



Stochastic Representative Agent

**Jose Apesteguia
Miguel A. Ballester**

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STOCHASTIC REPRESENTATIVE AGENT*

JOSE APESTEGUIA[†] AND MIGUEL A. BALLESTER[‡]

ABSTRACT. Consider the aggregation of a collection of individual stochastic behaviors that fit a given stochastic choice model. We say that such a model has a representative agent if their aggregate stochastic behaviour also fits the model. We show that the Luce model and several prominent extensions thereof do not have a representative agent. On the positive side, we show that the random utility model and several domain-specific restrictions thereof do have a representative agent.

Keywords: Representative agent; Stochastic choice.

JEL classification numbers: D0; D7; E1.

1. INTRODUCTION

The aggregation of heterogeneous individual behavior is one of the most important research topics in economic theory, and it has major implications for macroeconomics, political economics, and empirical and experimental research. Both classic and recent findings are negative. Only under very restrictive assumptions, do aggregate demand or aggregate preferences inherit the properties of their individual counterparts (see the classic works of Gorman, 1953; Eisenberg, 1961; Sonnenschein, 1973; Mantel, 1974; Debreu, 1974; and recent findings by Jackson and Yariv, 2016). The results of the above-cited studies seriously question the representative agent approach, by showing that aggregate behavior does not necessarily match the class of individual behaviors under consideration, and hence the use of a representative agent approach may create problems for welfare analysis and prediction purposes (see, e.g., Kirman, 1992).

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[†] ICREA, Universitat Pompeu Fabra and Barcelona GSE. E-mail: jose.apestegua@upf.edu.

[‡] University of Oxford. E-mail: miguel.ballester@economics.ox.ac.uk.

In this paper we revisit the study of the representative agent and, in a departure from the standard approach in the literature, take individual behavior to be stochastic. Psychologists and economists alike have long advocated the stochastic view of behavior.¹ This view has recently generated renewed interest because it enables the estimation of relevant preference parameters and provides a stylized way to introduce a number of behavioral considerations.² This paper examines the most influential stochastic choice models to determine whether they have a representative agent. Formally, we say that a model has a representative agent if for every collection of individual stochastic choice functions that fit the model, the corresponding aggregate stochastic behavior also fits it.

In line with the literature on the aggregation of individual deterministic behavior, we present impossibility results. Notably, we show that the Luce model (also known as the logit model) has no representative agent and that there is basically no way to overcome this shortcoming; the slightest individual heterogeneity places aggregate behavior outside the Luce model. These are very negative findings, since the Luce model is, arguably, the most influential stochastic choice model, both in theoretical and empirical terms. The implications are immediately obvious: even when the Luce model is our preferred model of individual behavior, it simply cannot be used to explain aggregate behavior, because it could yield biased estimates, turn out inaccurate predictions, and ultimately lead to misleading welfare implications.

We then show that these negative findings extend to several prominent generalizations of the Luce model, specifically, those with an additive i.i.d. error on the utility evaluations (see Train (2009) for an introduction); additive perturbed utility models (Fudenberg, Iijima, and Strzalecki, 2015); and the elimination by aspects model of Tversky (1972), all of which have no representative agent.

In addition, we provide a technical result which is useful in determining whether a model has a representative agent. We illustrate the use of the technique in an exercise to show that also in the random consideration stochastic choice model of Manzini and

¹Classic references in psychology and economics are Thurstone (1927), Luce (1959), Tversky (1972), Block and Marshak (1960), and McFadden and Richter (1990).

²See Agranov and Ortoleva (2016) for a recent experimental study showing that individual behavior is intrinsically stochastic. For recent theoretical papers see Gul and Pesendorfer (2006), Fudenberg and Strzalecki (2013), Gul, Natenzon, and Pesendorfer (2014), Manzini and Mariotti (2014), Fudenberg, Iijima and Strzalecki (2015) and Caplin and Dean (2016).

Mariotti (2014), which is not an extension of the Luce model, there is no representative agent.

Importantly, and in sharp contrast with the literature on the representative agent, we are also able to present some positive findings. We show that there is a representative agent in the influential random utility model of Block and Marshak (1960) and also in important subclasses of the random utility model, such as the random expected utility model of Gul and Pesendorfer (2006), the random inter-temporal choice model of Lu and Saito (2016), or the single-peaked random utility model of Apesteguia, Ballester and Lu (2016). We therefore offer a general model, the random utility model, which is widely applicable in the theory of individual decision-making and in microeconomics, and also a series of domain-specific versions of the model, all of which can be used for aggregation purposes. These are encouraging results, especially in light of the long history of negative or very restrictive results reported in the literature for the aggregation of individual deterministic behavior.

We close the introduction by relating our work to that part of the aggregation literature that has taken some form of stochastic approach. We start with the classic works of McFadden (1981) and Anderson, de Palma and Thisse (1988), which have strongly influenced the industrial organization literature. McFadden (1981) takes the data generated by a random utility model and shows that, under certain assumptions, there is a deterministic utility function, defined over fractions of consumption, which yields the same data. Relatedly, Anderson, de Palma and Thisse (1988) assume a logit model of a given price-based product evaluation, and find a deterministic utility function that is consistent with it. In both cases, the stochastic data are derived from aggregating a group of deterministic individuals, and the deterministic utility function is that of the representative agent. Note that this approach is very different from ours, where we take a group of stochastic individuals and study whether their aggregate stochastic behavior is of the same nature as their individual stochastic behaviors.

As far as we are aware, the only work that considers the problem of aggregating stochastic individuals is Golman (2011, 2012). In a game-theoretic setting, he uses a model of quantal response equilibrium with homogeneous utility evaluations and heterogeneous noise among agents, and basically shows that the noise structure of the aggregate behavior may be different from that of the individual behaviors. Hence, using our terminology, Golman shows that the i.i.d. additive random utility model where all individuals share the same utility evaluation, but potentially different noise

distributions, is not a representative-agent model. His result is related to our Corollary 1, where we show that the i.i.d. additive random utility model, with potential heterogeneity of utility evaluations and noise distributions across individuals, has no representative agent. Overall, we depart from Golman’s work by providing a systematic study of the aggregation of stochastic individual decision-making, which entails the analysis of all the relevant stochastic models in the literature. This allows us to demarcate models without a representative agent and, importantly, it enables us to contribute some positive findings concerning long-standing stochastic choice models.

2. STOCHASTIC CHOICE FUNCTIONS AND THE REPRESENTATIVE AGENT

Let X be a finite set of alternatives. A stochastic choice function is a mapping $\rho : X \times 2^X \setminus \emptyset \rightarrow [0, 1]$ such that the following properties hold: (i) $\rho(x, A) > 0$ implies that $x \in A$ and (ii) $\sum_{x \in A} \rho(x, A) = 1$. We interpret $\rho(x, A)$ as the probability of choosing alternative x from menu A . We denote by **SCF** the space of all stochastic choice functions.

A subset of **SCF** is called a model. We say that a model has a *representative agent* if, for every finite collection of stochastic choice functions $\{\rho_i\}_{i=1}^I$ conforming to the model, and every $\{\lambda_i\}_{i=1}^I$ with $\lambda_i > 0$ and $\sum_{i=1}^I \lambda_i = 1$, the *aggregate behavior* $\bar{\rho} = \sum_{i=1}^I \lambda_i \rho_i$ also conforms to the model. In other words, a model has a representative agent if it is a convex subset of **SCF**. Otherwise, we say that the model has no representative agent.

Notice that the unrestricted model **SCF** does have a representative agent. We now investigate to determine whether the most relevant models have or do not have a representative agent.

3. THE MODEL OF LUCE AND THREE EXTENSIONS

The most influential stochastic choice model is the Luce (1959) model, which has provided the basis for a substantial theoretical literature, and has proved instrumental in the micro-econometrics of discrete choice analysis. Luce choice probabilities are formally written as $\rho(x, A) = \frac{U(x)}{\sum_{y \in A} U(y)}$ where $U : X \rightarrow \mathbb{R}_{++}$ represents a utility evaluation of the alternatives. That is, alternatives from the relevant menu A are selected with probabilities proportional to their importance. Denoting by **Luce** the set of all Luce stochastic choice functions, the following example illustrates how **Luce** has no representative agent.

Example 1. Let $X = \{x, y, z\}$. Consider the following two Luce utility values: $U_1(x) = 2$, $U_1(y) = 1$, and $U_1(z) = 4$, and $U_2(x) = 1$, $U_2(y) = 2$, and $U_2(z) = 1$. Table 2 summarizes the corresponding Luce stochastic choice functions ρ_1 and ρ_2 , together with the aggregate behavior $\bar{\rho}$ implied by weights $\lambda_1 = \lambda_2 = \frac{1}{2}$.

TABLE 1. Two Luce individuals and their aggregate behavior

| | ρ_1 | ρ_2 | $\bar{\rho}$ |
|--------------------|---------------|---------------|-----------------|
| $(x, \{x, y\})$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ |
| $(x, \{x, z\})$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{5}{12}$ |
| $(y, \{y, z\})$ | $\frac{1}{5}$ | $\frac{2}{3}$ | $\frac{13}{30}$ |
| $(x, \{x, y, z\})$ | $\frac{2}{7}$ | $\frac{1}{4}$ | $\frac{15}{56}$ |
| $(y, \{x, y, z\})$ | $\frac{1}{7}$ | $\frac{1}{2}$ | $\frac{9}{28}$ |

Now suppose, by contradiction, that $\bar{\rho} \in \text{Luce}$, for some Luce values \bar{U} . Notice that $\bar{\rho}(x, \{x, y\}) = \frac{1}{2} = \bar{\rho}(y, \{x, y\})$ implies that $\bar{U}(x) = \bar{U}(y)$. However, $\bar{\rho}(x, \{x, y, z\}) = \frac{15}{56} < \frac{9}{28} = \bar{\rho}(y, \{x, y, z\})$ implies that $\bar{U}(x) < \bar{U}(y)$, which is a contradiction.

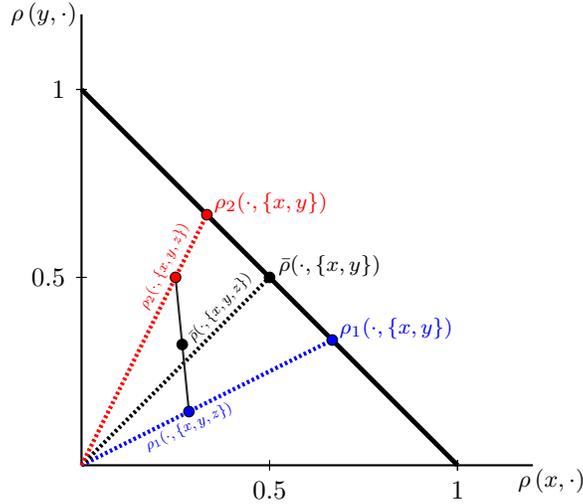


FIGURE 1. Geometric Representation of Luce's Example

Figure 1 depicts the geometry of the problem. The Luce choice probabilities in menu $\{x, y\}$ lie on the line segment connecting points $(1, 0)$ and $(0, 1)$. Luce implies that the choice ratios between x and y in $\{x, y, z\}$ are the same as those from $\{x, y\}$, which is represented by the dashed lines connecting the individual choice probabilities in $\{x, y, z\}$ and $\{x, y\}$ with the origin. However, given the asymmetry in the individual evaluations of z , the aggregate behavior of $\{x, y\}$ does not lie on the same ray as the

aggregate behavior of $\{x, y, z\}$, and hence the aggregate behavior does not fit the Luce model. \blacklozenge

The logic in Figure 1 suggests that the non-representative agent result can be formulated in much stronger terms. Notice in the figure that even the slightest asymmetry between individuals in terms of their evaluation of z means that aggregate behavior no longer falls on its corresponding ray. We use this argument in Proposition 1 to show formally that, for a subset of Luce to have a representative agent, it must contain no heterogeneity whatsoever. In other words, we cannot sensibly restrict the Luce model and obtain positive results, since, whenever we allow for two, or more, different Luce stochastic choice behaviors, there is no representative agent. This shows that the non-existence of a representative agent pervades the Luce model as a whole.

Proposition 1. *Let $|X| \geq 3$. If $L \subseteq \text{Luce}$ has a representative agent, then $|L| = 1$.*

Proof of Proposition 1: Let ρ_1 and ρ_2 in L . Since $L \subseteq \text{Luce}$, there exist Luce values U_1 and U_2 that generate ρ_1 and ρ_2 and, without loss of generality, we can assume that $\sum_{x \in X} U_1(x) = \sum_{x \in X} U_2(x) = K$. Consider the aggregate behavior $\bar{\rho} = \frac{1}{2}\rho_1 + \frac{1}{2}\rho_2$. If L has a representative agent, then $\bar{\rho} \in L \subseteq \text{Luce}$. Hence, for every triple of distinct alternatives x, y, z , it must be that

$$\frac{\bar{\rho}(x, X)}{\bar{\rho}(y, X)} = \frac{\bar{\rho}(x, X \setminus \{z\})}{\bar{\rho}(y, X \setminus \{z\})}$$

which is equivalent to

$$\frac{\frac{1}{2} \frac{U_1(x)}{K} + \frac{1}{2} \frac{U_2(x)}{K}}{\frac{1}{2} \frac{U_1(y)}{K} + \frac{1}{2} \frac{U_2(y)}{K}} = \frac{\frac{1}{2} \frac{U_1(x)}{K-U_1(z)} + \frac{1}{2} \frac{U_2(x)}{K-U_2(z)}}{\frac{1}{2} \frac{U_1(y)}{K-U_1(z)} + \frac{1}{2} \frac{U_2(y)}{K-U_2(z)}}.$$

This ultimately implies that $[U_1(z) - U_2(z)][U_1(y)U_2(x) - U_1(x)U_2(y)] = 0$, i.e., either $U_1(z) - U_2(z) = 0$ or $U_1(y)U_2(x) - U_1(x)U_2(y) = 0$.

Suppose that $U_1 \neq U_2$. Then, there must exist alternatives α and β in X such that $U_1(\alpha) > U_2(\alpha)$ and $U_1(\beta) < U_2(\beta)$. Trivially, $U_1(\beta)U_2(\alpha) - U_1(\alpha)U_2(\beta) \neq 0$ and, given the conclusion in the previous paragraph, it must be that $U_1(\gamma) = U_2(\gamma)$ for every $\gamma \in X \setminus \{\alpha, \beta\}$. But then, $[U_1(\alpha) - U_2(\alpha)][U_1(\beta)U_2(\gamma) - U_1(\gamma)U_2(\beta)] \neq 0$, which is a contradiction. Thus, we have proved that $U_1 = U_2$ and hence, $|L| = 1$. \blacksquare

We now show that the aggregate behavior in Example 1 is not only outside Luce, but also outside three important extensions of the Luce model. This indicates that these three models also fail to have a representative agent. Considering the simplicity of the

example, these results reveal the broad magnitude of the problem. Later, in Section 5, we will return to these, and other, models to formally delineate the commonalities underlying the non-existence results.

3.1. iid Errors. This subsection analyzes the most general version of this model, where $\rho(x, A) = \Pr\{U(x) + \gamma(x) > U(y) + \gamma(y) \text{ for all } y \in A \setminus x\}$ for a utility function U over X and for i.i.d. realizations from a continuous and strictly increasing distribution Γ over the reals.³ Denote by **iid** all the possible stochastic choice functions that are generated in this way. It is well-known that this class of models includes the Luce model, as emerges from the assumption that Γ is extreme type-I.

We now replicate the logic of Example 1 within the larger class of **iid**. Specifically, given that $\bar{\rho}(x, \{x, y\}) = \frac{1}{2} = \bar{\rho}(y, \{x, y\})$, the i.i.d. nature of the error implies that $\bar{U}(x) = \bar{U}(y)$, and hence it should also be that $\bar{\rho}(x, \{x, y, z\}) = \bar{\rho}(y, \{x, y, z\})$, which is not the case. This proves the following corollary.

Corollary 1. *iid has no representative agent.*

3.2. Additive perturbed utility. The additive perturbed utility model (APUM) contemplates a decision-maker maximizing expected utility and a convex perturbation function that can be interpreted as a desire for randomization, or the cost of sticking to a plan, etc (see Mattsson and Weibull, 2002; Fudenberg, Iijima, and Strzalecki, 2014). Adopting the formal specification of Fudenberg, Iijima, and Strzalecki (2014), in an APUM there is a utility function U over X representing the value of each alternative, and a cost function $c : [0, 1] \rightarrow \mathbb{R} \cup \{\infty\}$, strictly convex, C^1 on $(0, 1)$ and with $\lim_{q \rightarrow 0} c'(q) = -\infty$, representing the cost of choosing any alternative with a given probability. Denote by $\rho(A) = \{\rho(x, A)\}_{x \in A}$ the choice probabilities in A . Then, the APUM choice probabilities are $\rho(A) = \arg \max_p \sum_{x \in A} [U(x)p(x) - c(p(x))]$. Denote by **APUM** the set of all APUM stochastic choice functions. It is a well-known fact that, under a particular cost function, **APUM** includes **Luce** (see Fudenberg, Iijima, and Strzalecki, 2014).

Example 1 again serves to show that **APUM** has no representative agent. The argument is analogous to the previous one. Notice that, whenever $\bar{\rho}(x, \{x, y\}) = \frac{1}{2} = \bar{\rho}(y, \{x, y\})$, the symmetry and continuity of the cost function c again imply that $\bar{U}(x) = \bar{U}(y)$, and hence it should follow that $\bar{\rho}(x, \{x, y, z\}) = \bar{\rho}(y, \{x, y, z\})$, which is not the case. The following corollary is therefore proved.

³See Train (2009) for a textbook introduction of this model.

Corollary 2. *APUM has no representative agent.*

3.3. Elimination by Aspects. Tversky (1972) introduces the following attribute-based stochastic choice model, which he calls elimination by aspects (EBA). Alternatives in X are evaluated by their attributes or aspects. Faced with menu A , the individual randomly selects one aspect and eliminates all the alternatives in A that do not possess it. The individual continues in this way until only one alternative remains, and this is her choice. Aspects are selected à la Luce, that is, with probabilities proportional to their importance, and this determines the choice probabilities. Formally, let $W : 2^X \rightarrow \mathbb{R}_{++}$ represent the evaluation of aspects. Then, $\rho(x, A) = \frac{\sum_{B: B \supseteq A} W(B) \rho(x, A \cap B)}{\sum_{B: B \cap A \neq \{A, \emptyset\}} W(B)}$. Denote by **EBA** the set of all EBA stochastic choice functions.

An W consistent with the $\bar{\rho}$ of Example 1 should satisfy the following system of equations:

$$\begin{aligned} \bar{\rho}(x, \{x, y\}) &= \frac{W(\{x\}) + W(\{x, z\})}{W(\{x\}) + W(\{y\}) + W(\{x, z\}) + W(\{y, z\})} = \frac{1}{2} \\ \bar{\rho}(x, \{x, z\}) &= \frac{W(\{x\}) + W(\{x, y\})}{W(\{x\}) + W(\{z\}) + W(\{x, y\}) + W(\{y, z\})} = \frac{5}{12} \\ \bar{\rho}(y, \{y, z\}) &= \frac{W(\{y\}) + W(\{x, y\})}{W(\{y\}) + W(\{z\}) + W(\{x, y\}) + W(\{x, z\})} = \frac{13}{30} \\ \bar{\rho}(x, \{x, y, z\}) &= \frac{W(\{y\}) + W(\{x, y\}) \frac{1}{2} + W(\{x, z\}) \frac{5}{12}}{W(\{x\}) + W(\{y\}) + W(\{z\}) + W(\{x, y\}) + W(\{x, z\}) + W(\{y, z\})} = \frac{15}{56} \\ \bar{\rho}(y, \{x, y, z\}) &= \frac{W(\{y\}) + W(\{x, y\}) \frac{1}{2} + W(\{y, z\}) \frac{13}{30}}{W(\{x\}) + W(\{y\}) + W(\{z\}) + W(\{x, y\}) + W(\{x, z\}) + W(\{y, z\})} = \frac{9}{28} \end{aligned}$$

It is routine to check that this system of equations does not have any solution with strictly positive values, leading us to the following result.

Corollary 3. *EBA has no representative agent.*

4. RANDOM UTILITY MODELS

This section shows the presence of a representative agent in the general class of random utility models (RUMs), and in certain relevant domain restrictions. RUMs have proved very useful, both in theory and applications, and encompass a wide array of important models, such as that of Luce (see, e.g., Block and Marshak, 1960; McFadden, 1981).

In a RUM, the individual entertains preferences randomly. At the moment of choice, a preference is realized, and the maximal alternative for that preference is chosen from the menu of available alternatives. Formally, denote by \mathcal{P} the collection of all linear

orders over X .⁴ Consider the simplex $\Delta(\mathcal{P})$ of all probability distributions on \mathcal{P} . Any $\mu \in \Delta(\mathcal{P})$ defines an associated stochastic choice function by considering $\rho(x, A) = \sum_{P \in \mathcal{P}: x=m_P(A)} \mu(P)$, where $m_P(A)$ denotes the maximal element in A according to P .⁵ We denote by RUM the set of all RUM stochastic choice functions.

We now revisit Example 1. Table 2 provides the individual distributions, μ_1 and μ_2 , that generate the stochastic choice functions ρ_1 and ρ_2 . Notably, the average distribution $\bar{\mu} = \frac{1}{2}\mu_1 + \frac{1}{2}\mu_2$ does indeed generate aggregate behavior $\bar{\rho}$, thus showing that $\bar{\rho}$ is also present in RUM. To see the rationale behind this observation, notice that, for any alternative x and menu A , $\bar{\rho}(x, A) = \frac{1}{2}\rho_1(x, A) + \frac{1}{2}\rho_2(x, A) = \frac{1}{2} \sum_{P: x=m_P(A)} \mu_1(P) + \frac{1}{2} \sum_{P: x=m_P(A)} \mu_2(P) = \sum_{P: x=m_P(A)} (\frac{1}{2}\mu_1(P) + \frac{1}{2}\mu_2(P)) = \sum_{P: x=m_P(A)} \bar{\mu}(P)$.

TABLE 2. RUMs generating the stochastic choice of Example 1

| | xyz | xzy | yxz | yzx | zxy | zyx |
|-------------|------------------|------------------|------------------|------------------|-------------------|------------------|
| μ_1 | $\frac{2}{35}$ | $\frac{8}{35}$ | $\frac{1}{21}$ | $\frac{2}{21}$ | $\frac{8}{21}$ | $\frac{4}{21}$ |
| μ_2 | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{12}$ | $\frac{1}{6}$ |
| $\bar{\mu}$ | $\frac{47}{420}$ | $\frac{83}{420}$ | $\frac{25}{168}$ | $\frac{29}{168}$ | $\frac{117}{504}$ | $\frac{45}{252}$ |

Note: $\omega\eta\theta$ denotes the linear order P with $\omega P \eta P \theta$.

It is immediate that this observation extends to every collection of RUM stochastic choice functions, which shows that RUM is convex. Thus, the next result immediately follows.

Proposition 2. *RUM has a representative agent.*

The convexity of RUM had already been pointed out by Gul, Natenzon and Pendorfer (2014). Notice that the connection between convexity and the presence of a representative agent in RUM is novel.

We now extend the logic leading to the above result to particular RUMs that impose more behavioral structure, and are relevant to understanding specific settings, such as those involving risk preferences or time preferences, etc. The argument rests on the fact that the mapping from $\Delta(\mathcal{P})$ to SCF, which assigns to any probability distribution μ the corresponding RUM stochastic choice function, as suggested above, is a linear mapping. Thus, the next result immediately follows.

⁴A linear order is a transitive, complete and asymmetric binary relation.

⁵That is, $m_P(A)Py$ for every $y \in A \setminus \{m_P(A)\}$.

Corollary 4. *The set of RUMs associated to a convex subset of $\Delta(\mathcal{P})$ has a representative agent.*

Notice that, for every $\mathcal{Q} \subseteq \mathcal{P}$, $\Delta(\mathcal{Q})$ is a convex subset of $\Delta(\mathcal{P})$, and hence Corollary 4 implies that the following families of RUMs have a representative agent.

- (1) RUMs with expected utilities (Gul and Pesendorfer, 2006): Let X be a set of lotteries and consider all the probability distributions with support on the set of preferences which admit an expected utility representation.
- (2) RUMs with discounted utilities (Lu and Saito, 2016): Let X be a set of monetary streams and restrict RUMs to the set of probability distributions with support on preferences which admit a discounted utility representation.
- (3) RUMs with single-peaked (or single-dipped) preferences (Apestegua, Ballester and Lu, 2016): Let \prec be an order over the set of alternatives X and consider the set of probability distributions with support on preferences which are single-peaked or single-dipped with respect to \prec .

To conclude, notice also that the same logic can be applied to parametric restrictions of the models described above. For example, there is also a representative agent in RUMs with CRRA expected utilities and in RUMs with exponential discounted utilities. These results are of high practical relevance since both of which are often used in applied work.

5. THE INSTRUMENTAL VALUE OF RANDOM UTILITY MODELS⁶

Given the definition of RUM and the linearity argument used in Proposition 2, it is immediate to see that RUM can be written as the convex hull of the set of stochastic choice functions rationalized by some linear order, a class we denote by RAT.⁷ Hence, it follows that every model Δ containing RAT and having a representative agent must also contain RUM. In this section we offer a technical result which, by extending this idea, establishes sufficient conditions for a model not to have a representative agent. This result can be used to prove, alternatively, that the models studied in Section 3 have no representative agent and, also, to establish further negative results – in particular,

⁶We owe the main result of this section to Drew Fudenberg, Ryota Iijima and Tomasz Strzalecki, who kindly commented on a previous version of this paper.

⁷We formally write $\rho \in \text{RAT}$ whenever there exists $P \in \mathcal{P}$ such that, for every menu A and alternative x , $x = m_P(A)$ implies that $\rho(x, A) = 1$.

we show that the random consideration set model of Manzini and Mariotti (2014) is another model with no representative agent.

Formally, suppose we consider a model Δ that may fail to contain **RAT**, but find, when taking the closure of the model, $cl(\Delta)$, that, in fact, $\mathbf{RAT} \subseteq cl(\Delta)$. We can extend our previous reasoning to prove that a necessary condition for Δ to have a representative agent is that $\mathbf{RUM} \subseteq cl(\Delta)$. This immediately implies that, if one closed superset of Δ fails to contain **RUM**, the model Δ has no representative agent. The applicability of the result is facilitated by understanding a closed superset of Δ as the class of stochastic choice functions that satisfy a closed property, as shown below.

Proposition 3. *Let Δ be such that $\mathbf{RAT} \subseteq cl(\Delta)$. If there exists a closed model M with $\Delta \subseteq M$ and $\mathbf{RUM} \not\subseteq M$, then Δ has no representative agent.*

Proof of Proposition 3: Proceeding by contradiction, suppose that Δ has a representative agent. Let $\rho \in \mathbf{RUM}$ and $\mathbf{RAT} = \{r_1, r_2, \dots, r_{|\mathcal{P}|}\}$. By the logic in Proposition 2, we know that there are weights $\lambda_1, \dots, \lambda_{|\mathcal{P}|}$ such that $\rho = \sum_{i=1}^{|\mathcal{P}|} \lambda_i r_i$. Since $r_i \in \mathbf{RAT} \subseteq cl(\Delta)$, we can find a sequence $\{\rho_i^j\}$ such that $\{\rho_i^j\} \subseteq \Delta$ and $\lim_j \rho_i^j = r_i$. Hence, $\rho = \sum_{i=1}^{|\mathcal{P}|} \lambda_i r_i = \sum_{i=1}^{|\mathcal{P}|} \lambda_i \lim_j \rho_i^j = \lim_j \sum_{i=1}^{|\mathcal{P}|} \lambda_i \rho_i^j$. Since Δ has a representative agent, $\sum_{i=1}^{|\mathcal{P}|} \lambda_i \rho_i^j = \delta^j \in \Delta$ and hence, $\rho = \lim_j \delta^j \in cl(\Delta) \subseteq M$. We have proved that $\mathbf{RUM} \subseteq M$, which is a contradiction. ■

Proposition 3 can then be used to reproduce the negative results of Section 3. Take M to represent the set of stochastic choice functions satisfying weak stochastic transitivity, which clearly constitute a closed model.⁸ It is well-known that RUMs do not necessarily satisfy weak stochastic transitivity, but the i.i.d. errors models do.⁹ Furthermore, Fudenberg, Iijima, and Strzalecki (2014) show that APUM satisfies weak stochastic transitivity and Tversky (1972) argues that his EBA does also. Then, it is clear that Luce, iid, APUM and EBA are all subsets of M , unlike RUM, which is not. Now, let $r \in \mathbf{RAT}$ be rationalized by preference P , with $x_1 P x_2 P \dots P x_{|X|}$. Clearly, the sequence of Luce utility values defined by $U^j(x_i) = 2^{j(|X|-i)}$ creates a sequence of Luce stochastic choice functions converging to r , and hence $\mathbf{RAT} \subseteq cl(\mathbf{Luce})$. Since Luce is in the intersection of iid, APUM, and EBA, all the conditions of Proposition 3 hold, showing that none of these models has a representative agent.

⁸The stochastic choice function ρ satisfies weak stochastic transitivity if, for every three alternatives x, y and z , whenever $\rho(x, \{x, y\}) \geq \frac{1}{2}$ and $\rho(y, \{y, z\}) \geq \frac{1}{2}$, then $\rho(x, \{x, z\}) \geq \frac{1}{2}$.

⁹See for instance, Block and Marshak (1960)

Proposition 3 allows us to establish further results. We now use it to show that Manzini and Mariotti’s (2014) random consideration set model, which is independent of the Luce model, is another model without a representative agent. Here, the individual contemplates the available alternatives with a given probability, the attention parameter, and then selects the preference-maximal alternative among the considered alternatives. There are some technical details to be discussed. The model encompasses only strictly positive stochastic choice functions and, when no alternative is considered, a default alternative $a^* \notin X$ is always assumed to be contemplated and thus, chosen.¹⁰ Denote by **RCS** the set of all random consideration set choice functions and consider M to be the closure of all the strictly positive stochastic choice functions satisfying i-Independence.¹¹ Manzini and Mariotti (2014) show that i-Independence is one of the characterizing properties of **RCS** and hence, $\text{RCS} \subseteq M$. Also, to see that $\text{RAT} \subseteq \text{cl}(\text{RCS})$, one simply needs to consider a sequence of stochastic choice functions belonging to **RCS**, in which all the attention parameters approach 1. Finally, it can be seen that M does not contain **RUM** by showing a strictly positive **RUM** that fails to satisfy i-Independence. Let $X = \{x, y, z\}$, xP_1yP_1z , zP_2yP_2x , $\mu(P_1) = \mu(P_2) = \frac{1}{2} - 2\epsilon$ and $\mu(P) = \epsilon$ for every other $P \in \mathcal{P} \setminus \{P_1, P_2\}$, with ϵ sufficiently small. Then, $\frac{p(x, \{x, y\})}{p(x, \{x, y, z\})} = 1$, but $\frac{p(x, \{x\})}{p(x, \{x, z\})} = 2$ and the proof is complete.

Corollary 5. *RCS has no representative agent.*

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¹⁰This latter assumption restricts the domain of the stochastic choice functions by imposing that only menus including a^* are considered. Our main definitions extend naturally to this restricted domain.

¹¹A strictly positive stochastic choice function ρ satisfies i-Independence if, for every two menus A and B and alternatives $x, y \in A \cap B$, $\frac{\rho(x, A \setminus \{y\})}{\rho(x, A)} = \frac{\rho(x, B \setminus \{y\})}{\rho(x, B)}$ and $\frac{\rho(a^*, A \setminus \{y\})}{\rho(a^*, A)} = \frac{\rho(a^*, B \setminus \{y\})}{\rho(a^*, B)}$.

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