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Theoretical Notes on Bubbles and the Current Crisis

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Abstract

We explore a view of the crisis as a shock to investor sentiment that led to the collapse of a bubble or pyramid scheme in financial markets. We embed this view in a standard model of the financial accelerator and explore its empirical and policy implications. In particular, we show how the model can account for: (i) a gradual and protracted expansionary phase followed by a sudden and sharp recession; (ii) the connection (or lack of connection!) between financial and real economic activity and; (iii) a fast and strong transmission of shocks across countries. We also use the model to explore the role of fiscal policy.

JEL classification: E32, E44, G01, O40

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*We dedicate this research to the memory of Paul Samuelson, the best economist of the twentieth century, and the first one to understand that pyramid schemes are possible and might raise welfare even if we are all rational and well informed. We thank Fernando Broner, Francesco Caselli, Pierre-Olivier Gourinchas and Chris Telmer for comments on an earlier draft, and Nobu Kiyotaki, Hélène Rey and Stavros Panageas for very helpful discussions. We also thank Gonçalo Pina and Robert Zymek for excellent research assistance. Martin: CREI and Universitat Pompeu Fabra, amartin@crei.cat. Ventura: CREI and Universitat Pompeu Fabra, jventura@crei.cat. CREI, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005-Barcelona, Spain. We acknowledge financial support from the European Research Council, the Lamfalussy Program sponsored by the European Central Bank, the Spanish Ministry of Science and Innovation, the Generalitat de Catalunya, and the Barcelona GSE Research Network.

History shows that capitalist economies alternate between expansions and recessions. Thus, even in the heights of the expansion that went from the mid 1990s to the subprime mortgage crisis in the summer of 2007 it was widely understood that a recession would someday hit the world economy. But nobody anticipated what has happened since. The depth of the current recession and the blazing speed with which it has propagated across industries and countries far exceeds even the most pessimistic scenarios. In fact, we need to go back to the Great Depression of the 1930s to find a crisis of a similar magnitude and global scope. It is still not clear however that the lessons we learned from that earlier crisis are useful to understand what is going on today.

As everybody else, macroeconomists have been taken by surprise by the unfolding of events. Even worse, providing an accurate diagnosis of the problem and coming up with clear-cut policy prescriptions has proved to be a difficult challenge. Part of the reason for this, of course, is that state-of-the-art macroeconomic models used for policy analysis are poorly adapted to this task. These models typically emphasize nominal rigidities and labor market frictions, and downplay the role of financial frictions. As a profession, we must go back to the drawing board and reverse these priorities. To understand the current crisis we need models that place financial frictions at center stage.

Recent attempts to do this build on the seminal contributions by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) who developed models of the "financial accelerator" mechanism.[1] These models were designed to show how financial frictions amplify the impact of traditional macroeconomic shocks through their effects on net worth. The intuition is simple: the role of financial markets is to intermediate funds from those that have them (i.e. the savers or creditors) to those who know what to do with them (i.e. the entrepreneurs or borrowers). This intermediation is useful because it raises the average efficiency of the economy and thus the welfare of its inhabitants. In order for this intermediation to be feasible, however, savers need guarantees from entrepreneurs that the funds they lend them (plus an attractive enough return!) will be paid back once the investments give their fruits. The net worth of entrepreneurs, i.e. the amount of future funds that they can pledge today to creditors, is akin to those guarantees. When net worth is low, entrepreneurs cannot borrow enough and the economy operates at low levels of efficiency. When net worth is high, entrepreneurs can borrow enough and the economy operates at high levels of efficiency.

[1] Of course, these initial models were quite stylized. Carlstrom and Fuerst (1997) and Bernanke et al. (1999) developed more sophisticated versions for quantitative analysis. Recently, Gertler and Kiyotaki (2010) and Fernandez-Villaverde and Ohanian (2010) have used versions of this model to study the current crisis.

There are two alternative ways of using the financial accelerator model to think about the current crisis. The first one is based on the notion that, as a result of unprecedented changes in the financial system, the financial accelerator mechanism has become very powerful at amplifying traditional macroeconomic shocks. Consequently, small "real" shocks that affect the efficiency of investment or the productivity of financial intermediation can now unleash very large contractions of credit and deep recessions. This view thus stresses the amplifying role of financial markets, but it still requires us to identify the specific shock to economic fundamentals that pushed the world economy into such a severe recession.

A second and complementary way of using the model is based on the notion that, instead of a small macroeconomic shock of the traditional kind, the world economy has suffered a large shock to investor sentiment that has drastically reduced net worth. Although intuitively appealing, it is hard to articulate this view because we lack a formal model of such shocks. Our main goal in writing these notes is to provide such a model. We show how, within the financial accelerator framework, changes in investor sentiment affect the market valuation of firms and therefore their net worth. In particular, investor optimism gives rise to bubbles that increase the price of firms. These bubbles are useful because they raise net worth, leading to a credit expansion and a boom. When investors become pessimistic, these bubbles burst and net worth falls, leading to a credit contraction and a recession.

This alternative perspective amounts to more than just an academic exercise. On the empirical side, introducing bubbles in the financial accelerator model allows us to provide a simple unified narrative of the main macroeconomic developments of the recent past up to the current crisis as a bubbly episode that started in the early 1990s and ended in 2007-08. This narrative fits very well with the broad turn of events: a steady, protracted expansion phase that entailed significant increases in asset prices and in credit to the private sector, and a fast, severe downturn during which these variables collapsed (see Figure 1). It also provides a potential explanation to the speed and the strength with which shocks spread across sectors and countries. More generally, the introduction of bubbles in a model of financial frictions can provide answers to two burning questions for current macroeconomics: (i) Why do asset (stock, housing, ...) prices fluctuate so much and in ways that seem so unrelated to fundamentals? and (ii) How is it that these fluctuations in asset prices have such significant effects on real activity? The importance of these questions goes beyond understanding the events of recent years: as the postwar experience of industrialized economies shows, substantial fluctuations in asset prices are not uncommon and, when they happen, they are typically associated with substantial macroeconomic developments.[2]

On the policy side, viewing the crisis as the collapse of a bubble has far reaching implications for the role of fiscal policy as a stabilization tool.[3] The case for a fiscal stimulus package and its optimal design depend crucially on whether the shock that led to the crisis is a traditional macroeconomic shock or a shock to investor sentiment. If the latter, we describe the type of fiscal package that can restore the economy to its pre-crisis path. Whether this package is feasible, though, depends crucially on the credibility of the government. When credibility is low, attempts to undo the crisis through the use of fiscal policy might merely cause it to move across markets, from private financial markets to public-debt markets.

In thinking about the origin and consequences of the current crisis, there are different, but complementary, lines of research that can be pursued. One approach is to focus on the particular details and institutional arrangements of financial markets, emphasizing the role of specific features – like regulation or the incentives of certain market participants – in generating and fueling the crisis.[4] An alternative approach is to take a step back and think instead of the general features that have characterized financial markets, and more generally the macroeconomy, in recent years. This approach, which we adopt in these notes, is also followed in recent papers by Gertler and Kiyotaki (2009) and Caballero, Farhi and Gourinchas (2008). As mentioned already, Gertler and Kiyotaki draw on the insights of the financial accelerator literature in order to interpret the current crisis. We differ from them by modeling the crisis as a shock to investor sentiment that ended a bubbly episode. Caballero, Farhi and Gourinchas also view the crisis as the bursting of a bubble, although they do not provide a formal model of how bubbles can arise in equilibrium. They argue that the bubble was fueled by a shortage of financial assets in the world economy and focus on the chronology of events that followed its bursting.

Methodologically, we build on the traditional literature on rational bubbles that goes back to Samuelson (1958). Tirole (1985) analyzed the conditions for the existence of such bubbles in the context of a production economy. Our model is close to Tirole's with the difference that, in our

[2] In analyzing housing and equity prices in industrialized economies during the postwar period, IMF (2002) found that equity price busts occured on average once every 13 years and entailed price declines of about 45 percent, whereas housing busts occured on average every 20 years and involved price declines of about 30 percent. Both equity and housing price busts were associated with output losses reflecting declines in both consumption and investment.

[3] In this regard, this paper is related to Bernanke and Gertler (1999) who studied the design of monetary policy in a financial-accelerator model with bubbles. Although similar in spirit, both papers differ substantially on their emphasis. In the model of Bernanke and Gertler, the emphasis is on policy analysis: asset bubbles are introduced exogenously and not as an equilibrium phenomenon. In these notes, instead, the emphasis is in the development of a consistent framework to study the interaction between asset bubbles and the financial accelerator mechanism.

[4] For such an account, see Brunnermeier (2009).

setup, the presence of financial frictions: (i) relaxes the conditions for the existence of bubbles and; (ii) it implies that bubbles can be expansionary and increase credit and output. Azariadis and Smith (1993) were, to the best of our knowledge, the first to study the relationship between contracting frictions and the existence of rational bubbles. Our finding regarding the relationship between financial frictions and the possibility of expansionary bubbles is related to recent results by Caballero and Krishnamurthy (2006), Kraay and Ventura (2007), Farhi and Tirole (2009) and Kocherlakota (2009). Our framework differs from these last papers in two crucial respects, though. The first is that we study expansionary bubbles in the context of a standard production economy. The second is that, as in Martin and Ventura (2010), bubbles in our setting can arise even if all investments are dynamically efficient in the economy's fundamental equilibrium.[5]

These notes are organized as follows. Section 1 develops a stylized version of the financial accelerator model and explores the effects of traditional macroeconomic shocks. Section 2 shows that the model has additional equilibria with bubbly episodes and uses them to interpret the crisis. Sections 3 and 4 extend the framework to study how policy can react to the bursting of a bubble, and how bubbly episodes are transmitted across countries. Section 5 concludes.

1 A canonical model of financial frictions and business cycles

In a recent paper, Gertler and Kiyotaki (2009) develop a "canonical framework to help organize thinking about credit market frictions and aggregate economic activity in the context of the current crisis" (p.1). This framework is built around an agency cost that limits the ability of firms to pledge future resources to their creditors. This section develops a stripped-down version of this framework and uses it in the way that Gertler and Kiyotaki suggest.

1.1 Basic setup

Our model builds on Samuelson's two-period overlapping-generations structure. The world economy contains an infinite sequence of generations, indexed by $t \in (-\infty, +\infty)$. Each generation contains a continuum of individuals of size one, indexed by $i \in I_t$. Individuals maximize expected old-age consumption, i.e. $U_{it} = E_t \{c_{it+1}\}$; where U_{it} and c_{it+1} are the utility function when young and

[5] There is also a literature on bubbles and economic growth that is closely related to this paper. Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993) extend the Samuelson-Tirole model to economies with endogenous growth due to externalities in capital accumulation. In their models, bubbles slow down the growth rate of the economy. Olivier (2000) uses a similar model to show how, if tied to R&D firms, bubbles might foster technological progress and growth.

the old-age consumption of individual i of generation t. To finance their consumption, individuals supply one unit of labor when young. Since individuals only care about old age consumption, they save their entire labor income. Since individuals are risk-neutral, they always invest their savings so as to maximize their expected return.

The world economy also contains an infinite sequence of generations of firms, indexed by $j \in J_t$. The set J_t contains all firms that were created, in period t or before and are still operating. Firms produce output with a Cobb-Douglas technology: $F(l_{jt}, k_{jt}) = l_{jt}^{1-\alpha} \cdot k_{jt}^{\alpha}$; where l_{jt} and k_{jt} are the labor and capital used by firm j in period t. Firms also produce capital with a technology that uses one unit of output in period t to produce A_{jt} units of capital in period $t+1$. The capital stock of firm j evolves as follows:

$$k_{jt+1} = A_{jt} \cdot Z_{jt} + (1-\delta) \cdot k_{jt}, \tag{1}$$

where Z_{jt} is the investment of firm j, and $\delta \in [0,1]$ is the rate of depreciation. To motivate the need for intermediation, we make two assumptions about the life cycle of firms. The first one is that investment efficiency is high when a firm starts and then stabilizes at a lower level when it becomes mature:

$$A_{jt} = \begin{cases} \pi_t & \text{if } j \in J_t^N \\ 1 & \text{if } j \notin J_t^N \end{cases}, \tag{2}$$

where J_t^N is the set of "new" firms in period t, i.e. the set of firms that are created in period t and start producing output in period $t+1$. We refer to π_t as the investment efficiency and assume that it fluctuates randomly with $\pi_t > 1$. The second assumption is that only a subset I_t^E of generation t is capable of starting a firm. We refer to this subset as the "entrepreneurs" and assume that it has measure $\varepsilon \in [0,1]$. Everybody can manage an old firm.

Workers and savings are allocated to firms in the labor and financial markets. The labor market is competitive and all workers and firms can trade in it with zero or negligible transaction costs. Maximization then implies that:

$$l_{jt} = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} \cdot k_{jt}, \tag{3}$$

where w_t is the wage rate per unit of labor. Equation (3) is the labor demand of firm j, which results from hiring labor until its marginal product equals the wage. Since the aggregate supply of labor is one, market clearing implies that:

$$w_t = (1-\alpha) \cdot k_t^{\alpha}, \tag{4}$$

where $k_t \equiv \int_{j \in J_t} k_{jt}$ is the aggregate capital stock. Since all firms use the same capital-labor ratio, this must be the aggregate one. Thus, Equation (4) says that the wage equals the marginal product of labor evaluated at the aggregate capital-labor ratio.

We turn next to the key piece of the model, namely, the financial market. This market consists of a credit market where individuals lend to firms, and a stock market where individuals buy and sell old firms. Both markets are competitive and all savers and firms can trade in them with zero or negligible transaction costs. Firms can write contingent credit contracts, but there is an agency cost that limits the overall ability of firms to obtain credit. In particular, firms can commit or pledge to their creditors only a fraction ϕ_t of their resources in period t. We refer to ϕ_t as the financial friction and assume that it fluctuates randomly within the unit interval. We adopt the convention that, in period t, individuals know the realization of shocks with index t (i.e. π_t and ϕ_t), but they do not know the realizations of shocks with index $t+1$ (i.e. π_{t+1} and ϕ_{t+1}). The resources of the firm in period $t+1$ consist of the revenue from sales net of labor costs, i.e. $F(l_{jt+1}, k_{jt+1}) - w_{t+1} \cdot l_{jt+1}$, plus the firm's resale or market value, i.e. V_{jt+1}. Therefore, we have that in each possible state of nature in period $t+1$ the following constraint holds:

$$R_{t+1} \cdot f_{jt} \leq \phi_{t+1} \cdot \left[F(l_{jt+1}, k_{jt+1}) - w_{t+1} \cdot l_{jt+1} + V_{jt+1}\right], \tag{5}$$

where f_{jt} is the credit that firm j obtains in the credit market in period t, and R_{t+1} is the (gross) ex-post return to loans. Since R_{t+1} might be contingent on any variable which is known in period $t+1$, we refer to $E_t R_{t+1}$, as the interest rate. The right-hand side of Equation (5) captures the concept of net worth. That is, the amount of future resources that firms can use as a collateral to obtain credit today. The shock ϕ_t captures the quality of the legal system and other institutional arrangements that support credit.

Maximization implies that non-entrepreneurs will lend and buy old firms simultaneously if and only if the expected return to owning an old firm equals the interest rate:[6]

$$E_t R_{t+1} = \max_{\langle Z_{jt}, f_{jt} \rangle} \frac{E_t\left\{\alpha \cdot k_{t+1}^{\alpha-1} \cdot \left[A_{jt} \cdot Z_{jt} + (1-\delta) \cdot k_{jt}\right] - R_{t+1} \cdot f_{jt} + V_{jt+1}\right\}}{V_{jt} + Z_{jt} - f_{jt}} \quad \text{if } j \notin J_t^N, \tag{6}$$

where the maximization is subject to the constraint in Equation (5). To compute the return to owning an old firm, note that in period t the owner must spend the purchase price plus the cost of

[6] Here, we have used that Equations (3) and (4) imply that $F(l_{jt}, k_{jt}) - w_t \cdot l_{jt} = \alpha \cdot k_t^{\alpha-1} \cdot k_{jt}$.

new capital minus credit. Then, in period $t+1$ the owner obtains the revenue from sales net of labor and financing costs plus the resale value of the firm. Maximization also implies that entrepreneurs start new firms only if the expected return to doing so is not lower than the interest rate:

$$E_t R_{t+1} \leq \max_{\langle Z_{jt}, f_{jt} \rangle} \frac{E_t \left\{ \alpha \cdot k_{t+1}^{\alpha-1} \cdot A_{jt} \cdot Z_{jt} - R_{t+1} \cdot f_{jt} + V_{jt+1} \right\}}{Z_{jt} - f_{jt}} \quad \text{if } j \in J_t^N, \tag{7}$$

where the maximization is once again subject to the constraint in Equation (5). Unlike old firms, new firms start without capital and their owners, who are also their creators, do not have to pay a price for them, i.e. $k_{jt} = V_{jt} = 0$ if $j \in J_t^N$.

The next step is to determine the interest rate and firm prices that clear the credit and stock market. We are interested in equilibria in which firms are credit constrained. Our assumption that credit contracts can be fully state contingent implies that, in those equilibria, Equation (5) must hold with equality in all states of nature since firms have borrowed as much as possible against their future net worth. We conjecture that the following interest rate and firm prices clear the credit and stock market,

$$E_t R_{t+1} = \alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta, \tag{8}$$

$$V_{jt} = (1 - \delta) \cdot k_{jt}, \tag{9}$$

and then verify this conjecture. Equation (8) says that the interest rate equals the return to producing a unit of capital within an old firm. Equation (9) says that the price of a firm equals the cost of replacing the capital that it owns. Ideally, all investment should take place within new firms, as these have a technological advantage when producing new capital. This is not possible however if the financial friction is severe enough. The conjecture in Equations (8) and (9) turns out to be correct if the equilibrium is inefficient and some investment is carried out within old firms.

At the proposed interest rate and firm prices, entrepreneurs strictly prefer to start new firms than to lend or purchase old firms. Moreover, since the interest rate is below the return to investing in new firms the owners of these firms ask for as much credit as possible. Since the optimal financing contract ensures that Equation (5) is binding in all states of nature, we find credit by adding this constraint across states of nature:[7]

[7] Adding up Equation (5) across states of nature yields:

$$E_t R_{t+1} \cdot f_{jt} = E_t \left\{ \phi_{t+1} \cdot \left[\alpha \cdot k_{t+1}^{\alpha-1} \cdot \pi_t \cdot (w_t + f_{jt}) + V_{jt+1} \right] \right\},$$

where we have used that: (i) Equations (3) and (4) imply that $F(l_{jt}, k_{jt}) - w_t \cdot l_{jt} = \alpha \cdot k_t^{\alpha-1} \cdot k_{jt}$; and (ii) entrepreneurs put all of their savings in the firm and Equations (1) and (2) then imply that $k_{jt+1} = \pi_t \cdot (w_t + f_{jt})$. To obtain

$$f_{jt} = \frac{1}{1 - E_t\phi_{t+1} \cdot \pi_t} \cdot E_t\left\{\phi_{t+1} \cdot \pi_t \cdot w_t\right\}. \tag{10}$$

Not surprisingly, credit increases with the wealth of entrepreneurs and their investment efficiency, and decreases with the financial friction.

At the proposed interest rate and firm prices, non-entrepreneurs are indifferent between lending and purchasing old firms. If they choose the latter, they are also indifferent regarding the amount of investment and external financing of their firms. As a group, non-entrepreneurs purchase the stock of old firms, give credit to new firms and use any savings left to produce new capital within their old firms. To verify that markets clear, we must check that this group has enough savings to do all of this:

$$(1 - \varepsilon) \cdot w_t - f_t^N \geq V_t, \tag{11}$$

where $V_t \equiv \int_{j \notin J_t^N} V_{jt}$ and $f_t^N \equiv \int_{j \in J_t^N} f_{jt}$. We assume from now on that this condition holds and, as a result, the conjectured interest rate and firm prices are verified.[8]

Aggregating Equation (1) across firms, we find that:[9]

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1) \cdot \varepsilon}{1 - E_t\phi_{t+1} \cdot \pi_t}\right] \cdot (1 - \alpha) \cdot k_t^{\alpha}. \tag{12}$$

Equation (12) is the law of motion of the capital stock. The dynamics of this economy are akin to those of a Solow model with shocks to the average efficiency of investment. From any initial capital stock, the economy converges towards a steady state in which the capital stock fluctuates within a range that is defined by the support of the shocks. These shocks might originate in the investment technology (π_t) or the financial friction (ϕ_t), but have similar macroeconomic effects as they both work through the average efficiency of investment.

Equation (10), we substitute in the conjectured interest rate and firm prices and solve for f_{jt}.

[8] This requires that:

$$\frac{1 - E_t\phi_{t+1} \cdot \pi_t - \varepsilon}{1 - E_t\phi_{t+1} \cdot \pi_t} \cdot (1 - \alpha) \cdot k_t^{\alpha} \geq (1 - \delta) \cdot k_t.$$

In terms of the primitives of the model, this implies that: (i) $E_t\phi_{t+1} \cdot \pi_t < 1 - \varepsilon$ in all dates and states of nature, and (ii) δ is high enough. The first restriction ensures that the credit constraint is tight enough so that, after giving credit to new firms, non-entreprenurs still have some savings left in their hands. The second restriction ensures firm prices are sufficiently low so that these savings are sufficient to purchase the stock of old firms.

[9] Investment spending consists of the savings of the young minus their purchases of old firms, i.e. $w_t - V_t = (1 - \alpha) \cdot k_t^{\alpha} - (1 - \delta) \cdot k_t$. Of this total, new firms invest $\frac{\varepsilon}{1 - E_t\phi_{t+1} \cdot \pi_t} \cdot (1 - \alpha) \cdot k_t^{\alpha}$ with efficiency π_t, while the rest is invested by old firms with efficiency one.

1.2 Looking to the crisis through the lens of the canonical model

We are ready to use the canonical model in the way that Gertler and Kiyotaki suggest, namely, as a framework to help organize our thinking about the current crisis. The stylized facts are well known, of course (see Figure 1). The world economy entered a long and steady expansion around the mid 1990s, with increases in consumption and investment. The prices of stocks, real estate and other assets grew to unprecedented levels. Intermediation soared, while interest rates fell to historical lows. This expansion lasted more than a decade, leading many to think that the business cycle was over. This might have been too optimistic. But nobody anticipated what happened after the summer of 2007: a sudden and sharp drop in stock and real estate prices, a massive collapse in intermediation and the worse financial crisis since the Great Depression. Since then, investment has come to a halt and the world economy has experienced negative growth. We are only now starting to see the light at the end of the tunnel.

The key question, of course, is how did all this happen. Coming up with a convincing explanation for such a sharp and unexpected change in economic outcomes is a fascinating academic challenge with far reaching policy implications. At a deep level, explanations of the crisis fall into one of two rough categories. The first one includes explanations based on the notion that something fundamental or technological has happened. These explanations emphasize aggregate resource constraints and view the crisis as a negative shift of these constraints. A second set of explanations start from the premise that nothing fundamental has changed, and that we are only witnessing a massive coordination failure. This second set of explanations emphasize the role of expectations and view the crisis as a negative shift in those.

The canonical model described above offers two alternative, but complementary, explanations of the crisis: a shock to the investment technology, π_t; and a shock to the financial friction, ϕ_t. Both of these shocks are fundamental or technological, although they originate in different parts of the economy: the corporate or the financial sector, respectively. We consider each of them in turn.

Figure 2 shows the response of the economy to a transitory shock to the investment technology.[10,11] The different panels plot the assumed path for the shock (π_t) and the responses of the capital stock (k_t), consumption (c_t), the stock market (V_t), the interest rate ($E_t R_{t+1}$) and inter-

[10] In particular, we assume that $\pi_t = \bar{\pi}$ if $0 \leq t < T$ and $\pi_t = \pi$ for all $t < 0$ and $t \geq T$, with $\bar{\pi} > \pi$. To allow for a clean experiment, we assume that $\phi_t = \phi$ for all t, and that the economy was already in the steady state in period $t = 0$.

[11] Table 1 in the appendix contains all information regarding the parametrization of figures 2-3 and 6-8.

mediation (f_t^N).[12] All variables are shown as deviations from the steady state. The increase in π_t raises the average efficiency of investment through two channels. For a given allocation of investment, new firms become more efficient at investing. In addition, their net worth increases, relaxing their credit constraint and allowing them to undertake a larger share of the economy's investment. The increase in the average efficiency of investment shifts the law of motion of the capital stock upwards and the economy starts transitioning towards a higher steady state. As this happens, the capital stock and consumption increase. In the financial market, the interest rate declines, while intermediation and firm prices increase. When π_t goes back to its original level, all these changes unwind. The law of motion of the capital stock goes back to its original shape and the capital stock starts declining. Eventually, the economy goes back to its original steady state.

Figure 3 shows the response of the economy to a transitory shock to the financial friction.[13] We have calibrated the shocks so that the quantitative effect on the average efficiency of investment is the same in Figures 2 and 3. The most remarkable aspect of Figure 3 is that it is almost a carbon copy of Figure 2. The only difference between these figures is that Figure 3 shows a larger increase in intermediation. The reason is that shocks to the financial friction only affect the average efficiency of investment through one channel: the net worth of firms increases, relaxing their credit constraint and improving the allocation of investment. This is why a shock to ϕ_t requires a larger increase in intermediation than a shock to π_t to obtain the same increase in the average efficiency of investment. Since shocks to π_t and ϕ_t are observationally equivalent from a macroeconomic perspective, the only way to tell them apart would be to use microeconomic data to find out whether aggregate fluctuations in the average efficiency of investment are due to firms being more productive or having better access to credit.

The model is stylized and much work remains to be done to get it ready for serious quantitative analysis. In particular, it can be extended along various dimensions to strengthen the financial accelerator mechanism.[14] But Figures 2 and 3 already show that it is possible to write down a model based on fundamental or technological shocks to the corporate (i.e. π_t) and/or the financial sector (i.e. ϕ_t) that delivers dynamics that are qualitatively consistent with the evidence. Moreover, the notion that it is a drop in aggregate net worth that has caused a collapse in intermediation is

[12] The response of output and wages mimics that of the capital stock.

[13] In particular, we assume that $E_t\phi_{t+1} = \bar{\phi}$ if $0 \leq t < T$ and $E_t\phi_{t+1} = \phi$ for all $t < 0$ and $t \geq T$, with $\bar{\phi} > \phi$. To allow for a clean experiment, we assume that $\pi_t = \pi$ for all t, and that the economy was already in the steady state in period $t = 0$.

[14] Most prominently, fluctuations in the relative price of assets (i.e. capital) can feed back into borrowing constraints and exacerbate volatility (see Kiyotaki and Moore (1997)).

certainly appealing as it conforms to the perceptions of many observers and market participants. Not surprisingly then, much current research follows this research strategy.

Despite this, we remain unconvinced that the current crisis is the result of a technological or fundamental shock. Even accounting for the amplifying forces of the financial system, what particular shock could have caused such a dramatic downturn as the one suffered by the world economy? It seems difficult to identify a specific technological shock that could underlie such a large change in the investment opportunities faced by firms. Likewise, it seems difficult to identify a specific change in the institutional and/or technological framework of financial markets that has so suddenly left them so impaired to do their job. Neither the resources available for intermediation, nor the technology used for it seem to have changed much.

This is why we search for an alternative explanation of this crisis, one that can help us understand how output and wealth can fall so much even though resources and technology remain apparently unchanged. We would like to do so by complementing the financial accelerator framework, preserving its central feature, namely, the predominant role of financial frictions. If successful, such an explanation could shed light on more than the recent past. Even before the current crisis, the large and unpredictable fluctuations in the stock and housing markets of recent years hardly mirrored the evolution of technological or fundamental shocks.[15] And, with a longer-term perspective that encompasses the whole of the postwar period, it appears that large fluctuations in asset prices (and their macroeconomic implications) have hardly been uncommon. So we search for an explanation of (i) why asset prices move in ways that are unrelated to fundamentals, and (ii) how these movements in asset prices can lead to fluctuations in production with unchanged resources. This does not require changing the model, but only the way we look at it. We show this next.

2 Bubbles as pyramid schemes

What is the price of a firm? We showed that the canonical model has an equilibrium in which the price of a firm equals the cost that it would take to replace the capital it owns. This price is often referred to as the fundamental value of a firm, since it also equals the net present value of all the output that the capital owned by the firm will ever produce. But the canonical model has many other equilibria in which firm prices are above their fundamental value. It is customary to refer

[15] Although the recent evolution of real state prices is perhaps too close to us to draw any definitive conclusions, the stock price boom and bust of the late 1990s, which has been widely studied, seems hard to attribute to movements in fundamentals. For a detailed discussion on this last point, see LeRoy (2004).

to these equilibria as bubbly, since they capture the notion of firms being overvalued or having a bubble. We use these equilibria to sketch an alternative explanation of the current crisis.

2.1 Setup with bubbles

We solve the model again, conjecturing that the interest rate is still given by Equation (8) but that firm prices are now given by:

$$V_{jt} = (1 - \delta) \cdot k_{jt} + b_{jt}, \tag{13}$$

where b_{jt} is the overvaluation or bubble in firm j. The assumption that firm prices equal their fundamental value can be expressed as the restriction that $b_{jt} = 0$ for all j and t. This restriction cannot be justified on a priori grounds but there is always an equilibrium in which it is satisfied, as we showed in the previous section. Equation (13) already points out to the first macroeconomic effect of bubbles: since firm prices are high, the amount of savings devoted to purchase the stock of old firms increases and this reduces the funds available for investment.

At the proposed interest rate and firm prices, entrepreneurs strictly prefer to start new firms than to lend or purchase old firms and, just as before, they ask for as much credit as possible:

$$f_{jt} = \frac{1}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot E_t \left\{ \phi_{t+1} \cdot \left(\pi_t \cdot w_t + \frac{b_{jt+1}}{\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta} \right) \right\}. \tag{14}$$

Equation (14) points out to the second macroeconomic effect of bubbles: since future firm prices are high, entrepreneurs are able to obtain more credit and this improves the allocation of investments.

Of course, not any stochastic process for b_{jt} can be part of an equilibrium. Broadly speaking, there are two restrictions or requirements that bubbles must satisfy. The first one is that bubbles should grow fast enough to be attractive. At the proposed interest rate and firm prices, non-entrepreneurs are indifferent between lending and purchasing old firms if and only if:

$$E_t R_{t+1} = \frac{E_t b_{jt+1}}{b_{jt}}. \tag{15}$$

Equation (15) says that the expected growth rate of bubbles must equal the interest rate. If the growth rate of the bubble were less than the interest rate, owning firms with a bubble would not be attractive. This cannot be an equilibrium. If the growth rate of the bubble exceeded the interest rate, non-entrepreneurs would want to borrow to purchase bubbly firms. This cannot be an equilibrium either. The requirement that all bubbles have the same expected growth rate does not mean that all bubbles be correlated though.

The second requirement for a bubble to be part of the equilibrium is that it should not grow too fast. Otherwise, the aggregate bubble would eventually be too large for the young to be able to purchase it and markets would not clear. Knowing this, standard backward-induction arguments would rule out the bubble today. To verify that markets clear, we must check that non-entrepreneurs have enough savings to lend to entrepreneurs and purchase the stock of old firms. That, is, we must check that Equation (11) holds. We keep assuming that this condition holds and, as a result, the conjectured interest rate and firm prices are verified.[16]

Aggregating Equation (1) across firms, we find that:

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1)\cdot\varepsilon}{1 - E_t\phi_{t+1}\cdot\pi_t}\right]\cdot(1-\alpha)\cdot k_t^{\alpha} + \frac{\pi_t - 1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\frac{E_t\left\{\phi_{t+1}\cdot b_{t+1}^N\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta} - b_t - b_t^N, \quad (16)$$

where $b_t \equiv \int_{j\notin J_{t-1}/J_{t-1}^N} b_{jt}$ and $b_t^N \equiv \int_{j\in J_{t-1}^N} b_{jt}$.[17] A comparison of Equations (12) and (16) shows that, in principle, the effect of bubbles on capital accumulation is ambiguous. The last two terms of Equation (16) show that purchasing the existing bubble reduces capital accumulation by diverting resources away from investment. Since only non-entrepreneurs purchase bubbly firms and their investment efficiency is one, the existing bubble crowds out capital one to one. The second term of Equation (16) shows that the expected bubble expands capital accumulation by relaxing credit constraints, increasing intermediation and the average efficiency of investment. To understand this term, note that the expected bubble raises the net worth of efficient investors by $\frac{E_t b_{t+1}^N}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta}$, which enables them to expand borrowing by a factor of $\frac{E_t\phi_{t+1}}{1 - E_t\phi_{t+1}\cdot\pi_t}$, and each unit borrowed entails an efficiency gain of $\pi_t - 1$.[18]

To complete the description of the dynamics of the economy, we need to determine the evolution of the aggregate bubble. Aggregating Equation (15) across firms, we find that:

[16] This requires now that:

$$\frac{1 - E_t\phi_{t+1}\cdot\pi_t - \varepsilon}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot(1-\alpha)\cdot k_t^{\alpha} - \frac{1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\frac{E_t\left\{\phi_{t+1}\cdot b_{t+1}^N\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta} \geq (1-\delta)\cdot k_t + b_t + b_t^N,$$

where $b_t \equiv \int_{j\notin J_{t-1}^N} b_{jt}$ and $b_t^N \equiv \int_{j\in J_{t-1}^N} b_{jt}^N$. The presence of bubbles makes the condition more stringent. Bubbles raise both intermediation and the value of old firms, leaving less savings to produce capital within old firms.

[17] Investment spending consists of the savings of the young minus their purchases of old firms, i.e. $w_t - V_t = (1-\alpha)\cdot k_t^{\alpha} - (1-\delta)\cdot k_t - b_t - b_t^N$. Of this total, new firms invest $\frac{1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\left(\varepsilon\cdot(1-\alpha)\cdot k_t^{\alpha} + \frac{E_t\left\{\phi_{t+1}\cdot b_{t+1}^N\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta}\right)$ with efficiency π_t, while the rest is invested by old firms with efficiency one.

[18] This decomposition of the second term assumes that b_{t+1}^N and ϕ_{t+1} are uncorrelated.

$$E_t b_{t+1} = \left(\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta\right) \cdot \left(b_t + b_t^N\right). \tag{17}$$

That is, the aggregate bubble grows faster than the interest rate because of the creation of new firms and, with them, new bubbles too. Any sequence for k_t, b_t and b_t^N that satisfies Equations (16) and (17) is an equilibrium, provided that Equation (11) holds in all dates and states of nature. The dynamics of this economy depend on the dynamics of firm prices, and we turn to these next.

2.2 Bubbly episodes

Bubbly episodes can take place in the canonical model. Generically, the economy fluctuates between periods in which $b_t = b_t^N = 0$ and periods in which $b_t > 0$ and/or $b_t^N > 0$. We say that the economy is in the fundamental state if $b_t = b_t^N = 0$. We say instead that the economy is experiencing a bubbly episode if $b_t > 0$ and/or $b_t^N > 0$. A bubbly episode starts when the economy leaves the fundamental state and ends the first period in which the economy returns to the fundamental state. Let $z_t \in \{F, B\}$ be a sunspot variable that determines the state of the economy. We refer to z_t as investor sentiment. The transition probabilities $\Pr(z_{t+1} = F \,|z_t = B)$ and $\Pr(z_{t+1} = B \,|z_t = F)$ could be a function of any endogenous or exogenous variable of the model, and could fluctuate randomly over time.

In the fundamental state, firm prices equal their fundamental values. Each period, there is some probability that a bubble episode starts in the new generation of firms. When this happens, an aggregate bubble appears and starts to grow according to Equation (17). This growth in the bubble is due to two factors: (i) as the new firms become old, their bubble keeps growing at an expected rate that equals the interest rate; and (ii) new bubbles appear in the successive generations of new firms. Throughout the bubbly episode, there is some probability that the episode ends and the economy reverts to the fundamental state. When this happens, all bubbles burst and firm prices go back to their fundamental values.

It turns out that this simple model can give rise to a wide array of equilibrium dynamics with bubbly episodes of different sorts.[19] To simplify the discussion, consider the simple example in which the probability of an episode ending is constant, i.e. $\Pr(z_{t+1} = F \,|z_t = B) = p$; and the rate of bubbly creation is also constant, i.e. $b_t^N = b^N > 0$ when the episode starts and then $b_t^N = n \cdot b_t$ until the episode ends, with $n > 0$. We also assume that $\Pr(z_{t+1} = B \,|z_t = F)$ is small, so that the

[19] See Martin and Ventura (2010) for a full analysis of the set of equilibria in a related model.

fundamental state is similar to the equilibrium of section 1. We use this example throughout the paper for illustrative purposes. It nicely captures the notion of a shock to investor sentiment. In the fundamental state, investors expect bubbles to survive with a low probability and, as a result, investors do not purchase firms with bubbles. During a bubbly episode, investors expect bubbles to survive with a high probability and new bubbles to appear at a rate that is proportional to the stock of existing bubbles. Both sets of expectations are self-fulfilling and this allows us to interpret transitions between these two states as shocks to investor sentiment.

To be able to graphically describe the dynamics of the bubble during an episode, we further simplify by assuming that there are no other type of shocks, i.e. $\pi_t = \pi$ and $\phi_t = \phi$. Moreover, if the rate of depreciation is large, i.e. $\delta \approx 1$, we can make the model recursive through a simple transformation of variables. Define x_t as the bubble's share of wealth or savings, i.e. $x_t \equiv \dfrac{b_t}{(1-\alpha) \cdot k_t^{\alpha}}$. Then, during a bubbly episode, we can rewrite Equation (17) as follows:

$$x_{t+1} = \frac{\dfrac{\alpha}{1-\alpha} \cdot \dfrac{1+n}{1-p} \cdot x_t}{1 + \dfrac{(\pi-1) \cdot \varepsilon}{1-\phi \cdot \pi} + \left(\dfrac{(\pi-1) \cdot \phi \cdot n}{1-\phi \cdot \pi} - 1 \right) \cdot (1+n) \cdot x_t}, \tag{18}$$

if $z_{t+1} = B$ and $x_{t+1} = 0$ if $z_{t+1} = F$. Naturally, the derivation of Equation (18) assumes that Equation (11) holds. This condition can now be rewritten as follows:

$$x_t \leq \frac{1-\phi \cdot \pi - \varepsilon}{1 - \phi \cdot (\pi - n)} \cdot (1+n)^{-1} \equiv \bar{x}. \tag{19}$$

The key observation is that the capital stock does not appear in Equations (18) and (19). Any path for x_t that that satisfies Equations (18) and (19) in all dates and states of nature is an equilibrium of the economy. Since $x_t = 0$ does this, we trivially have that such a path always exists. Of course, the interesting question is whether more paths are possible and, if so, how do these paths look like. Knowing this, we can then use Equation (16) to determine the associated paths for the capital stock, which is given by

$$k_{t+1} = \left[1 + \frac{(\pi-1) \cdot \varepsilon}{1-\phi \cdot \pi} + \left(\frac{\phi \cdot (\pi-1) \cdot n}{1-\phi \cdot \pi} - 1 \right) \cdot (1+n) \cdot x_t \right] \cdot (1-\alpha) \cdot k_t^{\alpha}. \tag{20}$$

This allows us to interpret bubbly episodes literally as shocks to the law of motion of the economy.

Equations (18) and (19) embody the two requirements for bubbly episodes to be part of an equilibrium, and that we mentioned earlier. The first one is that the bubble must be expected to grow fast enough. Otherwise, holding the bubble would not be attractive and nobody would purchase it. This requirement is embodied in Equation (18), which is nothing but a restatement of Equation (15). The second requirement is that the bubble cannot be expected to grow too fast. Otherwise, it would eventually exceed available funds and it could not be purchased. Knowing this, standard backward-induction arguments would rule out the bubble today. This requirement is embodied in Equation (19) which is nothing but a restatement of Equation (11). Equations (18) and (19) can be used to show that bubbly episodes can happen if α is sufficiently low.

This example can generate two types of bubbly episodes. The first type is the conventional or contractionary bubbly episode emphasized by Tirole (1985). These episodes occur in economies where some investments are dynamically inefficient in the fundamental state, and they require that $\frac{(\pi-1)\cdot\phi\cdot n}{1-\phi\cdot\pi} < 1$.[20] This condition ensures that bubbles have a negative effect on capital accumulation, as the reduction in investment spending is not compensated by the increase in the average efficiency of investment. Bubbles raise the interest rate and reduce the capital stock. Figure 4 illustrates one of these contractionary episodes.[21] The thick line depicts Equation (18) and the thin one depicts the 45 degree line. The initial bubble must be in the interval $x_s \in [0, x^*]$. After the initial bubble appears, it declines as a share of wealth throughout. Only if the initial bubble is maximal, i.e. $x_s = x^*$, this rate of decline becomes zero.

The second type of bubbly episode is the non-conventional or expansionary one analyzed by Martin and Ventura (2010). These episodes arise in economies with financial frictions, and exist even if all investments are dynamically efficient in the fundamental state. These episodes require that $\frac{(\pi-1)\cdot\phi\cdot n}{1-\phi\cdot\pi} > 1$.[22] This condition ensures that bubbles have a positive effect on capital accumulation, as the reduction in investment spending is compensated by the increase in the average efficiency of investment. These bubbles reduce the interest rate and increase the capital stock. Figure 5 illustrates one of them. The initial bubble can be anywhere the interval $x_s \in [0, \bar{x}]$.

[20] Episodes of this type exist if Equation (18) is below the 45 degree line for some $x_t \leq \bar{x}$. This requires that:

$$\frac{\alpha}{1-\alpha} \leq 1 + \frac{(\pi-1)\cdot\varepsilon}{1-\phi\cdot\pi}.$$

[21] Since they naturally assume that $\delta = 1$ in order to illustrate the recursive characterization of x, Figures 4 and 5 are parametrized differently than the rest.

[22] Episodes of this type exist if Equation (18) is below the 45 degree line for some $x_t \leq \bar{x}$. This requires that:

$$\frac{\alpha}{1-\alpha} \leq \max_{p,n}\left\{\frac{1-p}{1+n}\cdot\left[1+\frac{(\pi-1)\cdot\varepsilon}{1-\phi\cdot\pi}+\left(\frac{(\pi-1)\cdot\phi\cdot n}{1-\phi\cdot\pi}-1\right)\cdot\frac{1-\phi\cdot\pi-\varepsilon}{1-\phi\cdot(\pi-n)}\right]\right\}.$$

Interestingly, these episodes might look quite different from the conventional ones. In particular, episodes might start with a small bubble that gains momentum over time. These bubbles can become very large before suddenly bursting.

2.3 Looking to the crisis through the lens of the canonical model, again

The canonical model therefore offers a third explanation of the crisis: a shock to investor sentiment. Since non-conventional or expansionary bubbles are the only ones that stand a chance to be empirically relevant in the present situation, we focus on them in what follows. We would like to stress once more that we are not changing the model of the economy, but only the way to use it. Rather than looking for fundamental or technological explanations such as shocks to π_t and ϕ_t, we instead look for an explanation that relies on a coordination failure by focusing on shocks to z_t.

Figure 6 shows the response of the economy to a shock to investor sentiment.[23] We have calibrated the shock so that its effects on the capital stock are roughly the same as those of the technological shocks in Figures 2 and 3. The behavior of the different macroeconomic variables is similar to those in these previous figures. The main difference is that financial variables tend to fluctuate much more in the case of a shock to z_t. One reason is that the shock has a direct effect on firm prices that is absent in the case of shocks to π_t and/or ϕ_t. In addition, high asset prices reduce investment spending and this requires even a larger increase in intermediation to generate the same increase in the capital stock.

The start of a bubble generates a positive wealth shock which can literally be described as a transfer from the future. This is a central feature of a pyramid scheme where the initiator claims that, by making him/her a payment now, the other party earns the right to receive a payment from a third person later. By successfully creating and selling a bubble, entrepreneurs assign themselves and sell the "rights" to the savings of a generation living in the very far future or, to be more exact, living at infinity. This appropriation of rights is a pure windfall or wealth gain for the entrepreneurs.

This wealth shock generates an efficiency gain, as it helps overcome the negative effects of the financial friction. The bubble increases the net worth of entrepreneurs and allows new firms to obtain more credit and invest more. In a very real sense, the bubble is like the oil that greases the machinery that moves financial markets. The rights to the future generated by the bubble

[23] In particular, we assume that $z_t = B$ if $0 \leq t < T$ and $z_t = F$ for all $t < 0$ and $t \geq T$. To allow for a clean experiment, we assume that $\pi_t = \pi$ and $\phi_t = \phi$ for all t, and that the economy was already in the steady state in period $t = 0$.

provide the collateralizable net worth that financial markets need to work efficiently. The bubble thus results in an increased average efficiency of investment. This is why the effects of a shock to investor sentiment resemble those of fundamental shocks.

As a research strategy, viewing the current crisis as the bursting of a macroeconomic pyramid scheme **or** bubble seems to overcome the shortcomings of alternatives that rely on technological shocks. In particular, it explains (i) why asset prices move in ways that are often unrelated to fundamentals; and (ii) why these movements in asset prices can lead to fluctuations in production with unchanged resources. Moreover, this alternative view of the crisis fundamentally affects the role of fiscal policy as a stabilization tool. We turn to this topic next.

3 Policy implications

We have modeled the current crisis as a negative shock to net worth that led to a collapse of intermediation and the average efficiency of investment. Is there anything that governments can do to reverse such a situation? If the shock is fundamental or technological, the canonical model cannot provide a meaningful answer to this question since it lacks a good description of the microeconomics of productivity and the financial friction. But if the shock is the bursting of a bubble, the canonical model turns out to be quite useful for policy analysis. Keeping with the exploratory spirit of these notes, we add a government to the framework developed above and draw some tentative results.

3.1 Setup with a government

Assume next that the world economy contains a government that gives subsidies to firms and finances these subsidies by taxing individuals and issuing debt. Unlike much of the recent literature on the crisis, we do not to give the government an advantage over the market as a lender. Instead, we assume the government enforces payments due by using the same legal system and related institutional arrangements as the private sector.[24] This implies that it is not possible to improve the allocation of investments without raising the net worth of new firms.[25]

[24] For instance, some of the policies advocated by Gertler and Kiyotaki (2010) and Curdia and Woodford (2010) are based on the assumption that (at least, after the crisis) the government is better at lending than the private sector.

[25] Consider a proposal for the government to lend to new firms. Since the total amount of resources that the legal system can extract from these firms is fixed, any lending done by the the government uses up an equivalent amount of net worth. If financed by issuing debt and/or taxing non-entrepreneurs, government lending crowds out private credit one-to-one. Even worse, if partly financed by taxing entrepreneurs, government lending crowds out private credit more than one-to-one. The reason is that taking away resources from entrepreneurs lowers the net worth of their firms.

Let T_{it} and S_{jt} be the tax levied on individual i and the subsidy given to firm j in period t. The government borrows by issuing one-period bonds which yield a (gross) return equal to R^d_{t+1}. As in the case of private debt, we allow this return to be fully contingent and therefore to vary across states of nature. This could reflect a contingent contractual rate, or the government's failure to keep with its contractual obligations. Let d_t be the payments made to debtholders in period t. Then, the government's budget constraint can be written as follows:

$$d_{t+1} = R^d_{t+1} \cdot (d_t + S_t - T_t)\,, \tag{21}$$

where $T_t \equiv \int_{i \in I_t} T_{it}$ and $S_t \equiv \int_{j \in J_t} S_{jt}$. Equation (21) says that the government borrows to make debt payments, i.e. d_t, and to finance the primary budget deficit, i.e. $S_t - T_t$.

The presence of the government has no effect on technology, i.e. Equations (1) and (2); or the functioning of the labor market, i.e. Equations (3) and (4). It does however affect the financial market in three specific ways: (i) there is now an additional market for government debt; (ii) taxes reduce the savings available to purchase financial assets; and (iii) subsidies improve the balance sheet of firms and therefore their net worth. This last effect means that Equation (5) should be replaced by the following one:

$$R_{t+1} \cdot f_{jt} \leq \phi_{t+1} \cdot \left[F\left(l_{jt+1}, k_{jt+1}\right) - w_{t+1} \cdot l_{jt+1} + S_{jt} + V_{jt+1}\right]. \tag{22}$$

Equation (22) recognizes that future subsidies also constitute a source of revenue for the firm. The conditions for maximization also need to be modified as follows:

$$E_t R_{t+1} = E_t R^d_{t+1} = \max_{\langle Z_{jt}, f_{jt}\rangle} \frac{E_t\left\{\alpha \cdot k^{\alpha-1}_{t+1} \cdot \left[A_{jt} \cdot Z_{jt} + (1-\delta) \cdot k_{jt}\right] - R_{t+1} \cdot f_{jt} + S_{jt} + V_{jt+1}\right\}}{V_{jt} + Z_{jt} - f_{jt}} \quad \text{if } j \notin J^N_t, \tag{23}$$

$$E_t R_{t+1} \leq \max_{\langle Z_{jt}, f_{jt}\rangle} \frac{E_t\left\{\alpha \cdot k^{\alpha-1}_{t+1} \cdot A_{jt} \cdot Z_{jt} - R_{t+1} \cdot f_{jt} + S_{jt} + V_{jt+1}\right\}}{Z_{jt} - f_{jt}} \quad \text{if } j \in J^N_t. \tag{24}$$

Equations (23) and (24) are natural generalizations of Equations (6) and (7). Equation (23) says that maximization by entrepreneurs requires that the expected return to owning an old firm and holding government debt must equal the interest rate. Equation (24) says that maximization by entrepreneurs implies that starting new firms must yield a return that is at least as high as the interest rate.

We conjecture that firm prices and the interest rate on private credit are still given by Equations

(13) and (8), respectively. In addition, we conjecture that the expected return on government debt is given by:

$$E_t R^d_{t+1} = \alpha \cdot k^{\alpha-1}_{t+1} + 1 - \delta. \tag{25}$$

Equation (25) says that government debt must offer the same expected return as private credit. This is a direct implication of risk neutrality.

At the proposed interest rate and firm prices, entrepreneurs strictly prefer to start new firms than to lend or purchase old firms and, just as before, they ask for as much credit as possible:

$$f_{jt} = \frac{1}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot E_t \left\{ \phi_{t+1} \cdot \left(\pi_t \cdot (w_t - T_{it}) + \frac{b_{jt+1} + S_{jt+1}}{\alpha \cdot k^{\alpha-1}_{t+1} + 1 - \delta} \right) \right\}, \tag{26}$$

where T_{it} are the taxes levied on the entrepreneur that starts and owns firm j. Intermediation decreases with taxes on entrepreneurs and increases with subsidies to new firms.

At the proposed interest rate and firm prices, non-entrepreneurs are indifferent among lending to new firms, buying government debt or purchasing old firms. If they choose the latter, they are also indifferent regarding the amount of investment and external financing of their firms. As a group, the non-entrepreneurs purchase the stock of old firms, give credit to new firms, buy the government debt and use any savings left to produce new capital within their old firms. To verify that markets clear, we must check now that:

$$(1 - \varepsilon) \cdot w_t - \left(T_t - T^E_t \right) - f^N_t - (d_t + S_t - T_t) \geq V_t, \tag{27}$$

where $T^E_t \equiv \int_{i \in I^E_t} T_{it}$. We keep assuming that this condition holds and our conjecture is verified.[26]

Aggregating Equation (1) across firms, we find that:

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1) \cdot \varepsilon}{1 - E_t \phi_{t+1} \cdot \pi_t} \right] \cdot (1 - \alpha) \cdot k^\alpha_t + \frac{\pi_t - 1}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot \left(\frac{E_t \left\{ \phi_{t+1} \cdot \left(b^N_{t+1} + S^N_{t+1} \right) \right\}}{\alpha \cdot k^{\alpha-1}_{t+1} + 1 - \delta} - T^E_t \right) - b_t - b^N_t - d_t - S_t, \tag{28}$$

where $S^N_t \equiv \int_{j \in J^N_{t-1}} S_{jt}$.[27] A comparison of Equations (16) and (28) shows that fiscal policy has two

[26] This requires now that:

$$\frac{1 - E_t \phi_{t+1} \cdot \pi_t - \varepsilon}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot (1 - \alpha) \cdot k^\alpha_t + \frac{1}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot \left(\frac{E_t \left\{ \phi_{t+1} \cdot \left(b^N_{t+1} + S^N_{t+1} \right) \right\}}{\alpha \cdot k^{\alpha-1}_{t+1} + 1 - \delta} - T^E_t \right) - (d_t + S_t) \geq (1 - \delta) \cdot k_t + b_t + b^N_t,$$

where $T^N_t \equiv \int_{j \in J^N_{t-1}} T_{jt}$. Note that taxes on entrepreneurs relax this condition while debt and subsidies tighten it.

[27] Investment spending consists of the savings of the young minus their purchases of old firms and government debt,

effects on capital accumulation. The first one is the conventional crowding-out effect, captured by the last two terms of Equation (28). As the debt grows, it absorbs a larger fraction of the savings of the young generation and this diverts resources away from capital accumulation. But there is also a second effect here that is due to the financial friction and is captured by the second term of Equation (16). Subsidies to new firms foster capital accumulation by relaxing credit constraints, increasing intermediation and the average efficiency of investment. For the opposite reasons, taxes to entrepreneurs reduce capital accumulation.

To complete the description of the dynamics of the economy, we still need Equation (17) describing the evolution of the aggregate bubble and, in addition, we need the following equation describing the evolution of fiscal variables:

$$E_t d_{t+1} = \left(\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta\right) \cdot \left(d_t + S_t - T_t\right). \tag{29}$$

Equation (29) follows from Equations (21) and (25). The equilibrium depends on the fiscal policy adopted by the government. A fiscal policy is a feasible sequence for taxes and subsidies, i.e. T_{it} and S_{jt}, and a return process R_{t+1}^{d} satisfying Equation (25). Once this policy has been specified, any sequence for k_t, d_t, b_t and b_t^N that satisfies Equations (17), (28) and (29) is an equilibrium, provided that Equation (27) holds in all dates and states of nature. We show next how fiscal policy works in some of these equilibria.

3.2 'Undoing' the crisis?

Let us start with a disclaimer: we do not search for the optimal fiscal policy. Instead, we focus on the more modest question of whether the government can use fiscal policy to reverse the situation and bring the economy back to the pre-crisis growth path. This might be a desirable goal for most individuals, but not necessarily for all as some might benefit from the crisis. Moreover, the pre-crisis path might not be the optimal path in any meaningful way. To determine the optimal path, we need to give weights to the welfare of different individuals by choosing a social welfare function. We do not do this here.

The key observation is that the bubble implements a series of intragenerational and inter-

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i.e. $w_t - T_t - V_t - (d_t + S_t - T_t) = (1-\alpha) \cdot k_t^{\alpha} - (1-\delta) \cdot k_t - b_t - b_t^N - d_t - S_t$. Of this total, new firms invest $\frac{1}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot \left(\varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha} + \frac{E_t\left\{\phi_{t+1} \cdot \left(b_{t+1}^N + T_{t+1}^N\right)\right\}}{\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta} - T_t^E \right)$ with efficiency π_t, while the rest is invested by old firms with efficiency one.

generational transfers that the government might be able to replicate with fiscal policy. In fact, Equations (17), (28) and (29) provide a simple blueprint for fiscal policy to undo the crisis. We now develop this blueprint for our example of section 2, assuming initially that government debt R_{t+1}^d is non-contingent so that Equation (29) holds ex-post and not just in expectation:

1. Suppose that the bubble has burst at time T. Set all fiscal variables equal to zero, i.e. $T_{it} = S_{jt} = d_t = 0$ for $t = T, T+1, ...$, $i \in I_t$ and $j \in J_t$, and use Equations (17) and (28) to describe the desired bubbly equilibrium. Let $\hat{b}_t$ and $\hat{b}_t^N$ describe this equilibrium.

2. Then, set the following targets for fiscal variables for $t = T, T+1, ...$: (i) $d_{t+1} = (1-p) \cdot \left(\hat{b}_{t+1} - b_{t+1}\right)$; (ii) $S_{t+1}^N = (1-p) \cdot n \cdot \left(\hat{b}_{t+1} - b_{t+1}\right)$; and (iii) $T_{t+1} = \left(b_t - \hat{b}_t\right) \cdot (1+n) \cdot p$, with $T_t^E = 0$. Finally, set $S_T = 0$ and distribute T_T randomly among the old.[28]

This simple algorithm describes the fiscal policy that replicates the desired bubbly equilibrium. When a bubbly episode ends and the economy reverts to the fundamental state with $\hat{b}_t = \hat{b}_t^N = 0$, this fiscal policy steps in and keeps the economy in the same growth path. The government issues debt each period with a market value of $\frac{d_{t+1}}{R_{t+1}^d} = \frac{E_t \hat{b}_{t+1}}{R_{t+1}^d} = \hat{b}_t \cdot (1+n)$, where the last equality follows from Equation (17). Of the revenue raised from the sale of this debt, the government devotes $d_t = (1-p) \cdot \hat{b}_t$ to repaying bondholders and $S_t^N = (1-p) \cdot \hat{b}_t \cdot n$ to subsidizing productive firms. This scheme is not only feasible but it turns out that $T_{t+1} = -\hat{b}_t \cdot (1+n) \cdot p < 0$ and the government makes profits from running it! Since the economy has not left the pre-crisis growth path, these profits can be used to raise consumption for each generation.

Where are these profits coming from? The proposed fiscal policy ensures that the market value of government debt equals that of the disappeared bubble in each period: hence, non-entrepreneurs devote the same resources to purchase the debt than they would have devoted to purchase the bubble. These resources strictly exceed those that are needed to pay maturing debt and to finance the subsidies that prevent a fall in the net worth of firms. This is because, unlike the bubble, the government implements these transfers without risk. Hence, the actual interest payments that the government must make to bondholders at each point in time are below the realized return to the bubble during the episode. Likewise, the subsidy that the government must make to productive firms in order to prevent a fall in their net worth is lower than the one implemented through bubble

[28] This implies that Equation (17) is valid throughout. Assume instead that the government used these resources to bail out bubble owners. Then, we should modify Equation (17) by setting $p = 0$ as bubble owners would not suffer any loss after the bubble collapse. This shows that bubble dynamics depend on expected bailout policies.

creation. Government debt is a Ponzi scheme and, as a result, it extracts a transfer from the future just like the bubble. Since government debt never bursts, it does so more efficiently.

Does this mean that undoing the crisis is too modest a goal for fiscal policy? Should government debt permanently substitute bubbles as a way to help the credit market to work better? A long history of sovereign debt crises around the world and the recent events in European sovereign debt markets strongly suggest a negative answer to this question. To see this, we generalize slightly the example and recognize that shocks to investor sentiment also affect the sovereign debt market. In particular, there are two states for this variable. With probability $1-q$, investors expect government debt will be rolled over with a high probability and are willing to purchase it. With probability q, investors expect government debt not to be rolled over and do not purchase it. We have not formally modeled the objectives of the government, and we will abstain from doing so. Instead, we simply assume that the government defaults on its debt if there is a rollover crisis. This makes rollover crises possible. When such a crisis occurs, the government debt vanishes just like bubbles burst at the end of a bubbly episode.

With rollover crises, the blueprint above is still valid provided we slightly generalize the second step as follows:

2'. Then, set the following targets for fiscal variables for $t = T, T+1, ...$: (i) $d_{t+1} = \frac{1-p}{1-q} \cdot \left(\hat{b}_{t+1} - b_{t+1}\right)$; (ii) $S_{t+1}^N = \frac{1-p}{1-q} \cdot n \cdot \left(\hat{b}_{t+1} - b_{t+1}\right)$; and (iii) $T_{t+1} = \left(b_t - \hat{b}_t\right) \cdot (1+n) \cdot \frac{p-q}{1-q}$, with $T_t^E = 0$. Finally, set $S_T = 0$ and distribute T_T randomly among the old.

The blueprint is basically the same as before, but fiscal policy now makes losses if $q > p$. The probability of a rollover crisis results in the need to pay high interest rates and promise large subsidies. Public promises are so risky that they require the government to raise taxes in every period to keep its policy running. In fact, the policy may become altogether unfeasible if q is sufficiently low, since the path of taxation required to sustain it would eventually violate Equation (27). In this case, government credibility is so low that it is impossible for it to replicate the bubble. Whenever possible, though, this policy is capable of undoing the crisis. It might certainly fail and generate a sovereign debt crisis. And even if it works, it is inferior to having a bubble. But the model still suggests that it can restore the pre-crisis growth path and this is better than doing nothing.[29]

[29] Another reason to think that government debt is inferior is that the ability of the government to target subsidies efficiently might be low. That is, the parameter n might be lower for government debt. This seems quite realistic and important, but we do not pursue it here.

This simple model therefore provides a useful perspective on the current situation of the world economy. As in the blueprint above, the policy response to the crisis has been a massive buildup in government debt. It remains to be seen, however, how much of this buildup has been productively used to raise the net worth of efficient firms. Overall, it seems clear though that government debt has not proved to be superior to the bubble. The world economy has not yet recovered its pre-crisis growth path and, after the initial buildup, fiscal policy has not yielded profits but losses. At the time of writing these note, events seem to have taken a turn for the worse. The market has lost confidence on governments and the crisis has moved across markets, from private financial markets to public-debt markets. The current outlook remains as uncertain as ever.

4 International transmission

Up to now we have looked at the effects of bubbles on the world economy as a whole, as if borders did not matter. To some extent, this approach seems quite appropriate. The current crisis has propagated across industrial countries with a speed and strength that suggest borders do not matter much anymore. But this is in itself an interesting observation. It raises the question of how shocks to investor sentiment are transmitted across countries. To tackle this question, we break the world economy into various countries.

4.1 A reinterpretation of the model

We shall think next of the world economy as containing C countries, indexed by $c = 1, ..., C$. These countries are split into two groups: high- and low-productivity. Countries in the high-productivity group have an investment efficiency equal to π_t, while countries in the low-productivity group have an investment efficiency equal to one. Thus, the countries in the high-productivity group contain a fraction ε of the world's population and these are the "entrepreneurs" of the world economy.

We need to make assumptions about the geographical extent of markets. It is natural to assume that labor markets are local so that workers can only be hired to work with capital located within the same country. This does not preclude however that goods trade arbitrage away wage differences across countries. In particular, we modify slightly the basic model by assuming that firms produce output with a Cobb-Douglas technology that uses capital and an intermediate input: $F(m_{jt}, k_{jt}) = m_{jt}^{1-\alpha} \cdot k_{jt}^{\alpha}$; where m_{jt} is the intermediate. To produce one unit of m_{jt}, one unit of labor is required. Free trade ensures that the price of the intermediate input is the same in all countries. Perfect competition ensures that the wage rate equals the price of the intermediate in each country. Thus, wages are equalized across countries even though labor markets are local.[30] Moreover, the equilibrium wage is still given by Equation (4).

Financial markets are global in nature so that individuals and firms can trade goods and assets with individuals and firms in other countries. Thus, there is a single world interest rate and set of firm prices, and it is straightforward to show that these are given by Equations (8) and (13). At these interest rate and firm prices, entrepreneurs in the group of high-productivity countries strictly prefer to start new firms than to lend or purchase old firms and, as a result, they ask for as much credit as possible. Assume country c belongs to the group of high-productivity countries. Then, this country will borrow from the rest of the world the following amount:

$$f_{ct} = \frac{E_t \phi_{t+1}}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot \left(s_c \cdot \pi_t \cdot w_t + \frac{E_t b^N_{ct+1}}{\alpha \cdot k^{\alpha-1}_{t+1} + 1 - \delta} \right) \tag{30}$$

where s_c is the fraction of the world population located in country c. In addition, all the old firms in country c will be sold to foreigners. At the proposed interest rate and firm prices, the group of low-productivity countries are indifferent between purchasing old firms at home or abroad, investing in them, and lending if Equation (15) holds. In this reinterpretation of the model bubbles thus help channel resources from low- to high-productivity countries. To verify the conjectured interest rate and prices, we keep assuming that Equation (11) holds.

We can now describe the dynamics of this economy. Aggregating Equation (1), the law of motion of the aggregate or world capital stock k_t is still given by Equation (16). Equation (17) describing the dynamics of the aggregate bubble still applies. But it is useful to disaggregate these dynamics at the country level:

$$E_t b_{ct+1} = \left(\alpha \cdot k^{\alpha-1}_{t+1} + 1 - \delta \right) \cdot \left(b_{ct} + b^N_{ct} \right), \quad \text{for } c = 1, ..., C, \tag{31}$$

where b_{ct} and b^N_{ct} be the set of old and new bubbles in country c. Note that, in the group of low-productivity countries, $b^N_{ct} = 0$ since no new firms are being created there.[31]

[30] This result is nothing but the factor-price equalization theorem of international trade. The intermediate input is labor-intensive and the final good is capital-intensive. Countries with high capital-labor ratios import the intermediate input and export the final good, while countries with low capital-labor ratios do the opposite.

[31] How can there be old bubbles in the group of low-productivity countries? We do not rule out the possibility that countries transition between groups during a bubbly episode. To keep things simple, we assume that the set of countries transitioning in both directions has the same size so that ε is constant. Not much would change if we allowed the relative size of the groups vary.

We have now reinterpreted our model of the world economy as one with many countries. Any sequence for k_t, b_{ct} and b_{ct}^N for $c = 1, ..., C$ that satisfies Equations (16) and (31) with $b_t = \sum_c b_{ct}$ and $b_t^N = \sum_c b_{ct}^N$ is an equilibrium, provided that Equation (11) holds in all dates and states of nature. We examine some of these equilibria next.

4.2 International transmission

Now a bubbly episode starts a whole system of country bubbles. We can study such an episode by using a variation of the example developed in section 2. In particular, we assume that during a bubbly episode: (i) bubble creation evolves according to $b_{ct}^N = n \cdot [\omega_c \cdot b_t + (1 - \omega) \cdot b_{ct}]$ in the set of high-productivity countries, where $\omega \in (0, 1)$ and ω_c is a nonnegative constant that must add up to ω among all productive countries; (ii) in any given period, there is a fixed probability p_n that bubble creation stops and n becomes zero thereby ending the bubbly episode, and (iii) in any given period, and for each high-productivity country c, there is a fixed probability p_s that the existing stock of bubble bursts, i.e. that $b_{ct} = 0$. Our process of bubble creation therefore implies that creation in each country depends on both local and global conditions, whereas total bubble creation remains a fixed fraction n of the world bubble. We can think of our example of section 2 as the particular case in which $p_s = 0$.[32]

In this generalization of our example, the world equilibrium is still formally described by Equations (18) to (20), with the only difference that $(1-p)$ now stands for the product $(1-p_n) \cdot (1-p_s)$. Any country bubble is now subject to two types of uncertainty. There is, as in section 2, the risk that investors become pessimistic regarding the worldwide bubble creation as captured by p_n: these expectations immediately end the bubbly episode. Beyond that, there is also the risk that investors become pessimistic regarding the value of existing firms in a particular country as captured by p_s: these expectations cause that country's bubble to burst even if the episode in itself continues. Formally, these country-specific changes in sentiment have no effects on the aggregate properties of the bubbly episode but they do affect the distribution of real and financial activity throughout the high-productivity world.

To see this, consider that there is a negative shock to investor sentiment that bursts the bubble in country c. On impact, this shock reduces the size of the world bubble. But we have already discussed how, in this type of episode, the current size of the bubble is positively related to expectations

[32] Note that in this example we are implicitly assuming that countries do not transition between the low- and high-productivity groups during the bubbly episode.

regarding bubble creation in the future. By reducing the world bubble, this negative shock is thus immediately transmitted to the value of productive firms and to their net worth all over the world. Investor pessimism regarding country c therefore brings about a global slowdown: financial intermediation contracts worldwide, there is a drop in total capital flows towards the set of developed countries and the efficiency of investment naturally falls as well. This slowdown is somewhat persistent because the initial fall in the capital stock depresses wages all over the world, which lends an additional blow to the net worth of firms and tightens their borrowing constraints even further. All of these effects are particularly acute in country c since it is there that the fall in net worth is most pronounced.

As painful as the collapse of a country bubble may be, its effects on the real and financial aggregates of the world economy are transitory. The reason is that all fundamental features of the bubbly process remain unaffected: worldwide bubble creation remains a fixed fraction of the aggregate bubble so that, as can be seen from Equation (18), a partial collapse of the bubble has no long-run effects on the dynamics of the episode. Such a collapse also has transitory implications for the distribution of real and financial activity. When the bubble bursts in a particular country, our process for b_{tc}^N implies that bubble creation is temporarily redirected away from that country towards the rest of the high-productivity world. Other high-productivity countries thus see their rate of bubble creation rise to partially occupy the space of the disappeared one. As wages and the interest rate return to their pre-crisis levels, however, country c eventually recovers from the collapse of its bubble and so does its share of the world bubble, of worldwide bubble creation and of intermediation and entrepreneurial rents.

Figures 7 and 8 show the response of the economy to a such a country-specific shock to investor sentiment. The example depicts a world divided into two productive countries of equal size, that we denote by Home (H) and Foreign (F), and an unproductive rest of the world. Figure 7 illustrates the response of world aggregates to the bursting of the bubble in H, whereas Figure 8 illustrates the responses of the country variables.[33] We have calibrated the bubbly episode so that it is exactly the same as the one in figure 6. Figure 7 illustrates how the collapse of the bubble in H brings about a worldwide recession. On impact, all financial and real indicators fall, including capital, consumption, the stock market and financial intermediation. From there, the world economy recovers as the bubble returns to its pre-crisis size. Figure 8 decomposes these effects at the country

[33] We assume that the world is in a bubbleless steady state in period $t = 0$. At that time, there is a shock to investor sentiment that starts a bubbly episode in the world. At time T, however, the bubble of H bursts.

level. It shows how the recession affects F even though the fall is naturally more severe in H. The figure also illustrates how the collapse of the bubble in H has transitory effects on the distribution of economic activity between both productive countries.

This example provides an illustration of how changes in investor sentiment regarding a particular country can have global effects.[34] This transmission operates partly through factor markets because the collapse of the bubble in one country depresses wages worldwide, which reduces the net worth of all productive firms. But it also operates through investor expectations, when changes in investor sentiment regarding one part of the world affect investor sentiment elsewhere. This is a new and powerful channel of transmission of shocks that is absent when, as is customary in macroeconomics, one insists on focusing exclusively on the fundamental equilibrium.[35]

Finally, it is worth mentioning that the presence of many countries does not affect the global blueprint for fiscal policy that we described in the previous section. It does however raise some new issues such as the sharing of costs and benefits from such a policy. We have little to say on this matter, since we have not formally modeled government objectives. But we shall mention two results that follow straight from the model. The first one is that government spending should increase in the group of high-productivity countries, but not in the group of low-productivity ones. The reason is that this spending consists of subsidies to new firms that raise their collateral. The second result is that, to the extent that countries have different credibility, government debt should be issued only by the countries with the highest credibility. The reason is that this lowers the cost of fiscal policy and the probability of a rollover crisis. This means a possible decoupling between the countries that spend and the ones that borrow and this is likely to lead to frictions among policymakers. This is, we think, as far as we can take the model in these notes in this direction. Further research on the effects of fiscal policy in a world of bubbles is certainly needed.

[34] Here we have referred to the international transmission of changes in investor sentiment at the country-level, but the same logic could be applied to study the transmission of sector-level shocks within an economy. The working-paper version of these notes contains an example along these lines.

[35] In our example, investor pessimism regarding the value of firms in one country reduces expected bubble creation all over the world, but there are other mechanisms through which changes in investor sentiment at the country or regional level could have global effects. A shock to investor sentiment at the country level could, for example, bring the world's bubbly episode to an end. One way in which this could happen is if bubble creation is sufficiently concentrated in one country. In this case, the end of this creation due to a change in investor expectations might make the world bubble unsustainable.

5 Concluding remarks

These notes have developed a model of the financial accelerator in which bubbly episodes arise in equilibrium. We have used this model to explore a view of the current crisis as a shock to investor sentiment that led to the collapse of a bubble or pyramid scheme in financial markets. According to this view, asset prices today depend on market expectations of future asset prices. When investor sentiment is high, asset prices are high and this raises the net worth of firms, relaxing their credit constraints and improving the allocation of investment. This fosters credit, capital accumulation and consumption. When investor sentiment is low, the opposite occurs: lower asset prices reduce the net worth of firms, tightening their credit constraints and worsening the allocation of investment. This leads to a reduction in credit, capital accumulation and consumption.

As a research strategy, viewing the current crisis as the collapse of a bubble is more appealing than alternatives that rely on fundamental or technological shocks. It provides a simple unified narrative of the main macroeconomic developments of the recent past and the current crisis. Namely, the crisis was caused by the collapse of a bubbly episode that had sustained a steady expansion in net worth, output and consumption since the 1990s. This narrative is consistent with the fact that the expansionary phase was gradual and protracted while the recessionary phase has been sudden and sharp. It does not require us to identify a large and negative fundamental or technological shock to blame for the current state of the world economy. It can also account for the connection (or lack of connection!) between financial and real economic activity, and the speed and strength with which shocks are transmitted across different sectors or countries. Finally, it provides us with a simple blueprint for the design of fiscal policies to 'undo' the crisis, although it also highlights that these policies rely on government commitment for their success. In the absence of such commitment, these policies might simply move the crisis across markets, from private financial markets to public-debt markets.

The analytical framework developed in these notes is useful because it allows us to think through various aspects of the current crisis. Moreover, it can be fruitfully extended in various relevant directions. The first one is to introduce a more realistic description of labor markets. The crisis has led to a significant increase in unemployment throughout the world. Our model, with flexible wages and a fully inelastic labor supply, has nothing to say about the connection between bubbles and unemployment. The second extension is to introduce money and explore the role of monetary policy in counteracting the crisis. As it stands now, the only role for money in our model would be as a store of value, and there would be little formal distinction between monetary and fiscal policy. To follow current practice in monetary economics, we would need to introduce money as a unit of account and allow for nominal rigidities. A third and final extension is to explicitly introduce government objectives and constraints. Political economy issues have played an important role in the unfolding and handling of this crisis. A particularly important observation is that, even though the current crisis has a global nature, fiscal and monetary policies around the world are decided at the country or regional level. It seems crucial to analyze the implications of this mismatch between economic and political borders.

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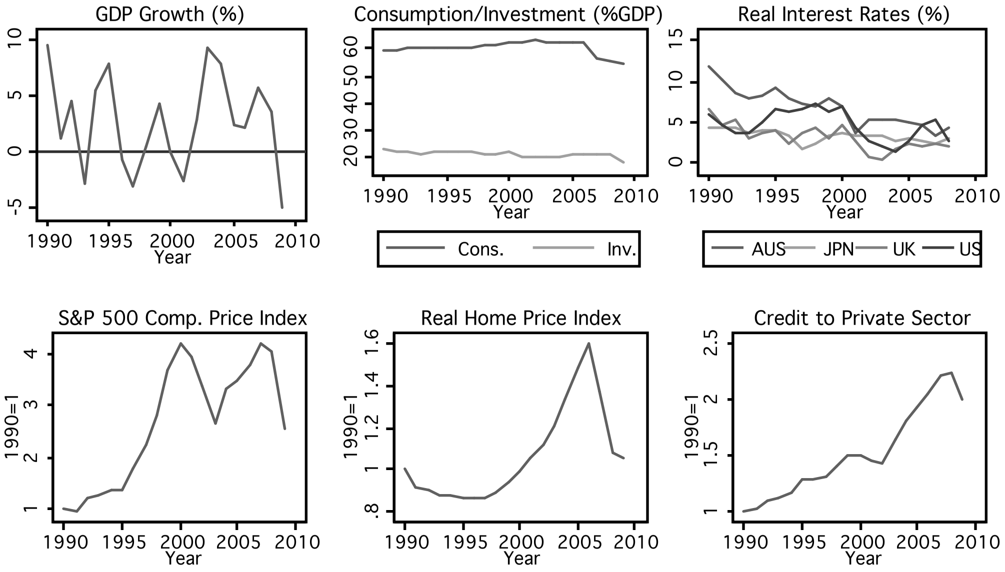
Figure 1: Recent Macroeconomic Trends in Advanced Economies



Notes:
Real variables constitute aggregates of data from Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Spain, Sweden, Switzerland, United Kingdom, United States. Data is taken from the IMF's WEO and the World Bank's WDI. Financial indeces (for the United States only) are based on Shiller (2005).

Table 1: Parameter Values for Figures

Figures	Parameter	Description	Value	Shock
2–3 6–8	α	Capital share	1/3	- -
	δ	Rate of depreciation	0.78	- -
	ε	Measure of entrepreneurs	0.05	- -
	ϕ	Financial friction	0.18	- -
	π	Investment efficiency	4.20	- -
2-3	ϕ	Financial friction	- -	0.02
	π	Investment efficiency	- -	0.3
6	p	Probability of bubbly episode ending	0.2	- -
	n	Rate of bubbly creation	0.7	- -
	$b_0/(1-\alpha)k_0^{\alpha}$	Initial bubble as share of savings	0.2	$b_T=0$
7-8	p_n	Probability of bubbly episode ending	0.11	- -
	p_s	Probability of each country bubble bursting	0.11	- -
	s_c	Measure of entrepreneurs in country $c \in \{H,F\}$	0.025	- -
	ω_c	Bubble creation as a share of world bubble in country $c \in \{H,F\}$	0.425	- -
	$1-\omega$	Bubble creation as a share of own bubble in country $c \in \{H,F\}$	0.15	- -
	$b^c_0/(1-\alpha)k_0^{\alpha}$	Initial bubble as share of savings in country $c \in \{H,F\}$	0.1	$b^H_T=0$

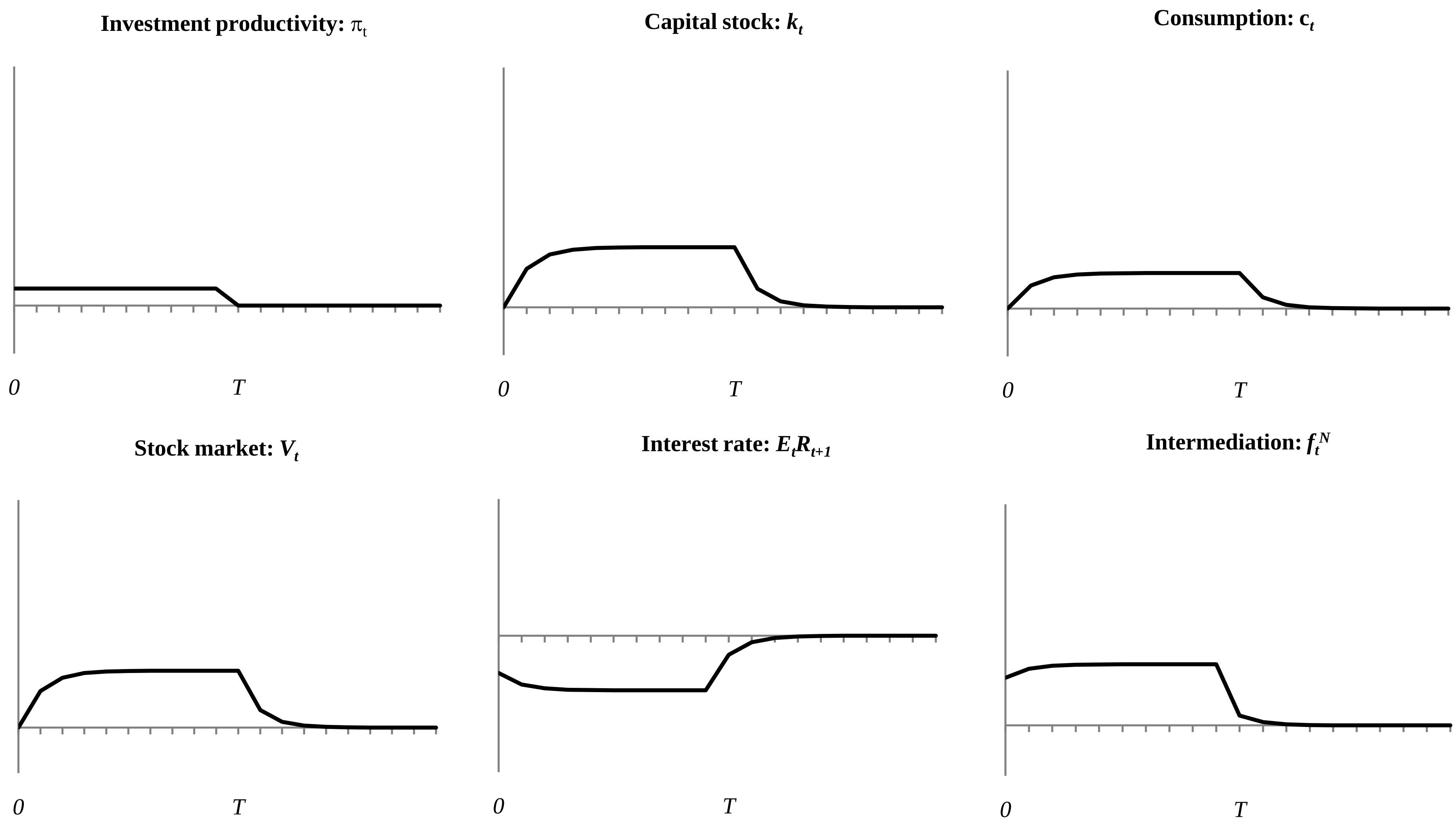


Figure 2: Transitory Shock to Investment Productivity in the Canonical Model

Percentage deviations from the steady-state, $\pi'=4.5$

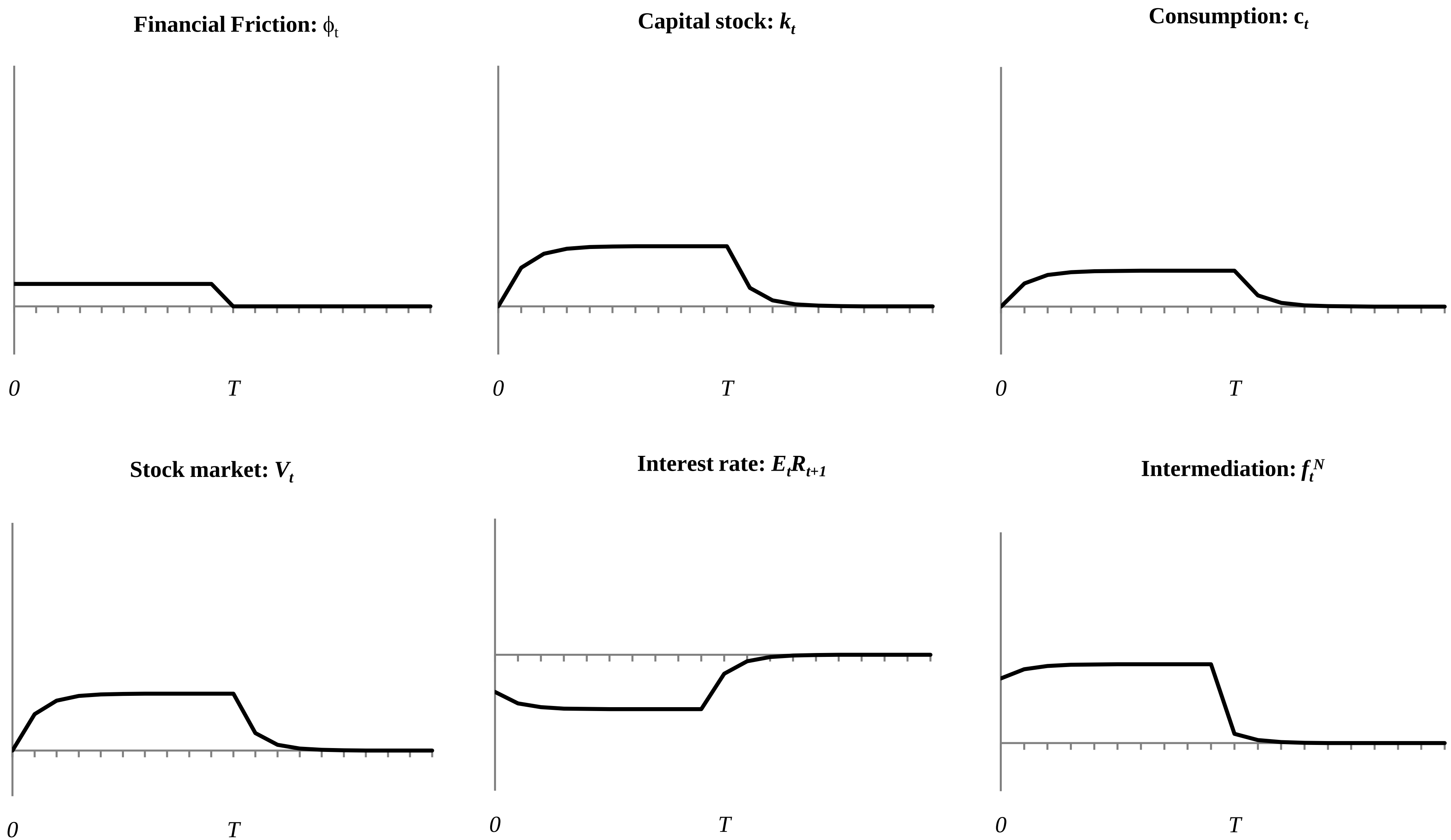


Figure 3: Transitory Shock to the Financial Friction in the Canonical Model

Percentage deviations from the steady-state, $\phi' = 0.2$

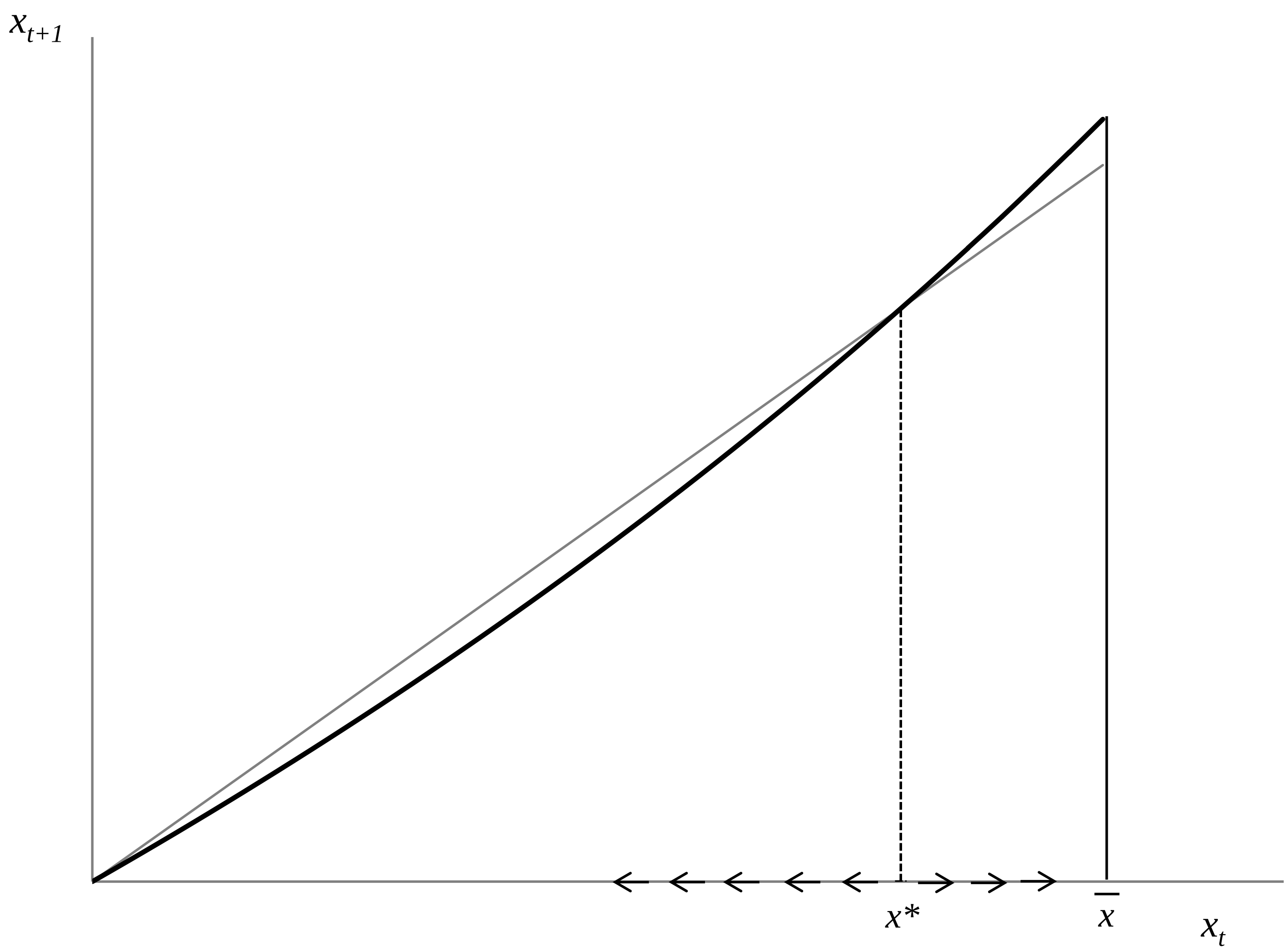


Figure 4: Contractionary Bubble

Initial bubble must be in the interval $0 \leq x_s \leq x^*$, where $\bar{x}$ represents the largest admissable bubble, as given by equation (19), and x* the steady-state as given by equation (18).

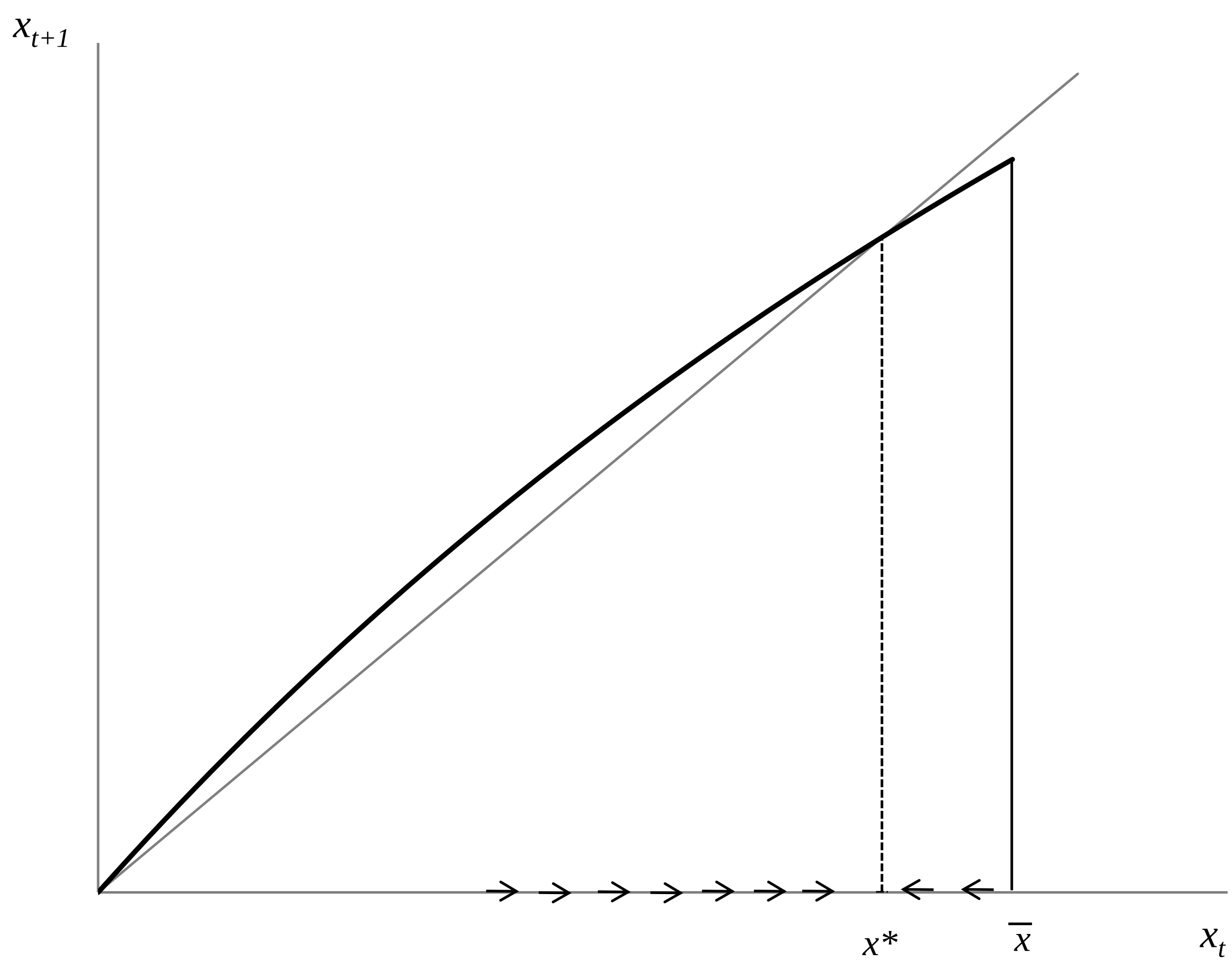


Figure 5: Expansionary Bubble

Initial bubble must be in the interval $0 \leq x_s \leq \overline{x}$, where $\overline{x}$ represents the largest admissable bubble, as given by equation (19), and x^* the steady-state as given by equation (18).

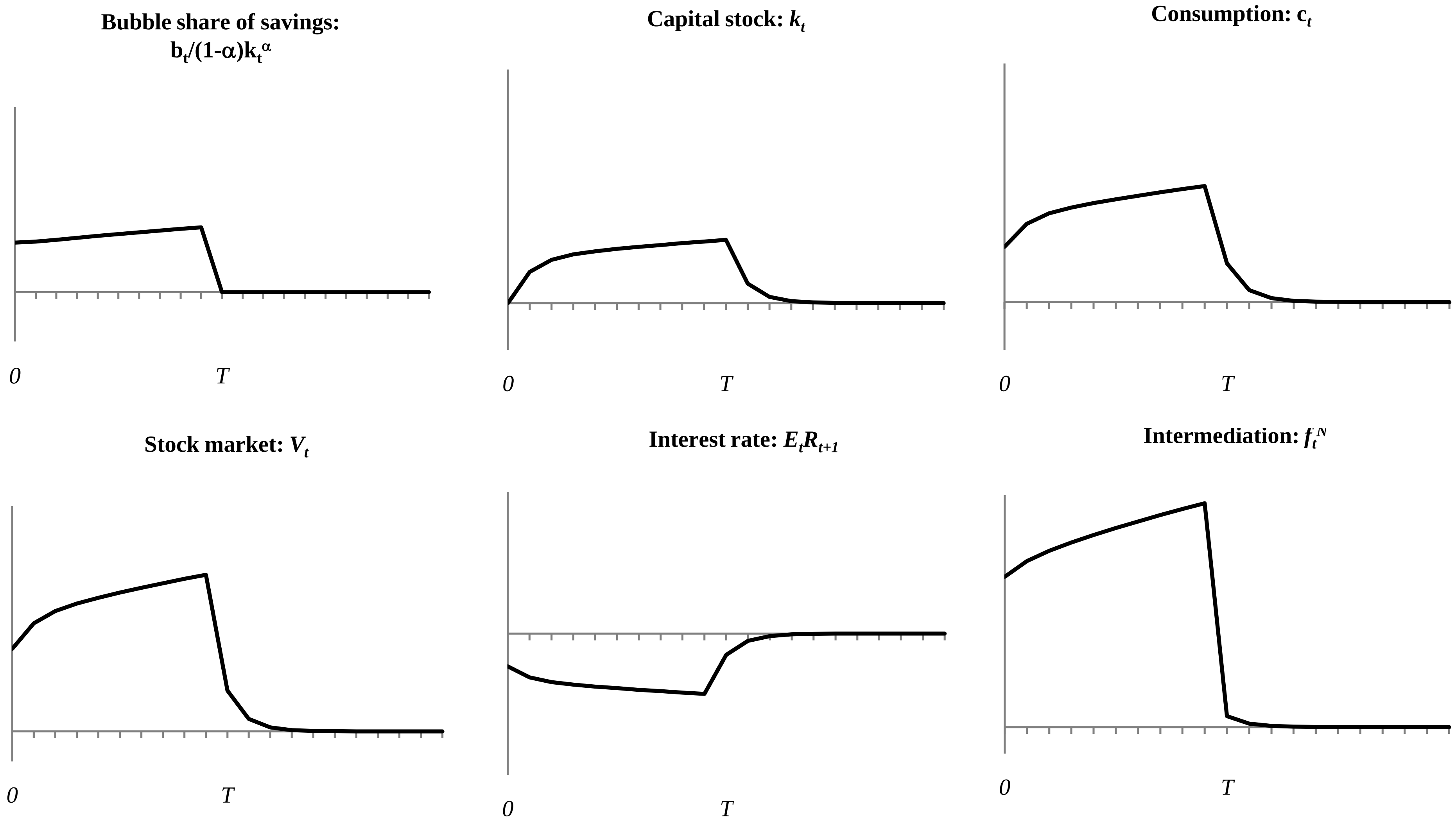


Figure 6: Transitory Expansionary Bubble

Percentage deviations from the steady-state, $p = 0.2$, $n=0.7$, $b_0/(1-\alpha)k_0^{\alpha}=0.2$

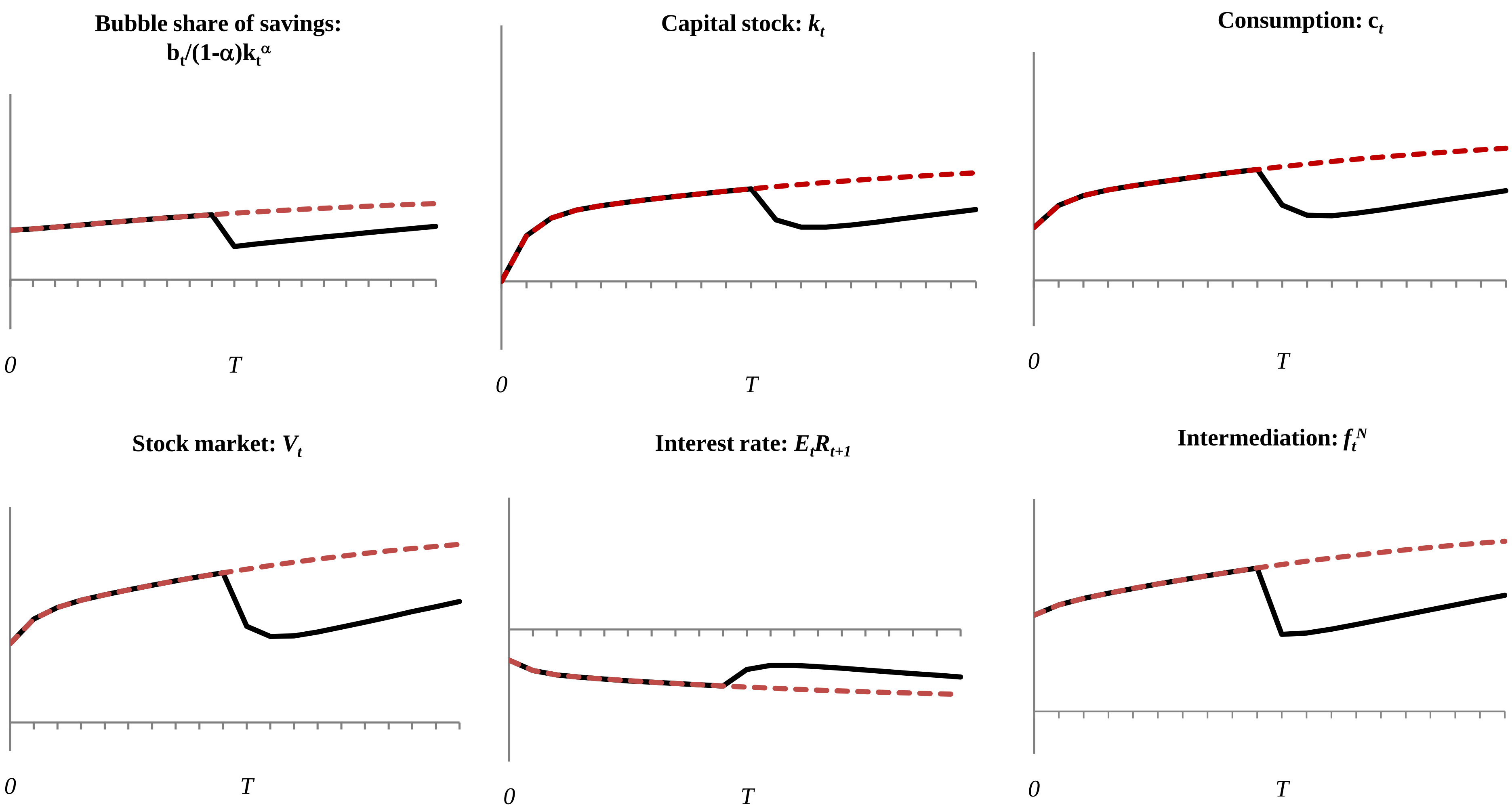


Figure 7: Bursting of Home Bubble: Evolution of World Aggregates

Percentage deviations from the steady-state

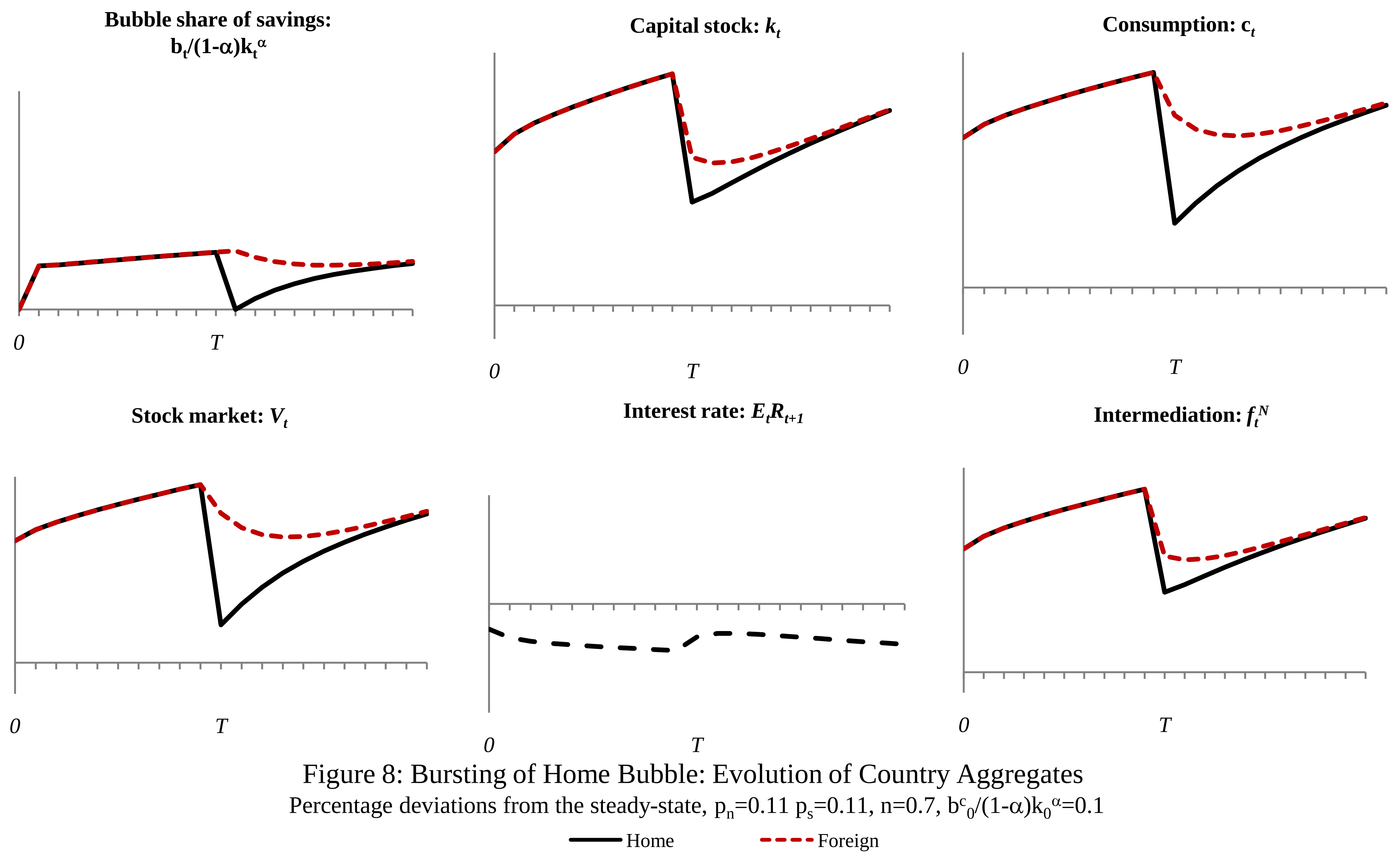


Figure 8: Bursting of Home Bubble: Evolution of Country Aggregates

Percentage deviations from the steady-state, p_n=0.11 p_s=0.11, n=0.7, $b^c{}_0/(1-\alpha)k_0{}^{\alpha}$=0.1