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# Fiscal policy, composition of intergenerational transfers, and income distribution\*

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## Abstract

In this paper, we characterize the relationship between the initial distribution of human capital and physical inheritances among individuals and the long-run distribution of these two variables. In a model with indivisible investment in education, we analyze how the initial distribution of income determines the posterior intergenerational mobility in human capital and the evolution of intragenerational income inequality. This analysis enables us in turn to characterize the effects of fiscal policy on future income distribution and mobility when the composition of intergenerational transfers is endogenous. We find that a tax on inheritance results in both less intergenerational mobility and smaller investment in human capital. However, a tax on labor income may promote human capital accumulation if the education premium is sufficiently high.

*JEL classification codes:* D64, E21, E13, E62

*Keywords:* Altruism, Intergenerational Transfers, Human Capital, Income Distribution.

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## 1. Introduction

The question of how inequality is generated and how it evolves over time is one of the major concerns in economic analysis. In the last decades a large number of studies have provided evidence supporting the presumption that intergenerational transfers are key to explain the empirical distribution of income and wealth.<sup>1</sup> As intergenerational transfers may take the form of physical capital (bequest) or human capital (investment in education), that empirical evidence also documents that both types of transfers affect the distribution of relevant economic variables among individuals. In this paper, we follow this line of research and show analytically how the joint initial distribution of bequest and human capital, as well as fiscal policy, determines the stationary composition of intergenerational transfers that individuals leave to their offspring.

Investment in education is a key factor of income inequality.<sup>2</sup> As was pointed out by Galor and Zeira (1993), there are two main features that give rise to this relationship. On the one hand, the technology of human capital accumulation exhibits a non-convexity since the investment in education is indivisible. This technological feature implies that access to education by the poorest individuals depends on whether they can borrow or not. On the other hand, there are capital market imperfections resulting in borrowing constraints so that those individuals with an income below some threshold value can not afford the cost of education.<sup>3</sup> Therefore, the initial distribution of income determines the number of individuals who can acquire education and, thus, it determines the aggregate stock of human capital and the rate of economic growth. This mechanism linking education with income distribution and growth was already widely analyzed in the literature by authors like Galor and Zeira (1993), García-Peñalosa (1995), Galor and Tsiddon (1997), and Owen and Weil (1998), among others.

Intergenerational transfers from parents to children account for a part of the observed inequality since these transfers help to reduce the negative effects of borrowing constraints on the accumulation of human capital. When parents do not pay the education cost and, thus, only leave physical bequest to their offspring, only those individuals who receive a sufficiently large inheritance and, thus, do not need to borrow can acquire human capital (see Becker and Tomes, 1976; Eckstein and Zilcha, 1994; or Behrman et al., 1995). Galor and Zeira (1993) show that, if one assumes credit market imperfections and a non-convex education technology, then the inherited distribution of wealth determines the accumulation of human capital and the dynamics of the distribution of income. Note however that, when education is financed by parents, only those individuals whose parents have a sufficiently high level of income have access to education.

The literature that we have reviewed above has not considered simultaneously the two types of intergenerational transfers we have mentioned: (i) transfers of physical capital by means of bequests; and, (ii) transfers of human capital by means of the

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<sup>1</sup>See, for instance, Becker and Tomes (1986), Gokhale et al. (1999), Gokhale and Kotlikoff (2002), Laitner (2002) or Wolf (2002), among many others.

<sup>2</sup>García-Peñalosa (1994) or Aghion et al. (1999) review the literature that examines the role of education on the link between distribution and growth.

<sup>3</sup>See, for instance, Dynarski (2002) or Keane (2002) for a discussion of the role of borrowing constraints on decisions concerning human capital acquisition.

parents' investment in the education of their children. In this paper, we consider the interaction between the composition of intergenerational transfers and income distribution. We will show that the initial mix of these two types of transfers is a key variable to understand the resulting investment in education, intergenerational earnings mobility, and intragenerational income distribution. In a related paper, Zilcha (2003) also characterizes the previous relation through a model where the interaction between those variables is based on an "ad-hoc" mechanism. Since this author intends to show that differences in the composition of intergenerational transfers may explain at least part of the observed differences in growth and inequality across countries, he assumes that this composition is exogenously given. In particular, he assumes a "joy of giving" motive for intergenerational transfers where parents' marginal utilities from physical bequests and from transfers of human capital are different. However, this way of modelling the link between the composition of intergenerational transfers and income distribution imposes a rigid constraint on the analysis of the determinants of both income distribution and mobility. In contrast, we consider that the composition of intergenerational transfers is endogenously determined by other economic factors like education costs, borrowing constraints or fiscal policies without introducing any differential treatment at the preference level between these two types of transfers.

Our paper develops a model of a small open economy populated by overlapping generations of individuals who differ in the amount and composition of inherited transfers from parents. In this economy the disposable lifetime income of an individual is fully determined by the bequest and human capital inherited from his parent. These intergenerational transfers arise because individuals take into account the disposable income of their offspring as they care about the starting opportunities of their children. More precisely, we assume that parents derive utility from their contribution to the future lifetime income of their children without discriminating between the two types of intergenerational transfers used for making such a contribution.<sup>4</sup> Thus, the income of parents determines the total contribution to the future lifetime income of their children. The composition of this contribution between the two types of transfers is endogenous in our model and depends on the relative returns of these transfers. However, since we assume that the investment in education is indivisible and that parents cannot force their children to give them transfers, if the cost of education is sufficiently large parents will not finance the cost of education and, thus, they will only leave bequest to their offspring. Obviously, this occurs to those individuals with an income level below some threshold. In this way, the initial distribution of income determines the evolution of the composition of intergenerational transfers and, thus, the size of the educated population along the equilibrium path. This simple mechanism explains how the initial distribution of income determines the posterior evolution of intragenerational income inequality and of intergenerational mobility in terms of human capital. As the income of an individual depends on the value of his inheritance, we will see that an individual finances the education of his children if he has received an inheritance that is larger than some threshold value. This threshold amount of bequest is smaller for educated

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<sup>4</sup>Becker and Tomes (1986) defend this formulation of altruism. Our notion of altruism lies thus between the "joy-of-giving" motive, where individuals receive direct utility from the act of giving, and "family altruism", where individuals' felicity depends on the disposable income of their children. See Michel et al. (2006) for a comparison between different forms of altruism.

than for non-educated individuals since educated individuals earn a higher labor income as a result of the education premium. Therefore, the access to education of individuals does not depend on the transfers received from their parents, but on the transfers that their parents have received.

A natural question to ask in our model is how different fiscal policies affect the evolution of both income distribution and mobility. In this paper we analyze the effects of the following government interventions: a pay-as-you-go social security system, a tax on inheritance, a tax on capital income, a tax on labor income, and a subsidy on education investment. Note that our analysis is purely positive as we focus exclusively on the effects on human capital accumulation, inequality, and mobility of traditional macroeconomic tax instruments.

Among other results, we obtain that raising the inheritance tax rate results in an increase of the fraction of non-educated individuals in the total population and reduces both the individual amount of bequest and the inequality in initial wealth. A higher labor income tax also reduces the amount of bequest but may raise the fraction of educated individuals if the education premium is sufficiently large. Therefore, inheritance taxation has some equalizing effects but reduces the accumulation of human capital as individuals enjoy less disposable income to pay for the indivisible cost of education of their children. Moreover, contrary to the conventional wisdom, an increase in the labor income tax may end up promoting the accumulation of human capital since it may reduce the minimum level of inheritance required to invest in children education. In this case, labor income taxes will promote upward mobility in human capital.

The paper is organized as follows. Section 2 presents the model of overlapping generations with altruistic individuals. Section 3 solves the intertemporal choice problem faced by an individual. In section 4 we describe the dynamics of the joint distribution of bequest and human capital following a given initial distribution. Section 5 analyzes the effects of fiscal policy on the intergenerational mobility in human capital and on the stationary distribution of income. Section 6 concludes the paper.

## 2. The model

We consider a small open economy populated by overlapping generations of individuals who live for three periods. There is a continuum of dynasties distributed on the interval  $[0, 1]$ . A new generation of individuals is born in each period within each dynasty. Each individual has offspring at the beginning of the second period of his life and the number of children per parent is  $n \geq 1$ . An agent makes economic decisions only during the last two periods of his life. In every period, the youngest individuals neither consume nor work, but they can accumulate human capital by attending formal school. Individuals work and supply inelastically one unit of labor when they are adult (second period of life) and are retired when they are old (third period of life). Individuals are assumed to care about the future income of their children and they can give two kinds of transfers to them: physical bequest and education. We will use the convention that the generation  $t$  is composed of the individuals who are adult (workers) in period  $t$ . As we will see next, all the individuals belonging to the same dynasty  $i \in [0, 1]$  and to the same generation  $t$  are identical in all respects.

Individuals derive utility from both their own lifetime consumption and their con-

tribution to the lifetime income of their children. Preferences of an individual belonging to dynasty  $i$  and generation  $t$  are represented by the utility function:

$$U_t^i = \ln c_t^i + \rho \ln x_{t+1}^i + \beta \ln I_{t+1}^i, \quad (2.1)$$

where  $\rho > 0$  is the temporal discount factor, the coefficient  $\beta > 0$  measures the intensity of altruism,  $c_t^i$  and  $x_{t+1}^i$  are the amounts of consumption in the second and third periods of life, respectively, and  $I_{t+1}^i$  is the after-tax contribution to the future lifetime income of each of their children. We assume that individuals do not discriminate among their children so that they make the same contribution  $I_{t+1}^i$  for all their direct descendants.<sup>5</sup> The parental contribution to the income of an individual belonging to dynasty  $i$  and generation  $t + 1$  is then given by

$$I_{t+1}^i = (1 - \tau_w)w_{t+1}\Delta_{t+1}^i + (1 - \tau_b)b_{t+1}^i, \quad (2.2)$$

where  $w_{t+1}$  is the wage per efficiency unit of labor at period  $t + 1$ ,  $\tau_w \in [0, 1]$  is the tax rate on labor income,  $\Delta_{t+1}^i$  is the increase in the number of efficiency units of labor supplied by an individual belonging to dynasty  $i$  and generation  $t + 1$  thanks to the investment in education made by his parent,  $b_{t+1}^i$  is the amount of inheritance that an individual of dynasty  $i$  and generation  $t + 1$  receives from his parent, and  $\tau_b \in [0, 1]$  is the tax rate on inheritances.

Let  $e_t^i$  denote the income that the adult individual of dynasty  $i$  and generation  $t$  devotes to finance the education of each of their children. We assume that the level of human capital  $h_{t+1}^i$  of an adult individual belonging to dynasty  $i$  and generation  $t + 1$  is entirely determined by his parent's investment  $e_t^i$  in his education. In particular, the human capital level of an individual can take two values depending on whether his parent investment in his education is below or above the fixed cost of education  $\mu$ . Thus, the level of human capital at period  $t + 1$  of an adult individual belonging to dynasty  $i$  who is born at period  $t$  is given by the following equation:

$$h_{t+1}^i = 1 + \Delta_{t+1}^i, \quad (2.3)$$

with

$$\Delta_{t+1}^i = \begin{cases} 0 & \text{if } e_t^i < \mu \\ \varepsilon & \text{if } e_t^i \geq \mu, \end{cases}$$

where  $\varepsilon > 0$  and  $\mu > 0$ . Obviously, the optimal investment in education for the individuals who want to have uneducated children (with  $h_{t+1}^i = 1$ ) is  $e_t^i = 0$ , whereas those individuals who want educated children (with  $h_{t+1}^i = 1 + \varepsilon$ ) will choose  $e_t^i = \mu$ . The number of efficiency units of labor supplied by an individual belonging to dynasty  $i$  and generation  $t$  is equal to his level  $h_t^i$  of human capital.

There is a single commodity that can be devoted to either consumption or investment, and the investment can be either in physical or in human capital. Adult individuals distribute their income, which is composed of wage earnings plus the amount inherited from their parents, between consumption, investment in education of their

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<sup>5</sup>The altruism parameter  $\beta$  can thus be rewritten as  $\beta = n\rho\beta'$ , where  $\beta'$  would denote the pure altruism factor per descendant.

children, and saving. Thus, the budget constraint faced by an adult individual belonging to dynasty  $i$  and generation  $t$  is

$$(1 - \tau_w)w_t h_t^i + (1 - \tau_b)b_t^i = c_t^i + s_t^i + (1 - s_e)ne_t^i + \lambda, \quad (2.4)$$

where  $s_t^i$  is the amount saved by this individual,  $\lambda$  is the lump-sum tax faced by an adult individual, and  $s_e \in [0, 1]$  is the subsidy rate on education spending. The left hand side of (2.4) is the after tax income of the adult individual under consideration. When individuals are old, they receive a return on their saving, which is distributed between consumption and bequest for their children. Therefore, the budget constraint at period  $t + 1$  of an old individual of dynasty  $i$  and generation  $t$  (i.e., born at period  $t - 1$ ) will be

$$[1 + (1 - \tau_k)r_{t+1}]s_t^i - \theta = x_{t+1}^i + nb_{t+1}^i, \quad (2.5)$$

where  $r_t$  is the before-tax rate of return on saving at period  $t$ ,  $\tau_k \in [0, 1]$  is the tax rate on capital income, and  $\theta$  is the lump-sum tax faced by an old individual.

We also impose the constraint that parents cannot force their children to give them gifts when they (the parents) are old,

$$b_{t+1}^i \geq 0. \quad (2.6)$$

Note also that negative voluntary bequests will never arise in equilibrium given our assumption of one-sided altruism (from parents to children).

In this economy there is a government that selects the different tax and subsidy rates and that spends the corresponding net revenue to finance its own consumption. The government faces a balanced budget constraint in each period so that it is subject to the following constraint at period  $t$ :

$$n \times \left[ \int_{[0,1]} (\tau_w w_t h_t^i + \tau_b b_t^i - s_e n e_t^i + \lambda) di \right] + \int_{[0,1]} (\tau_k r_t s_{t-1}^i + \theta) di = G_t, \quad (2.7)$$

where  $G_t$  denotes average government consumption per old individual at period  $t$ . We assume that government consumption is unproductive and does not affect directly individuals' welfare. Since  $G_t$  is endogenous, fiscal policy exhibits income effects at the individual and aggregate levels. However, under our small open economy assumption, government spending does not directly crowd out private saving.

Let us assume that the good of this economy is produced by means of a production function displaying constant returns to scale and that the stock of physical capital fully depreciates after one period. As firms behave competitively, they choose the ratio of physical to human capital, such that their marginal productivity equal the rental rates of both capitals. Because of the small open nature of this economy, the interest rate is exogenously given at the constant international level  $r$ , which under constant return to scale determines the value of the ratio of physical to human capital. The value of this ratio of capitals determines in turn the wage  $w$  per efficiency unit of labor in equilibrium. Thus,  $r_t = r$  and  $w_t = w$  for all  $t$ .

### 3. The individual problem

In this section, we will solve the problem that a generic individual belonging to dynasty  $i$  and generation  $t$  faces in order to choose the levels of consumption at adult and old ages and the transfers to his immediate descendants. Note first that the amount that an individual receives as inheritance and his level of human capital are the state variables determining his optimal choice.<sup>6</sup> Thus, an individual belonging to dynasty  $i$  and generation  $t$  maximizes (2.1) with respect to  $\{c_t^i, x_{t+1}^i, e_t^i, b_{t+1}^i\}$  subject to (2.2), (2.3), (2.4), (2.5), (2.6) and the non-negative constraints  $c_t^i \geq 0$  and  $x_{t+1}^i \geq 0$ , by taking as given the amount  $b_t^i$  inherited from his parent and his level  $h_t^i$  of human capital. Recall that in this intertemporal maximization problem, the optimal value of the control variable  $e_t^i$  will be either zero or  $\mu$  because of the functional form adopted by the technology producing human capital. Thus, we will solve the individual problem by following a two-stage procedure: first, we take the value of  $e_t^i$  as given, and then solve for the amounts of saving  $s_t^i$  and bequest  $b_{t+1}^i$ ; and second, we find the optimal amount of  $e_t^i$  given the values of  $s_t^i$  and  $b_{t+1}^i$  obtained in the previous stage.

We now proceed by presenting the details of the solution procedure. From the first order conditions of the individual problem, we obtain in Appendix A the following optimality conditions:

$$x_{t+1}^i = \rho R(\tau_k) c_t^i, \quad (3.1)$$

and

$$\frac{\beta(1-\tau_b)}{I_{t+1}^i} \leq \frac{n(1+\rho)/R(\tau_k)}{(1-\tau_w)wh_t^i + (1-\tau_b)b_t^i - n(1-s_e)e_t^i - \frac{nb_{t+1}^i}{R(\tau_k)} - \Omega}, \quad (3.2)$$

with

$$\Omega = \lambda + \frac{\theta}{R(\tau_k)},$$

where the condition (3.2) holds with equality if  $b_{t+1}^i > 0$ , and where  $R(\tau_k)$  will denote from now on the after-tax gross rate of return on saving, i.e.,  $R(\tau_k) = 1 + (1 - \tau_k)r$ . Equation (3.1) yields the optimal allocation of consumption along the lifetime of an individual belonging to dynasty  $i$  and generation  $t$ . Equation (3.2) characterizes the optimal amount of bequest. This condition tells us that, when the bequest  $b_{t+1}^i$  is positive, the marginal variation in the utility of parents arising from a larger amount of bequest must be equal to zero. On the one hand, the right hand side of this equation is the utility loss experienced by the individual from the decrease in his lifetime income devoted to own consumption due to a marginal increase in the amount of bequest left to their children. On the other hand, the left hand side of (3.2) is the utility gain obtained by the individual from the marginal contribution of his bequest to the future lifetime income of their children.

Combining (3.1) with the budget constraints (2.4) and (2.5), we can derive the amount  $s_t^i$  of saving as a function of the amount of intergenerational transfers. Thus,

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<sup>6</sup>Human capital is a state variable because the individuals' education was decided and financed by their parents.

we obtain the following expression:

$$s_t^i = \frac{\rho R(\tau_k) [(1 - \tau_w)wh_t^i + (1 - \tau_b)b_t^i - n(1 - s_e)e_t^i - \lambda] + nb_{t+1}^i + \theta}{(1 + \rho)R(\tau_k)}. \quad (3.3)$$

Moreover, from (3.2) we can also compute the optimal amount of bequest that parents leave to their children when the constraint (2.6) is not binding, i.e., when  $b_{t+1}^i > 0$ . By taking the condition (3.2) with equality, we directly obtain  $b_{t+1}^i$  as a function of the investment in the education of children  $e_t^i$  and of the endowments  $b_t^i$  and  $h_t^i$ , i.e.,

$$\begin{aligned} b_{t+1}^i &\equiv B(b_t^i, h_t^i, e_t^i) \\ &= \left[ \frac{\beta}{n(1 + \beta + \rho)} \right] \left\{ R(\tau_k) [(1 - \tau_w)wh_t^i + (1 - \tau_b)b_t^i - (1 - s_e)ne_t^i - \Omega] \right. \\ &\quad \left. - \left[ \frac{n(1 + \rho)(1 - \tau_w)}{\beta(1 - \tau_b)} \right] w\Delta_{t+1}^i \right\}. \end{aligned} \quad (3.4)$$

In the second stage of our solution procedure we will choose the investment in education  $e_t^i$  that solves the individual's problem. Since the investment in education is indivisible, individuals must actually decide whether they invest  $\mu$  units of income or do not invest at all. Observe that this decision is subject to the following restrictions. First, a positive investment in education for individuals with low levels of income may imply a negative optimal amount of bequest, which is not allowed in our economy by assumption. In this case, individuals will not invest in the education of their children. Therefore, the investment in education will be possible only if the individuals' income is sufficiently large so that this investment does not force individuals to leave a negative bequest. Second, if the amount  $b_{t+1}^i$  of bequest is positive when the individual invests  $\mu$  units of income in the education of their children, we have to analyze whether this amount  $\mu$  is the optimal amount  $e_t^i$  of investment in education. We next analyze these two issues separately.

### 3.1. Optimal investment in education

Let us first assume that the individual has a sufficiently large level of income so that the optimal amount  $b_{t+1}^i$  of bequest is positive even if he decides to invest in the education of his children. We will now analyze whether to invest in education is an optimal strategy in this case. In Appendix A we obtain that the optimal levels  $c_t^i$  and  $x_{t+1}^i$  of consumption are given by

$$c_t^i = \left[ \frac{n}{\beta(1 - \tau_b)R(\tau_k)} \right] I_{t+1}^i, \quad (3.5)$$

$$x_{t+1}^i = \left[ \frac{n\rho}{\beta(1 - \tau_b)} \right] I_{t+1}^i, \quad (3.6)$$

when (2.6) is not binding, i.e., when  $b_{t+1}^i > 0$ . Observe that conditions (3.5) and (3.6) yield the optimal levels  $c_t^i$  and  $x_{t+1}^i$  as increasing functions of  $I_{t+1}^i$ . Therefore,

the choice of education investment that maximizes the utility (2.1) at period  $t$  is the one that maximizes the contribution of parents to the future income of their children,  $I_{t+1}^i$ . Since the investment in education is indivisible, individuals must actually decide whether they invest  $\mu$  units of income or do not invest at all. An individual will be willing to invest in the education of their children if and only if this action increases the after-tax lifetime income of their offspring. Thus, in order to determine the optimal education decision we must compare the benefit of investing in education with the associated opportunity cost.

Note that an adult individual at period  $t$  can either invest the amount  $\mu$  in the education of his children or save this amount in order to leave a larger bequest in the next period. On the one hand, if he decides to invest in the education of their offspring, he must spend  $(1 - s_e)\mu$  units of income per child because  $\mu$  is the cost of education and the government subsidizes the investment in education at the rate  $s_e$ . We obtain from (2.3) that this investment in education raises the after-tax lifetime income of each child by  $(1 - \tau_w)\varepsilon w$  units. On the other hand, if that individual decides to save the amount  $(1 - s_e)\mu$  in order to make a physical transfer to his children in the next period, then the after-tax lifetime income of the latter will increase by  $(1 - \tau_b)(1 - s_e)\mu R(\tau_k)$  units since  $R(\tau_k)$  is the after-tax return on saving. Therefore, an individual born at  $t - 1$  would like to invest in the education of his children at period  $t$  if and only if the following condition holds:

$$(1 - \tau_w)\varepsilon w \geq (1 - \tau_b)(1 - s_e)\mu R(\tau_k). \quad (3.7)$$

From the previous equation we see that the optimality of investing in education does not depend on the individual's choices, but on the aggregate variables of the economy. We also observe that the optimality of investing in the education of children depends on fiscal policy. From condition (3.7), we directly obtain that the inheritance tax, the capital income tax and the education subsidy raise the willingness of individuals to invest in the education of their direct descendants, whereas the labor income tax reduces this willingness.

When condition (3.7) does not hold, individuals adopt the corner solution  $e_t^i = 0$ . From now on we will assume that condition (3.7) holds. Under this condition individuals will invest in the education of their offspring if they can afford the minimum after-tax cost of education given by  $(1 - s_e)\mu$ . Note however that, even when condition (3.7) holds, individuals will not invest in the education of their children if this investment gives raise to a negative optimal amount of bequest. In the next subsection we derive the levels of income above which individuals invest in the education of their direct descendants.

### 3.2. Human capital policy

Condition (2.6) is in fact the feasibility condition on investment in education. Given that parents cannot force their children to make transfers to them, they effectively invest in the education of their children if and only if the parents' income is sufficiently large so as to leave a non-negative bequest after making the investment in education. By imposing that parents invest  $\mu$  in the education of their children at period  $t$  (so that

$h_{t+1}^i = 1 + \varepsilon$ ), we obtain from (3.2) that  $b_{t+1}^i \geq 0$  if and only if the following condition holds:

$$\frac{\beta(1 - \tau_b)}{(1 - \tau_w)w\varepsilon} > \frac{n(1 + \rho)/R(\tau_k)}{(1 - \tau_w)wh_t^i + (1 - \tau_b)b_t^i - n(1 - s_e)\mu - \Omega}. \quad (3.8)$$

This condition says that those parents who have invested in the education of their children can satisfy the optimality condition (3.2) without violating the non-negativity constraint on bequests (2.6).

The feasibility condition (3.8) can be rewritten as a threshold level for the inheritance  $b_t^i$  received by parents. This threshold depends on the human capital level  $h_t^i$  of parents. On the one hand, if parents are non-educated (i.e.,  $h_t^i = 1$ ), then the threshold amount of bequest is

$$\tilde{b} = \left( \frac{1}{1 - \tau_b} \right) \left\{ (1 - s_e)n\mu + \left[ \frac{n(1 + \rho)\varepsilon}{\beta(1 - \tau_b)R(\tau_k)} - 1 \right] (1 - \tau_w)w + \Omega \right\}. \quad (3.9)$$

Thus, an individual with a level  $h_t^i = 1$  of human capital effectively invests in the education of his offspring if and only if he has received an inheritance  $b_t^i$  that satisfies  $b_t^i \geq \tilde{b}$ . On the other hand, if parents are educated (i.e.,  $h_t^i = 1 + \varepsilon$ ), then the threshold amount of bequest in this case is given by

$$\hat{b} = \left( \frac{1}{1 - \tau_b} \right) \left\{ (1 - s_e)n\mu + \left[ \frac{n(1 + \rho)\varepsilon}{\beta(1 - \tau_b)R(\tau_k)} - (1 + \varepsilon) \right] (1 - \tau_w)w + \Omega \right\}. \quad (3.10)$$

Thus, an individual with a level  $h_t^i = 1 + \varepsilon$  of human capital effectively invests in the education of their children if and only if his inheritance  $b_t^i$  satisfies  $b_t^i \geq \hat{b}$ .

The threshold levels (3.9) and (3.10) of bequest were obtained by eliminating those situations where a positive investment in the education of children would imply a positive transfer from children to parents. Those threshold values determine the dynamics of the human capital level within each dynasty. In particular, the dynamics of human capital inside a dynasty is given by the following dynamic equation:

$$h_{t+1}^i = \begin{cases} 1 & \text{if either } h_t^i = 1 \text{ and } 0 \leq b_t^i < \tilde{b} \text{ or } h_t^i = 1 + \varepsilon \text{ and } 0 \leq b_t^i < \hat{b}; \\ 1 + \varepsilon & \text{if either } h_t^i = 1 \text{ and } b_t^i \geq \tilde{b} \text{ or } h_t^i = 1 + \varepsilon \text{ and } b_t^i \geq \hat{b}. \end{cases} \quad (3.11)$$

By comparing (3.10) and (3.9), we directly obtain that  $\tilde{b} > \hat{b}$ . Since the labor income of educated individuals is larger than that of non-educated, the threshold amount of inheritance above which parents are willing to pay for the education of their children is smaller for educated parents.

In our economy, when individuals do not invest in the education of their offspring, they always leave a strictly positive amount of bequest. This follows from the fact that the utility function (2.1) satisfies the Inada condition at origin with respect to the parents contribution to the lifetime income of their children  $I_{t+1}^i$ , that is, the marginal utility with respect to  $I_{t+1}^i$  goes to infinity when this contribution tends to zero.

From the threshold levels of bequest defined in this section and equation (3.4), we

get the following equation characterizing the dynamics of bequests within a dynasty:

$$b_{t+1}^i = \begin{cases} B^1(b_t^i) & \text{if } h_t^i = 1 \text{ and } 0 \leq b_t^i < \tilde{b}; \\ B^2(b_t^i) & \text{if } h_t^i = 1 \text{ and } b_t^i > \tilde{b}; \\ B^3(b_t^i) & \text{if } h_t^i = 1 + \varepsilon \text{ and } 0 \leq b_t^i < \hat{b}; \\ B^4(b_t^i) & \text{if } h_t^i = 1 + \varepsilon \text{ and } b_t^i > \hat{b}; \end{cases} \quad (3.12)$$

where  $B^1(b_t^i) \equiv B(b_t^i, 1, 0)$ ,  $B^2(b_t^i) \equiv B(b_t^i, 1, \mu)$ ,  $B^3(b_t^i) \equiv B(b_t^i, 1 + \varepsilon, 0)$ , and  $B^4(b_t^i) \equiv B(b_t^i, 1 + \varepsilon, \mu)$ .

The dynamic equations (3.11) and (3.12) fully describe the policy functions for human capital and bequest, respectively, within a dynasty when condition (3.7) holds. In other words, these two equations determine the amount of human capital and bequest for the next cohort of the dynasty given the human capital and bequest of the present cohort.

#### 4. The dynamics of dynastic income

In this section we study the dynamics of the joint distribution of bequest and human capital. Under our assumptions, this dynamics follows directly from the dynamic equations of bequest and human capital (3.11) and (3.12), respectively. Since we have considered a small open economy, the evolution of each dynasty does not depend on the aggregate distribution. Thus, in this section we analyze the evolution of bequest and human capital for a given dynasty along time. In this sense, observe that individuals within a cohort differ in two respects: first, individuals have different levels of income in their second period of life since they have received different transfers from their parents; and, second, individuals also differ in the composition of income due to the different composition of the transfers received from their parents. Thus, the amount that a dynasty initially receives as bequest and the initial level of human capital fully determine the entire posterior path of bequest, human capital, and income.

##### 4.1. Stationary distribution of income

We will now characterize the stationary distribution of bequest and human capital. For that purpose, we will prove that the dynamic system composed of equations (3.11) and (3.12) has at most two stationary solutions. We will see that there are three candidates for these steady states: a corner solution, where the amount of bequest is zero; and two interior solutions given by the two possible fixed points of (3.12), which we will denote by  $\bar{b}^1$  and  $\bar{b}^2$ . The point  $\bar{b}^1$  is a fixed point of  $B^1(b_t^i)$ , whereas  $\bar{b}^2$  is a fixed point of  $B^4(b_t^i)$ . Thus, we get from (3.4) that

$$\bar{b}^1 = \frac{\beta R(\tau_k) [(1 - \tau_w) w - \Omega]}{n(1 + \beta + \rho) - \beta R(\tau_k)(1 - \tau_b)}, \quad (4.1)$$

and

$$\bar{b}^2 = \frac{\beta R(\tau_k) \left[ (1 - \tau_w) w (1 + \varepsilon) - (1 - s_e) n \mu - \frac{n(1+\rho)(1-\tau_w)w\varepsilon}{\beta R(\tau_k)(1-\tau_b)} - \Omega \right]}{n(1 + \beta + \rho) - \beta R(\tau_k)(1 - \tau_b)}. \quad (4.2)$$

Obviously, a necessary condition for an amount of bequest being an interior steady state is that an educated (non-educated) parent who has received this level of inheritance does (not) actually invest in the education of their children. We will see next that the fixed points of the functions  $B^2(b_t^i)$  and  $B^3(b_t^i)$  can not be stationary values of bequest. In order to prove that a fixed point  $\bar{b}$  of  $B^2(b_t^i)$  is not a steady state for bequest, let us assume that  $b_t^i = \bar{b} > \hat{b}$  and  $h_t^i = 1$ . As follows from (3.11) and (3.12), this individual leaves a bequest per capita equal to  $b_{t+1}^i = B^2(\bar{b}) = \bar{b}$  and invests in the education of their children so that  $h_{t+1}^i = 1 + \varepsilon$ . Thus, a son of the previous individual will enjoy an endowment vector  $(h_{t+1}^i, b_{t+1}^i)$  equal to  $(1 + \varepsilon, \bar{b})$  so that he will also invest in the education of their children and will leave them a bequest equal to  $b_{t+2}^i = B^4(\bar{b}) \neq \bar{b}$ .<sup>7</sup> This proves that the fixed point of  $B^2(b_t^i)$  is not a steady state because it is not a rest point of the dynamic equation (3.11).

We can follow similar arguments to prove that a fixed point  $\bar{b}$  of  $B^3(b_t^i)$  cannot be a steady state. For this purpose, assume that  $b_t^i = \bar{b} < \hat{b}$  and  $h_t^i = 1 + \varepsilon$ . As follows from (3.11) and (3.12), this individual leaves a bequest per capita equal to  $b_{t+1}^i = B^3(\bar{b}) = \bar{b}$ . However, he does not invest in the education of their children so that  $h_{t+1}^i = 1$ . Thus, a son of the previous individual will enjoy an endowment  $(h_{t+1}^i, b_{t+1}^i)$  equal to  $(1, \bar{b})$  and, thus, he will not invest either in the education of their children and will leave them a bequest equal to  $b_{t+2}^i = B^1(\bar{b}) \neq \bar{b}$ .<sup>8</sup> This proves in turn that the fixed point of  $B^3(b_t^i)$  is not a rest point of the dynamic equation (3.11).

As a summary, we conclude that mobility in human capital across generations prevents the fixed points of  $B^2(b_t^i)$  and  $B^3(b_t^i)$  from being steady states for bequest. However, by the same reason, the fixed points of  $B^1(b_t^i)$  and  $B^4(b_t^i)$  may be steady states. The amount  $\bar{b}^1$  of bequest is stationary because those non-educated individuals who have received this level of inheritance do not invest in the education of their children, whereas  $\bar{b}^2$  is a stationary amount of bequest since the educated individuals who have received this level of inheritance do finance the education of their offspring.

Observe that  $\bar{b}^1$  can be either smaller or larger than  $\bar{b}^2$ . In the first case the educated individuals leave a larger amount of bequest to their children than the non-educated, whereas the opposite is true in the second case. By using (4.1) and (4.2), we obtain that  $\bar{b}^1 < \bar{b}^2$  if and only if

$$(1 - \tau_w) w \varepsilon - (1 - s_e) n \mu - \frac{n(1 + \rho)(1 - \tau_w) w \varepsilon}{\beta R(\tau_k)(1 - \tau_b)} > 0. \quad (4.3)$$

The left-hand side of (4.3) collects the three forces driving the relationship between  $\bar{b}^1$  and  $\bar{b}^2$ . This relationship depends first on how large is the labor income of educated parents with respect to the income of non-educated parents (the education premium). Second, the education cost reduces the amount of bequest that the parents investing in

<sup>7</sup> As  $\bar{b} > \hat{b}$  and  $\tilde{b} > \hat{b}$  then  $\bar{b} > \tilde{b}$ , which together with  $h_{t+1} = 1 + \varepsilon$  implies that  $b_{t+2}^i = B^4(\bar{b})$ .

<sup>8</sup> Similarly, as  $\bar{b} < \hat{b}$  and  $\tilde{b} > \hat{b}$  then  $\bar{b} < \tilde{b}$ , which together with  $h_{t+1} = 1 + \varepsilon$  implies that  $b_{t+2}^i = B^1(\bar{b})$ .

education are willing to leave to their children. Finally, the larger is the contribution of education to the labor income of children, the smaller is the amount of bequest that educated parents must leave to achieve the optimal amount of the contribution to the future income of their children.

In order to simplify the exposition and keep the length of the paper within a reasonable bound, we will only analyze the dynamics of the more empirically plausible case where  $\bar{b}^1 < \bar{b}^2$ . For instance, Nordblom and Ohlsson (2005) estimate that the education level of parents in Sweden increases the probability that they transfer both human and physical capital to their children. That is, intergenerational transfers of human capital and physical wealth are complements. Therefore, we will assume that (4.3) holds from now on.<sup>9</sup>

We are interested in those parameter configurations for which the interior steady states for bequest  $\bar{b}^1$  and  $\bar{b}^2$  exist and are stable. In this case, the economy exhibits heterogeneity among individuals at the steady state and, thus, we can analyze how the initial composition of the intergenerational transfers and the fiscal policy parameters affect income inequality and human capital mobility. In order to establish the existence and stability of  $\bar{b}^1$  and  $\bar{b}^2$ , we must impose some assumptions on the fundamentals of our economy. In particular, the existence and stability of the stationary amounts of bequest depend on whether the non-negative constraint on bequests (2.6) is binding.

First, the existence of two interior and stable fixed points  $\bar{b}^1$  and  $\bar{b}^2$  requires the functions  $B^j(b_t^i)$  in (3.12) to have slope smaller than one for all  $j = 1, 2, 3, 4$ , and to satisfy that  $B^1(0) > 0$  and  $B^4(0) > 0$ . On the one hand, as was pointed in the previous section, the Inada condition of the utility function (2.1) with respect to  $I_{t+1}^i$  ensures that  $B^1(b_t^i) > 0$  for all  $b_t^i \geq 0$ . Moreover, the condition (4.3) ensures that  $B^4(b_t^i) > B^1(b_t^i)$  for all  $b_t^i \geq 0$  so that, in this case, it is also true that  $B^4(0) > 0$ . On the other hand, given these properties, interior fixed points exist if and only if the functions  $B^j(b_t^i)$  have slope smaller than one. This property of  $B^j(b_t^i)$  holds under the following condition:

$$\frac{\beta(1 - \tau_b)R(\tau_k)}{n(1 + \beta + \rho)} < 1, \quad (4.4)$$

which also ensures the stability of the interior steady states  $\bar{b}^1$  and  $\bar{b}^2$  provided they exist.

Second, since condition (4.4) holds, the interior steady states  $\bar{b}^1$  and  $\bar{b}^2$  exist if and only if the threshold values  $\tilde{b}$  and  $\hat{b}$  are positive and negative, respectively. On the one hand, if  $\tilde{b} < 0$  then the function  $B^1(b_t^i)$  is not defined for positive values of  $b_t^i$  and, thus, the steady state  $\bar{b}^1$  does not exist, which means that the non-educated individuals always decide to invest in the education of their children in this case. On the other hand, since  $B^4(\hat{b}) = 0$  and the function  $B^4(b_t^i)$  has slope smaller than one, the steady state  $\bar{b}^2$  does not arise if  $\hat{b} > 0$ . The following two conditions ensure that  $\tilde{b} > 0$  and  $\hat{b} < 0$ , respectively:

$$(1 - s_e)n\mu + \left[ \frac{n(1 + \rho)\varepsilon}{\beta(1 - \tau_b)R(\tau_k)} - 1 \right] (1 - \tau_w)w + \Omega > 0, \quad (4.5)$$

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<sup>9</sup>The analysis of the case with  $\bar{b}^1 > \bar{b}^2$  becomes just a mechanical exercise that replicates the same arguments that we will use in the rest of the paper for the case under consideration.

and

$$(1 - s_e)n\mu + \left[ \frac{n(1 + \rho)\varepsilon}{\beta(1 - \tau_b)R(\tau_k)} - (1 + \varepsilon) \right] (1 - \tau_w)w + \Omega < 0. \quad (4.6)$$

Finally, condition (4.5) guarantees that  $\bar{b}^1 > 0$  but is not sufficient to ensure that this fixed point of (3.4) is a steady state for the amount of bequest. In order to do so, we also need to impose that

$$\bar{b}^1 < \tilde{b}. \quad (4.7)$$

If (4.7) does not hold, the amount  $\bar{b}^1$  of bequest could not be a steady state because the function  $B^1(b_t^i)$  does not characterize the dynamics of bequest left by non-educated individuals who have received an inheritance larger than  $\tilde{b}$ . Given the definitions of  $\tilde{b}$  and  $\bar{b}^1$  in (3.9) and (4.1), respectively, the equation (4.7) ends up being just a condition on the fundamentals of the economy.

From now on we will also assume that the conditions (4.4), (4.5), (4.6) and (4.7) hold. Under these conditions, our economy converges to a two-point distribution with appealing empirical properties.<sup>10</sup> Under this distribution some dynasties leave a positive bequest to each of their children equal to  $\bar{b}^1$  and do not invest in their education, whereas other dynasties do invest in the education of their children and leave a larger bequest per capita equal to  $\bar{b}^2$ .<sup>11</sup> As we have already mentioned, this property of the stationary distribution agrees with the empirical evidence provided by Nordblom and Ohlsson (2005). The case under consideration is thus depicted in Figure 1, which plots the relationship between the bequest left to children and the inheritance received from parents given by the dynamic equation (3.12). Note that this relationship is piecewise linear.

[Insert Figure 1]

Since the initial distribution of bequest and human capital determines both the stationary income distribution and the intergenerational mobility in human capital, we will characterize in the following subsection the entire path of bequest and human capital for all types of dynasties when the economy converges to the aforementioned two-point distribution. We will use this analysis to study in the next section the impact of fiscal policy in the distribution dynamics for an arbitrary initial distribution.

## 4.2. Dynamic analysis

In this subsection we analyze the dynamics of an economy that converges to the two-point distribution having the steady-state values  $\bar{b}^1$  and  $\bar{b}^2$ . As was shown in the previous subsection, this occurs when the conditions (4.3), (4.4), (4.5) and (4.6) hold. We will next analyze how the initial distribution of bequest and human capital determines the number of dynasties converging to a situation with  $h_t^i = 1$  and  $b_t^i = \bar{b}^1$  and those converging to another with  $h_t^i = 1 + \varepsilon$  and  $b_t^i = \bar{b}^2$ .

<sup>10</sup>In the case with  $\bar{b}^1 > \bar{b}^2$  (i.e., when (4.3) does not hold), we must also impose that  $B^4(0) > 0$  to ensure the existence of a two-point interior stationary distribution.

<sup>11</sup>When at least one of these conditions does not hold, then the economy converges to either a degenerate distribution or a distribution defined by corner steady state values of bequest. In Appendix B we present all possible configurations of the stationary distribution.

We first observe that those dynasties whose members have the initial human capital  $h_t^i = 1$  and who have received an inheritance  $b_t^i$  smaller than  $\tilde{b}$  converge to a steady state given by  $h^i = 1$  and  $b^i = \bar{b}^1$ , whereas all the dynasties with members having the initial human capital  $h_t^i = 1 + \varepsilon$  converge to a steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . This conclusion directly follows from the dynamics of human capital and bequest described by (3.11) and (3.12), and after using condition (4.4) ensuring the stability of the steady states  $\bar{b}^1$  and  $\bar{b}^2$ . On the one hand, the members of a dynasty with  $h_t^i = 1$  and  $b_t^i$  smaller than  $\tilde{b}$  do not invest in the education of their children and leave an amount  $b_{t+1}^i$  of bequest satisfying  $b_{t+1}^i < \tilde{b}$ . On the other hand, the members of a dynasty with a level  $h_t^i = 1 + \varepsilon$  of human capital always invest in the education of their children because condition (4.6) ensures that  $\hat{b} < 0$ . Condition (4.4) ensures that the bequests of the former and the latter dynasties will converge to  $\bar{b}^1$  and  $\bar{b}^2$ , respectively.

Less trivial is the dynamic adjustment of human capital and bequest for the other group of dynasties, i.e., those with  $h_t^i = 1$  and  $b_t^i$  larger than  $\tilde{b}$ . The members of these dynasties decide to finance the education of their children as dictated by equation (3.11). However, it is necessary to know how large is the amount  $b_{t+1}^i = B^2(b_t^i)$  of bequest that they leave since this amount determines in turn the behavior of their offspring. From condition (4.6), they always leave a bequest per capita  $b_{t+1}^i$  larger than  $\tilde{b} < 0$ . The children of those individuals will then decide to leave a bequest per capita equal to  $b_{t+2}^i = B^4(b_{t+1}^i)$  and, thus, the dynasty will converge to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . Therefore, the threshold value of bequest  $\tilde{b}$  determines the dynamics of the initially non-educated dynasties and, in particular, their upward mobility in terms of human capital. The non-educated dynasties will converge to the steady state associated with  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$  if their members are initially endowed with an inheritance  $b_t^i$  larger than  $\tilde{b}$ , whereas these non-educated dynasties with an inheritance  $b_t^i$  smaller than  $\tilde{b}$  will converge to the steady state given by  $h^i = 1$  and  $b^i = \bar{b}^1$ .

We can thus summarize the dynamic behavior of the economy considered in this subsection as follows. The dynasties with an initial level  $h_0^i = 1$  of human capital will converge: (i) to the steady state  $h^i = 1$  and  $b^i = \bar{b}^1$  if  $b_0^i < \tilde{b}$ ; and (ii) to the steady state  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$  if  $b_0^i > \tilde{b}$ . The dynasties with an initial level  $h_0^i = 1 + \varepsilon$  of human capital will always converge to the steady state  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . Therefore, we have shown that the initial distribution of bequest and human capital determines the stationary distribution of these variables, and thus the stationary distribution of income. In particular, our model predicts that the propensity to invest in the education of children depends positively on the education level of parents.

With respect to human capital mobility, we observe that only the non-educated dynasties may experience intergenerational mobility. In this case, the inheritance is the variable that determines this mobility. In particular, the dynasties with an initial level  $h_0^i = 1$  of human capital educate their children in the first period if  $b_0^i > \tilde{b}$ , and then they remain as educated dynasties forever. Therefore, for these dynasties the human capital adjusts instantaneously to the level  $1 + \varepsilon$  and the amount of bequest converge monotonously along time to the steady state  $\bar{b}^2$ . Moreover, the threshold  $\tilde{b}$  in (3.9) contains all the information about the determinants of this one-shot upward

mobility.

## 5. Effects of fiscal policy on the stationary distribution

In this section we will analyze how fiscal policy affects the stationary distribution towards which an economy converges given an initial distribution. We will assume a parametric configuration ensuring that the economy converges to the empirically plausible two-point distribution considered in Subsection 4.2. Given an initial distribution of bequest and human capital, we will analyze how non-anticipated permanent marginal shocks on the fiscal parameters alter this stationary distribution. In particular, we will develop balanced-budget incidence analyses where government consumption will accommodate the permanent fiscal shocks in order to satisfy the constraint (2.7).

Fiscal policy can alter the stationary amounts  $\bar{b}^1$  and  $\bar{b}^2$  of bequest and can also affect the proportion of dynasties converging to each of these steady states by distorting the intergenerational mobility in human capital. The distance between  $\bar{b}^1$  and  $\bar{b}^2$  measures the inequality between the income of educated and non-educated adult individuals due to the different amounts of inheritance they receive. As the education premium  $\varepsilon$  on wage earnings is fixed, the variation in the distance between  $\bar{b}^1$  and  $\bar{b}^2$  provides all the relevant information about the effects of fiscal policy on income inequality. To obtain the effects on  $\bar{b}^1$  and  $\bar{b}^2$  we must analyze the impact of fiscal policies on the functions  $B^1(b_t^i)$  and  $B^4(b_t^i)$ , respectively.<sup>12</sup> Moreover, the effect of these policies on intergenerational upward mobility in human capital is given instead by their impact on the threshold amount  $\tilde{b}$  of bequest. A small value of  $\tilde{b}$  means that the set of initial amounts of bequest for which non-educated dynasties end up being educated becomes larger. In other words, a small value of  $\tilde{b}$  makes easier intergenerational upward mobility.<sup>13</sup>

We next study separately the effects of each of the tax instruments under consideration.

### 5.1. Inheritance taxation

We will first show that the tax on inheritance reduces the stationary values  $\bar{b}^1$  and  $\bar{b}^2$  of bequest. Differentiating the function (3.4) with respect to  $\tau_b$  we obtain

$$\frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \tau_b} = - \left[ \frac{\beta R(\tau_k) b_t^i}{n(1 + \beta + \rho)} + \frac{(1 + \rho)(1 - \tau_w)w \Delta_{t+1}^i}{(1 + \beta + \rho)(1 - \tau_b)^2} \right], \quad (5.1)$$

which is clearly negative. Since  $\bar{b}^1$  and  $\bar{b}^2$  are fixed points of  $B^1(b_t^i) = B(b_t^i, 1, 0)$  and  $B^4(b_t^i) = B(b_t^i, 1 + \varepsilon, \mu)$ , respectively, the effects of the inheritance tax on these stationary solutions immediately follow from (5.1) and (2.3). Moreover, the marginal increase in the rate of this tax reduces the gap between  $\bar{b}^1$  and  $\bar{b}^2$  because the negative

<sup>12</sup>In fact, the effect of fiscal policies on  $\bar{b}^1$  and  $\bar{b}^2$  can also be directly derived by applying respectively the implicit function theorem to the equation  $\bar{b}^1 = B^1(\bar{b}^1)$  and  $\bar{b}^2 = B^4(\bar{b}^2)$  and using Assumption (4.4).

<sup>13</sup>The fiscal policy can also affect the threshold level  $\hat{b}$ . However, since we will only consider marginal shocks in fiscal policy, we can maintain the assumption that this threshold level remains negative after the shocks.

impact of this permanent policy shock on  $B^4(b_t^i)$  is larger than in  $B^1(b_t^i)$  as follows from (5.1) after substituting the corresponding value of  $\Delta_{t+1}^i$ . Thus, the increase in the tax rate on inheritances reduces the stationary differences in the income per capita between educated and non-educated individuals.

We now analyze how the marginal permanent shock in  $\tau_b$  affects intergenerational mobility. For that purpose, we study the effects of the tax on the threshold amount  $\tilde{b}$  of bequest that determines the steady state towards which each dynasty converges. From equation (3.9), we directly obtain that

$$\frac{\partial \tilde{b}}{\partial \tau_b} = \frac{\tilde{b}}{(1 - \tau_b)} + \frac{n(1 + \rho)(1 - \tau_w)w\varepsilon}{\beta(1 - \tau_b)^3 R(\tau_k)},$$

which is positive, i.e., the marginal increase in  $\tau_b$  pushes the value of  $\tilde{b}$  up. Therefore, the tax on inheritances reduces the number of initially non-educated dynasties converging to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ . As was expected from the dynamic analysis of the previous section, this tax does not alter the number of initially educated dynasties converging to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$ .

The effects of inheritance taxation on the stationary distribution can then be summarized as follows. An increase in the rate of this tax raises the fraction of individuals converging to the steady state given by  $h^i = 1$  and  $b^i = \bar{b}^1$ . Thus, the stationary fraction of non-educated individuals increases and the amount of bequest becomes smaller in the long run. This reduction in the amount of bequest is larger for the group of educated adult individuals (i.e., for the richest people). The aggregate adult income at the steady state then goes down and, moreover, the proportion of aggregate income enjoyed by the poorest adult individuals rises. Therefore, all these effects of the inheritance taxation translate into a reduction in the inequality between educated and non-educated people at the stationary distribution of income at the cost of a reduction in both aggregate income and human capital accumulation. Figure 2 presents the change in the relative frequencies of the stationary distribution of bequests after an increase in the tax rate on inheritances.

[Insert Figure 2]

Inheritance taxes makes physical bequests less attractive as an instrument to increase the lifetime income of children. This explains why an inheritance tax reduces the amount of bequest that parents leave to their offspring. Moreover, this distortion of the tax on the bequest's decision margin also explains why this tax raises the amount of inheritance that an individual must receive from his parents in order to be willing to invest in the education of their children. Therefore, an increase in the inheritance tax ends up being an impediment for upward mobility as the reduction in initial wealth prevents a larger fraction of individuals from jumping the hurdle associated to the indivisible cost of education.

Given the effect of the inheritance taxation on the stationary distribution of bequests, we can derive the effect of this tax on the stationary amount of saving. From (3.3), and using (5.1), we get that

$$\frac{\partial s_t^i}{\partial \tau_b} = - \frac{\rho R(\tau_k) b_t^i - n \left[ \frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \tau_b} \right]}{(1 + \rho) R(\tau_k)},$$

which is negative. Thus, we conclude that in our small open economy, inheritance taxes reduce the saving of each individual at the steady state. Since in our economy leaving bequest is a motive for saving, a tax on inheritances affects negatively savings and bequest. Obviously, from this result and from the fact that the stationary proportion of non-educated individuals raises, we can conclude that an inheritance tax affects negatively the aggregate amount of saving at the steady state.

## 5.2. Labor income taxation

By following the same procedure as in the previous subsection, we obtain the effects of labor income taxation. We first get from (3.4) that

$$\frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \tau_w} = \left[ \frac{\beta w R(\tau_k)}{n(1 + \beta + \rho)} \right] \left[ \frac{n(1 + \rho) \Delta_{t+1}^i}{\beta(1 - \tau_b) R(\tau_k)} - h_t^i \right]. \quad (5.2)$$

By using (2.3) and (3.12), we get that  $\frac{\partial B^1}{\partial \tau_w} < 0$  and  $\frac{\partial B^2}{\partial \tau_w} < 0$ , where the last inequality follows from condition (4.3). Hence, a marginal increase in the rate of the labor income tax reduces the stationary amounts  $\bar{b}^1$  and  $\bar{b}^2$  of bequest. Moreover, by using condition (4.3) we also obtain that an increase in the rate of the labor income tax pushes the gap between  $\bar{b}^1$  and  $\bar{b}^2$  down. In other words, the labor income taxation reduces the stationary differences in the income per capita between educated and non-educated individuals.

We now analyze how a marginal increase in  $\tau_w$  affects the upward mobility in human capital. From (3.9), we get that

$$\frac{\partial \tilde{b}}{\partial \tau_w} = \left( \frac{w}{1 - \tau_b} \right) \left[ 1 - \frac{n(1 + \rho) \varepsilon}{\beta(1 - \tau_b) R(\tau_k)} \right]. \quad (5.3)$$

Let us define

$$\varepsilon^* = \frac{\beta(1 - \tau_b) R(\tau_k)}{n(1 + \rho)}. \quad (5.4)$$

If  $\varepsilon < \varepsilon^*$  then  $\frac{\partial \tilde{b}}{\partial \tau_w} > 0$ , whereas  $\frac{\partial \tilde{b}}{\partial \tau_w} < 0$  when  $\varepsilon > \varepsilon^*$ . Therefore, the labor income tax raises the number of initially non-educated dynasties converging to the steady state given by  $h^i = 1 + \varepsilon$  and  $b^i = \bar{b}^2$  if and only if the education premium  $\varepsilon$  is sufficiently large. In this case, a positive permanent shock in the tax rate raises the stationary fraction of population that is educated ( $h^i = 1 + \varepsilon$ ) and leaves a bequest equal to  $\bar{b}^2$ . Clearly, the opposite conclusion is derived from the case with a education premium smaller than the threshold  $\varepsilon^*$ . In this case, the labor income tax has the same effect as the inheritance tax. Figure 2 and 3 illustrate the change in the relative frequencies of the stationary distribution of bequests after an increase in the tax rate on labor income.

[Insert Figure 3]

Note that the ambiguity of the effect of labor income taxation on the threshold amount  $\tilde{b}$  of bequest did not appear when we considered inheritance taxation. The intuition of these different effects can be easily understood by looking at the condition (3.2) holding with equality. Consider thus the marginal uneducated individual who has

received the amount  $\tilde{b}$  of inheritance and is thus indifferent between these two policies: (i) leaving the bequest  $B_1(\tilde{b})$  and not investing in the education of their children and (ii) leaving zero bequest and investing the amount  $\mu$  in the education of each of his children. Therefore, the optimality condition (3.2) for this marginal individual is

$$\frac{\beta(1-\tau_b)}{(1-\tau_w)\varepsilon w} = \frac{n(1+\rho)/R(\tau_k)}{(1-\tau_w)w + (1-\tau_b)\tilde{b} - (1-s_e)n\mu - \Omega}. \quad (5.5)$$

We clearly see that an increase in the inheritance tax  $\tau_b$  lowers the benefit from leaving physical bequest, which is collected in the left hand side of (5.5), and raises the cost from leaving them, which is collected in the right hand side of (5.5). In this case,  $\tilde{b}$  always decreases as  $\tau_b$  increases. However, when  $\tau_w$  increases, the benefit from leaving some physical bequest rises since the return from the investment in human capital is more heavily taxed. Moreover, a larger tax on labor income raises also the cost of leaving physical bequest as individuals will have less disposable lifetime income. This explains the ambiguity of the effect on the bequest threshold  $\tilde{b}$  of changes in the tax rate  $\tau_w$ . As we see from (5.3),  $\tilde{b}$  is increasing (decreasing) in the value  $\tau_w$  of the tax rate on labor income if the education premium  $\varepsilon$  is sufficiently small (large). More precisely, if the education premium is larger than  $\varepsilon^*$ , then the labor income tax raises the willingness to invest in the education of their children for the marginal individuals we are considering. These marginal individuals will need a smaller inheritance to invest in education and this explains why the labor income tax raises the stationary fraction of educated population when the education premium is sufficiently large. In this case, a tax on labor income acts as an instrument promoting human capital accumulation since the induced increase in the marginal cost of leaving physical bequest triggers a change in the composition of transfers in favor of education investment.

At this point, one should investigate what are the empirically plausible value for the education premium  $\varepsilon$ . Note that conditions (4.3) and (4.4) imply that

$$\varepsilon^* \in \left(1, 1 + \frac{\beta}{1+\rho}\right).$$

Thus, in an economy with  $\varepsilon > \varepsilon^*$  educated individuals obtain a wage per efficiency unit of labor that is more than twice as much the wage perceived by non-educated individuals. Several empirical studies provide evidence supporting the existence of a large education premium (see, e.g., Bound and Johnson, 1992; or Barro and Lee, 2000), and show a dramatic increase in this premium from the middle of the past century (see, e.g., Autor et al., 1998). For instance, Barro and Lee (2000) estimate that the wage of individuals who have completed the higher level of education relative to those with an incomplete primary level is around 2.18.

In terms of the policy functions appearing in Figure 1, we can summarize the previous results by saying that an increase in the tax rates on inheritance and labor income shift the policy functions  $B^1$  and  $B^4$  downwards, which results in a decrease of the stationary values  $\bar{b}^1$  and  $\bar{b}^2$ . However, a rise in the tax rate on inheritance always shifts the policy function  $B^2$  upwards, which results in a decrease of the threshold value  $\tilde{b}$  of inheritance and thus in less intergenerational upward mobility, whereas the direction of the shift of  $B^2$  is generally ambiguous when the tax rate on labor income rises.

Finally, we can see that labor income taxation reduces individual saving at the steady state of both educated and non-educated individuals. We obtain this result from differentiating (3.3) and using the derivative (5.2). The effect of this tax on the amount of aggregate saving then depends on the education premium because  $\varepsilon$  determines whether the proportion of educated individuals goes up or down. If  $\varepsilon < \varepsilon^*$  the aggregate saving depends negatively on the tax rate because in this case the tax raises the proportion of non-educated individuals. However, the effect of the tax on aggregate saving is ambiguous when  $\varepsilon > \varepsilon^*$  because in this case the proportion of educated individuals rises and the saving of the two types of individuals decreases.

### 5.3. Capital income taxation

We will show in this subsection that the capital income tax displays the same qualitative marginal effects on the stationary levels of bequest and on the stationary distribution of human capital as the tax on inheritance analyzed in Subsection 5.1. On the one hand, we get from (3.4) that

$$\frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \tau_k} = - \frac{\beta r [(1 - \tau_w) w h_t^i + (1 - \tau_b) b_t^i - n(1 - s_e) e_t^i - \lambda]}{n(1 + \beta + \rho)}. \quad (5.6)$$

Observe that the expression inside the bracket in (5.6) is the individual's disposable income at the adult age (see the budget constraint (2.4)). Hence, the derivative (5.6) is negative. Therefore, the tax on capital income reduces the stationary amounts  $\bar{b}^1$  and  $\bar{b}^2$  of bequest and, moreover, condition (4.3) implies that this reduction is larger for the amount  $\bar{b}^2$  of bequest of educated individuals.

On the other hand, the marginal increase in the tax rate  $\tau_k$  raises the threshold amount  $\tilde{b}$  of bequest. Clearly, we obtain from (3.9)

$$\frac{\partial \tilde{b}}{\partial \tau_k} = \frac{n(1 + \rho)r(1 - \tau_w)w\varepsilon}{\beta [(1 - \tau_b)R(\tau_k)]^2} + \frac{\theta r}{(1 - \tau_b)[R(\tau_k)]^2}, \quad (5.7)$$

which is positive. This means that this fiscal reform leads to an increase in the fraction of individuals converging to the steady state given by  $h^i = 1$  and  $b^i = \bar{b}^1$ . Thus, the non-educated adult individuals increase their stationary weight in the total population. Therefore, the aggregate income of adult individuals at the steady state goes down and, moreover, the proportion of aggregate income enjoyed by the poorest adult individuals rises. Figure 2 depicts also the effect of a rise in the capital income tax.

The condition (3.2) also provides the economic mechanism underlying the previous effects of capital income taxation. Note that this tax reduces the value of old age consumption for a given amount of bequest left to children. Hence, the tax increases the marginal utility loss derived from reducing consumption to increase the amount of bequest left to children. However, the tax does not alter the contribution of parents to the lifetime income of their children and, hence, the marginal utility gain associated to this contribution does not depend on the tax rate. Therefore, the capital income tax reduces the bequest per capita left by individuals to their offspring. Moreover, this tax reduces the fraction of educated population since individuals require a larger amount of inheritance to invest in the education of their children.

Finally, we also obtain from (3.3) that the marginal increase in the rate of the capital income tax has an ambiguous impact on the individual's saving at the steady state. Since the capital income tax reduces the after-tax returns on saving, this tax displays an income effect and a substitution effect on saving. In other words, an increase in the tax rate stimulates saving to compensate the reduction in the disposable income at the old age, whereas the tax raises the amount of consumption at the adult age that the individual must sacrifice to obtain a unit of consumption at old age.

#### 5.4. Education subsidies

In this subsection, we analyze the marginal effects of a change in the subsidy to education investment. Obviously, this policy can affect only the bequest left by those individuals who invest in the education of their children since only those individuals are entitled to receive the subsidy. In particular, an increase in the subsidy rate raises the disposable income of these individuals and does not alter their contribution to the lifetime income of their children. Hence, the subsidy stimulates the willingness of parents to leave bequest, which reduces the amount of inheritance that they must receive in order to invest in the education of their children. This conclusion can directly be proved by using conditions (3.2) and (3.9). Observe that a permanent increase in the subsidy rate raises the stationary fraction of population that is educated ( $h^i = 1 + \varepsilon$ ) and leave bequest equal to  $\bar{b}^2$  at the steady state. Moreover, the amount  $\bar{b}^2$  of bequest left by each educated individual goes up, whereas the amount  $\bar{b}^1$  of bequest left by non educated (i.e., the poorest) individuals does not change. Therefore, the aggregate income of adult individuals rises at the steady state and, moreover, the proportion of aggregate income enjoyed by poorest adult individuals decreases. Figure 4 illustrates the change in the relative frequencies of the stationary distribution of bequests after an increase in the subsidy rate on education.

[Insert Figure 4]

Before closing this subsection, we should note that an increase in the subsidy to education has an impact qualitatively similar to a reduction in the cost of education  $\mu$  and to an increase in the education premium  $\varepsilon$ . The education premium could increase as a result of skill-biased technical change while the reduction in the education cost faced by parents could be achieved through the partial provision by the government of education. The two parameters  $\mu$  and  $\varepsilon$  characterize the technology for human capital accumulation as they determine the productivity of the education system. This productivity is obviously an increasing function of the education premium  $\varepsilon$  and a decreasing function of the education cost  $\mu$ . Therefore, a more productive education sector results in a larger upward mobility in human capital so that there is a larger fraction of population that is educated, and in a larger income inequality between educated and non-educated individuals (see Acemoglu, 2002).

#### 5.5. Social security system

We now analyze the effects of a pay-as-you-go social security system where old individuals receive lump-sum benefits that are financed by the contributions of adult

individuals. If the contributions of adult individuals take the form of labor income taxes, the social security system has two distortionary effects on our economy. On the one hand, since the labor supply in efficiency units is endogenous, the labor income tax distorts the individual choice as was showed in Subsection 5.2. On the other hand, the social security system implies an ex-ante intergenerational redistribution from adult to old individuals. Since we have already studied the distortionary effects of labor income taxation, we will exclusively focus on the effects of intergenerational redistribution achieved through lump-sum taxes. To do so, we consider a marginal variation of the lump-sum tax  $\lambda$  satisfying

$$\frac{d\theta}{d\lambda} = -n \quad (5.8)$$

in order to fulfil the budget constraint (2.7). Therefore, we will assume that the government increases the lump-sum tax  $\lambda$  paid by the adult individuals, and the additional revenues are entirely devoted to finance an increase in the lump-sum subsidy  $\theta$  to the old individuals.

The effects of a pay-as-you-go social security system on income distribution and on intergenerational mobility in human capital depend on whether the economy is dynamically efficient or inefficient as defined by Cass (1979). In particular, an increase in the lump-sum transfer from adult to old individuals (an increase in  $\lambda$ ) has qualitatively the same stationary effects as a rise in the capital income tax if  $R(\tau_k) > n$ , whereas this variation on the intergenerational transfer has qualitatively the opposite effects to a rise in the capital income tax when  $R(\tau_k) < n$ . This is true except for the relative effect on the stationary amounts  $\bar{b}^1$  and  $\bar{b}^2$  of bequest. Variations in the social security transfers never alter the gap between these two stationary levels of bequest. The previous conclusions are easily derived by differentiating (3.4) and (3.9) with respect to  $\lambda$ . Using (5.8), we obtain

$$\frac{\partial B(b_t^i, h_t^i, e_t^i)}{\partial \lambda} = \frac{\beta(n - R(\tau_k))}{n(1 + \beta + \rho)},$$

and

$$\frac{\partial \tilde{b}}{\partial \lambda} = 1 - \frac{n}{R(\tau_k)}.$$

Following then the same procedure used for the other fiscal instruments, we directly derive the previous characterization of the effects of the social security system.

The social security program distorts the decisions on bequest and education investment by affecting the marginal utility loss derived from reducing consumption in order to increase the amount of bequest left to children. In particular, there may exist a wedge between the returns on saving given by  $R(\tau_k)$  and the returns of social security program given by  $n$ . Thus, this policy can alter the present value to the lifetime income that individuals devote to own consumption given the amount of bequest that they will leave to their children. If  $R(\tau_k) > n$ , then the social security program reduces the present value of lifetime income of individuals. Therefore, in this case this policy reduces the willingness of parents to leave bequest and to invest in the education of their children. Evidently, the opposite conclusion is derived when  $R(\tau_k) < n$ .

## 6. Conclusion

In this paper we have analyzed how the initial distribution and composition of wealth between bequest and human capital characterize the evolution of both the distribution of income and the intergenerational mobility in human capital. There are three main assumptions that give rise to our result. First, the education of individuals can only be financed by their parents, who derive satisfaction from their contribution to the lifetime income of their children with independence of the type of the intergenerational transfers used for that purpose. Second, we assume that the acquisition of human capital is indivisible and requires thus a minimum amount of investment. Finally, parents cannot force children to give them transfers, so that those parents with a sufficiently small income will not invest in the education of their children. Hence, intragenerational income distribution and intergenerational mobility in human capital are affected by the percentage of individuals who inherited a sufficiently large amount of physical wealth to enable them to invest in the education of their offspring. Furthermore, the minimum amount of inheritance required by educated parents to give education to their children differs from the minimum amount for non-educated parents.

From our results we can conclude that the cross-country differences in the composition of intergenerational transfers, income inequality, and mobility are determined by: (i) differences in the distribution of initial wealth; (ii) differences in the distribution of the composition of initial wealth; (iii) differences in the fiscal policy set by the governments; and (iv) differences in the process of human capital accumulation. Concerning the latter determinant, the first candidate for generating cross-country differences is the education technology. In particular, countries can differ in the education costs, in the number and length of education levels, or in the productivity of the technology used to accumulate human capital. Moreover, the cross-country differences in the process of human capital accumulation can also arise from the amount of public resources invested in education.

A natural and promising extension of our research is to analyze the implications that our results have for economic growth and development. This requires a generalization of the process of human capital accumulation in order to allow for some intergenerational transmission of embodied human capital. Moreover, the assumption of constant interest and wage rates should be modified accordingly along the lines of Owen and Weil (1998). The analysis would face then the challenge of dealing with an evolution within each dynasty that will depend on the aggregate income distribution.

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# Appendix

## A. Optimality conditions of the individual problem

We derive in this appendix the optimal conditions on  $c_t^i$ ,  $x_t^i$ ,  $s_t^i$  and  $b_{t+1}^i$ . To this end we take the value of  $e_t^i$  as given. First, by combining (2.4) and (2.5) we obtain the following intertemporal budget constraint:

$$(1 - \tau_w)w_t h_t^i + (1 - \tau_b)b_t^i - \Omega = c_t^i + (1 - s_e)n e_t^i + \frac{x_{t+1}^i + n b_{t+1}^i}{R(\tau_k)}. \quad (\text{A.1})$$

Second, consider the problem consisting on maximizing (2.1) with respect to  $\{c_t^i, x_{t+1}^i, b_{t+1}^i\}$  subject to (A.1) and (2.6). Denote by  $\psi$  the Lagrangian multiplier associated with the constraint (A.1). The first order conditions of the previous problem are given by

$$c_t^i = \frac{1}{\psi}, \quad (\text{A.2})$$

$$x_{t+1}^i = \frac{\rho R(\tau_k)}{\psi}, \quad (\text{A.3})$$

and

$$\frac{\beta(1 - \tau_b)}{I_{t+1}^i} \leq \frac{n\psi}{R(\tau_k)}. \quad (\text{A.4})$$

By combining (A.2) and (A.3), we directly get (3.1). Moreover, from (A.2) and (A.3) we obtain

$$c_t^i + \frac{x_{t+1}^i}{R(\tau_k)} = \frac{1 + \rho}{\psi}. \quad (\text{A.5})$$

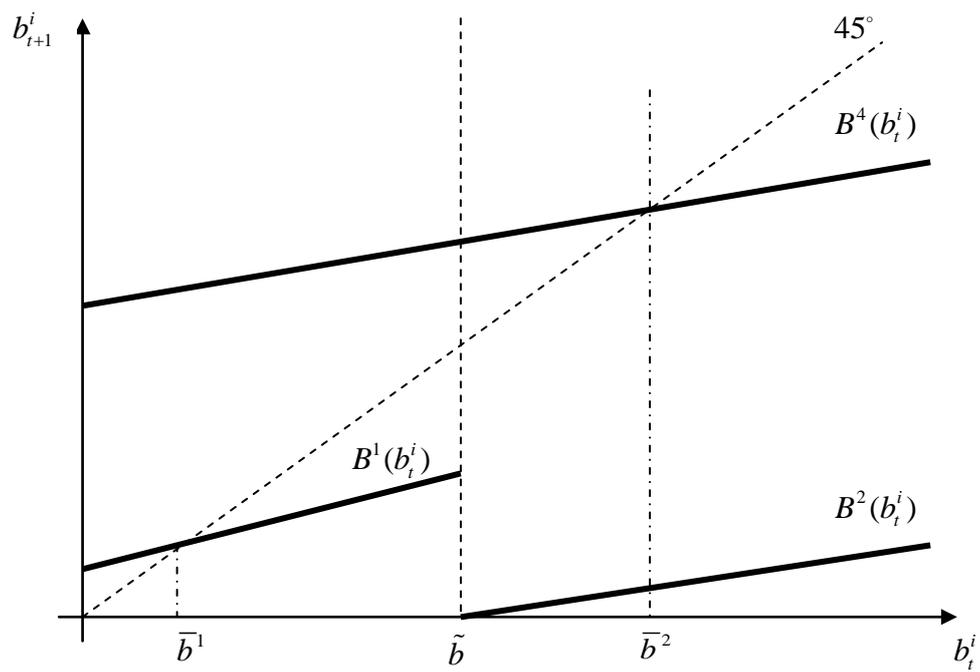
Combining (A.1), (A.4), and (A.5) we can easily derive equation (3.2). Finally, after solving for  $\psi$  in condition (A.4) when it holds with equality and substituting the result in (A.2) and (A.3), we obtain conditions (3.5) and (3.6).

## B. Different configurations of the stationary distribution

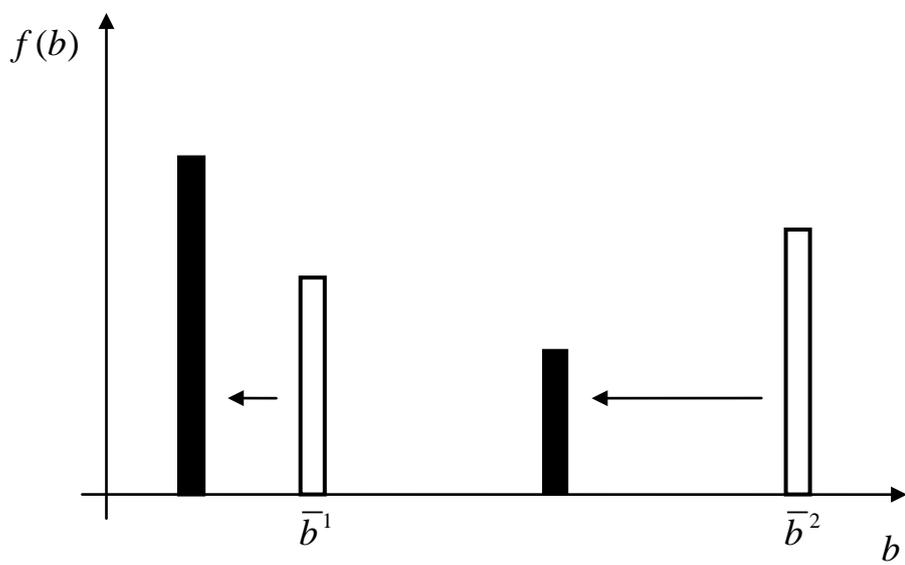
The conditions (4.4), (4.5), (4.6) and (4.7) determine the configuration of the stationary distribution of physical bequest and human capital. In particular, the following configurations of the stationary distribution can emerge in our economy:

1. When condition (4.4) does not hold, then no stable stationary distribution exists.
2. When condition (4.4) holds, the economy converges to a degenerate distribution if at most one of the following situations occurs: (i) condition (4.6) does not hold; or (ii) at least one of the conditions (4.5) and (4.7) is not satisfied. First, if condition (4.6) does not hold, then the fixed point  $\bar{b}^1$  is the unique interior steady-state. In this situation all dynasties leave an amount of bequest equal to  $\bar{b}^1$  and do not invest in the education of their children. Second, if at least one of the conditions (4.5) and (4.7) does not hold, then the fixed point  $\bar{b}^2$  is the unique interior steady-state. In this case all the dynasties invest in their children's education and leave an amount of bequest equal to  $\bar{b}^2$ .

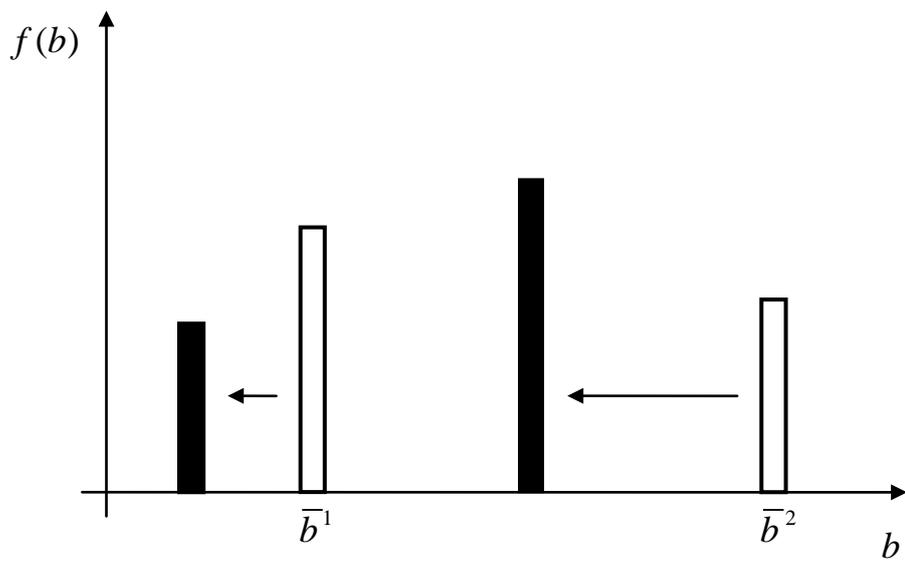
3. When condition (4.4) holds, and condition (4.6) together with at least one of conditions (4.5) and (4.7) does not, then no stationary distribution exists.
4. Finally, when all the conditions (4.4), (4.5), (4.6) and (4.7) hold, then the economy converges to a two-point distribution where some dynasties leave a bequest per children equal to  $\bar{b}^1$  and do not invest in their education, whereas other dynasties do invest in the education of their children and leave a bequest per capita equal to  $\bar{b}^2$ .



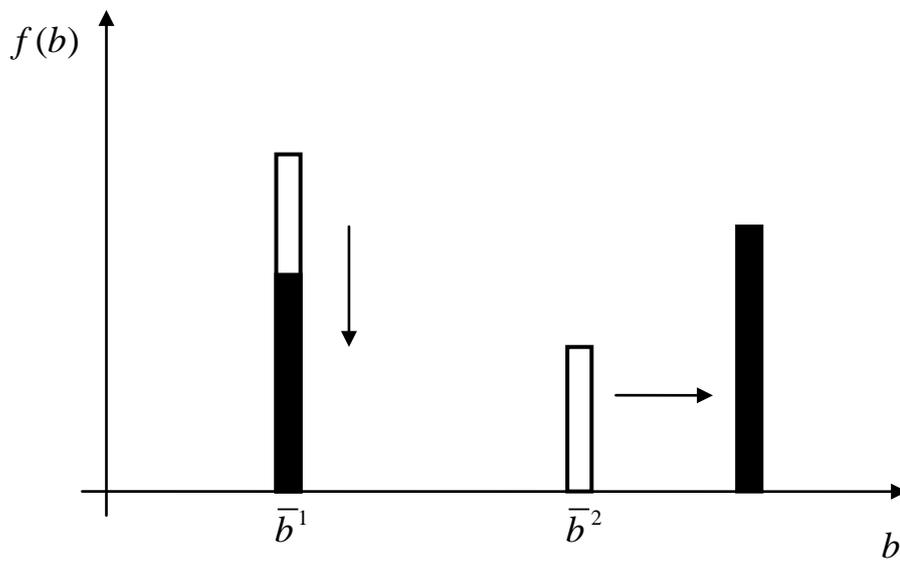
**Figure 1.** The dynamics of bequests.



**Figure 2.** The effect on the distribution of bequests of a rise in the inheritance tax, in the capital income tax, or in the labor income tax when  $\varepsilon < \varepsilon^*$ .



**Figure 3.** The effect of a rise in the labor income tax on the distribution of bequests when  $\varepsilon > \varepsilon^*$ .



**Figure 4.** The effect of a rise in the education subsidy rate.