

The Design and Efficiency of Loyalty Rewards

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The design and efficiency of loyalty rewards

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Abstract

The goal of this paper is to reexamine the optimal design and efficiency of loyalty rewards in markets for final consumption goods. While the literature to date has emphasized the role of loyalty rewards as endogenous switching costs (which distort the efficient allocation of consumers), I analyze instead the ability of alternative designs to foster consumer participation and increase total surplus. First, the efficiency of loyalty rewards depend on their specific design. A commitment to the price of repeat purchases can involve substantial efficiency gains by reducing price-cost margins. However, discount policies imply higher future regular prices and are likely to reduce total surplus. Second, firms may prefer to set up inefficient rewards (discounts), especially in circumstances where a commitment to the price of repeat purchases triggers Coasian dynamics.

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1 Introduction

In a large array of markets firms use a variety of pricing schemes that reward consumer loyalty; they are usually labelled "loyalty programs" or "loyalty rewards" (LRs). Although their specific design vary a lot across firms and industries, in all cases suppliers precommit to treat repeat buyers better than newcomers. The goal of this paper is to examine the efficiency implications of LRs in markets for final consumption goods. In particular, I ask whether or not LRs foster consumer participation and hence reduce the inefficiency associated with firms' market power.

All major airlines currently run frequent flyer programs (FFPs) that offer travellers free tickets and class upgrades according to the number of miles or trips flown with them. The rules of FFPs are relatively complex. On the one hand, individual airlines form partnerships with other airlines and with firms active in independent markets, which expand travelers' ability to earn and redeem miles. On the other hand, airlines restrict consumers' ability to enjoy their privileges by setting expiration dates for miles earned, limiting the number of seats available for certain flights, and blackout dates. Hotels, supermarkets and other retailers have also launched similar programs. In contrast, some other firms offer much simpler schemes to reward consumer loyalty. Exemplary cases include the classic repeat purchase coupon offered by some manufacturers or the popular buy-ten-and-get-one-free type of program, very common among service suppliers, such as golf courts or car washing networks.

Some observers have interpreted these pricing schemes as merely quantity discounts (non linear prices). However, this interpretation clouds over the time dimension that is essential in all these examples. Firstly, in some cases the seller commits to the future transaction price (airline tickets at zero price), while in others (like repeat purchase coupons) the net transaction price is left undetermined. Hence, it is not immediately obvious whether by handing out coupons firms are inducing a lower transaction price for repeat purchases, or a higher price for new customers. Secondly, consumer preferences may change during the time interval between purchases. Thirdly, setting up LRs may affect rivals' future pricing behavior.

The theoretical literature has emphasized the role of LRs as endogenous switching costs.¹ Most authors have analyzed the role of LRs in some version

¹A completely different view of FFPs has recently been put forward by Basso et al.

of the Hotelling framework where consumers' relative preferences are subject to shocks. In this set up LRs allow firms to retain previous customers, even when rival firms offer goods or services that better match their current preferences. As a result, LRs are welfare reducing because they cause a mismatch in the allocation of consumers. However, in this view it is unclear whether LRs tend to relax or exacerbate price competition (See, for instance, Banerjee and Summers, 1987; Caminal and Matutes, 1990; Cairns and Gailbraith (1990), Bulkley (1992), Fudenberg and Tirole, 2000, Kim et al., 2001, Caminal and Claici, 2007, Ackerman (2007) , Chen and Percy (2007), and Fong and Liu, 2009).² It has been shown that the effect of LRs on average prices largely depends on market structure and the particular design of the reward programs. It is important to emphasize that this literature has overlooked the effect of LRs on consumer participation, by assuming that in equilibrium the entire consumer base is served. Moreover, very little attention has been paid to the endogenous design of LRs, as most papers simply assume that firms are only allowed to use a particular specification.³ In contrast, the goal of this paper is to contribute to our understanding of LRs beyond the endogenous switching costs motive and studying the differential effects of alternative designs.

The closest precedent of this paper I am aware of was published a long time ago: Crémer (1984) studied the incentives of a monopolist to use repeat purchase coupons in a market for an experience good. He showed that, under certain conditions, the ability to commit to a future discount for repeat buyers could increase both profits and total welfare.⁴

(2009). These programs are alternatively understood as instruments to exploit the agency relationship between employers (who pay for the tickets) and employees (who book travel and enjoy the benefits of LRs). In other words, FFPs are the "bribes" offered to employees to book flights at higher prices.

²Armstrong (2006) and Fudenberg and Villas-Boas (2006) discuss how this literature fits into the broader theme of behavior-based price discrimination, which in turn is a particular form of dynamic price discrimination.

³Two exceptions are the duopoly models of Kim et al. (2001) and Caminal and Matutes (1990). In the first paper firms are allowed to choose between cash rewards and product rewards. The latter are more efficient in the sense that they imply a lower cost for firms for the same consumer benefit. In either case it is assumed that firms cannot commit to the future transaction price, only to the magnitude of the discount enjoyed by repeat buyers. Caminal and Matutes (1990) show that when firms can choose between committing to the price of repeat purchases and a discount (a coupon), they choose the first option.

⁴See also Bulkley (1992).

Unfortunately, the available empirical evidence on the effects of LRs is scarce. In the marketing literature one can find weak evidence of the positive effect of introducing LRs on a firm's market share, although its effect on profitability is less clear (See, for instance, Sharp and Sharp, 1997, Bolton et al, 2000, Lal and Bell, 2003, and Lewis, 2004). More recently, Hartman and Viard (2008) have empirically challenged the view of LRs as endogenous switching costs. Lederman (2007, and 2008) points to the airline industry and provides convincing evidence on the positive effects of FFPs on sales and prices (excluding free tickets enjoyed by frequent flyers). Such evidence is compatible with the more conventional view of LRs as endogenous switching costs, but also with the approach taken in this paper that emphasizes the effect on consumer participation.⁵ In a recent paper, Liu and Yang (2009) examined the behavior of a sample of travelers with respect to the top 11 US airlines, all of which offered a loyalty program. They provide evidence favorable to the hypothesis that LRs expand aggregate consumption of air travel, and suggest this additional consumption comes from two sources: travelers who previously used other modes of transportation, and the average consumer traveling more frequently.⁶ Clearly, more work is needed in order to separate empirically the role of LRs as switching costs and as devices fostering market participation.

In this paper I present a simple, tractable model, which is able to shed some light on the channels by which different designs of LRs affect total welfare. However, it is a highly stylized, two-period monopoly model and only a limited number of designs of LRs are considered. Hence, it will be important to discuss how the predictions of the model are affected when these restrictive assumptions are relaxed. In any case, the goal is not to explain the particular characteristics of LRs in a specific market, but to illustrate how a small set of principles are likely to apply to a variety of market types.⁷ Nevertheless, this simple model is able to provide important insights that are likely to extend to a much larger set of environments. In particular: (i) LRs can induce

⁵In order to provide empirical evidence of the role of FFPs in fostering consumer participation we would need to check that the negative impact on rivals' sales is lower than the positive impact on own sales.

⁶See also Kopalle and Neslin (2003).

⁷Some of the features of FFPs mentioned above and that are not captured by the formal model are discussed in the last two sections. However, the bundling component of some LRs is a completely different issue and I do not deal with it in this paper. See Gans and King (2006).

substantial efficiency gains by fostering consumer participation. The reason is that consumers pay up-front for the promise of future low prices. Under uncertain preferences, consumers are more homogeneous ex-ante, which improves the trade-off between profit maximization and consumer participation. However, the size and even the sign of these gains heavily depend on the specific design; (ii) private and social incentives may not always be well aligned and, as a result, firms may choose relatively inefficient designs. The latter is particularly relevant in those scenarios in which the most efficient designs trigger Coasian dynamics.

As in the previous literature, an essential feature of this model is that consumers face some uncertainty about future preferences. In particular, consumers are uncertain about their future valuation of the good. I will also argue that in a competitive setting with differentiated products it will be important whether consumers are uncertain about their common valuation of the good or about the relative valuation of different varieties. Overall, the model predicts both the design of the reward program and its welfare implications crucially depend on the amount of uncertainty faced by consumers.

In this sense the subject of this paper is closely related to Chen and Percy (2007). While they examine the role of preference uncertainty on dynamic pricing, they have a different goal and set up. In particular, they assume consumers are uncertain about their future relative valuation, and hence LRs become endogenous switching costs. Also, under some circumstances firms may initially find it optimal not to make any commitment, but then subsequently treat newcomers better than repeat buyers: play a *poaching strategy*. In our set up it is always optimal to commit to reward loyalty, and the focus is exclusively on the design and efficiency of LRs.

As a benchmark I consider the case where the monopolist enjoys unlimited commitment capacity. In this case, the optimal policy exhibits the following features. First, the price for repeat purchases is equal to marginal cost. This maximizes the value of the firm-customer relationship. In fact, the seller is able to appropriate the efficiency gains created in the second period through a higher first period price. Second, the price for newcomers in the second period is higher than the static monopoly price. The intuition is that the loss in second period profits caused by distorting the price upwards is more than compensated by the positive effect on first period demand and profits. Third, the gap between the price for newcomers and for repeat buyers raises consumers' willingness to pay in the first period (and causes a parallel shift in the first period demand function). Hence, the monopolist finds it optimal

to serve a larger number of consumers in the first period. Clearly, the first and the third effect lead to efficiency gains, although the second effect has a negative impact on efficiency.

The full commitment case is of course highly unrealistic. Firms may be able to sign implicit or explicit contracts with their current customers, but it is nearly impossible to do it with consumers they have not met yet. Thus, a somewhat more realistic scenario would be one where the monopolist enjoys unlimited commitment power with respect to current customers, but no commitment power at all with respect to future customers.⁸ In this context the monopolist must still make a design choice. It can either commit to the price of repeat purchases, or to a discount policy. In the latter case the level of transaction prices to be paid by loyal consumers is left undetermined. For simplicity I restrict attention to lump-sum discounts.

In order to illustrate the role of uncertainty on the optimal pricing scheme, it is sufficient to restrict attention to the two extreme scenarios. In the first scenario (Section 3) preferences of individual consumers are independent over time. This implies the second period demand functions of repeat and first time buyers are identical. If the seller commits to a lower price for repeat buyers then in the second period, it will charge the static monopoly price to newcomers. Expectations of such a high price for newcomers makes it in fact profitable for the seller to commit to a price equal to marginal costs for repeat purchases. Such a pricing policy unambiguously raises welfare with respect to the case of no price discrimination: lower price for repeat buyers, same price for first time buyers in the second period, and higher consumer participation in the first.

Alternatively, the seller may use a discount policy. In this case, it can achieve a rigid combination of a high price for newcomers (higher than the static monopoly price) and a low price for repeat buyers (lower than the static monopoly price). It is shown that the optimal discount policy may generate higher or lower profits than the optimal price commitment policy depending on the shape of demand function. However, from a social point of view, a prime commitment policy is always preferred. It may even be the case that the monopolists chooses a discount policy even though such a policy involves a lower level of total surplus than in the absence of price discrimination.

⁸Unlimited commitment power regarding current consumers can also be an optimistic description of the real world. In any case, I will not deal here with the issue of how reputation and/or consumer protection laws can help enforcing firms' promises.

In the second scenario (Section 4) the correlation between preferences of an individual consumer over time is almost perfect.⁹ In this case, most of the potential first time buyers in the second period have a relatively low willingness to pay. As a result, if the firm decides to commit to a low price for repeat purchases, then in the second period it will also choose a relatively low price for newcomers. In other words, commitment to the price of repeat purchases triggers Coasian dynamics. In the first period the anticipation of a low price for newcomers more than compensates for the profitable effects of a low price for repeat buyers. In other words, commitment to a low price of repeat purchases will have little effect on first period profits, while drastically reducing second period profits.

In this context, discounts are still profitable. In the second period, the seller finds it optimal to set a relatively high regular price (significantly higher than the static monopoly price), which involves a price for repeat buyers approximately equal to the static monopoly price. The reason is that there are very few newcomers with a high willingness to pay, and very few repeat buyers with a relatively low willingness to pay. In the first period, the demand function slightly shifts upwards because of the expected price gap between repeat buyers and newcomers, and the seller finds it optimal to serve a slightly higher number of consumers than in the static case. In other words, the first is a second order effect, but the first and third are first order effects with opposite signs. Thus, the net welfare implications of the optimal discount policy is in general ambiguous, although negative in some important examples, like the linear case.

In Section 5 I discuss how these insights would carry over to more general setups. In particular, I discuss the role of inter-firm competition, a multi-period framework, and a larger set of feasible pricing schemes.

Some concluding remarks (Section 6) close the paper.

2 The model

A profit maximizing monopolist produces a non-durable good at constant marginal cost, which for simplicity is normalized to zero. The market duration is for two periods, labeled 1 and 2. There is a continuum of consumers with mass one; in each period every consumer purchases either one unit or

⁹Under perfect correlation, individual consumers face no uncertainty and firms have no incentive to set up LRs.

zero. Consumers are heterogeneous in their valuation of the good. More specifically, in the first period each consumer's valuation is a realization of the random variable, r , distributed over the interval $[0, 1]$ with probability density $f(r)$ and cumulative density function $F(r)$, which is assumed to be twice continuously differentiable.¹⁰ I also assume that the density function is strictly positive anywhere in the interval $[0, 1]$, and it does not decrease too quickly, $-2f(r) - f'(r)r < 0$. This assumption implies the one period profit function is strictly concave.

Consumer preferences are imperfectly correlated across periods. In particular, consumer i 's second period valuation, r_{i2} , is equal to her first period valuation, r_{i1} , with probability μ , and an independent random draw from the common density function, $f(r)$, with probability $1 - \mu$. The parameter μ is common knowledge and lies in the interval $[0, 1]$.

If consumer i purchases one unit of the good at price p , she obtains a net utility of $r_i - p$. If she does not consume her utility is zero.

If the firm is unable to discriminate between repeat buyers and newcomers then it will set the static monopoly price, p^m , in both periods. That is, p^m is the solution to the problem of choosing p in order to maximize $\pi(p) \equiv [1 - F(p)]p$. The first order condition characterizes the solution:

$$\pi'(p^m) = 1 - F(p^m) - f(p^m)p^m = 0$$

Also, the static total surplus obtained from transacting at price p is $TS(p) \equiv \int_p^1 r f(r) dr$, and consumer surplus is $CS(p) \equiv \int_p^1 (r - p) f(r) dr$.

Some of the welfare results hold for general functional forms. In those cases where the sign cannot be determined (as is often the case in the literature on price discrimination), it is useful to consider the uniform distribution (linear demand): $f(r) = 1$.

Both the firm and consumers are forward looking, and in period 1 maximize the expected discounted value of their payoffs using the same factor δ , $\delta \in [0, 1]$.¹¹

Most of the insights of the model can be obtained by analyzing two extreme values of μ : $\mu = 0$ (zero correlation in individual preferences) and

¹⁰More generally, I could have assumed that r is distributed over the interval $[r^L, r^H]$. The assumption that $r^H = 1$ is just a normalization. However, allowing r^L to be strictly positive could lead to some technical difficulties.

¹¹As long as consumers and the firm share the same discount factor, its value does not affect the qualitative results.

$\mu = 1 - \epsilon$, where ϵ is a positive but arbitrarily small number (almost perfect correlation). It is well-known (See Armstrong, 2006, Section 2) that if consumer preferences are perfectly stable then LRs are irrelevant. This approach will enable us to examine the optimal pricing policy as we approach the scenario with perfectly stable preferences (as ϵ goes to zero).

A crucial feature of the model is the firm's ability in the second period to price discriminate between repeat buyers and newcomers. On top of that, in the first period the firm has access to alternative commitment devices. It has been shown (see, for instance, Chen and Percy, 2007) that, in some circumstances, firms may choose not to reward consumer loyalty and rather offer better deals to non-customers (*poaching strategy*). In this model, poaching is never part of the monopolist's optimal plan, and hence we can focus on the optimal design of LRs.

As a benchmark I will consider the case where the firm in the first period can commit to all future prices. A more realistic scenario will be one where the firm has unlimited commitment capacity regarding their current customers, but cannot commit to the price charged in the second period to new customers. Moreover, I will focus attention on two designs of LRs: the firm may choose either the price of repeat purchases or a lump-sum discount with respect to the regular price. The firm can also choose not to commit in the first period, and yet remain free to price discriminate in the second period.

3 Monopoly pricing under independent preferences

In this section I analyze monopoly pricing when current and future preferences are independent ($\mu = 0$). Let p_1 be the price charged in the first period, and p_2^r and p_2^n the prices charged in the second period to repeat buyers and newcomers, respectively. In the first period, consumer i 's expected utility of purchasing the good is $r_{i1} - p_1 + \delta CS(p_2^r)$, and the expected utility of not purchasing is $\delta CS(p_2^n)$. Thus, all consumers with $r_{i1} \geq \bar{r}_1$ will choose to purchase, where \bar{r}_1 is given by:

$$\bar{r}_1 = p_1 + \delta [CS(p_2^n) - CS(p_2^r)] \quad (1)$$

Hence, the demand from repeat buyers and newcomers is $q_2^r = [1 - F(\bar{r}_1)][1 - F(p_2^r)]$ and $q_2^n = F(\bar{r}_1)[1 - F(p_2^n)]$, respectively. The discounted value of profits at

time 1 can be written as:

$$\Pi_1(p_1, p_2^r, p_2^n) = [1 - F(\bar{r}_1)] p_1 + \delta \{ [1 - F(\bar{r}_1)] \pi(p_2^r) + F(\bar{r}_1) \pi(p_2^n) \} \quad (2)$$

3.1 The benchmark case

Suppose that the firm has full commitment capacity. That is, it can set all prices: p_1, p_2^r, p_2^n at the beginning of the first period. Thus, the monopolist chooses (p_1, p_2^r, p_2^n) in order to maximize (2) subject to (1). The main characteristics of the optimal pricing policy are summarized in the following proposition.¹²

Proposition 1 *Under $\mu = 0$ and full commitment capacity, the monopoly solution involves: (a) $p_2^r = 0$, (b) $p_2^n > p^m$, and (c) $\bar{r}_1 < p^m$. More specifically, p_2^n and \bar{r}_1 are given respectively by:*

$$1 - F(p_2^n) - F(\bar{r}_1) f(p_2^n) p_2^n = 0 \quad (3)$$

$$1 - F(\bar{r}_1) - f(\bar{r}_1) \bar{r}_1 - f(\bar{r}_1) \delta [TS(0) - TS(p_2^n)] = 0 \quad (4)$$

First, the monopolist finds it optimal to commit to a price for repeat buyers equal to marginal cost, which is the price that maximizes the value of the firm-customer relationship. The firm is willing to efficiently price the repeat purchase because it can appropriate these gains by charging a higher price in the first period. Second, the monopolist sets the future price for newcomers above the monopoly price. Starting at $p_2^n = p^m$, a small increase in p_2^n implies a second order loss in second period profits, but it involves a first order gain in first period profits. The reason is that a higher p_2^n makes the first period purchase relatively more attractive for consumers. Third, the discrimination between repeat buyers and newcomers raises consumers' willingness to pay in the first period (the demand function shifts upwards), which leads the monopolist to serve a larger number of consumers in the first period. Clearly the first and the third effects are welfare enhancing, but the second is welfare reducing.

In fact, we can compute the increase in total welfare associated to the optimal commitment policy with respect to the case of no discrimination

¹²The option of no commitment in the first period is equivalent to commit to $p_2^r = p_2^n = p^m$. The same comment applies to the other cases considered in this section.

(when the firm charges p^m in both periods):

$$\Delta W = \delta [1 - F(\bar{r}_1)] \int_0^{p^m} r f(r) dr - \delta F(\bar{r}_1) \int_{p^m}^{p_2^n} r f(r) dr + \int_{\bar{r}_1}^{p^m} r f(r) dr \quad (5)$$

The three terms of the right hand side correspond to the three effects discussed above.¹³

3.2 Commitment to the price of repeat purchases

In the real world most firms seem able to commit to their future pricing policy with respect to their current customers, or at least are able to lay down significant restrictions on their future pricing behavior. In the extreme case, sellers can sign legally enforceable, long-term contracts with current buyers. However, it is much less plausible that firms do this with consumers they still have not interacted with.¹⁴ In this and the next subsections I take the view that the monopolist can commit to a particular scheme to reward consumer loyalty, but it must leave undetermined the price to be paid by newcomers. In particular, in this subsection the monopolist chooses (p_1, p_2^r) in the first period and p_2^n in the second. The optimal pricing policy is summarized in the following proposition:

Proposition 2 *Under $\mu = 0$, and if the firm can only commit to the price for repeat buyers, the monopoly solution includes: (a) $p_2^r = 0$, (b) $p_2^n = p^m$, and (c) $\bar{r}_1 < p^m$. More specifically, \bar{r}_1 is given by:*

$$1 - F(\bar{r}_1) - f(\bar{r}_1)\bar{r}_1 - f(\bar{r}_1)\delta [TS(0) - TS(p^m)] = 0 \quad (6)$$

¹³If we restrict attention to the uniform distribution case, then the first two terms of equation (5) can be written as:

$$\Delta W = \delta \left\{ \int_0^{\frac{1}{2}} r dr - \bar{r}_1 \int_0^{\frac{1}{1+\bar{r}_1}} r dr \right\} = \frac{\delta(1-\bar{r}_1)^2}{8(1+\bar{r}_1)^2} > 0$$

Since the third effect is always positive, the overall welfare effect is also positive.

¹⁴In the real world firms sometimes maintain some discretionary power over the specification of LRs. For instance, airlines unilaterally change from time to time the conditions to earn and redeem frequent flyer miles. In this paper I entirely ignore any limitation to firms' commitment power with respect to their current customers.

The intuition behind these results is rather straightforward. In the second period, the monopolist finds it optimal to set the static monopoly price to newcomers, since p_2^n is chosen in order to maximize $F(\bar{r}_1) [1 - F(p_2^n)] p_2^n$. However, the incentives to adopt a marginal cost pricing rule for repeat purchases remain constant. Since $p_2^n = p^m > p_2^r = 0$, the demand function in the first period shifts upwards with respect to the static case and, as a result, the optimal p_1 involves higher sales in the first period than in the absence of commitment. In other words, in this case we obtain both the first and third effects identified in the previous subsection, each with positive welfare implications, but the second effect is null. More specifically, the increase in total welfare associated with the optimal commitment policy (with respect to the case of no discrimination) can be written as:

$$\Delta W = \delta [1 - F(\bar{r}_1)] \int_0^{p^m} r f(r) dr + \int_{\bar{r}_1}^{p^m} r f(r) dr > 0$$

Summarizing:

Proposition 3 *Under $\mu = 0$, and if the firm can only commit to the price for repeat buyers, the monopoly solution raises total welfare with respect to the case of no price discrimination.*

3.3 Discounts

Suppose now that in the first period the seller offers its customers a fixed discount (coupon) of face value c . This implies that in the second period the net transaction price for repeat buyers is $p_2^r = p_2^n - c$, where p_2^n is selected in the second period. In particular, given (\bar{r}_1, c) , the firm chooses p_2^n in order to maximize:

$$\pi_2 = [1 - F(\bar{r}_1)] \pi(p_2^n - c) + F(\bar{r}_1) \pi(p_2^n)$$

In case an interior solution exists, it is given by:

$$[1 - F(\bar{r}_1)] \pi'(p_2^n - c) + F(\bar{r}_1) \pi'(p_2^n) = 0 \quad (7)$$

Thus, provided $c > 0$, $p_2^n > p^m > p_2^n - c$. The firm might have incentives to choose a corner solution where newcomers are not served: $p_2^n \geq 1$. In such a corner solution the firm finds it optimal to set the static monopoly price to

repeat buyers: $p_2^n - c = p^m$. Thus, the equilibrium value of p_2^n will be given by equation (7), provided the following constraint is satisfied:

$$[1 - F(\bar{r}_1)] \pi(p_2^n - c) + F(\bar{r}_1) \pi(p_2^n) \geq [1 - F(\bar{r}_1)] \pi(p^m)$$

Thus, by using discounts the monopolist can achieve a certain commitment to a lower price for repeat buyers and a higher price for newcomers (first and second effects), provided the solution is interior. If the monopolist finds it optimal to exclude newcomers in the second period, then a discount policy involves the second effect, but not the first. Finally, because of the different expected consumer surplus obtained by repeat buyers and newcomers, the monopolist sets a first period price so it serves more first period consumers than in the no price discrimination case (the third effect). More specifically, in the first period, if the monopolist anticipates an interior solution in the second period, then it chooses p_1 and c in order to maximize:

$$\Pi_1 = [1 - F(\bar{r}_1)] p_1 + \delta \{F(\bar{r}_1) \pi(p_2^n) + [1 - F(\bar{r}_1)] \pi(p_2^n - c)\}$$

subject to $\bar{r}_1 = p_1 + \delta [CS(p_2^n) - CS(p_2^n - c)]$ and subject to p_2^n being determined by equation (7). The first order conditions characterizes the optimal values of \bar{r}_1 and c :

$$1 - F(\bar{r}_1) - f(\bar{r}_1) \bar{r}_1 - f(\bar{r}_1) \delta [TS(p_2^n - c) - TS(p_2^n)] = 0 \quad (8)$$

$$f(p_2^n - c)(p_2^n - c) - [F(p_2^n - c) - F(p_2^n)] \frac{dp_2^n}{dc} = 0 \quad (9)$$

where $\frac{dp_2^n}{dc}$ comes from applying the implicit function theorem to equation (7). Since $\frac{dp_2^n}{dc} > 0$ equation (9) implies that $p_2^n - c > 0$. Also, equation (8) implies that $\bar{r}_1 < p^m$.

Alternatively, if in the second period newcomers are excluded, then the first period objective function becomes:

$$\Pi_1 = [1 - F(\bar{r}_1)] [\bar{r}_1 + \delta TS(p^m)]$$

In this case, the monopoly solution would involve $\bar{r}_1 < p^m$, $p_2^n - c = p^m$, and $p_2^n \geq 1$.

The following proposition summarizes this discussion.

Proposition 4 *If $\mu = 0$ the monopolist's optimal discount policy implies:*

- (i) $0 < p_2^n - c \leq p^m$,
- (ii) $p_2^n > p^m$,
- and (iii) $\bar{r}_1 < p^m$.

It is not possible to provide a full characterization of the circumstances under which an optimal discount policy raises or reduces total welfare. In general, it is hard to compare the size of the second (negative) effect to the size of the first and third effects (positive). If we restrict attention to the uniform density case, then we can offer an unambiguous answer (For the proof see Appendix 1):

Result 1 *If $f(r) = 1$ the optimal discount policy reduces total surplus.*

The main reason a discount policy reduces total surplus is the concavity of total surplus as a function of price. In the absence of commitment in the second period, a monopolist sets a price equal to $\frac{1}{2}$. In contrast, under the optimal discount policy the price charged to newcomers is raised above $\frac{1}{2}$, more than the reduction obtained by repeat buyers. As a result, the total surplus generated in the second period is lower than in the absence of commitment (the sum of the first and second effects has a negative sign). Finally, the positive welfare effect generated in the first period, associated to higher consumer participation, is not capable of reversing this result.¹⁵

3.4 Comparing alternative pricing schemes

After characterizing the optimal price commitment and discount policies in the last two subsections, we can now compare the preferred policy from a private (monopoly profits) and social (total surplus) viewpoints.

Let us first look at private incentives. It turns out that the relative profitability of these policies is highly dependent on the shape of the demand function. As shown in Appendix 2, the linear demand is a borderline case:

Result 2 *If $f(r) = 1$ the monopolist is indifferent between committing to the price of repeat purchases and a discount policy.*

In fact, it is relatively easy to find examples in which the monopolist strictly prefers one policy over the other (See Appendix 3). Thus,

¹⁵As discussed in the Appendix, the impact of the optimal discount policy on welfare is negative, but second order. This suggests that may there exist alternative functional forms under which a discount policy may increase welfare.

Proposition 5 *There exist density functions for which the monopolist strictly prefers committing to the price of repeat purchases over a discount policy. There also exist other density functions for which the ordering is reversed. Thus, if $\mu = 0$ it is not possible to predict which design of LRs will be chosen by a monopolist.*

However, the social planner has unambiguous preferences over these two alternative designs. From Propositions 2 and 4 it follows that:

Proposition 6 *The monopoly's optimal price commitment policy is more efficient than the optimal discount policy.*

Under a discount policy p_2^r falls in the interval $(0, p^m]$. Thus, the first welfare effect of a discount policy is lower than the first effect of a price commitment policy. Also, under a discount policy p_2^n is unambiguously higher than p^m and hence the second welfare effect is strictly negative, while it is null in the case of a price commitment policy. Finally, comparing equations (6) and (8), we note that \bar{r}_1 is higher in the case of a discount policy (the third welfare effect is higher in the case of a price commitment policy).

Besides the relative performance of these two alternative designs, it is as well important to emphasize that a price commitment policy raises welfare with respect to the benchmark of no price discrimination (Proposition 3), while a discount policy may actually decrease it (Result 1)

4 Monopoly pricing under almost perfectly correlated preferences

This section deals with the case where the preferences of an individual consumer are almost perfectly correlated: $\mu = 1 - \epsilon$, where ϵ is an arbitrarily small, positive number.

The demand in the second period from repeat buyers, q_2^r , is given by:¹⁶

¹⁶Take a particular realization $r, r \geq \bar{r}_1$. A mass of consumers equal to $(1 - \epsilon) f(r)$ had a valuation r in the first period and keep it in the second. However, a mass of $\epsilon [1 - F(\bar{r}_1)]$ consumers also purchased in the first period and get a new realization in the second. Hence, $\epsilon [1 - F(\bar{r}_1)] f(r)$ also have a second period valuation equal to r . If we add up these two groups and compute the integral from p_2^r to 1, we obtain q_2^r , provided $p_2^r \geq \bar{r}_1$. The rest of expressions for q_2^r and q_2^n are obtained analogously.

$$q_2^r = \begin{cases} [1 - \epsilon F(\bar{r}_1)] [1 - F(p_2^r)] & \text{if } 1 \geq p_2^r \geq \bar{r}_1 \\ [1 - F(\bar{r}_1)] [1 - \epsilon F(p_2^r)] & \text{if } 0 \leq p_2^r \leq \bar{r}_1 \end{cases}$$

and the demand from newcomers, q_2^n :

$$q_2^n = \begin{cases} \epsilon F(\bar{r}_1) [1 - F(p_2^n)] & \text{if } 1 \geq p_2^n \geq \bar{r}_1 \\ F(\bar{r}_1) - [1 - \epsilon + \epsilon F(\bar{r}_1)] F(p_2^n) & \text{if } 0 \leq p_2^n \leq \bar{r}_1 \end{cases}$$

Thus, at a price equal to \bar{r}_1 these demand functions are not differentiable. The price elasticity of the demand from repeat buyers is higher for price increases than for price decreases, but the reverse in the demand from newcomers.

In order to simplify the presentation, I assume that the market is semi-anonymous (Fudenberg and Tirole, 1998). That is, price discrimination requires the voluntary participation of consumers. For instance, the seller may be able to distinguish repeat buyers from newcomers if they present a coupon obtained with their first period purchase, or if they voluntarily registered in the reward program. Therefore, we can restrict attention to price schemes that truly reward loyalty ($p_2^r \leq p_2^n$). Also, if the firm does not make any commitment in the first period, then it would set the static monopoly price in each period. I argue below that the same qualitative results are obtained in the case of an identified customers market (in which case the seller can price discriminate in the second period against former customers).

4.1 The benchmark case

Let us reconsider the full commitment case in which the monopoly can choose all prices in the first period. The net expected gains from purchasing in the first period can be written as follows:

$$\begin{aligned} G(r_{i1}) &= r_{i1} - p_1 + \delta [(1 - \epsilon) \max(0, r_{i1} - p_2^r) + \epsilon CS(p_2^r)] - \\ &\quad - \delta [(1 - \epsilon) \max(0, r_{i1} - p_2^n) + \epsilon CS(p_2^n)] \end{aligned}$$

Since these gains are strictly monotone in the first period valuation, r_{i1} , then there exists a threshold value, \bar{r}_1 , that separates consumers into two groups, as in the case of independent preferences. There exists three possible cases: (i) $\bar{r}_1 \leq p_2^r \leq p_2^n$, (ii) $p_2^r < \bar{r}_1 \leq p_2^n$ and (iii) $p_2^r \leq p_2^n < \bar{r}_1$. In Appendix 4 I prove the following lemma:

Lemma 1 *If μ is arbitrarily close to 1 and under full commitment capacity, the monopoly solution satisfies: $p_2^r < \bar{r}_1 \leq p_2^n$.*

Thus, in case (ii) we have:¹⁷

$$\bar{r}_1 = \frac{1}{1 + \delta(1 - \epsilon)} \{p_1 + \delta(1 - \epsilon)p_2^r + \delta\epsilon[CS(p_2^n) - CS(p_2^r)]\}$$

The firm's present value of profits in the first period is:

$$\Pi_1(p_1, p_2^r, p_2^n) = [1 - F(\bar{r}_1)]p_1 + \delta \{[1 - F(\bar{r}_1)][1 - \epsilon F(p_2^r)]p_2^r + \epsilon F(\bar{r}_1)\pi(p_2^n)\}$$

The next proposition characterizes the optimal monopoly strategy under full commitment.

Proposition 7 *If μ is arbitrarily close to 1 and under full commitment capacity, the monopoly solution involves: (a) $p_2^r = 0$, (b) $p_2^n > p^m$, and (c) $\bar{r}_1 < p^m$. More specifically, p_2^n and \bar{r}_1 are respectively given by:*

$$1 - F(p_2^n) - F(\bar{r}_1)f(p_2^n)p_2^n = 0$$

$$1 - F(\bar{r}_1) - f(\bar{r}_1)\bar{r}_1 - f(\bar{r}_1)\frac{\delta\epsilon}{1 + \delta(1 - \epsilon)}[TS(0) - TS(p_2^n)] = 0$$

If we take the limit as ϵ goes to zero, then \bar{r}_1 goes to p^m and p_2^n goes to p^s , where p^s is implicitly given by:

$$1 - F(p^s) - F(p^m)f(p^s)p^s = 0 \quad (10)$$

Given that $F(p^m) < 1$, equation (10) implies that $p^s > p^m$. Thus, the monopolist's optimal policy for $\mu = 1 - \epsilon$ is qualitatively identical to the one for $\mu = 0$.

If we take the limit as ϵ goes to zero, then the monopoly solution becomes $p_1 = (1 + \delta)p^m$, $\bar{r}_1 = p^m$, $p_2^r = 0$, $p_2^n = p^s$. In fact, if $\epsilon = 0$ the monopolist is indifferent between the previous solution and the repetition of the static monopoly price: $p_1 = \bar{r}_1 = p_2^r = p_2^n = p^m$. However, only the former is robust to a small amount of uncertainty ($\epsilon > 0$).

¹⁷In an identified customers market other cases are also feasible. However, it can be shown that the seller does not find it optimal to commit to a policy with $p_2^n < p_2^r$. Hence, the lemma would still hold.

4.2 Commitment to the price of repeat purchases

The seller chooses (p_1, p_2^r) in the first period and p_2^n in the second.

When individual preferences are highly correlated, the firm's commitment to the price of repeat purchases generates Coasian dynamics. Almost all potential new customers in the second period have a willingness to pay lower than \bar{r}_1 . As a result the firm has strong incentives to set a relatively low p_2^n , in particular lower than \bar{r}_1 . More specifically, p_2^n is the solution of the following optimization problem: choose p in order to maximize:

$$\pi_2^n(p) = \{F(\bar{r}_1) - [1 - \epsilon + \epsilon F(\bar{r}_1)] F(p)\} p$$

subject to $\bar{r}_1 \geq p \geq p_2^r$. The first order condition of an interior solution is:

$$F(\bar{r}_1) - [1 - \epsilon + \epsilon F(\bar{r}_1)] [F(p_2^n) + f(p_2^n) p_2^n] = 0 \quad (11)$$

Since ϵ is an arbitrarily small number, then $0 < p_2^n < \bar{r}_1$. Suppose that in the first period the firm chooses to commit to the price for repeat purchases. That is, p_1 and p_2^r are chosen in order to maximize:

$$\Pi_1(p_1, p_2^r, p_2^n) = [1 - F(\bar{r}_1)] p_1 + \delta \{[1 - F(\bar{r}_1)] [1 - \epsilon F(p_2^r)] p_2^r + \pi_2^n(p_2^n)\}$$

where

$$\bar{r}_1 = p_1 + \delta \epsilon [CS(p_2^n) - CS(p_2^r)] + \delta (1 - \epsilon) (p_2^r - p_2^n)$$

The solution includes $p_2^r = 0$ and \bar{r}_1 is implicitly given by:

$$1 - F(\bar{r}_1) - f(\bar{r}_1) \{\bar{r}_1 + \delta \epsilon [TS(0) - TS(p_2^n)] + \delta (1 - \epsilon) p_2^n\} = 0 \quad (12)$$

Note that $0 < \bar{r}_1 < p^m$ and, as ϵ goes to zero, \bar{r}_1 converges to a value strictly below p^m . In the limit as ϵ goes to zero the firm's profits become:

$$\Pi_1 = \pi(\bar{r}_1) + \delta \pi(p_2^n) \ll (1 + \delta) \pi(p^m)$$

where \bar{r}_1 and p_2^n are given by equations (11) and (12) evaluated at $\epsilon = 0$.

Therefore, in this case if the firm commits to the price of repeat purchases, it obtains a lower level of profits than if it chooses not to commit.

All this discussion is encapsulated in the following proposition:¹⁸

¹⁸In an identified customers market, the monopolist may prefer to commit to the price for repeat purchase over no commitment. However, as discussed below, both options are dominated by a discount policy.

Proposition 8 *If μ is arbitrarily close to 1 and the firm's commitment capacity is restricted to the price of repeat purchases, then the monopolist prefers not to use such a commitment capacity and set the static monopoly price in both periods.*

4.3 Discounts

The seller sets (p_1, c) in the first period and p_2^n in the second, in the understanding that $p_2^r = p_2^n - c$.

Given the discount c , in the second period the firm chooses p_2^n in order to maximize second period profits. In principle there are three possible cases: (i) $\bar{r}_1 \leq p_2^r \leq p_2^n$, (ii) $p_2^r < \bar{r}_1 \leq p_2^n$ and (iii) $p_2^r \leq p_2^n < \bar{r}_1$. The next Lemma (See Appendix 5) allows us to concentrate on case (i):

Lemma 2 *The optimal discount policy involves $\bar{r}_1 \leq p_2^r = p_2^n - c$.*

The intuition here is the following. Most repeat buyers have a reservation price higher than \bar{r}_1 . Thus, if the firm lowers p_2^n below $c + \bar{r}_1$ it will attract very few repeat buyers; hence it cannot be an optimal strategy.

Thus, it follows that in the second period p_2^n is the solution to maximize:

$$\Pi_2 = [1 - \epsilon F(\bar{r}_1)] \pi(p_2^n - c) + \epsilon F(\bar{r}_1) \pi(p_2^n)$$

subject to $p_2^n - c \geq \bar{r}_1$.

The first order condition of an interior solution is:

$$[1 - \epsilon F(\bar{r}_1)] \pi'(p_2^n - c) + \epsilon F(\bar{r}_1) \pi'(p_2^n) = 0 \quad (13)$$

Thus, $p_2^n - c$ is below but arbitrarily close to p^m and $p_2^n > p^m$.

Suppose that in the second period the constraint $\bar{r}_1 \leq p_2^n - c$ is not binding. Then, the firm's optimization problem in the first period consists of choosing (p_1, c) in order to maximize:

$$\Pi_1 = [1 - F(\bar{r}_1)] p_1 + \delta \{ [1 - \epsilon F(\bar{r}_1)] \pi(p_2^n - c) + \epsilon F(\bar{r}_1) \pi(p_2^n) \}$$

where

$$\bar{r}_1 = p_1 + \delta \epsilon [CS(p_2^n) - CS(p_2^n - c)]$$

and p_2^n is given by equation (13). The first order conditions with respect to p_1 and c can be written, respectively:

$$1 - F(\bar{r}_1) - f(\bar{r}_1) \{ \bar{r}_1 + \delta \epsilon [TS(p_2^n - c) - TS(p_2^n)] \} = 0$$

$$F(\bar{r}_1) \pi'(p_2^n) - [1 - F(\bar{r}_1)] \left\{ [1 - F(p_2^n - c)] \frac{d(p_2^n - c)}{dc} - [1 - F(p_2^n)] \frac{dp_2^n}{dc} \right\} = 0$$

If we take the limit as ϵ goes to zero, then $p_2^n - c$ and \bar{r}_1 go to p^m , $\frac{dp_2^n}{dc}$ goes to 1, and hence p_2^n goes to p^s , defined in equation (10), $p^s > p^m$.

Suppose now that in the second period the constraint is binding: $p_2^n = \bar{r}_1 + c$. Then the optimization problem can be reformulated as choosing (p_1, c) in order to maximize:

$$\Pi_1 = [1 - F(\bar{r}_1)] p_1 + \delta \{ [1 - \epsilon F(\bar{r}_1)] \pi(\bar{r}_1) + \epsilon F(\bar{r}_1) \pi(\bar{r}_1 + c) \}$$

where

$$\bar{r}_1 = p_1 + \delta \epsilon [CS(\bar{r}_1 + c) - CS(\bar{r}_1)]$$

In this case the solution shares the same qualitative properties. This discussion is summarized in the next proposition.

Proposition 9 *If μ is arbitrarily close to 1 the monopolist's optimal discount policy involves: (a) p_2^r is below but arbitrarily close to p^m , (b) p_2^r is arbitrarily close to p^s and hence higher than p^m , and (c) $\bar{r}_1 < p^m$.*

It is important to note that c converges to a strictly positive number ($p^s - p^m$) as ϵ goes to zero. This implies that for any $\epsilon > 0$ the optimal discount policy involves higher profits than in the case of no commitment ($c = 0$). Thus,¹⁹

Proposition 10 *If μ is arbitrarily close to 1, the monopolist strictly prefers a discount policy over a commitment to the price of repeat purchases.*

The impact of the optimal discount policy on welfare can be written as:

$$\Delta W = \delta \epsilon \int_{p_2^n - c}^{p^m} r f(r) dr - \delta \epsilon F(\bar{r}_1) \int_{p^m}^{p_2^n} r f(r) dr + \int_{\bar{r}_1}^{p^m} r f(r) dr$$

The first (lower price for repeat purchases) is a second order effect. However, the second and the third are first order effects and have different signs.

¹⁹In an identified customers market, if the monopolist does not make any commitment in the first period then he will have incentives to price discriminate in the second period in favor of newcomers. This reduces ex-ante profits far below the level obtained under no discrimination. Consequently, the monopolist still prefers to set $c > 0$ in the first period.

As in the case of $\mu = 0$, a full characterization of the welfare implications of an optimal discount policy is not possible.

In the case of a uniform distribution $p^m = \frac{1}{2}, p^s = \frac{2}{3}$. Thus, disregarding all quadratic and higher order terms we can write:

$$\Delta W \approx -\delta\epsilon\bar{r}_1 \int_{\frac{1}{2}}^{\frac{2}{3}} r dr + \int_{\bar{r}_1}^{\frac{1}{2}} r dr$$

Since $\bar{r}_1 \approx \frac{1}{2} - \delta\epsilon\frac{7}{144}$, then $\Delta W \approx -\frac{7}{288}\delta\epsilon < 0$.

Result 3 *If $f(r) = 1$, then the optimal discount policy reduces total surplus.*

Summarizing, if individual preferences exhibit very high serial correlation, then the monopolist prefers not to commit to the repeat purchase price because this triggers Coasian dynamics. However, the monopolist finds it optimal to commit to a relatively large lump-sum coupon, which does not significantly reduce the repeat purchase price but does raise the price charged to newcomers. Under a uniform distribution, the monopolist's favorite design of LRs actually reduces total welfare.

5 Discussion

In this section I discuss how the results of the previous model might extend to more general environments. In particular, I comment on alternative pricing schemes, more general discount functions, the effects of inter-firm competition, and the properties of equilibria in a fully dynamic framework with overlapping generations of consumers.

5.1 Alternative pricing schemes

In the paper I assume that the monopolist offers a single contract in the first period, at a price p_1 , which bundles two items: the first period good and the right to buy the good in the second period at a price p_2^r (the future's contract). Is such a bundling strategy optimal? Or, on the contrary, can the monopolist do better by selling the two items separately?

In the case $\mu = 1 - \epsilon$ selling the two items separately is never profitable, for the same reasons a bundling strategy is not optimal either. That is, in the first period only consumers with a relatively high valuation would be willing

to purchase the future's contract. Thus, once in the second period, the firm has incentives to set a relatively low regular price since most potential buyers have a low willingness to pay (Coasian dynamics). The details are discussed in the Appendix.

The case $\mu = 0$ is a bit more complicated. In fact, in the Appendix it is shown there exists an equilibrium where the firm makes higher profits by selling the future's contract separately. However, this is a very extreme scenario, since all consumers are perfectly homogeneous with respect to the second period. Thus, if consumers expect that the spot market will be closed in the second period ($p_2^n \geq 1$) then they will all be willing to buy the future's contract at a price equal to $\delta CS(p_2^x)$. In this context, the firm's optimal policy consists of setting $p_1 = p^m$, and $p_2^r = 0$. Once in the second period, there is no potential demand in the spot market, which is consistent with consumers' expectations in the first period.

This result is not robust to small perturbations of the environment. Suppose, for instance, that in the first period there are two groups of consumers. Type *A* consumers, with mass $1 - \varepsilon$, have independent valuations over time. In contrast, type *B* consumers, with mass ε , have constant valuations. Suppose ε is an arbitrarily small, positive number. In this case, Coasian dynamics are triggered again, and a strategy of selling separately the first period good and the future's contract becomes unprofitable. The reason is that a strategy of selling the future's contract separately can only be profitable if the contract is bought by type *A* consumers. However, in the second period the firm has the incentive to sell the good at a relatively low price in order to attract type *B* consumers with a low willingness to pay (See the Appendix 6 for details).²⁰

5.2 A general discount function

In this paper I have considered two alternative designs of LRs: commitment to the price of repeat purchases and a lump-sum discount. Two natural questions arise. First, what are the consequences of allowing a more general set of designs? Second, are there any reasons, not made explicit heretofore, that restrict the set of feasible designs?

²⁰It is important to note that, in contrast the previous discussion, results of Section 3 are robust to this type of perturbations: when the monopolists offers the bundled contract, only a fraction of type *A* purchase the contract, and hence in the second period there is a sufficient mass of consumers with high valuations.

In the first period the firm could announce the first period price, p_1 , and a discount function $p_2^r = f(p_2^n)$. Thus, in the second period, the firm chooses the regular price, p_2^n , and given such a price the discount function determines p_2^r . The two schemes considered in previous sections are particular cases. Commitment to the price of repeat purchases is equivalent to set $f(x) = a$, where a is a constant, and a lump-sum discount is equivalent to set $f(x) = x - c$. In fact, one can find examples in which neither of these two options is the firm's optimal discount function, and therefore our formulation was indeed restrictive. For instance, if $\mu = 0$ and $f(r) = 1$, it can be shown that it is optimal to set $f(r) = -a + br$, with $a > 0$ and $b > 1$.²¹ Such a discount function is a bit difficult to interpret: a repeat buyer receives, first, an extra charge of $(b - 1)$ per cent (we might call it a negative proportional discount), and next it enjoys a lump sum discount of a .

However, the issue of the optimal design of the discount function is likely to be less important in practice than other factors not captured in the model. One concern is that consumers are likely to dislike complicated computations in order to figure out the actual transaction price. Probably more important are the informational requirements implicit in the discount function. That is, repeat buyers must be able to observe the actual transaction price that new consumers pay in the second period. In some cases (manufactured products sold in supermarkets) this is straightforward since the posted price is likely to be the actual price paid by regular consumers. However, in other cases identifying the actual regular price may be much more complicated. In fact, in the absence of reputation concerns, a firm could post a very high price (which translates into a relatively high price for repeat buyers), but at the same time offer ad hoc discounts to newcomers. This would be an implicit form of default, which sometimes may be difficult to observe. Thus, in markets where old customers cannot easily observe the price paid by new consumers, committing to a discount function is not an option and all the firm can do is to commit to a fixed price for repeat purchases. If the informational requirements are met, firms may still opt for a simple specification of the discount function. This may explain why in the real world lump-sum discounts are highly appreciated by both firms and consumers.

²¹Details are available upon request.

5.3 Inter-firm competition

Most of the insights of the current model are likely to extend to a multi-firm environment, provided consumers are uncertain only about their absolute valuation, and hence their participation in the market, and not about their relative preference for the differentiated products offered by rival firms. In particular, the incentives to set up LRs analyzed in this paper will still be present, but inter-firm competition will bring about additional effects.

Consider a monopolistic competitive market. Even though, there is no strategic price effect, individual incentives to offer a LR will depend on what the other firms do. Suppose rivals firms do not reward loyalty. If the firm commits to the price of repeat purchases, this will trigger Coasian dynamics for two complementary reasons: (1) in order to lure those consumers with a high relative preference for the firm's product, but a relatively low willingness to pay (like in the model of this paper), but also (2) in order to attract customers of rival firms (low relative preference but high willingness to pay). In contrast, if rival firms actually commit to a low price for their repeat buyers it becomes more difficult in the second period to steal business away from rival firms. Consequently, the incentives to commit to a low price for repeat purchases are enhanced. LRs programs are thus very likely to be strategic complements and a model like this may exhibit a multiplicity of equilibria.

In an oligopolistic market each firm will try to anticipate the rivals' reaction to its LRs program. Such strategic effect may also influence equilibrium decisions in ways that are far more difficult to visualize.

5.4 A multi-period framework

In the baseline model both consumers and the firm live for two periods. A natural extension would be to consider a fully dynamic model where a long-lived firm sells to overlapping generations of consumers. If the firm can discriminate between consumers belonging to different generations then the model would be equivalent to extending the current model to an arbitrary number of periods (same time horizon for both consumers and the firm). Clearly, the set of profitable designs of LRs would expand considerably. In the two-period framework it was shown that a monopolist may not find it optimal to commit to a low price for repeat buyers, due to the fact that such a commitment will imply a low regular price in the future. The two-

period framework in a way exacerbates the strength of Coasian dynamics. In contrast, in a multi-period framework a monopolist could set up a loyalty program that promises consumers the right to purchase the good at a low price after n purchases at the regular price (buy-ten-and-get-one-free kind of program.) In this case most repeat buyers pay the regular price in a given period, since only a fraction $\frac{1}{n+1}$ (in average) are exempted. As a result, incentives to set a low regular price are significantly reduced (Coasian dynamics are avoided). Consequently, firms may find such a design profitable, at least compared with the no commitment case.²²

Suppose that the firm cannot discriminate between consumers belonging to different generations.²³ In particular, suppose that the firm faces an infinite horizon but consumers live for two periods. In each period consumers born at the beginning of the period (young) coexist with consumers born in the previous period (old). Suppose that in period t the firm sets the "regular" price, p_t , and the repeat purchase price for the next period, p_{t+1}^r . Note that p_{t+1}^r is paid by young consumers as well as old consumers who did not buy in period t . Even if the latter group is largely composed by consumers with a low willingness to pay (μ high), incentives to set a low p_{t+1}^r are drastically reduced due to the high willingness to pay of the young (enhanced by the prospective of enjoying loyalty rewards in the next period). Thus, in this particular example, commitment to the price of repeat purchases may not trigger Coasian dynamics.²⁴ However, this is exclusively due to the fact that the rate of arrival of new consumers is large relative to the entire pool of consumers (fifty per cent in the example). I conjecture that in an overlapping generations framework where the rate of arrival is sufficiently low, then the firm would be tempted again to set a low regular price from time to time in an scenario where the pool of previous consumers enjoy a separate pricing

²²Some of the insights of a multi-period framework might also be illustrated in the two-period set up by allowing for random rewards. Suppose we let the monopolist commit to marginal cost pricing for repeat purchases, only for a fraction α of first period customers. By choosing α properly Coasian dynamics can be avoided, and such a policy could become profitable even if μ is close to 1. Unfortunately, a full characterization of the optimal value of α presents some technical problems.

²³See, for instance, Villas-Boas (2004) and Caminal and Claiici (2007).

²⁴Blind to differential generations, firms will no longer be willing to commit to marginal cost pricing for repeat buyers. This will be because they will not be able to capture the entire surplus through the first period price, which in this case will be closer to the static monopoly price. Caminal and Claiici (2007) obtain the same result in a different framework.

regime.

6 Concluding remarks

In this paper I show that the design of LRs significantly matters for efficiency. In particular, a commitment to the price of repeat purchases may bring about large efficiency gains by fostering consumer participation. However, this type of LRs may not be incentive compatible, in the sense that firms may prefer to use discount policies, especially when consumer preferences are relatively stable. Discount policies are always less efficient than price commitment, and may even imply lower total surplus than in the absence of behavior-based price discrimination. Thus, the model can explain the heterogeneity of designs observed in the real world, and conveys simultaneously a clear prediction about the welfare implications of these pricing schemes. On the one hand, FFPs and programs of the type "buy-ten-and-get-one-free" do commit to the transaction price (set equal to zero) repeat buyers pay when they enjoy the benefits of the program. According to the model, these classes of LRs are welfare enhancing. On the other hand, repeat purchase coupons do not involve a commitment to the transaction price and are likely to reduce welfare.

The model presented in this paper is highly stylized. The previous section has discussed how the predictions of the model may change in more general environments. In any case, it aims at capturing the nature of LRs in a variety of markets. Hence, it cannot contemplate the particular characteristics of a specific market. However, some features of LRs observed in the real world can only be explained by introducing these market-specific characteristics. For instance, FFPs restrict the set of flights that a frequent flyer can purchase at zero price. One possible explanation is that the demand for airline tickets is composed by business and leisure travellers, and the latter have much more flexibility with respect to their travel dates. Moreover, some flights are more likely to hit the capacity limit than others. Since most frequent flyers use their free tickets for their leisure trips, then it is indeed efficient to allow them to use their privileges in flights with a low probability of hitting the capacity limit. In this case, marginal costs are close to zero, which in turn would explain why rewards take the form of free tickets.

This paper emphasizes the effect of LRs on consumer participation. The competing explanation proposed in the literature is endogenous switching

costs, which assumes aggregate sales are fixed and concludes LRs are unambiguously welfare decreasing (they distort the efficient allocation of consumers). Thus, the motivation of LRs is crucial from an efficiency point of view. In principle, we can empirically separate these two theories by measuring the effect of LRs on total sales. However, as discussed in the introduction, most studies have focused on the performances of individual programs. A notable exception is the recent paper by Liu and Yang (2009), which provides evidence of a positive effect of FFPs on air travel consumption. I hope the current paper contributes to bring the empirical relation between loyalty programs and aggregate consumption into the spotlight, which is essential to assess the welfare implications of these pricing schemes.

If taken literally some of the predictions of the model seem exaggerated when compared to real world observations. In particular, LRs have a larger effect on the variance of transaction prices that a consumer experiences over time. For instance, the commitment to the price for repeat purchases looks very much like a two-part tariff. In the first period consumers pay a high price, and in the second a price equal to marginal cost. In some cases, the optimal discount policy also involves a discount equivalent to the static monopoly price-cost margin.

The "quantitative" predictions appear more moderate when we take into account two elements discussed in the previous section: consumers' longer horizons and the firm's inability to discriminate between consumers belonging to different generations. Other reasons favorable to more modest LRs are the temptation to default if the credibility of the program relies on reputation, and the presence of myopic consumers. Also, if we let product quality in the second period be endogenous, then LRs may exacerbate the moral hazard problem. More specifically, if the firm commits to a price equal to marginal cost for repeat buyers, but cannot commit to future quality standards, then the incentives to maintain those standards in the second period will most likely deteriorate. As a result, the firm may prefer to reduce the magnitude of LRs in order to alleviate the quality underinvestment problem.

I have argued that all these considerations tend to reduce the magnitude of LRs, at least if we restrict ourselves to a particular design (commitment to the price of repeat purchases.) Future research should address how all these considerations affect the optimal design.

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8 Appendix

8.1 Result 1

If $F(r) = r$ then equation (7) in the main text implies that:

$$p_2^n = \frac{1 + (1 - \bar{r}_1) 2c}{2}$$

Second period profits evaluated at the interior solution are given by:

$$\pi_2 = \frac{1}{4} [1 - 4c^2\bar{r}_1 (1 - \bar{r}_1)] \quad (14)$$

In the second period, if c is sufficiently high, the seller may consider deviating from the interior solution, disregarding newcomers ($q_2^n = 0$) and choosing p_2^n such that $p_2^n - c = p^m = \frac{1}{2}$. Such a deviation involves second period profits equal to:

$$\pi_2 = \frac{1}{4} (1 - \bar{r}_1) \quad (15)$$

Thus, the seller will choose the interior solution if and only if (14) is higher than (15), which can be written as:

$$4c^2 (1 - \bar{r}_1) \leq 1 \quad (16)$$

Suppose that constraint (16) is not binding. Then the first period optimization problem consists of choosing (p_1, c) in order to maximize the present value of profits:

$$\Pi = (1 - \bar{r}_1) p_1 + \frac{\delta}{4} [1 - 4c^2\bar{r}_1 (1 - \bar{r}_1)]$$

subject to:

$$\bar{r}_1 = p_1 - \frac{\delta c}{2} [1 - c(1 - 2\bar{r}_1)]$$

From the first order conditions we obtain that $c = \frac{1}{2}$ and $\bar{r}_1 = \frac{8-\delta}{16}$. Note that constraint (16) is satisfied. If the monopolist follows such a policy then it makes $\Pi = \frac{64+80\delta+\delta^2}{256}$. Alternatively, if the monopolist sets an arbitrarily high c , which implies that in the second period $q_2^n = 0$ and $p_2^n - c = \frac{1}{2}$, then it also sets $p_1 = \frac{1}{2} - \frac{\delta}{16}$ ($\bar{r}_1 = \frac{1}{2} - \frac{3\delta}{16}$), which implies that $\Pi = \frac{8+3\delta}{32} < \frac{64+80\delta+\delta^2}{256}$. Hence, $c = \frac{1}{2}$ and $\bar{r}_1 = \frac{8-\delta}{16}$ is the monopolist's optimal discount policy.

From equation (5) in the main text:

$$\Delta W = \delta (1 - \bar{r}_1) \int_{\frac{1-\bar{r}_1}{2}}^{\frac{1}{2}} r dr - \delta \bar{r}_1 \int_{\frac{1}{2}}^{1-\bar{r}_1} r dr + \int_{\bar{r}_1}^{\frac{1}{2}} r dr$$

Evaluating this expression at $\bar{r}_1 = \frac{8-\delta}{16}$ we determine that $\Delta W = -\frac{\delta^2}{64} (1 - \frac{\delta}{4}) < 0$.

8.2 Result 2

If the firm can commit to the price for repeat purchases, and $F(x) = x$, then it chooses $p_2^r = 0$ (in the first period) and $p_2^n = \frac{1}{2}$. Therefore, second period profits are $\pi_2 = \frac{\bar{r}_1}{4}$. The firm chooses in the first period a price p_1 in order to maximize:

$$\Pi = (1 - \bar{r}_1) p_1 + \frac{\delta}{4} \bar{r}_1$$

subject to $\bar{r}_1 = p_1 - \frac{3\delta}{8}$. Thus, at the optimal policy $\bar{r}_1 = \frac{8-\delta}{16}$ and $\Pi = \frac{64+80\delta+\delta^2}{256}$, which coincide with those results obtained in the optimal discount policy.

8.3 Proposition 5

For simplicity, let us consider the case $\delta = 1$.

EXAMPLE 1

Suppose that $r = \frac{1}{2}$, with probability $\frac{1}{2}$, and r is a random draw from a uniform distribution on $[0, 1]$, with the complementary probability. Then the static monopoly price is $p^m = \frac{1}{2}$, and monopoly profits per period are $\frac{3}{8}$.

For the moment suppose that in the first period, the monopolist sets p_1 and LRs such that all consumers with $r_1 \geq \frac{1}{2}$ purchase in the first period. Hence, total sales are $\frac{3}{4}$.

If the monopolist commits to $p_2^r = 0$, then $p_2^n = \frac{1}{2}$, and second period profits are $\pi_2 = \frac{1}{4} \frac{3}{4} \frac{1}{2} = \frac{3}{32}$.

In the first period, a consumer with $r_i = \frac{1}{2}$ is willing to pay a price $p_1 = r_i + CS(0) - CS(\frac{1}{2}) = \frac{15}{16}$.

Therefore, the present value of profits at time 1 is $\Pi = \frac{3}{4} \frac{15}{16} + \frac{3}{32} = \frac{51}{64}$.

If the monopolist sets a discount $c \leq 1$, then in the second period it will set $p_2^n = \frac{1}{2} + c$. First of all, it cannot be optimal to set $p_2^n < \frac{1}{2}$. Second, if the firm sets $p_2^n > \frac{1}{2}$ such that $p_2^n - c < \frac{1}{2}$, then second period profits are $\pi_2 = \frac{1}{4} \frac{1}{2} (1 - p_2^n) p_2^n + \frac{3}{4} [\frac{1}{2} + \frac{1}{2} (1 - p_2^n + c)] (p_2^n - c)$. In this case, the optimal price is $p_2^n = \frac{7+6c}{8}$, which implies that $p_2^n - c > \frac{1}{2}$. We have reached a contradiction. Third, if the firm sets p_2^n such that $p_2^n - c \geq \frac{1}{2}$, then it finds it optimal to set $p_2^n = \frac{1}{2} + c$, which implies second period profits are $\pi_2 = \frac{5-2c^2}{16}$.

In the first period, a consumer with $r_i = \frac{1}{2}$ is willing to pay a price $p_1 = r_i + CS(\frac{1}{2}) - CS(\frac{1}{2} + c) = \frac{2+c-c^2}{4}$. Hence, the present value of profits is $\Pi = \frac{3}{4} \frac{2+c-c^2}{4} + \frac{5-2c^2}{16}$. Thus, the optimal discount is $c = \frac{3}{10}$, which implies

the maximum amount of profits that can be obtained using a discount policy is lower than what might be obtained under a price commitment policy.²⁵

EXAMPLE 2

Suppose that $f(r) = 2$ if $r \in [\frac{1}{2}, 1]$, and $f(r) = 0$, otherwise. In this case, $p^m = \frac{1}{2}$ and profits per period are also $\frac{1}{2}$ (present value of profits is equal to 1). It is noteworthy that under such a policy total welfare is maximized.

Clearly, monopolists cannot benefit from a price commitment policy, since there are not further gains from trade. However, a discount policy can increase profits with respect to the no commitment case. In particular, consider $p_1 = 1, c = \frac{1}{2}$. In the first period consumers will anticipate that $p_2^n = 1$, and hence all consumers with $r_i \geq \frac{1}{2}$ will be willing to buy the good in the first period. As a result the present value of profits is $\Pi = \frac{5}{4} > 1$.

8.4 Lemma 1

Let us first consider case (i) $\bar{r}_1 \leq p_2^r \leq p_2^n$. In this case, the firm's present value of profits in the first period is:

$$\Pi_1(p_1, p_2^r, p_2^n) = [1 - F(\bar{r}_1)] p_1 + \delta \{ [1 - \epsilon F(\bar{r}_1)] [1 - F(p_2^r)] p_2^r + \epsilon F(\bar{r}_1) \pi(p_2^n) \}$$

$$\bar{r}_1 = p_1 + \delta \epsilon [CS(p_2^n) - CS(p_2^r)]$$

From the first order conditions of the firm's optimization problem, we can characterize the candidate to the monopoly solution:

$$1 - F(p_2^n) - F(\bar{r}_1) f(p_2^n) p_2^n = 0$$

$$1 - F(p_2^r) - \frac{1 - \epsilon F(\bar{r}_1)}{1 - \epsilon} f(p_2^r) p_2^r = 0$$

$$1 - F(\bar{r}_1) - \bar{r}_1 f(\bar{r}_1) - f(\bar{r}_1) \delta \epsilon [TS(p_2^r) - TS(p_2^n)] = 0$$

Note that as ϵ goes to zero, both p_2^r and \bar{r}_1 go to p^m . Consider the following deviation: $p_2^r = 0$ and p_1 such that \bar{r}_1 and p_2^n remain unchanged. The firm can charge a higher p_1 and the difference is equal to $\delta(1 - \epsilon)\bar{r}_1 +$

²⁵It can easily be checked the monopolist cannot do better by setting p_1 such that $\bar{r}_1 \neq \frac{1}{2}$.

$\delta\epsilon [CS(0) - CS(p_2^r)]$, where \bar{r}_1 and p_2^r are the candidate of the monopoly solution. Thus, the change in profits associated with the deviation can be written as a function of ϵ :

$$\Delta\Pi^d(\epsilon) = \delta(1 - \epsilon) [\pi(\bar{r}_1) - \pi(p_2^r)] + \delta\epsilon [1 - F(\bar{r}_1)] [TS(0) - TS(p_2^r)]$$

Note that $\Delta\Pi^d(0) = 0$, since at $\epsilon = 0$ $p_2^r = \bar{r}_1 = p^m$. Also,

$$\frac{d\Delta\Pi^d(\epsilon)}{d\epsilon} = \delta(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4)$$

where

$$\Omega_1 = -[\pi(\bar{r}_1) - \pi(p_2^r)]$$

$$\Omega_2 = [1 - F(\bar{r}_1)] [TS(0) - TS(p_2^r)]$$

$$\Omega_3 = \{-\epsilon f(\bar{r}_1) [TS(0) - TS(p_2^r)] + (1 - \epsilon) \pi'(\bar{r}_1)\} \frac{d\bar{r}_1}{d\epsilon}$$

$$\Omega_4 = \{-(1 - \epsilon) \pi'(p_2^r) - \epsilon [1 - F(\bar{r}_1)] f(p_2^r) p_2^r\} \frac{dp_2^r}{d\epsilon}$$

Note that evaluated at $\epsilon = 0$, $\Omega_1 = \Omega_3 = \Omega_4 = 0$ and $\Omega_2 > 0$. Hence, $\frac{d\Delta\Pi^d(\epsilon=0)}{d\epsilon} > 0$. Therefore, provided ϵ is a strictly positive, arbitrarily small number, such a deviation is profitable and hence we can rule out case (i).

Ruling out a solution of type (iii) is much simpler. Suppose we have a solution such that $p_2^r \leq p_2^n < \bar{r}_1$. In this case:

$$\bar{r}_1 = p_1 + \delta[(1 - \epsilon)(p_2^r - p_2^n) + \epsilon CS(p_2^r) - CS(p_2^n)]$$

The firm's profits in period 1 are:

$$\Pi_1 = [1 - F(\bar{r}_1)] p_1 + \delta \{ [1 - F(\bar{r}_1)] [1 - \epsilon F(p_2^r)] p_2^r + \{ F(\bar{r}_1) - [1 + \epsilon + \epsilon F(p_2^n)] \} p_2^n \}$$

The first order condition with respect to p_2^n evaluated at $p_2^n = p^m$ is:

$$\frac{\partial \Pi_1}{\partial p_2^n}(p_2^n = p^m) = \delta(1 - \epsilon) [1 - F(\bar{r}_1)] [1 - F(p^m)] > 0$$

This inequality together with the second order condition implies $p_2^n > p^m$. However, the first order condition with respect to \bar{r}_1 implies $\bar{r}_1 < p^m$. We have reached a contradiction.

8.5 Lemma 2

Let us first consider case (ii) $p_2^n - c < \bar{r}_1$. Then in the second period the firm chooses p_2^n in order to maximize:

$$\Pi_2 = [1 - F(\bar{r}_1)] [1 - \epsilon F(p_2^n - c)] (p_2^n - c) + \epsilon F(\bar{r}_1) \pi(p_2^n)$$

In this case, provided ϵ is sufficiently small, $\frac{d\Pi_2}{dp_2^n} > 0$.

Next, suppose the solution satisfies (iii) $p_2^n < \bar{r}_1$. Then in the second period the firm chooses p_2^n in order to maximize:

$$\begin{aligned} \Pi_2 = & [1 - F(\bar{r}_1)] [1 - \epsilon F(p_2^n - c)] (p_2^n - c) + \\ & + \{F(\bar{r}_1) - [1 - \epsilon + \epsilon F(\bar{r}_1)] F(p_2^n)\} p_2^n \end{aligned}$$

It turns out that:

$$\frac{d\Pi_2}{dp_2^n} (p_2^n = p^m) = \epsilon [1 - F(\bar{r}_1)] [1 - F(p_2^n - c) - f(p_2^n - c) (p_2^n - c)] > 0$$

Hence, $p_2^n > p^m > \bar{r}_1$. We have reached a contradiction.

8.6 Bundling versus non-bundling

Consider the case $\mu = 1 - \epsilon$. Suppose that in the first period the firm offers a contract, for a price x , that gives the consumer the right to buy the good in the second period at a price p_2^r . A consumer with first period valuation r_1 will be willing to pay x if and only if:

$$\begin{aligned} \Psi(r_1) &= \delta [(1 - \epsilon) \max\{r_1 - p_2^r, 0\} + \epsilon CS(p_2^r)] - x - \\ -\delta [(1 - \epsilon) \max\{r_1 - p_2^n, 0\} + \epsilon CS(p_2^n)] &\geq 0 \end{aligned}$$

The first term represents the expected gains from buying the contract, and the third the gains from arriving to the second period without a contract. Clearly, x can only be positive if $p_2^n > p_2^r$. Also, note that $\Psi(r_1)$ is flat if either $r_1 \leq p_2^r$ or $r_1 \geq p_2^n$, and strictly increasing if $r_1 \in [p_2^r, p_2^n]$. As a result, the monopolist will never consider charging a price x such that $\Psi(p_2^n) < 0$. Similarly, it will never charge a price x such that $\Psi(p_2^r) > 0$. Therefore, there always exists a threshold value $\bar{r}_1 \in [p_2^r, p_2^n]$ such that consumers purchase the

contract if and only if $r_1 \geq \bar{r}_1$. Thus, when we move to the second period the monopolist encounters a pool of consumers, most of them with valuations lower than \bar{r}_1 , and as a result has incentives to set $p_2^n < \bar{r}_1$ (the details coincide with the discussion in Section 4.2 for the case of bundling.) Hence, we reach a contradiction: there is no way the monopolist can make money by selling the future's contract.

Consider the case $\mu = 0$. In this case, the consumers' valuation of the contract is given by:

$$\Psi(r_1) = \delta [CS(p_2^r) - CS(p_2^n)] - x$$

Thus, the consumers' willingness to pay is independent of r_1 . It is easy to check that if consumers expect $p_2^n \geq 1$, i.e., $CS(p_2^n) = 0$, then the monopolist finds it optimal to set $p_1 = p^m, p_2^r = 0, x = \delta CS(0)$. Under these prices all consumers purchase the contract, so in the second period there are no newcomers and hence $p_2^n \geq 1$ is in fact optimal. The profits associated with such strategy are higher than in the bundling case.

Suppose, instead, that in the first period there are two groups of consumers. Type *A* consumers, with mass $1 - \varepsilon$, have independent valuations over time. In contrast, type *B* consumers, with mass ε , have constant valuations. Suppose ε is an arbitrarily small, positive number. In this case, Coasian dynamics are triggered again and a strategy of selling separately the first period good and the future's contract becomes unprofitable.

In order to check this, note that all type *A* consumers have the same willingness to pay for the contract, $\delta [CS(p_2^r) - CS(p_2^n)]$. However, a type *B* consumers will buy the contract if and only if $\delta r_1 - x \geq \max\{\delta(r_1 - p_2^n), 0\}$. The monopolist can only make money if he sells the contract to type *A* consumers. Hence, $x \leq \delta [CS(p_2^r) - CS(p_2^n)]$. Consider the following function in the range $p_2^n > p_2^r$:

$$\Omega(p_2^n) = CS(p_2^r) - CS(p_2^n) - p_2^n$$

First, $\Omega(p_2^r) = -p_2^r \leq 0$. Second, $\Omega'(p_2^n) = -F(p_2^n) < 0$. Thus, $x - \delta p_2^n \leq \delta \Omega(p_2^n) < 0$. Hence, type *B* consumers purchase the contract if and only if $r_1 \geq \bar{r}_1$, where $\bar{r}_1 = \frac{x}{\delta}$. In the second period, the monopolist finds it optimal to set $p_2^n < \bar{r}_1 = \frac{x}{\delta}$, and we reach a contradiction. Hence, there is no way the monopolist can make money by selling the future's contract separately.