the relative volatility of deliveries to fixed capital remains high for these firms. When w_0 further increases, the ratio of the volatility of deliveries and fixed capital decreases faster. This is because more firms become very wealthy and with low probability of facing future financing constraints. On the whole figure 3(II) shows that the fraction of firms with a binding financing constraint and the volatility of deliveries relative to fixed capital are positively correlated. On the contrary we observe a weaker correlation in the industry with only the financing constraint (figure 5(I)). In the latter case the volatility of deliveries relative to fixed capital is initially as high as 1.94, but it quickly drops to 1.45 at low values of w_0, despite the fraction of financially constrained firms is still higher than 30%. Subsequently the relative volatility of deliveries decreases only by 0.2 as the fraction of constrained firms decreases from 30% to 0. The high initial value in the relative volatility of deliveries is also caused by the effect of future expected financing constraints on fixed capital investment. But this effect quickly disappears as w_0 further increases, despite there is still a large fraction of constrained firms. This is because in the industry with only the financing constraint there is no interaction between the financing and the irreversibility constraint. Following a negative shock, the firms can quickly sell fixed capital and readjust it to the optimal level. This makes them much less cautious in investing in fixed capital, as soon as their wealth increases above the constrained region.

In figure 4(I) we analyze the asymmetry in the procyclicality of deliveries as a function of w_0 for the industry with both constraints. As w_0 increases the asymmetry in the procyclicality of deliveries disappears. Figure 4(II) shows that the difference in the pro-

Figure 4: economy with the financing and the irreversibility constraint: correlation between $\frac{deliveries_t}{sales_t}$ and $sales_t$
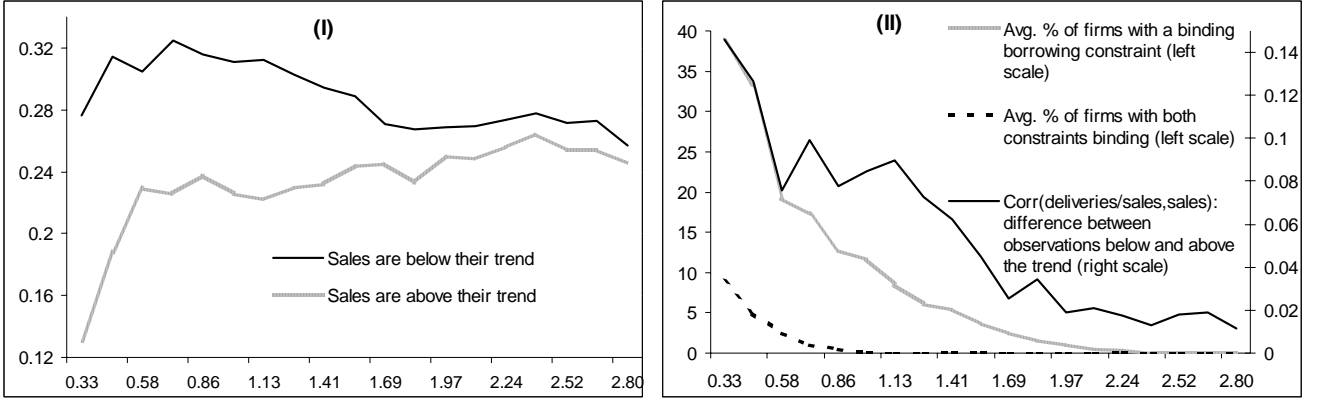

cyclicality of deliveries is strongly correlated with the intensity of financing constraints. This correlation is also driven by the interactions between the financing and the irreversibility constraint. This is demonstrated by the fact that the picture looks different in the industry with only the financing constraint (figure 5(III)). Here deliveries are initially more procyclical when output is above than when it is below its trend, but this asymmetry disappears as w_0 increases above the initial level. Further increases in w_0 and declines in the intensity of financing constraints have a small effect on the asymmetry in the procyclicality of deliveries.

Finally, table 13 reports the sensitivity analysis of the results in table 9 for different values of τ_k and τ_l. The first column of table 13 reports the simulation results obtained with the benchmark parameters. In the other columns τ_l is gradually increased and at the same time τ_k is reduced in order to keep the average collateral capacity of the firms approximately constant. Table 13 shows that the main results of the simulations are confirmed for the different values of τ_k and τ_l. This can be explained by looking back at section 3.3. An increase in τ_l reduces the downpayment necessary to buy variable capital. This increases the right hand side of equation (23). But at the same time the reduction in τ_k reduces the ability to borrow upfront the irreversible fixed capital. This reduces the right hand side of equation (23). The two effects more or less compensate each other in the simulated industries illustrated in table 13.

The asymmetry in the procyclicality of deliveries is confirmed in all simulations of table 13, even though it is significantly reduced in the last column. However we believe that this simulation is not realistic because, even though it is reasonable to assume that variable capital has some collateral value, the value of $\tau_l = 0.479$ implies that nearly all the residual value of input inventories can be pledged as collateral to obtain credit at the benchmark interest rate. In reality the value of inventories is the collateral for trade credit, which usually is implicitly priced well above the market interest rate (Ng et al,

Figure 5: economy with the financing constraint only
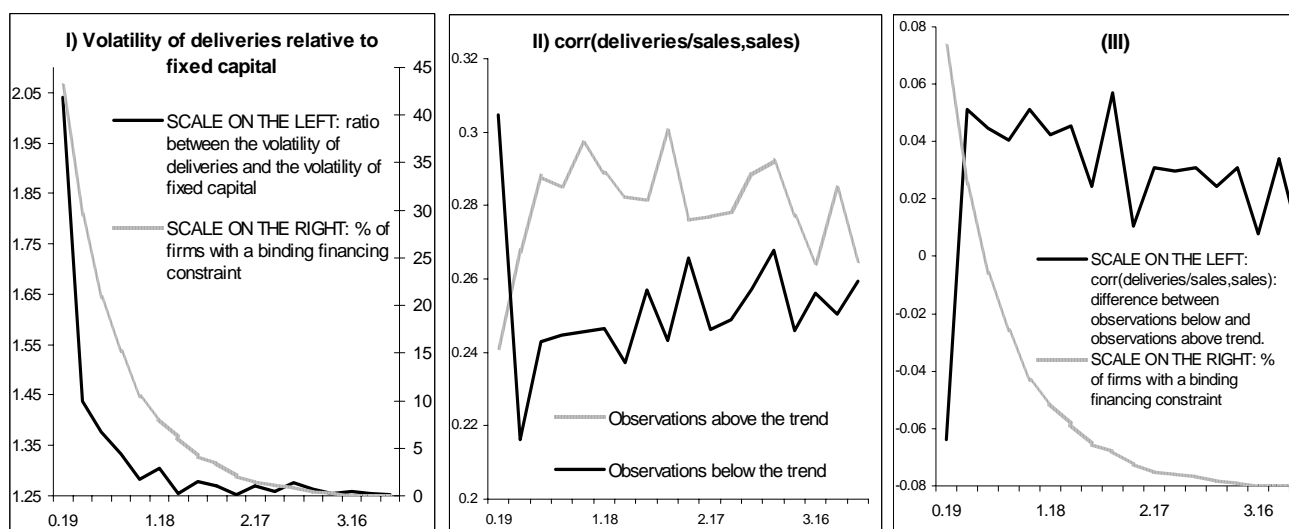
1999).

4.4 Irreversibility and aggregate output volatility

Veracierto (2002), in a general equilibrium real business cycle model with heterogeneous firms, shows that the presence of the irreversibility constraint at firm level has negligible effects on aggregate investment and output dynamics. In this section we show that the irreversibility of fixed capital strongly reduces the volatility of output of the simulated industry when financing constraints are also present. This particular finding is sensitive to the assumption that relative prices are constant in our simulated industries. But still it suggests that the irrelevance result obtained by Veracierto (2002) may change if financing constraints were to be introduced in a simulated general equilibrium economy.

In table 14 we show the results of four simulated industries. The first column replicates the statistics of the industry with both constraints, previously reported in table 9. The second column illustrates the statistics of an industry that has the same parameters of the industry of the first column, but that is not subject to irreversibility constraints. The fourth column replicates the results of the industry with only the irreversibility constraints, previously reported in table 9. Finally the fifth column uses the same parameters of the industry in the fourth columns, but once again it relaxes the irreversibility constraint. Therefore the comparison of columns one and two is an estimation of the partial equilibrium effect on aggregate output of eliminating the irreversibility constraint in an industry with financing imperfections. The comparison between columns four and five is the same exercise on an industry without financing imperfections. In the latter case output volatility increases by 17%. Also the relative volatilities of fixed capital and deliveries do

Table 13: Simulated statistics, industries with different values of τ_k and τ_l

	Empirical data	Simulated industry with both constraints			
		τ_k=.679 τ_l=0	τ_k=.5 τ_l=.179	τ_k=.35 τ_l=.329	τ_k=.2 τ_l=.479
st.dev. Sales	0.0429	0.0434	0.0447	0.0476	0.0472
$\frac{st.dev.(deliveries)}{st.dev.(sales)}$	1.38 - 1.16[1]	1.26	1.26	1.24	1.26
$\frac{st.dev.(fixedcap.)}{st.dev.(sales)}$	0.41[1]	0.37	0.36	0.35	0.35
$\frac{st.dev.(deliv.)}{st.dev(fixedcap.)}$	2.79[1]	3.42	3.46	3.52	3.63
corr($deliveries_t/sales_t,sales_t$)					
All periods	0.18**	0.24	0.25	0.25	0.26
Below trend	0.28**	0.28	0.27	0.27	0.26
Above trend	0.03	0.13	0.15	0.16	0.19
Decrease in sales	0.23*	0.30	0.29	0.30	0.26
Increase in sales	0.12	0.04	0.08	0.06	0.12
average % of binding fin. const.	N/A	39%	29%	29%	38%
average % of binding irrev. constraints	N/A	29%	28%	24%	24%
average % of both constraints binding	N/A	9%	6.3%	7.2%	7.3%
Quarterly turnover rate	3.67%	3.75%	3.88%	3.9%	4.07%

**significant at the 99% confidence level; *significant at the 95% confidence level

not change much when the irreversibility constraint is eliminated. The former case yields very different results. If we eliminate the irreversibility constraint from an industry with financing imperfections, then the volatility of fixed capital relative to output increases by 97%. This huge increase is due to the fact that by eliminating the irreversibility constraint we also eliminate the interactions with the financing constraint. The consequence is that the volatility of output increases by as much as 116%.

Table 14: irreversibility and output volatility

	Set of parameters 1			Set of parameters 2		
	Both const.	Only fin. constr.	$\Delta\%$	Only Irr.	No const.	$\Delta\%$
st.dev. Sales	0.0434	0.0939	116%	0.0482	0.0563	17%
$st.dev.(deliveries)/st.dev.(sales)$	1.26	1.33	5.5%	1.26	1.33	5.8%
$st.dev.(fixed\ cap.)/st.dev.(sales)$	0.37	0.73	97%	0.93	1.04	11.3%
$st.dev.(deliv.)/st.dev(fixed\ cap.)$	3.42	1.84	-46%	1.35	1.28	-4.9%

Set of parameters 1: parameters used in the benchmark calibration of the industry with both constraints. Set of parameters 2: parameters used in the benchmark calibration of the industry with only the irreversibility constraints.

5 Conclusions

In this paper we have illustrated a structural model of a profit maximizing firm subject to both borrowing constraints and irreversibility of fixed capital. The solution of the optimal investment problem shows that not only expected productivity but also current and future expected financing constraints affect investment decisions. Despite the firm being risk neutral, future expected financing constraints may reduce current investment in fixed capital. This "precautionary" reduction in investment may substantially affect aggregate investment dynamics in a way similar to the effect of convex adjustment costs.

More importantly, we have shown that the irreversibility and the financing constraint are complementary. Notably, the irreversibility of fixed capital amplifies the effects of the financing constraint on variable capital. By simulating an artificial industry with many heterogeneous firms we have shown that this amplification effect explains why aggregate investment in input inventories and deliveries of US durable manufacturing firms are very volatile (relative to capital) and procyclical, and why such procyclicality is highly asymmetrical, so that it disappears in periods in which aggregate output is above its trend. Our model is also consistent with the stylized fact that the procyclicality of aggregate inventories during downturns is driven by small firms.

Although we calibrate the model to match one specific US two digits durable manufacturing sector, we show that a similar behaviour of inventories and deliveries is present also in most other durable good sectors. More generally, the implications of the model could be useful in understanding firm dynamics in any productive sector that satisfies the following assumptions: i) both financing and irreversibility constraints are binding for a non negligible share of firms in equilibrium; ii) firms produce output using a combination of reversible and irreversible inputs.

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Appendix A: solution of the problem with endogenous exit:

In section 3 of the paper we illustrated the first order conditions of the firm's problem for the simplified case in which endogenous exit does not happen in equilibrium. Here we illustrate the full solution of the problem. If we substitute recursively $V_{t+1}^{stay}(w_{t+1}, \theta_{t+1}, k_{t+1})$ in equation (9) and we add constraints (2), (3) and (6) at times t and $t+1$, with the associated Lagrangian multipliers μ, λ and ϕ, we can represent the problem in the following way:

$$V_t^{stay}(w_t, \theta_t, k_t) = (1 + \phi_t)\left(w_t + \frac{b_t}{R} - k_{t+1} - l_t\right) + \mu_t[k_{t+1} - (1 - \delta_k)k_t] + \lambda_t[\tau_k k_t + \tau_l l_t - b_t] +$$
$$+ \frac{1}{R}E_t\{(1 - \gamma S_{t+1})w_{t+1} + \gamma S_{t+1}\{(1 + \phi_{t+1})\left(w_{t+1} + \frac{b_{t+1}}{R} - k_{t+2} - l_{t+1}\right) +$$
$$+ \mu_{t+1}[k_{t+2} - (1 - \delta_k)k_{t+1}] + \lambda_{t+1}[\tau_k k_{t+1} + \tau_l l_{t+1} - Rb_{t+1}]\}\} +$$
$$+ \frac{\gamma}{R^2}E_t\left\{S_{t+1}\left\{E_{t+1}\left[\gamma S_{t+2}V_{t+2}^{stay}(w_{t+2}, \theta_{t+2}, k_{t+2}) + (1 - \gamma S_{t+2})w_{t+2}\right]\right\}\right\}$$
$$(33)$$

The first order conditions of the problem are the following:

$$1 + \phi_t = R\lambda_t + E_t\left(1 + \gamma S_{t+1}\phi_{t+1}\right) + \gamma E_t(\Gamma_{t+1}) \tag{34}$$

$$\frac{\partial y_t}{\partial l_t} = UL + \frac{R(R - \tau_l)\lambda_t}{E_t\left(1 + \gamma S_{t+1}\phi_{t+1} + \gamma \Gamma_{t+1}\right)} \tag{35}$$

$$E_t\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}\right) = \frac{R^3\lambda_t + RUKE_t\left(1 + \gamma S_{t+1}\phi_{t+1} + \gamma \Gamma_{t+1}\right) - R^2\mu_t}{\gamma^2 E_t(S_{t+1}\Psi_{t+1})} +$$
$$- \frac{\gamma RE_t\left\{S_{t+1}\left[\lambda_{t+1}\tau_k - \mu_{t+1}(1 - \delta_k)\right]\right\} + \gamma RE_t[\Omega_{t+1}] + \gamma^2 covar_{t+1}}{\gamma^2 E_t(S_{t+1}\Psi_{t+1})} \tag{36}$$

Where:

$$\Gamma_{t+1} \equiv \frac{\partial S_{t+1}}{\partial w_{t+1}}\left\{\frac{\gamma}{R}\left\{E_{t+1}\left[[V_{t+2}(w_{t+2}, \theta_{t+2}, k_{t+2})] + \frac{(1 - \gamma)}{\gamma}w_{t+2}\right]\right\} - w_{t+1}\right\} \tag{37}$$

$$\Omega_{t+1} \equiv \frac{\partial S_{t+1}}{\partial k_{t+1}}\left\{\frac{\gamma}{R}\left\{E_{t+1}\left[[V_{t+2}(w_{t+2}, \theta_{t+2}, k_{t+2})] + \frac{(1 - \gamma)}{\gamma}w_{t+2}\right]\right\} - w_{t+1}\right\} \tag{38}$$

$$\Psi_{t+1} = S_{t+1}\left[\frac{1}{\gamma} + E_{t+1}\left(S_{t+2}\phi_{t+2}\right)\right] \tag{39}$$

$$covar_{t+1} = cov\left(MPK_{t+1}, S_{t+1}\Psi_{t+1}\right) \tag{40}$$

Equations (14), (15) and (16) are a special case of (34), (35) and (36) for $S_t = 1$ for any t. The terms Γ_{t+1} and Ω_{t+1} can be shown to be always equal to zero if inefficient liquidation never happens in equilibrium.

Appendix B: inefficient liquidation

The firm inefficiently liquidates in period t when it faces a combination of low wealth, low revenues and a binding irreversibility constraint, so that if it continues activity it would be forced to sell the assets and liquidate in period $t+1$ in order to be able to repay the debt, even if its business has a positive net present value. If wealth is very low, in order to satisfy constraint (17) the firm can set both l_t and d_t equal to zero, while k_{t+1} cannot be set below $(1 - \delta_k) k_t$. By substituting $d_t = l_t = 0$ and $k_{t+1} = (1 - \delta_k) k_t$ in equation (17) we get:

$$F \leq w_t - \left[(1 - \delta_k) - \frac{\tau_k}{R}\right] k_t \tag{41}$$

Condition (41) may not hold if w_t is very low. In order to see this, we substitute w_t in condition (41) using equation (8), and we consider the "worst case scenario" in which the firm was lacking the funds to invest in variable capital in the previous period: $y_{t-1} = 0$, $l_{t-1} = 0$, $b_{t-1} = \tau_k k_{t-1}$ and $k_t = (1 - \delta_k) k_{t-1}$. In this case condition (41) becomes:

$$F \leq \tau_k \left[\frac{1}{R}(1 - \delta_k) - 1\right] k_{t-1} \tag{42}$$

If τ_k is greater than zero, condition (42) cannot be satisfied even if $F = 0$. But with $F = 0$ any firm, no matter how small its scale of activity is, has a positive net present value of the discounted stream of future profits. Therefore equation (42) implies that in the worst case scenario a firm may decide to liquidate even though, if capital was reversible, it would have been profitable to continue.

Appendix C: numerical solution

In order to obtain a numerical solution of the dynamic nonlinear system defined by equations (2), (34), (35), (36) and (17), plus the standard complementary slackness conditions on λ_t, μ_t and ϕ_t, we discretise the state space as follows: k_t and w_t are both discretised in a 60 points grid, while θ_t is discretised in 16 elements, which correspond to the eight states of the aggregate shock and the two states of the idiosyncratic shock. The solution of the problem is simplified by the fact that, for all the set of parameters chosen in the paper, inefficient liquidation never happens in equilibrium, so that we can calculate the solution for the special case in which the terms Γ_{t+1} and Ω_{t+1} are always equal to zero. In order to compute the solution, first we formulate an initial guess of the forward terms $E_t\left(S_{t+1}\phi_{t+1}\right)$, $E_t\left(S_{t+1}\Psi_{t+1}\right)$, $E_t\left(S_{t+1}\lambda_{t+1}\right)$, $E_t\left(S_{t+1}\mu_{t+1}\right)$ and $covar_{t+1}$. Second, we solve the static optimization problem conditional on this guess, for each discrete value of the state variables $w_{i,t}, k_{i,t}$ and $\theta_{i,t}$. Third, we update the guess of $E_t\left(S_{t+1}\phi_{t+1}\right)$, $E_t\left(S_{t+1}\Psi_{t+1}\right)$, $E_t\left(S_{t+1}\lambda_{t+1}\right)$, $E_t\left(S_{t+1}\mu_{t+1}\right)$ and $covar_{t+1}$. We repeat these steps until the value function converges.