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Debt Dilution and Debt Overhang*

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Abstract

We introduce risky long-term debt (and a maturity choice) to a dynamic model of firm financing and production. This allows us to study two distortions which are absent from standard models of short-term debt: (1.) *Debt dilution* distorts firms' choice of debt which has an indirect effect on investment; (2.) *Debt overhang* directly distorts investment. In a dynamic model of production, leverage, and debt maturity, we show that the two distortions interact to reduce investment, increase leverage, and increase the default rate. We provide empirical evidence from U.S. firms that is consistent with the model predictions. Debt dilution and debt overhang can overturn standard results: A financial reform which increases investment, employment, output, and welfare in a standard model of short-term debt can have the opposite effect in a model with short-term debt and long-term debt.

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1. Introduction

This paper starts out from a simple observation. Empirically, most firm debt is long-term. About 67% of the average U.S. corporation's total stock of debt does not mature within the next year. This fact is missing from many economic models. The standard assumption is that all firm debt is short-term, i.e. all debt issued in period t fully matures in period $t+1$. In this paper, we ask whether this assumption is innocuous or if standard models miss something important by leaving out long-term debt. To answer this question, we introduce long-term debt (and a maturity choice) to a standard model of firm financing and production.[1] This allows us to study two problems which are absent from standard short-term debt models: debt dilution and debt overhang.

In a model with long-term debt, firms take decisions in the presence of previously issued outstanding debt. If a firm decides to increase its stock of debt, this raises the risk of default and lowers the price at which the firm can sell new debt. Through the credit market, the firm fully internalizes the reduction in the value of newly issued debt. But a higher risk of default also lowers the value of existing debt. This dilution of the value of existing debt is not internalized by the firm (*debt dilution*). The value of existing debt is also affected by the firm's investment decision. If investment increases the value of existing debt, this benefit is not internalized by the firm and reduces the gains which accrue to shareholders (*debt overhang*).

Individually, these two mechanisms are well understood. Debt dilution induces firms to increase leverage and the risk of default (Bizer and DeMarzo, 1992). Through higher credit spreads, debt dilution also raises firms' cost of capital and has a negative effect on investment. Debt overhang directly reduces investment (Myers, 1977). In the first part of our analysis, we derive analytical results that highlight the respective roles of *debt dilution* and *debt overhang* in a simple two-period production economy.

We then proceed to study debt dilution and debt overhang in a fully dynamic general equilibrium model of production, leverage, and debt maturity. This is the key contribution of the paper. In the dynamic model, firms issue both short-term debt and long-term debt. A higher share of long-term debt saves roll-over costs but increases the future amount of outstanding debt. This, in turn, renders future debt dilution and debt overhang more severe. In principle, firms' maturity choice allows them to minimize future debt dilution and debt overhang by using primarily short-term debt. One important result of our dynamic model, however, is that firms do not internalize all costs of long-term debt. In equilibrium, the share of long-term debt is high and the effects of debt dilution and debt overhang are large.

The model produces testable predictions for the cross-section of firms. Using firm-level data from Compustat and Moody's Default & Recovery Database, we construct an empirical proxy for the severity of debt dilution and debt overhang. Reduced-form empirical evidence is in line with our model predictions. Just as in the model, our firm-specific proxy for debt dilution and debt overhang is positively related to firm leverage

[1] In contrast to Gomes, Jermann, and Schmid (2016), we focus on real debt and do not explore the implications of nominal debt. There is a long tradition in corporate finance of modeling long-term debt. In Section 2, we explain how our results relate to that literature.

and default risk, and negatively related to a firm's rate of asset growth. In the model and in the data, these relationships are stronger for firms with a smaller distance to default.

In the last part of the paper, we use our dynamic model to study a number of counterfactual experiments. In a first set of experiments, we isolate the distinct roles of debt dilution and debt overhang. Both distortions are the result of a commitment problem. Because firms cannot credibly promise to act in the interest of long-term creditors in the future, creditors demand high credit spreads on long-term debt. By allowing firms to choose the total stock of debt, capital, or both with full commitment, we selectively eliminate either debt dilution, debt overhang, or both.

We find that debt dilution produces large negative effects on employment, GDP, and welfare, while debt overhang matters mainly for investment. Because of diminishing returns at the firm level, the last unit of capital adds little to firm value. This is why the large effect of debt overhang on investment does not translate into a large impact on firm value. Debt dilution, on the other hand, is less important for investment, but it strongly raises credit spreads. These spreads are inframarginal costs which firms need to pay on each unit of debt. For this reason, debt dilution has a stronger effect on firm value. In general equilibrium, this translates into large negative effects on firm entry, employment, GDP, and welfare.

In the last model experiment, we return to our initial question of whether the standard assumption of short-term debt is innocuous. We study a financial reform which lowers bankruptcy costs. While this reform increases investment, employment, output, and welfare in a standard model of short-term debt, we find that the same reform can have the opposite effect in a model with long-term debt. This is because the reduction in bankruptcy costs leads firms to increase leverage which renders debt dilution and debt overhang more severe.

Because models of firm financing play a key role in various applications (e.g. firm dynamics and misallocation, amplification and propagation of aggregate shocks, transmission of monetary policy), we argue that it is important to take the role of long-term debt into account.

In Section 2, we survey the related literature. Section 3 provides analytical results on debt dilution and debt overhang in a two-period setup. In Section 4, we compute the global solution to the problem of a firm which dynamically chooses production, leverage, and debt maturity. We test the model predictions in Section 5 using firm-level evidence from the U.S. corporate sector. In Section 6, we carry out a number of model-based experiments. Concluding remarks follow.

2. Related Literature

The paper most closely related to ours is Gomes et al. (2016). Their main result is that shocks to inflation change the real burden of outstanding nominal long-term debt and thereby distort investment. The key difference to our paper is that Gomes et al. (2016) focus on cyclical fluctuations while we study the effect of debt dilution and debt overhang on steady state quantities. Their model solution describes linear deviations from a deterministic steady state whereas we calculate a fully non-linear global solution to the dynamic firm problem. Another difference is that they do not allow for short-term debt issuance. This assumption is restrictive because a maturity choice allows firms to respond to and mitigate distortions from debt dilution and debt overhang.[2]

Other models of long-term debt rule out debt dilution and debt overhang a priori, either by assuming that debt is riskless (Alfaro, Bloom, and Lin, 2016) or that firms need to retire all outstanding debt before investing and issuing new debt (Caggese and Perez, 2015). Discrete-time models with one-period debt share this feature by construction (e.g. Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999; Cooley and Quadrini, 2001; Hennessy and Whited, 2005; Covas and Den Haan, 2012; Katagiri, 2014).

In corporate finance, debt dilution has previously been identified as a mechanism which generates excessive leverage and default risk (e.g. Bizer and DeMarzo, 1992; Admati, DeMarzo, Hellwig, and Pfleiderer, 2018).[3] Closely related to our work is the model of debt dilution by DeMarzo and He (2016) which includes an extension with endogenous investment. Brunnermeier and Oehmke (2013) show that debt dilution influences the maturity choice even if a firm's debt level is fixed. In their setup, creditors learn about a firm's default risk over time. In our setup, the "rat race" mechanism is absent because all creditors can exactly predict a firm's current and future default risk.

Debt overhang is a key concept in corporate finance since the seminal contribution by Myers (1977). Subsequent studies of debt overhang include Hennessy (2004), Moyen (2007), Titman and Tsyplakov (2007), Diamond and He (2014), and Occhino and Pescatori (2015). Debt dilution is not a concern in this literature, either because debt is exogenous, chosen with full commitment, or fully retired before the issuance of new debt.

Degryse, Ioannidou, and von Schedvin (2016) provide empirical evidence on debt dilution. Empirical studies of debt overhang are Lang, Ofek, and Stulz (1996), Hennessy (2004), and Hennessy, Levy, and Whited (2007).

The literature on sovereign default has found that debt dilution helps generating realistic levels of sovereign debt and credit spreads. A non-exhaustive list includes Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Hatchondo et al. (2016), and Aguiar, Amador, Hopenhayn, and Werning (2016). Because these are models of endowment economies, there is no effect of debt dilution on investment and there is no debt overhang.

Our model abstracts from financial instruments like debt covenants or secured debt.

[2] Also related are Crouzet (2016) and Poeschl (2017). Their focus lies on firms' debt maturity choice and they do not discuss the respective roles of debt dilution and debt overhang for investment, employment, output, and welfare.

[3] A remark on terminology: In corporate finance, sometimes the term 'debt dilution' is only used for the specific situation that an increased number of creditors needs to share a given liquidation value of a bankrupt firm. We use the term in a more general sense as the same mechanism is at work even if the liquidation value is zero or if existing debt is fully prioritized (as in Bizer and DeMarzo, 1992). In our usage of the term 'debt dilution' we therefore follow the literature on sovereign debt (Hatchondo, Martinez, and Sosa-Padilla, 2016).

The empirical corporate finance literature finds that less than 25% of investment grade corporate bonds include covenants which address debt dilution, and less than 20% feature restrictions with the potential to limit debt overhang.[4] Costs resulting from reduced flexibility might help explain why firms do not use these covenants more intensively in practice.

The theoretical literature finds that secured debt and seniority structures have opposing effects on the two distortions. Stulz and Johnson (1985) and Hackbarth and Mauer (2011) show that debt overhang is more severe if existing debt is prioritized (or secured). Newly issued debt should be prioritized (or secured) to reduce debt overhang. On the other hand, Chatterjee and Eyigungor (2015) find that debt dilution is reduced if existing debt has priority. Granting priority to newly issued debt renders debt dilution more severe. These opposing effects may explain why the empirical use of secured debt and seniority structures by U.S. corporations is limited.[5]

We conclude that a model without debt covenants, secured debt, and seniority structure can approximate the situation of the typical U.S. corporation. In the future, it may be of interest to explicitly study the trade-offs involved when firms choose between these financial instruments.

3. Two-period Model

We begin our analysis by studying a two-period model of a firm which produces output using capital and labor. The firm's stock of capital is financed through equity and debt. The optimal capital structure solves a trade-off between the tax advantage of debt and the expected costs of default. Importantly, the firm decides on its scale of production and the preferred capital structure in the presence of previously issued long-term debt. This variable is exogenous in the two-period setup. It will be endogenized in the fully dynamic economy of Section 4.

The stock of previously issued long-term debt matters for firm behavior through two channels: (1.) *Debt dilution* affects the firm's incentive to borrow because not all costs from additional debt are internalized by the firm. This has an indirect effect on investment. (2.) *Debt overhang* directly affects investment because the firm does not internalize all associated benefits. We use the simple two-period setup to derive analytical results on the effects of debt dilution and debt overhang on investment and output.

3.1. Setup

There are two periods: $t = 1, 2$. Consider a firm owned by risk neutral shareholders. In period 2 the firm uses capital k and labor l to produce output y using a technology with

[4] See Nash, Netter, and Poulsen (2003), Begley and Freedman (2004), Billett, King, and Mauer (2007), and Reisel (2014). We discuss this evidence in more detail in Appendix A.

[5] Secured debt is less than 20% of corporate debt (Compustat), and less than 20% of the number of bond issues (Billett et al., 2007). Subordinate bonds account for less than 5% of corporate debt (Gomes et al., 2016), and less than 25% of the number of bond issues (Billett et al., 2007).

diminishing returns:

$$y = \left(k^{\psi}\, l^{1-\psi}\right)^{\zeta}\,, \quad \text{with} \quad \zeta, \psi \in (0,1) \tag{1}$$

The firm chooses capital and labor in the initial period $t = 1$. Firm earnings in period 2 are uncertain because of an earnings shock ε. Earnings before interest and taxes in $t = 2$ are given as

$$y + \varepsilon k - wl - \delta k\,, \tag{2}$$

where w is the wage rate and δ is depreciation. At t=1, ε is a random variable with probability density $\varphi(\varepsilon)$.

There are two ways to finance capital in period 1: equity and debt.

Definition: Debt. A debt security is a promise to pay one unit of the numéraire good together with a fixed coupon payment c at the end of period 2.

In the two-period model, the firm issues new debt only once. Because we are interested in the question of how existing debt affects the firm's behavior, we introduce an exogenous variable b which denotes the quantity of bonds outstanding at the beginning of period 1. These bonds mature in period 2 just like the one-period bonds which the firm can issue in period 1. One may think of b as long-term debt which has been issued before period 1.

Let p be the market price of a one-period bond sold by the firm in period 1. If the firm sells an amount Δ of new bonds, it raises an amount $p\Delta$ on the bond market. This brings its stock of debt to $b + \Delta = \tilde{b}$.

An alternative to debt financing is equity issuance. Let e denote equity issuance in period 1, that is, the net cash flow from shareholders to the firm. If q is the stock of firm capital in place at the beginning of period 1, then the firm's period 2 stock of capital k is

$$k = q + e + p\Delta = q + e + p\,(\tilde{b} - b)\,. \tag{3}$$

Firm earnings are taxed at rate τ. Debt coupon payments are tax deductible. The firm's stock of equity (and firm assets) after production and repayment of debt in period 2 is

$$\tilde{q} = k - \tilde{b} + (1-\tau)[y + \varepsilon k - wl - \delta k - c\tilde{b}]\,. \tag{4}$$

The fact that coupon payments are tax-deductible lowers the total tax payment by the amount $\tau c\tilde{b}$. This is the benefit of debt. The downside is that the firm cannot commit to repaying its debt after production in period 2.

Definition: Limited Liability. Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. Default is costly. A fixed fraction ξ of firm assets is lost in this case.

The timing can be summarized as follows.

t=1 Given an existing stock of debt b and capital q, the firm chooses capital k and labor l. Investment is financed through equity issuance e and the revenue from the sale of additional bonds $p\,(\tilde{b}-b)$.

t=2 The firm's stock of debt is $\tilde{b}$. Firm earnings are realized. The firm decides whether to default.

3.2. Firm Problem

The firm maximizes shareholder value. Because shareholders are risk neutral, the firm's objective is the expected present value of net cash flows to shareholders.

We can solve the firm's problem using backward induction, beginning with the default decision after the realization of firm earnings in period 2. Limited liability protects shareholders from large negative realizations of the earnings shock ε. Given a firm's stock of capital k and debt $\tilde{b}$, there is a unique threshold realization $\bar{\varepsilon}$ which sets the firm's equity stock after production $\tilde{q}$ equal to zero:

$$\bar{\varepsilon}: \quad \tilde{q} = 0 \quad \Leftrightarrow \quad k - \tilde{b} + (1-\tau)[y + \bar{\varepsilon}k - wl - \delta k - c\tilde{b}] = 0 \tag{5}$$

If ε is smaller than $\bar{\varepsilon}$, full repayment would result in negative equity $\tilde{q}$ while default provides an outside option of zero. In this case, the firm optimally defaults on its liabilities.

In period 1, the firm decides on its scale of production and its preferred financing mix of equity and debt. The firm anticipates that shareholders receive $\tilde{q}$ whenever $\varepsilon \geq \bar{\varepsilon}$ and zero otherwise:

$$\max_{k,l,e,\tilde{b},\bar{\varepsilon}} \quad -e + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty}\Big[k - \tilde{b} + (1-\tau)[y + \bar{\varepsilon}k - wl - \delta k - c\tilde{b}]\Big]\,\varphi(\varepsilon)\,d\varepsilon \tag{6}$$

$$\text{subject to:} \quad y = \left(k^{\psi}\,l^{1-\psi}\right)^{\zeta}$$

$$\bar{\varepsilon}: \quad 0 = k - \tilde{b} + (1-\tau)[y + \bar{\varepsilon}k - wl - \delta k - c\tilde{b}]$$

$$k = q + e + p\,(\tilde{b}-b)\,,$$

where r is the risk-free interest rate. The optimal firm policy crucially depends on the bond price p. A high bond price implies a low credit spread which reduces the firm's cost of capital. We derive the firm-specific bond price from the creditors' optimization problem.

3.3. Creditors' Problem

Creditors are risk neutral and discount the future at the same rate $1/(1+r)$ as shareholders. In period 1 the firm can sell new bonds to them. If the firm does not default in period 2, creditors receive full repayment. In case of default they receive the firm's liquidation value $(1-\xi)\,\tilde{k}$, where

$$\tilde{k} \;\equiv\; k + (1-\tau)[y + \varepsilon k - wl - \delta k]\,. \tag{7}$$

Competitive creditors break even on expectation. The break-even price p of firm debt in period 1 depends on the probability $\Phi(\bar{\varepsilon})$ that the firm defaults in period 2:

$$p = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}}\int_{-\infty}^{\bar{\varepsilon}} \tilde{k}\,\varphi(\varepsilon)\,d\varepsilon\right] \tag{8}$$

The credit spread is defined as the excess return on firm debt (conditional on full repayment) over the riskless rate: $(1+c)/p-(1+r)$. If creditors expect a positive risk of default, they will charge a positive spread.[6]

3.4. Equilibrium

We solve for the partial equilibrium allocation given the wage w and the risk-free rate r. In equilibrium, the firm maximizes shareholder value (6) subject to creditors' break-even condition (8). We can simplify this problem by re-writing it in terms of only two endogenous variables: the scale of production k, and the default threshold $\bar{\varepsilon}$.

3.4.1. Consolidated Problem

Given the wage rate w, a necessary and sufficient condition for optimal labor demand is

$$l^* = \frac{\zeta(1-\psi)y}{w}\,. \tag{9}$$

Using this condition, we can express output net of wage payments as a function of capital only:

$$y^* - wl^* \;=\; A\,k^{\alpha}\,, \tag{10}$$

where A and α are functions of the wage rate w and the technology parameters ζ and ψ. There are diminishing returns to scale: $\alpha \in (0,1)$.[7]

[6]Sometimes more than one bond price satisfies creditors' break-even condition (8). In this case, different default probabilities correspond to different bond prices. Multiplicity may arise if the price is low because default is likely, and if default is likely because the price is low. See Calvo (1988). The conditions which introduce multiplicity are described in Nicolini, Teles, Ayres, and Navarro (2015). In our setup, this kind of multiplicity is absent. By allowing the firm in (6) to directly select the default probability through $\bar{\varepsilon}$, we implicitly assume that the firm sells its bonds to creditors by making a take-it-or-leave-it offer specifying both a price p and a quantity $\tilde{b}$ of bonds. In this way, we allow the firm to select the preferred (i.e. the lower) default probability.

[7]In particular

$$A \;\equiv\; (1-\zeta(1-\psi))\left(\frac{\zeta(1-\psi)}{w}\right)^{\frac{\zeta(1-\psi)}{1-\zeta(1-\psi)}} \quad \text{and} \quad \alpha \equiv \frac{\zeta\psi}{1-\zeta(1-\psi)}\,.$$

We continue by expressing the stock of debt $\tilde{b}$ in terms of k and $\bar{\varepsilon}$. Applying (10) to the definition of $\bar{\varepsilon}$ in (5) yields

$$\tilde{b}\,[1+(1-\tau)c] = k+(1-\tau)[Ak^{\alpha}+\bar{\varepsilon}k-\delta k] \;\Leftrightarrow\; \tilde{b} = \frac{k+(1-\tau)[Ak^{\alpha}+\bar{\varepsilon}k-\delta k]}{1+(1-\tau)c}\,. \tag{11}$$

Consider first the left hand side of the first equation in (11). Creditors are entitled to a fixed payment of $\tilde{b}[1+c]$ in period 2. But effectively the firm only pays $\tilde{b}[1+(1-\tau)c]$ because it can deduct $\tilde{b}\tau c$ from its tax bill. The right hand side of the first equation states that this payment consists of two parts. The safe part of firm assets after production, $k+(1-\tau)[Ak^{\alpha}-\delta k]$, and a fixed promised amount of the risky part of earnings, $(1-\tau)\bar{\varepsilon}k$.

Using (3), we can express equity issuance e in terms of k, $\tilde{b}$, and p:

$$e = k - q - p\,(\tilde{b}-b) \tag{12}$$

Combining these two expressions, the firm's problem can be re-written as

$$\max_{k,\tilde{b},\bar{\varepsilon},p} \quad q \,-\, k \,+\, p\,(\tilde{b}-b) \,+\, \frac{1-\tau}{1+r}\,k\int_{\bar{\varepsilon}}^{\infty}[\varepsilon-\bar{\varepsilon}]\;\varphi(\varepsilon)\,d\varepsilon\,. \tag{13}$$

The firm's objective is to maximize shareholder value. But in (13), the firm maximizes the total return to capital k including the value of newly issued debt $p(\tilde{b}-b)$. Shareholders benefit from a high value of newly issued debt as less equity issuance e is required for a given level of capital k (see equation (12)). Because creditors break even on expectation, shareholders appropriate the entire surplus created by the investment of k.

If labor demand is chosen according to equation (9), we know from creditors' break-even condition (8) that p depends on k, $\bar{\varepsilon}$, and $\tilde{b}$. From (11), $\tilde{b}$ only depends on k and $\bar{\varepsilon}$. It follows that the firm problem (13) characterizes the equilibrium allocation in terms of k and $\bar{\varepsilon}$ only. The firm maximizes (13) subject to the break-even condition (8) and the constraint pinning down the default threshold (11).

3.4.2. Debt Dilution

As shown above, the equilibrium allocation is pinned down by two endogenous variables: The scale of production k, and the default threshold $\bar{\varepsilon}$. Accordingly, an interior solution is characterized by two first order conditions.

For analytical tractability, in this part we consider the special case of $\xi = 1$. This means that the liquidation value of the firm is zero in case of default and the bond price in (8) only depends on $\bar{\varepsilon}$:

$$p = \frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})] \tag{14}$$

The same is true for the credit spread: $(1+c)/p-(1+r) = (1+r)\,\Phi(\bar{\varepsilon})/[1-\Phi(\bar{\varepsilon})]$.

The derivative of (13) with respect to k yields a first order condition for the optimal scale of production:

$$\underbrace{-1}_{\substack{Marginal\\cost\,of\\capital}} + \underbrace{\frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{1+(1-\tau)\left[A\alpha k^{\alpha-1}+\bar{\varepsilon}-\delta\right]}{1+(1-\tau)c}}_{\substack{Marginal\,increase\\in\,value\,of\,newly\,issued\,debt}} + \underbrace{\frac{1-\tau}{1+r}\int_{\bar{\varepsilon}}^{\infty}[\varepsilon-\bar{\varepsilon}]\;\varphi(\varepsilon)\,d\varepsilon}_{\substack{Marginal\,increase\\in\,value\,of\,equity}} = 0 \tag{15}$$

A marginal increase in k has an opportunity cost of one. The benefit consists of an increase in the value of newly issued debt and equity. Because of diminishing returns, the marginal increase in the value of newly issued debt is falling in k. Note that the number of previously issued bonds b does not appear in the first order condition for k. Conditional on the firm's choice of $\bar{\varepsilon}$, the existing stock of debt b does not affect investment. In this sense, there is no debt overhang if $\xi = 1$.[8]

A first order condition for an optimal choice of $\bar{\varepsilon}$ is

$$\underbrace{[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c}}_{Marginal\,tax\,benefit\,of\,\bar{\varepsilon}} \quad - \quad \underbrace{\varphi(\bar{\varepsilon})(1+c)(\tilde{b}-b)}_{\substack{Marginal\,increase\,in\\expected\,costs\,of\,default\\internalized\,by\,the\,firm}} = 0 \tag{16}$$

The first term is the marginal benefit of an increase in $\bar{\varepsilon}$. It is weighted with the repayment probability $[1-\Phi(\bar{\varepsilon})]$. If default is avoided, a higher value of $\bar{\varepsilon}$ increases the fixed amount promised to creditors by $(1+c)\partial\tilde{b}/\partial\bar{\varepsilon}$, and reduces the expected value of equity by $(1-\tau)k$.[9] Since coupon payments are tax deductible, it costs shareholders only $1+(1-\tau)c$ to increase the promised payment to creditors by $1+c$. Because competitive creditors break even, the entire tax benefit generated from substituting equity with debt is captured by shareholders.

The second term in (16) plays a key role in this model. It is the marginal cost of an increase in $\bar{\varepsilon}$. The probability of default increases by $\varphi(\bar{\varepsilon})$ and creditors lose the entire amount of $(1+c)\tilde{b}$ in this case (because $\xi = 1$). The firm only takes into account the default costs for the buyers of newly issued debt $(1+c)\Delta = (1+c)(\tilde{b}-b)$. While the firm fully internalizes the tax benefit of the entire stock of debt $\tilde{b}$, it does not internalize all associated costs. The firm takes into account that an increase in $\bar{\varepsilon}$ lowers the value of newly issued debt $p(\tilde{b}-b)$. But it disregards the fact that this also lowers the value of previously issued debt pb. This *dilution* of the value of previously issued debt through the sale of additional debt is not internalized by the firm.

The optimal value of $\bar{\varepsilon}$ is pinned down by the trade-off between the tax advantage of

[8]The assumption that debt is freely adjustable is important as well. See Section 3.4.3 below.

[9]The marginal tax benefit of $\bar{\varepsilon}$ can be written as

$$[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c} = [1-\Phi(\bar{\varepsilon})]\left[(1+c)\frac{\partial\tilde{b}}{\partial\bar{\varepsilon}} - (1-\tau)k\right].$$

debt and the internalized part of the expected costs of default. Proposition 3.1 describes the effect of debt dilution on the firm's behavior at an interior solution characterized by the two first order conditions (15) and (16).

Proposition 3.1. ***Debt Dilution:*** *Assume $\xi = 1$.*

1. *The default rate $\Phi(\bar{\varepsilon})$ is increasing in the stock of existing debt b.*
2. *For $b < \underline{b}$, capital k is increasing in b. For $b > \underline{b}$, capital is falling in b. The threshold value $\underline{b}$ is*
$$\underline{b} \equiv \frac{(1-\tau)(1-\alpha)Ak^{\alpha}}{1+(1-\tau)c}.$$
3. *If $b > \underline{b}$, leverage $\tilde{b}/k$ is increasing in b.*

A proof can be found in Appendix B. The first part of Proposition 3.1 is an immediate consequence of debt dilution. If $b = 0$, the entire stock of debt $\tilde{b}$ is issued in period 1 and the firm fully internalizes all expected default costs through the break-even price of debt. But with positive b, a part of the expected costs of default is not borne by the firm but by the holders of previously issued debt. This allows the firm to enjoy a given amount of the tax benefits of debt at a lower private cost. As a result, the firm utilizes the tax benefit of debt more intensively by raising $\tilde{b}$ and $\bar{\varepsilon}$. This effect of debt dilution on borrowing and default rates is well understood in corporate finance (Bizer and DeMarzo, 1992).

The increase in $\bar{\varepsilon}$ has an ambiguous effect on investment as described by the second part of Proposition 3.1. A higher value of $\bar{\varepsilon}$ reduces the effective tax rate as a larger part of firm earnings is paid out in the form of tax deductible debt coupons. This encourages investment. The downside is that the bond price p in (14) falls in $\bar{\varepsilon}$ which raises the cost of capital and discourages investment. Once b rises above $\underline{b}$, the second effect dominates. This effect of debt dilution on firm investment is different from debt overhang. The stock of existing debt b does not appear in the first order condition for k. If the firm did not respond to the increase in b by choosing a higher value of $\bar{\varepsilon}$, there would be no effect on investment. It is the endogenous response of borrowing which has implications for investment.

The third part of Proposition 3.1 characterizes the effect on leverage. As can be seen from (11), leverage $\tilde{b}/k$ is increasing in $\bar{\varepsilon}$ and in the average product of capital Ak^{α}/k. If both $\bar{\varepsilon}$ and k increase, the joint effect on leverage is ambiguous because the average product of capital is falling in k. But once $b > \underline{b}$ and k begins to fall in b, this theoretical ambiguity disappears and leverage necessarily increases in b.

3.4.3. Debt Overhang

In the previous subsection, we deliberately ruled out any role for debt overhang by setting $\xi = 1$. Now we do the opposite. We neutralize debt dilution by assuming that the firm's period 2 amount of debt $\tilde{b}$ is exogenous. The firm cannot dilute the value of existing debt by choosing the number of additional bonds. Any remaining effect from the existing stock of debt b on firm investment must be due to debt overhang.

Formally, we study the firm problem (13) subject to the two constraints (8) and (11), and some exogenous value $\tilde{b}$. Because $\tilde{b}$ is fixed, the definition of the default threshold (11) imposes a unique functional relationship between k and $\bar{\varepsilon}$.

$$\frac{d\bar{\varepsilon}}{dk} = -\frac{1 + (1-\tau)[A\alpha k^{\alpha-1} + \bar{\varepsilon} - \delta]}{(1-\tau)k} \tag{17}$$

Constraining the firm's choice of $\tilde{b}$ in this way leaves only one choice variable and only one first order condition. We obtain it from the derivative of the firm's objective (13) with respect to k:

$$\underbrace{-1}_{\substack{Marginal\\ cost\, of\\ capital}} + \underbrace{\frac{dp}{dk}(\tilde{b} - b)}_{\substack{Marginal\, increase\\ in\, value\, of\\ newly\, issued\, debt}} + \underbrace{\frac{1-\tau}{1+r}\left[\int_{\bar{\varepsilon}}^{\infty}[\varepsilon - \bar{\varepsilon}]\varphi(\varepsilon)d\varepsilon - \frac{d\bar{\varepsilon}}{dk}[1 - \Phi(\bar{\varepsilon})]k\right]}_{\substack{Marginal\, increase\\ in\, value\, of\, equity}} = 0 \tag{18}$$

With $\tilde{b}$ fixed, the firm's choice of k simultaneously controls $\bar{\varepsilon}$ and therefore the risk of default $\Phi(\bar{\varepsilon})$. The key difference to the first order condition in the previous section (15) is that now the firm's choice of k affects the bond price p. The firm takes into account that an increase in k affects the value of newly issued bonds $p(\tilde{b} - b)$. But it does not internalize the effect on the value of existing debt pb.

Proposition 3.2 describes the consequences for firm behavior at an interior solution if the period 2 amount of debt $\tilde{b}$ is fixed. These results hold for any value of $\xi \in (0, 1]$.

Proposition 3.2. ***Debt Overhang:*** *Assume that the period 2 amount of debt $\tilde{b}$ is fixed.*

1. *Capital k is falling in b if and only if the bond price p is increasing in k.*

2. *Leverage $\tilde{b}/k$ is increasing in b if and only if k is falling in b.*

3. *The default rate $\Phi(\bar{\varepsilon})$ is falling in k if and only if $1 + (1-\tau)[A\alpha k^{\alpha-1} + \bar{\varepsilon} - \delta] > 0$.*

The proof is deferred to Appendix B. The first part of Proposition 3.2 is an application of the classic debt overhang result from Myers (1977), p. 164-165. Because $\tilde{b}$ is fixed, the marginal unit of capital comes from an increase in equity. The firm internalizes that an increase in capital raises the value of both equity and newly issued debt. But shareholders do not benefit if an increase in capital also raises the value of existing debt. In this case, a part of the benefit from investing constitutes a transfer from shareholders to the holders of existing debt. The size of this transfer is increasing in the stock of existing debt b. The larger this transfer, the lower the firm's incentive to increase capital.[10]

[10]The opposite is true if the bond price p is falling in k. This can be the case if the increase in k raises the variance of earnings and makes default more likely. In this case, investment transfers value from the holders of existing debt to shareholders which increases the firm's incentive to invest.

With $\tilde{b}$ fixed, the effect of b on leverage $\tilde{b}/k$ directly follows from the behavior of capital k. The effect of an increase in k on the default rate $\Phi(\bar{\varepsilon})$ is ambiguous. An increase in k lowers leverage which reduces the risk of default. At the same time, it also raises the variance of earnings. If the latter effect dominates, a higher value of k may imply a higher default rate.

3.4.4. Summary of Analytical Results

Propositions 3.1 and 3.2 describe two different channels through which investment is affected by the stock of existing debt b. In Section 3.4.2, the period 2 amount of debt $\tilde{b}$ is endogenous and we assume $\xi = 1$. In this special case, an increase in capital has no effect on the value of existing debt. Either default is avoided and the value of existing debt is $b(1 + c)$, or default occurs and the value of debt is zero, independently of the amount of capital. As shown in Section 3.4.2, this implies that an increase in the stock of existing debt b has no effect on the firm's capital choice k for a given value of $\bar{\varepsilon}$. In this sense, there is no debt overhang if $\xi = 1$. It is only through the endogenous response of $\bar{\varepsilon}$ and $\tilde{b}$ induced by debt dilution that b affects k.

In Section 3.4.3, the period 2 amount of debt $\tilde{b}$ is exogenous. This is similar to Myers (1977) or other studies of debt overhang. In this special case, debt dilution does not play any role for k because the firm is unable to dilute existing debt by choosing a high value of $\tilde{b}$. For a given value of k, an increase in existing debt b has no effect on $\bar{\varepsilon}$. But k responds to the increase in b because of debt overhang.

In practice, the liquidation value of a firm is usually positive $(0 < \xi < 1)$ and firms may not be able to commit to a fixed value of debt $\tilde{b}$. This implies that both debt dilution and debt overhang simultaneously affect firm behavior. Furthermore, the stock of existing debt b is the key variable which determines the severity of debt dilution and debt overhang in this model. It is important to endogenize firms' dynamic choice of b. For these reasons, we now develop a fully dynamic model of production, leverage, and debt maturity.

4. Dynamic Model

The main additional feature of the dynamic model is that the stock of existing debt is endogenous. Firms can sell short-term bonds and long-term bonds. The amount of long-term debt issued today determines the stock of existing debt next period.

We also introduce an issuance cost for new bonds. Short-term debt needs to be constantly rolled over which implies high issuance costs. Long-term debt allows maintaining a given stock of debt at a lower level of bond issuance. The downside of long-term debt is that it gives rise to debt dilution and debt overhang in the future.

In all other respects, the firm problem is kept as close as possible to the two-period model from above. This means that we abstract from several model elements which are empirically relevant for firm behavior (e.g. persistent firm-level shocks, equity issuance costs). This keeps our analysis of debt dilution and debt overhang as clear and transparent as possible.

The model economy is populated by a stationary distribution of firms and a representative household. All agents take factor prices w and r as given. Since there is no aggregate risk, factor prices are constant over time as in Hopenhayn and Rogerson (1993).

4.1. Firm Setup

There is a continuum of atomistic firms. As in the two-period economy, a firm i uses capital k_{it} and labor l_{it} to produce output y_{it} using a technology with diminishing returns:

$$y_{it} = \left(k_{it}^{\psi}\, l_{it}^{1-\psi}\right)^{\zeta}, \quad \text{with:} \quad \zeta, \psi \in (0,1) \tag{19}$$

Earnings before interest and taxes are given as

$$y_{it} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f \tag{20}$$

The firm-specific idiosyncratic earnings shock ε_{it} is i.i.d. and follows a probability distribution $\varphi(\varepsilon)$. Firms pay a fixed cost f as in Hopenhayn and Rogerson (1993).

In contrast to the two-period economy, the firm can now choose between short-term debt and long-term debt.

Definition: Short-term Debt. A short-term bond issued at the end of period $t-1$ is a promise to pay one unit of the numéraire good together with a fixed coupon payment c in period t. The quantity of these short-term bonds sold by firm i at the end of period $t-1$ is $\tilde{b}_{it}^{S}$.

Definition: Long-term Debt. A long-term bond issued at the end of period $t-1$ is a promise to pay a fixed coupon payment c in period t. In addition, the firm repays a fraction $\gamma \in (0,1)$ of the principal in period t. In period $t+1$, a fraction $1-\gamma$ of the bond remains outstanding. The firm pays a coupon payment $(1-\gamma)c$ and repays the fraction γ of the remaining principal: $(1-\gamma)\gamma$. In this manner, payments decay geometrically over time. The maturity parameter γ controls the speed of decay. The quantity of long-term bonds chosen by the firm at the end of period $t-1$ is $\tilde{b}_{it}^L$.

This computationally tractable specification of long-term debt goes back to Leland (1994). Short-term debt and long-term debt are of equal seniority.

Definition: Issuance cost. The firm pays an amount η for each bond sold (or repurchased). The total issuance cost $H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it})$ is therefore

$$H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it}) \;=\; \eta\,(\,\tilde{b}_{it}^S + |\tilde{b}_{it}^L - b_{it}|\,)\,, \tag{21}$$

where b_{it} is the stock of previously issued long-term bonds outstanding before the firm decides on its investment and financing policy at the end of period $t-1$.

Altınkılıç and Hansen (2000) provide micro-evidence on the cost of debt issuance. This cost renders long-term debt attractive because it does not need to be rolled over each period at high issuance costs as it is the case for short-term debt. In the absence of the issuance cost, firms in our model would never issue any long-term debt. In practice, there are additional motives for using long-term debt. Even though the maturity choice in our model is simplistic, we find it important to allow firms to use both long-term debt and short-term debt since the latter is the standard debt instrument used in economic models.

The firm finances its capital stock by injecting equity and selling new short- and long-term bonds. Let q_{it-1} be the stock of assets in place before the firm decides on equity and debt issuance, and let e_{it} denote net equity issuance at the end of period $t-1$. A positive value of e_{it} indicates an injection of new equity into the firm, e.g. through the sale of new shares. A negative value indicates a net dividend payment from the firm to shareholders. It follows for the new stock of capital in period t:

$$k_{it} = q_{it-1} + e_{it} + p_{it}^S \tilde{b}_{it}^S + p_{it}^L(\tilde{b}_{it}^L - b_{it}) - H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it}) \tag{22}$$

The stock of firm assets in period t after production and repayment of debt is

$$q_{it} = k_{it} - \tilde{b}_{it}^S - \gamma \tilde{b}_{it}^L + (1-\tau)\left[y_{it} + \varepsilon_{it}k_{it} - wl_{it} - \delta k_{it} - f - c(\tilde{b}_{it}^S + \tilde{b}_{it}^L)\right]\,. \tag{23}$$

Definition: Limited Liability. As in the two-period economy, shareholders are free to default and hand over the firm's assets to creditors for liquidation. A fixed fraction ξ of firm assets is lost in this case.

The timing can be summarized as follows.

End of period $t-1$: Firm i has an amount b_{it} of long-term debt outstanding and a stock of assets q_{it-1}. Given b_{it} and q_{it-1}, the firm chooses capital k_{it} and labor l_{it}. Capital is financed through equity issuance e_{it}, and by selling short-term bonds $\tilde{b}_{it}^S$ and additional long-term bonds $\tilde{b}_{it}^L - b_{it}$.

Beginning of period t: The firm's draw of ε_{it} is realized. This determines firm earnings. The firm decides whether to default. If it decides not to default, it pays corporate income tax on its earnings net of depreciation and coupon payments. This leaves the firm with a stock of assets after production of q_{it}. Next period's amount of long-term debt is $b_{it+1} = (1-\gamma)\tilde{b}^L_{it}$.

4.2. Firm Problem

As in the two-period economy, a firm maximizes shareholder value taking as given the constant factor prices w and r. Because there are no equity issuance costs, the amount of assets in place q_{it-1} has no influence on the optimal firm policy. In contrast, the stock of existing debt b_{it} will matter for firm behavior. We can express shareholder value at the end of period $t-1$ as the sum of assets in place and a part which depends on firm behavior: $q_{it-1} + V(\, b_{it}\,)$.

The amount of assets after production q_{it} in (23) is an increasing function of ε_{it}. There is a unique threshold realization $\bar{\varepsilon}_{it}$ which sets shareholder value to zero at the end of period t:

$$\bar{\varepsilon}_{it}: \quad q_{it} + V\Big((1-\gamma)\tilde{b}^L_{it} \Big) \;=\; 0 \tag{24}$$

If ε_{it} is smaller than $\bar{\varepsilon}_{it}$, the firm optimally decides to default.

We assume that the firm has no ability to commit to future actions. This lack of commitment not only affects the firm's default choice, but also its decision of how much to produce and how to finance capital. The firm must therefore take its own future behavior as given. The only way in which it can influence $V(\,(1-\gamma)\tilde{b}^L_{it}\,)$ is through today's choice of long-term debt $\tilde{b}^L_{it}$.

Given a stock of assets q_{it-1} and existing debt b_{it}, at the end of period $t-1$ the firm solves

$$\max_{\substack{k_{it}, l_{it}, e_{it} \geq \underline{e},\\ \tilde{b}^S_{it}, \tilde{b}^L_{it}, \bar{\varepsilon}_{it}}} \quad -e_{it} + \frac{1}{1+r}\int_{\bar{\varepsilon}_{it}}^{\infty} \Big[q_{it} + V\Big((1-\gamma)\tilde{b}^L_{it} \Big) \Big] \varphi(\varepsilon) d\varepsilon \tag{25}$$

$$\begin{aligned}
\text{subject to:} \quad & q_{it} = k_{it} - \tilde{b}^S_{it} - \gamma \tilde{b}^L_{it} + (1-\tau)[y_{it} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f - c(\tilde{b}^S_{it} + \tilde{b}^L_{it})] \\
& y_{it} \;=\; \Big(k_{it}^{\psi}\, l_{it}^{1-\psi} \Big)^{\zeta} \\
& \bar{\varepsilon}_{it}: \quad q_{it} + V\Big((1-\gamma)\tilde{b}^L_{it} \Big) = 0 \\
& k_{it} = q_{it-1} + e_{it} + p^S_{it}\tilde{b}^S_{it} + p^L_{it}(\tilde{b}^L_{it} - b_{it}) - H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it})\,.
\end{aligned}$$

The firm's choice of e_{it} is bounded from below: $e_{it} \geq \underline{e}$, with $\underline{e} < 0$. This is an upper limit for net dividend payments. If the stock of existing debt b_{it} is sufficiently large, the firm may find it optimal to choose a corner solution and pay out the entire asset value of the firm as dividend: $e_{it} = -q_{it-1}$. In practice, it is illegal to pay dividends which substantially exceed firm earnings and deplete a firm's stock of capital. We choose the value of the constraint $\underline{e}$ such that it rules out this corner solution but is not binding in equilibrium. The exact value of $\underline{e}$ does not affect equilibrium variables.

4.3. Creditors' Problem

As in the two-period setup, the optimal firm policy crucially depends on the two bond prices p_{it}^S and p_{it}^L. Competitive creditors break even on expectation. In case of default, the value of the firm's assets is

$$\tilde{k}_{it} \equiv k_{it} + (1-\tau)[y_{it} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f]\,. \tag{26}$$

At this point, creditors liquidate the firm's assets and receive $(1-\xi)\tilde{k}_{it}$. Because short-term debt and long-term debt have equal seniority, the price of short-term debt is

$$p_{it}^S = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon}_{it})]\,(1+c) + \frac{(1-\xi)}{\tilde{b}_{it}^S + \tilde{b}_{it}^L}\int_{-\infty}^{\bar{\varepsilon}} \tilde{k}_{it}\,\varphi(\varepsilon)\,d\varepsilon\right]. \tag{27}$$

The break-even price of short-term debt p_{it}^S only depends on today's firm behavior, in particular today's default risk $\Phi(\bar{\varepsilon}_{it})$. The price of long-term debt p_{it}^L not only depends on today's firm behavior, but also on the future price of long-term debt p_{it+1}^L.

$$\begin{aligned} p_{it}^L = \frac{1}{1+r}\Bigg[&[1-\Phi(\bar{\varepsilon}_{it})]\left[\gamma + c + (1-\gamma)\,p_{it+1}^L\Big((1-\gamma)\tilde{b}_{it}^L\Big)\right] \\ &+ \frac{(1-\xi)}{\tilde{b}_{it}^S + \tilde{b}_{it}^L}\int_{-\infty}^{\bar{\varepsilon}} \tilde{k}_{it}\,\varphi(\varepsilon)\,d\varepsilon\Bigg] \end{aligned} \tag{28}$$

Because the future price of long-term debt depends on future firm behavior, it is a function of the future state of the firm: $p_{it+1}^L(\,(1-\gamma)\tilde{b}_{it}^L\,)$. Because today the firm cannot directly control future firm behavior, the only way in which it can influence the future bond price is through today's choice of long-term debt $\tilde{b}_{it}^L$.

4.4. Equilibrium Firm Policy

In equilibrium, a firm maximizes shareholder value (25) subject to creditors' two break-even conditions (27) and (28). Because we assume that the firm has no ability to commit to future actions, it takes its future behavior as given and chooses today's policy as a best response. In other words, the firm plays a game against its future selves. We restrict attention to the Markov Perfect equilibrium, i.e. we consider strategies which are functions of the current state of the firm.

The value $V(\,b_{it}\,)$ can be computed recursively. It is convenient to define the sum of assets in place q_{it-1} and equity issuance e_{it} as a choice variable: $\tilde{e}_{it} \equiv q_{it-1} + e_{it}$. In each period, the firm chooses a policy vector $\phi(b) = \{k, l, \tilde{e}, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}\}$ which solves

$$V\big(\,b\,\big) = \max_{\phi(b)=\left\{\substack{k,l,\tilde{e}\geq\underline{\tilde{e}},\\ \tilde{b}^S,\tilde{b}^L,\bar{\varepsilon}}\right\}} \quad -\tilde{e} + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty}\Big[\tilde{q} + V\big(\,(1-\gamma)\tilde{b}^L\,\big)\Big]\varphi(\varepsilon)d\varepsilon \tag{29}$$

$$\begin{aligned}
\text{subject to:}\quad \tilde{q} \;&=\; k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)\big[y + \varepsilon k - wl - \delta k - f - c(\tilde{b}^S + \tilde{b}^L)\big] \\
y \;&=\; \big(k^{\psi}\, l^{1-\psi}\big)^{\zeta} \\
\bar{\varepsilon}:\quad \tilde{q} \;&+\; V\big(\,(1-\gamma)\tilde{b}^L\,\big) \;=\; 0 \\
k \;&=\; \tilde{e} \,+\, p^S(b)\,\tilde{b}^S \,+\, p^L(b)\,(\tilde{b}^L - b) \,-\, H(\tilde{b}^S, \tilde{b}^L, b) \\
p^S(b) =&\; \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})]\,(1+c) \,+\, \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \tilde{k}\,\varphi(\varepsilon)\,d\varepsilon\right] \\
p^L(b) =&\; \frac{1}{1+r}\Bigg[[1-\Phi(\bar{\varepsilon})]\,\Big[\gamma + c + (1-\gamma)\,p^L\Big(\,(1-\gamma)\tilde{b}^L\,\Big)\Big] \\
&\qquad\qquad + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \tilde{k}\,\varphi(\varepsilon)\,d\varepsilon\Bigg].
\end{aligned}$$

Because a firm's policy $\phi(b) = \{k, l, e, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}\}$ only depends on its state b and because the firm's future state only depends on the firm policy today, the equilibrium bond prices $p^S(b)$ and $p^L(b)$ likewise only depend on the firm's state b.

4.5. Firm Entry & Exit

Defaulting firms leave the economy. There is free entry so new firms enter the economy as long as the value of a new firm $V(0)$ is positive.

4.6. Households

We close the model by introducing a representative household who owns all equity and debt claims issued by firms and receives all income in the economy. The government's revenue from the corporate income tax is paid out to the household as a lump-sum transfer. The household works, consumes, and invests its savings in equity and debt.

Future utility is discounted at rate β. We assume GHH preferences over consumption and labor, that is, period utility is

$$u\left(C - \chi\,\frac{L^{1+\theta}}{1+\theta}\right)\,, \tag{30}$$

where $u(\cdot)$ is increasing and concave, C is consumption, and L is labor supply.

4.7. General Equilibrium

We study the steady state of the economy with a stationary distribution of firms as in Hopenhayn and Rogerson (1993). All aggregate variables (factor prices, aggregate output, capital, consumption) are constant over time.

Let $\mu(b)$ be the mass of firms in state b, let ν be the mass of entrants, and let $\tilde{b}^L(b)$ and $\bar{\varepsilon}(b)$ be a firm's choice of long-term debt and the default threshold as a function of its state. The law of motion for the firm distribution is

$$\mu_{t+1}(b') = \int_0^\infty \mathbb{1}_{\{b'=(1-\gamma)\tilde{b}^L(b)\}} \Big[1 - \Phi\big(\bar{\varepsilon}(b)\big)\Big] \mu_t(b)\, db \; + \; \mathbb{1}_{\{b'=0\}}\, \nu_t \tag{31}$$

The equilibrium mass of entrants ν_t is pinned down by the free entry condition: $V(0) = 0$. A stationary distribution is a distribution μ^* satisfying $\mu_{t+1} = \mu_t = \mu^*$.

Definition: Stationary Equilibrium. A stationary equilibrium consists of *(i)* a policy vector $\phi(b) = \{k, l, \tilde{e}, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}\}$ and a value function $V(b)$, *(ii)* a stationary distribution μ^* and a mass of entrants ν^*, *(iii)* aggregate labor supply L^* and household consumption C^*, and *(iv)* a wage w^* and an interest rate r^*, such that:

1. $\phi(b)$ and $V(b)$ solve the firm problem (29).
2. The free entry condition holds: $V(0) = 0$.
3. The representative household chooses C^* and L^* optimally.
4. The labor market and the goods market clear.

In the steady state, we have: $1/(1+r^*) = \beta$. There is no aggregate risk and any given firm has zero weight in the representative household's portfolio. For this reason, only the expected return of a given firm's equity or debt claim matters and these claims are priced as if households were risk neutral. The return on the representative household's aggregate portfolio of equity and debt claims is certain and equal to the riskless rate r^*.

With GHH preferences, labor supply is a simple function of the wage w and the labor supply elasticity $1/\theta$:

$$L(w) = \left(\frac{w}{\chi}\right)^{\frac{1}{\theta}} \tag{32}$$

Let $l(b)$ be a firm's labor demand. Labor market clearing implies that

$$L(w^*) = \int_0^\infty l(b)\, \mu^*(b)\, db \tag{33}$$

Goods market clearing implies that

$$Y \equiv \int_0^\infty \Big[y(b) - f - H\big(\tilde{b}^S(b), \tilde{b}^L(b), b\big) - \xi \int_{-\infty}^{\bar{\varepsilon}(b)} \tilde{k}\, \varphi(\varepsilon)\, d\varepsilon\Big] \mu^*(b)\, db \; = \; C \, + \, I\,, \tag{34}$$

where C is household consumption and aggregate investment I is

$$I = \delta \int_0^\infty k(b)\, \mu^*(b) db \tag{35}$$

4.8. Quantitative Analysis

The Markov Perfect equilibrium in (29) can only be computed using numerical methods. Before choosing parameter values, we briefly describe our solution method.

4.8.1. Solution Method

Following the literature on sovereign default with long-term debt (e.g. Hatchondo and Martinez, 2009), we compute the Markov Perfect equilibrium of a finitely-lived firm. Starting from a final date, we iterate backward in time until the firm's value function and the two bond prices have converged. We then use the first-period equilibrium policy and value functions as the equilibrium of the infinite-horizon firm problem.

Common practice in the literature on risky debt is to compute the complete bond price schedules for all possible actions: $p^S(k, l, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon})$ and $p^L(k, l, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon})$. These price schedules from the 'outer loop' are then used to compute the optimal policy in an 'inner loop'. We find this 'inner-loop-outer-loop' procedure to be highly costly in terms of computing time.

We resort to an alternative solution method. Similar to the approach used in the consolidated problem of Section 3.4.1, we express equilibrium bond prices as a function of today's choice variables. Given the firm's future policy, both bond prices are pinned down by the firm's choices today. This allows us to compute equilibrium bond prices and today's firm policy in a single step. This reduces the number of necessary computations and allows for a faster and more precise solution.

4.8.2. Parametrization

The model period is one year. We set $\beta = 0.97$ which implies an annual rate of return on a riskless asset $r^* = 3.09\%$. We also specify $c = r^*$, which implies that the price of a riskless short-term bond and a riskless long-term bond are both equal to one. We set δ to match a 10% annual capital depreciation rate. Chetty, Guren, Manoli, and Weber (2011) report a steady state (Hicksian) labor supply elasticity of about 0.5 which implies $\theta = 2$. The fixed cost f is chosen to generate a unit mass of firms. We set the preference parameter χ to normalize profitability A to one.[11]

The repayment rate of long-term debt γ is a key parameter. We set it to match the Macaulay duration of U.S. corporate bonds with remaining term to maturity above one

[11] Profitability A is defined as in equation (10):

$$A\, k^\alpha \;=\; y^* - wl^*\,, \quad \text{with:} \quad l^* = \zeta(1-\psi)y/w$$

Table 1: Empirical Moments

	Data	Model
1984-2015:		
Capital-output ratio	1.81	1.81
Labor share	61.8%	61.8%
Leverage: Debt / Assets	27.2%	27.2%
Long-term debt share	67.4%	67.4%
1998-2010:		
Credit spread	2.65%	2.66%

Note: The capital-output ratio and the labor share are from the Flow of Funds for the aggregate of non-financial corporate businesses. The labor share is compensation of employees divided by revenue from sales of goods and services. The capital-output ratio is non-financial assets (marked-to-market, excluding inventories) divided by revenues. Leverage and the long-term debt share are from Compustat (excluding financial firms and utilities). Leverage is the average (across all firm-year observations) of the ratio of the book value of debt to the book value of total firm assets. The long-term debt share is the average ratio of debt due more than one year from today to total firm debt. The credit spread on corporate bonds is from Adrian, Colla, and Shin (2013). The model counterpart is the issuance-weighted average of the credit spread on short-term bonds and long-term bonds.

year. Gilchrist and Zakrajsek (2012) calculate an average duration of 6.47 years. This implies $\gamma = 0.1284$. We follow Gomes et al. (2016) in setting the tax rate $\tau = 0.4$.[12]

The firm-specific earnings shock is Normal with zero mean and standard deviation σ_ε. This leaves us with five unspecified parameters: The return to scale parameter ζ, the parameter pinning down the capital share ψ, the standard deviation of idiosyncratic shocks σ_ε, the debt issuance cost η, and the default cost ξ. We choose these values to replicate five key moments: The capital-output ratio, the labor share, leverage, the long-term debt share, and the average credit spread. These moments and their model counterparts are shown in Table 1.[13]

In our model, firms differ with respect to the stock of existing debt b. Given the stationary distribution μ^*, model statistics are constructed as weighted averages of firm policies. Table 2 reports our choice for the full set of parameter values.

Untargeted Moments

To assess our parametrization, we consider a number of untargeted empirical moments. The parameter ζ controls the degree of diminishing returns. Empirical estimates by Blundell and Bond (2000) suggest a value close to one. The parameter $\zeta = 0.8945$ is broadly in line with these estimates. Altınkılıç and Hansen (2000) provide micro-evidence for the debt issuance cost η. They calculate an average debt issuance cost of 0.0109, close to our value of $\eta = 0.008$.

[12] Hennessy and Whited (2005) suggest a value of 0.3. Gomes et al. (2016) argue that τ should be thought of as capturing additional relative benefits of using debt rather than equity (e.g. equity issuance costs).

[13] We choose Flow of Funds data for the capital-output ratio because Compustat records assets at historical cost which makes it hard to compare with output. Model counterparts of empirical moments are derived in Appendix C.

Table 2: Parametrization

Par.	Description	Value	Target/Source
β	time preference	0.97	
c	debt coupon	r^*	
δ	depreciation	0.1	
θ	preference parameter	2	Chetty et al. (2011)
χ	preference parameter	4.3416×10^{-6}	Normalization: $A = 1$
f	fixed cost	17.1031	Normalization: $\int \mu^*(b)\, db = 1$
γ	repayment rate LTD	0.1284	Gilchrist and Zakrajsek (2012)
τ	corporate income tax rate	0.4	Gomes et al. (2016)
ζ	technology parameter	0.8945	Capital-output ratio 1.81
ψ	technology parameter	0.309	Labor share 61.8%
σ_ε	st. dev. idiosyncratic shock	0.6875	Leverage 27.2%
η	debt issuance cost	0.008	Long-term debt share 67.4%
ξ	default cost	0.48	Credit spread 2.65%

The annual default rate generated by the model is 3.07%. This is high compared to an empirical value of around 1.05% reported by Duan, Sun, and Wang (2012). It is well known that empirical credit spreads are not fully explained by expected default losses (e.g. Elton, Gruber, Agrawal, and Mann, 2001). In our model, credit spreads are exclusively driven by default losses. This means that we have to decide whether we want our model to match the default rate and generate unrealistically low credit spreads, or if want to match credit spreads at the cost of generating unrealistically high default rates. In our model, the bond price schedule is key to understanding firm behavior. We therefore choose to match the average credit spread rather than the default rate.

4.9. Policy Functions

We now present the numerical solution to the dynamic firm problem described above. Because we use a global method, our solution describes equilibrium firm behavior over the entire state space. Figures 1 and 2 show firms' equilibrium policies as functions of the existing stock of debt b on the x-axis. Debt is normalized by the optimal capital stock k^* in a frictionless economy (without taxation and default or debt issuance costs). Where convenient, the y-axes have been normalized in a similar way.

In contrast to the two-period setup, the stock of existing debt b is now endogenous. Today's choice of long-term debt $\tilde{b}^L$ determines the amount of existing debt next period $b' = (1-\gamma)\tilde{b}^L$. Debt dilution and debt overhang jointly affect firms' equilibrium behavior. We will isolate and compare the respective roles of debt dilution and debt overhang in Section 6.

The role of existing debt is the same as in the two-period model. Through the bond market, the firm fully internalizes the effect of its actions on the value of newly issued

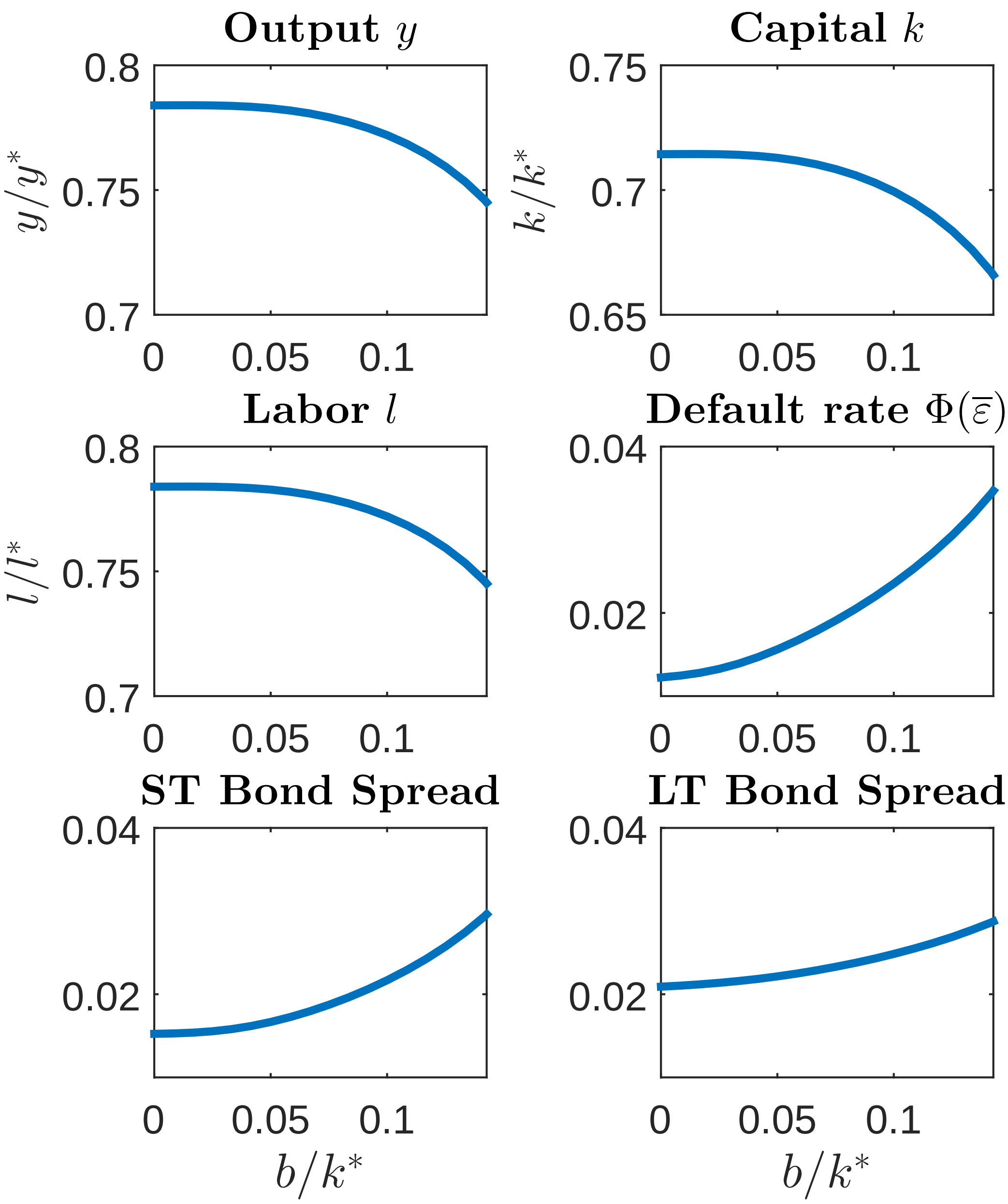

Figure 1: Policy Functions Part I

Output, capital, and labor are normalized by their respective frictionless values y^*, k^*, and l^*. The x-axes show the stock of existing debt b normalized by k^*.

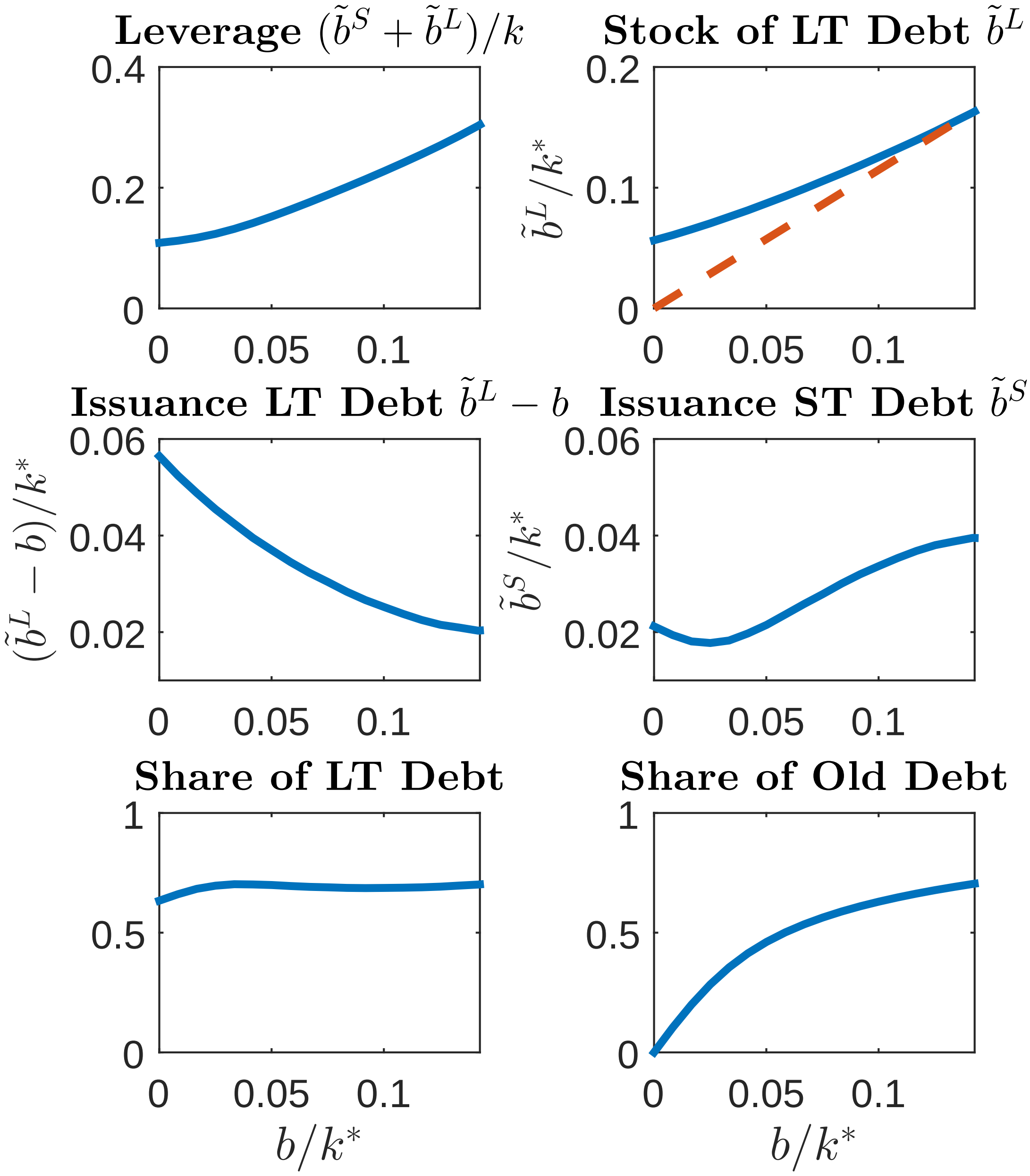

Figure 2: Policy Functions Part II

Debt is normalized by k^*. The x-axes show the stock of existing debt b normalized by k^*.

debt but it disregards the effect on the value of existing debt. As the stock of existing debt b increases, we observe in Figures 1 and 2 that the default rate, credit spreads, and leverage rise, and capital, labor demand, and output fall.

Consider the firm's default rate in the right panel of the middle row of Figure 1. As the stock of existing debt rises, the firm internalizes a smaller part of the expected costs of default and therefore chooses a higher default risk. Higher default risk is reflected by higher credit spreads (bottom row of Figure 1).

Higher credit spreads lead to higher costs of capital. This lowers the firm's demand for capital as the stock of existing debt increases (top right panel). In the two-period setup, the credit spread increased in b only because of the corresponding increase in today's default risk. In the dynamic model the long-term debt spread also rises when default risk in the future is expected to increase.

Because of complementarity in production, the firm's labor demand falls together with capital (left panel in the middle row). The decline in capital and labor is reflected by the fall in output (top left panel). The stock of existing debt has real economic effects.

The firm's capital structure decisions are displayed in Figure 2. The top left panel shows that leverage is increasing in b. As b rises, the firm internalizes fewer of the total expected costs of default and therefore chooses higher levels of total debt. The effect of b on the default rate, capital, and leverage becomes stronger as b and the risk of default increase. We will test this model prediction empirically in Section 5.

The key feature of the dynamic model is that the state variable b is endogenous. By choosing an amount of long-term debt $\tilde{b}^L$ today, a firm determines next period's stock of existing debt $b' = (1-\gamma)\tilde{b}^L$. Indirectly, this choice affects future firm behavior, e.g. default risk and investment. This, in turn, affects today's price of long-term debt p^L.

A firm is free to choose the mix between short-term debt and long-term debt. Any given stock of total debt can be implemented through a variety of maturity choices. By issuing primarily short-term debt, a firm can reduce the future stock of debt and thereby minimize future debt dilution and debt overhang. This increases the price of long-term debt. The benefit of long-term debt is that it saves debt issuance costs. Fewer new bonds have to be issued each period for a given level of leverage.

The firm's choice of the level of long-term debt $\tilde{b}^L$ increases in b (top right panel of Figure 2). As b rises, the choice of long-term debt is affected by two opposing forces. On the one hand, the effect of b on the default rate and capital becomes stronger as b rises. This means that the marginal cost of long-term debt increases in b.

On the other hand, not all costs of long-term debt are internalized by the firm. Future debt dilution and debt overhang lower today's price of long-term debt. A part of this loss is borne by the owners of existing long-term debt and is not internalized by the firm. Importantly, this part is increasing in b. The higher is b, the lower is the fraction of the total cost of long-term debt which is internalized by the firm. This explains why the firm's choice of long-term debt is increasing in b. A high level of long-term debt in the previous period implies a high level of long-term debt today. In the words of Gomes et al. (2016), long-term debt is "sticky".[14]

[14]The endogenous "stickiness" of long-term debt is also studied by Admati et al. (2018) in a model

New entrants without existing debt initially choose low levels of long-term debt. In the absence of default, they subsequently build up long-term debt until they converge towards a stable level (indicated by the intersection of the red dashed line with the policy function $\tilde{b}^L$). Even though it is feasible in this model to minimize future debt dilution and debt overhang by primarily using short-term debt, firms do not internalize all the costs of long-term debt. They drift into a range of the state space in which the stock of long-term debt is high and the effects of debt dilution and debt overhang are large.

The dynamic behavior of long-term debt is key to understanding the term structure of credit spreads (bottom row of Figure 1). For low values of b, creditors correctly anticipate that the firm will increase the amount of long-term debt in the future. This will increase the risk of default. Because the future risk of default only affects the value of long-term debt, the long-term spread is higher than the short-term spread for low values of b. If b is very high, creditors anticipate that the firm will lower the amount of long-term debt in the future. Because this will lower default risk in the future, the long-term spread is lower than the short-term spread for high values of b.[15]

In the bottom right panel of Figure 2, we plot the share of old debt, i.e. the existing stock of debt b over total debt $(\tilde{b}^S + \tilde{b}^L)$. We will use this measure as an indicator for debt dilution and debt overhang in the empirical analysis below.

5. Empirical Evidence

In the previous section, we studied the role of long-term debt in a dynamic model of firm financing and production. Our model produces novel predictions which are empirically testable using firm-level data.

To construct our sample, we merge Compustat data for publicly traded U.S. firms from 1984-2015 with default events recorded in Moody's Default & Recovery Database.[16] We exclude financial firms and utilities. For every year, our sample accounts for about one fourth of total U.S. employment and more than one third of total assets of non-financial firms. Additional details can be found in Appendix E.

We define the `OLD-Share` as

$$\texttt{OLD-Share} = \frac{b}{\tilde{b}^S + \tilde{b}^L} \ ,$$

where b is debt which has been issued in year $t-1$ or before and is outstanding at the end of year t, and $\tilde{b}^S + \tilde{b}^L$ is total firm debt at the end of year t. In the theoretical model,

of corporate finance, and by Aguiar et al. (2016) in a model of sovereign debt.

[15]Note that for high values of b the firm issues more short-term debt than long-term debt even though the long-term spread is lower and increasing more slowly in b than the short-term spread. This reflects the fact that credit spreads indicate the average cost of debt and not its true marginal cost (Aguiar et al., 2016).

[16]Jermann and Quadrini (2012) suggest 1984 as a starting date because of the beginning of the Great Moderation and regulatory changes in U.S. financial markets at that time. Moody's default data is available from 1988 onwards.

this variable is an indicator of debt dilution and debt overhang as it co-moves positively with default risk and leverage, and negatively with capital (see Figure 2). We use the `OLD-Share` as an empirical proxy for the severity of debt dilution and debt overhang and estimate its relationship with leverage, default risk, and asset growth.

To be clear, the goal of this exercise is not to establish causality. The variables `OLD-Share`, default risk, leverage, and asset growth are all choice variables. Correlations between these variables do not readily admit conclusions about causality (see e.g. Roberts and Whited, 2013). Nevertheless, these correlations provide reduced-form evidence which allows us to test if the data displays patterns which are consistent with the model of debt dilution and debt overhang developed in the previous section.

We focus on the cross-section of firms, i.e. the `OLD-Share` of firm j in our empirical analysis is the median of firm j's `OLD-Share` for all years in which firm j appears in our sample. With all other variables we proceed in the same way. We choose to ignore the time dimension because our model deliberately abstracts from several factors which are likely to be important in explaining short-run variations (e.g. adjustment costs to capital, equity issuance costs). Our model is designed to capture slow-moving or time-invariant patterns in firm behavior.[17] All of our regressions include sector-fixed effects and robust standard errors.

5.1. Leverage

Table 3 shows regression results with `Leverage` as the dependent variable. `Leverage` is the ratio of the book value of total firm debt to the book value of total assets. The first column gives results from a standard leverage regression. Similar regressions have been run by numerous studies in empirical corporate finance (e.g. Rajan and Zingales, 1995). Our results are standard. `Leverage` is negatively related to `Profitability`, and positively related to `Tangibility` and firm size (measured by `Log Sales`). Also the positive relationship between book leverage and `Tobin's q` is in line with existing evidence (e.g. Frank and Goyal, 2009, p. 22).

In column (2), we add the `OLD-Share` as an additional control variable. We note two results. First, the coefficients of the other explanatory variables are barely affected. This suggests that the `OLD-Share` adds genuinely new information to the statistical model. Second, the estimated coefficient of `OLD-Share` is positive and significant.

A one percentage point-increase in `OLD-Share` is associated to an increase in `Leverage` of 0.035 percentage points. The standard deviation of `OLD-Share` across firms is 36 percentage points. Accordingly, a one-standard deviation increase in `OLD-Share` is associated to an increase in `Leverage` of 1.3 percentage points.

Our parametrized dynamic model predicts that the elasticity of `Leverage` with respect to `OLD-Share` is higher for firms with a high risk of default. To test this prediction, we split the sample using the Altman Z-score. A low Z-score is commonly used as an

[17]We remove year-fixed effects from all variables before creating the cross-section of firms. Lemmon, Roberts, and Zender (2008) document that the majority of empirical variation in capital structure is explained by between-firm variation of time-invariant target leverage ratios.

Table 3: Leverage

OLS-Regression	(1) Leverage	(2) Leverage	(3) Leverage (low Z-score)	(4) Leverage (high Z-score)
OLD-Share		0.0354***	0.0602***	0.0222*
		(3.33)	(3.56)	(2.51)
Tobin's q	0.0145***	0.0146***	0.0528***	0.00405
	(4.46)	(4.43)	(8.76)	(1.10)
Profitability	-0.215***	-0.218***	-0.0702**	-0.0770**
	(-11.60)	(-11.57)	(-3.05)	(-2.98)
Tangibility	0.214***	0.207***	0.252***	0.0608
	(6.91)	(6.51)	(6.56)	(1.75)
Firm Age	-0.00404***	-0.00414***	-0.00336**	-0.00692***
	(-6.91)	(-7.11)	(-2.91)	(-13.03)
Log Sales	0.0121***	0.0107***	0.0108***	0.0187***
	(8.83)	(7.52)	(4.74)	(10.39)
Constant	-0.0102***	-0.00635***	0.0713***	-0.101***
	(-8.54)	(-3.93)	(16.25)	(-33.87)
4-digit Sector FE	YES	YES	YES	YES
Adjusted R^2	0.233	0.235	0.202	0.283
Observations	9,398	9,398	4,548	4,564

t statistics in parentheses. Standard errors are clustered at the 4-digit sector level.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

empirical indicator for a high risk of default. Column (3) reports results for firms with a Z-score below the sample median (i.e. high risk of default). We find that the estimated coefficient of `OLD-Share` is more than twice as large for the high-default-risk sample in column (3) than for the low-default-risk group in column (4).

5.2. Default

Moody's Default & Recovery Database contains information about firm default events. We create a dummy variable `Default` which takes the value of one if a firm defaults at least once during the sample period, and zero otherwise. There are 431 firms in our sample with at least one default event.[18] Because defaults are rare in certain sectors, we include sector-fixed effects at the 1-digit level only.

Results are displayed in Table 4. The first column shows the estimated coefficients of a logit model using the same set of control variables as above (with the addition of `Leverage`). Firms that default have higher `Leverage` and lower values of `Tobin's q`.

In column (2), we add the `OLD-Share`. The estimated coefficient for `OLD-Share` is significant and positive. This is true even though `Leverage` and other control variables are likely to pick up some of the effect of debt dilution and debt overhang on default risk. A one percentage point-increase in `OLD-Share` is associated to an increase in the default probability of 0.01 percentage points. In this sample, the standard deviation of `OLD-Share` across firms is 43 percentage points.[19] This implies that a one-standard deviation increase in `OLD-Share` corresponds to an increase in the probability of default of 0.43 percentage points. This is a sizeable amount given that the unconditional probability of having at least one default event during the sample period is about 4 percent.

In columns (3) and (4), we split the sample using the Altman Z-score. In the high-default-risk sample (with a low Z-score), there are 235 firms with at least one default event. In the low-default-risk sample (with a high Z-score), there are only 89 firms with a default event. This suggests that the Z-score is indeed a useful predictor of default. Comparing columns (3) and (4), we find that the positive relationship between the `OLD-Share` and default risk is much stronger in the high-default-risk sample than in the low-default-risk group. This is consistent with the convex policy function for the default rate calculated in Section 4.

5.3. Asset Growth

In Table 5 we use `Asset Growth` as the dependent variable. These regressions test the model predictions with respect to capital. `Asset Growth` is the median value of a firm's annual growth rate of total assets. As above, column (1) shows the results

[18] In the regressions we lose some default events because Compustat does not report all control variables for all firms.

[19] Because Moody's Default & Recovery Database has information about default events from 1988 onwards, we construct the cross-section of firms using data 1988-2015 for the `Default` regressions. The sample therefore differs from the one used in the `Leverage` and `Asset Growth` regressions.

Table 4: Default

Logit-Regression	(1) Default	(2) Default	(3) Default (low Z-score)	(4) Default (high Z-score)
OLD-Share		0.564***	1.241***	0.0491
		(5.09)	(6.69)	(0.17)
Leverage	3.224***	3.196***	2.784***	3.529***
	(13.26)	(13.06)	(8.76)	(5.53)
Tobin's q	-0.808***	-0.812***	-0.634**	-0.431
	(-5.21)	(-5.08)	(-3.03)	(-1.87)
Profitability	-0.393	-0.500	0.297	-3.023***
	(-1.33)	(-1.56)	(0.64)	(-3.61)
Tangibility	-0.0676	-0.133	-0.194	-0.592
	(-0.27)	(-0.53)	(-0.62)	(-0.98)
Firm Age	0.0247**	0.0238**	0.0109	0.0268
	(2.68)	(2.58)	(0.67)	(1.66)
Log Sales	0.317***	0.306***	0.336***	0.212***
	(12.81)	(12.19)	(10.47)	(4.19)
Constant	-3.571***	-3.516***	-3.342***	-3.258***
	(-16.08)	(-15.78)	(-11.14)	(-6.49)
1-digit Sector FE	YES	YES	YES	YES
Pseudo R^2	0.156	0.161	0.198	0.117
Observations	8,723	8,704	4,193	4,152

t statistics in parentheses. Robust standard errors.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

of a regression which does not include `OLD-Share` as an explanatory variable. `Asset Growth` is positively related to `Tobin's q` and `Profitability`. Older firms and firms with higher `Leverage` grow more slowly.

In column (2), we add the `OLD-Share`. The estimated coefficient is significant and negative. A one percentage point-increase in `OLD-Share` is associated to a decrease in `Asset Growth` of 0.07 percentage points. A one-standard deviation increase in `OLD-Share` corresponds to a decrease in annual asset growth of 2.6 percentage points. This is a sizeable amount given that the median annual asset growth rate is 1.7 percent.

In columns (3) and (4), we repeat this regression separately for the high-default-risk and the low-default-risk sample. In line with our model, the negative relationship between the `OLD-Share` and `Asset Growth` is stronger for firms with a higher probability of default.

5.4. Discussion of Empirical Results

The empirical results are consistent with firm behavior in the dynamic model studied in Section 4. In the cross-section of firms, the `OLD-Share` is positively correlated with leverage and default risk, and negatively correlated with asset growth. In line with the model predictions, these correlations are stronger for firms with a higher default risk.[20]

We carry out a number of robustness checks. Results are qualitatively unchanged if we include additional control variables (e.g. dividend payments, non-debt tax shields), or if we exclude the control variable `Leverage` from the default and investment regressions.

The relationship between `Default` and `OLD-Share` is unchanged if we only consider severe default episodes (e.g. Chapter 11) or if we construct the cross-section of firms excluding all firm-year observations that follow a default episode. As an alternative to `Asset Growth`, we use the ratio of investment expenditures to firm assets as the dependent variable and obtain qualitatively identical results.

We also construct an alternative to the Z-score. In this exercise, we use the full panel of firm-year observations and regress a default dummy on a set of explanatory variables (excluding the `OLD-Share`). This gives us a statistical model which allows to relate observable firm characteristics to default probabilities. We use this statistical model to split the firms from the pure cross-section into two groups with high and low default risk. All of our results go through.

The empirical results described above are not readily explained by alternative theoretical mechanisms. For instance, firms with a high share of long-term debt face low refinancing risk (as in Diamond, 1991) which may allow them to choose higher leverage. This mechanism can generate a positive relationship between `OLD-Share` and `Leverage`. This is plausible and might be relevant in the data. However, it cannot rationalize the empirical result that firms with a high share of long-term debt (i.e. a high `OLD-Share`) face a higher risk of default and invest less than other firms.

[20] This result is consistent with Hennessy (2004) who estimates that debt overhang is more severe for firms with low credit ratings.

Table 5: Asset growth

OLS-Regression	(1) Asset Growth	(2) Asset Growth	(3) Asset Growth (low Z-score)	(4) Asset Growth (high Z-score)
OLD-Share		-0.0726***	-0.0981***	-0.0563***
		(-8.38)	(-8.21)	(-6.12)
Leverage	-0.0866***	-0.0802***	-0.0424*	-0.0530*
	(-6.33)	(-5.81)	(-2.28)	(-2.39)
Tobin's q	0.0502***	0.0499***	0.0416***	0.0451***
	(10.20)	(10.09)	(7.97)	(5.10)
Profitability	0.265***	0.271***	0.215***	0.240***
	(12.87)	(13.33)	(8.52)	(4.87)
Tangibility	0.0213	0.0340	0.0198	0.0358
	(1.17)	(1.88)	(0.83)	(1.21)
Firm Age	-0.00851***	-0.00826***	-0.00804***	-0.00713***
	(-9.70)	(-9.55)	(-6.26)	(-8.85)
Log Sales	0.00311	0.00591***	0.00922***	0.000217
	(1.95)	(3.74)	(3.90)	(0.11)
Constant	-0.0341***	-0.0420***	-0.0694***	-0.0143***
	(-18.35)	(-18.75)	(-15.89)	(-3.47)
4-digit Sector FE	YES	YES	YES	YES
Adjusted R^2	0.170	0.184	0.123	0.162
Observations	9,398	9,398	4,548	4,564

t statistics in parentheses. Standard errors are clustered at the 4-digit sector level.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

6. Model Experiments

We now come back to the dynamic model of firm financing and production studied in Section 4. As shown above, the model's predictions are in line with reduced-form firm-level evidence. We can use our model to run a number of counterfactual experiments.

First, we conduct comparative statics. We study how changes in parameter values affect the steady state of the economy. This sheds further light on the key economic mechanisms at work.

In a second series of experiments, we isolate the distinct roles of debt dilution and debt overhang. By allowing firms to choose the total stock of debt, capital, or both with full commitment, we selectively eliminate either debt dilution, debt overhang, or both.

In a final experiment, we compare our benchmark model to a standard model of short-term debt. We find that debt dilution and debt overhang can overturn standard results. A financial reform which lowers bankruptcy costs and increases investment, employment, output, and welfare in a standard model of short-term debt can have the opposite effect in a model with long-term debt.

6.1. Comparative Statics

The qualitative features of firms' equilibrium behavior in the dynamic model are highly robust across different parametrizations. The default rate, credit spreads, and leverage are increasing and convex in b, and capital, labor, and output are decreasing and concave. The stock of long-term debt $\tilde{b}^L$ is always increasing in b. In this section, we study the quantitative sensitivity of steady state variables with respect to several model parameters. Table 6 summarizes the results.

The first change we consider is a higher repayment rate of long-term debt, $\gamma = 0.5$ (instead of $\gamma = 0.1284$ as in the benchmark case). Any given choice of $\tilde{b}^L$ in the current period now translates into a lower value of existing debt next period $b' = (1 - \gamma)\tilde{b}^L$. This reduces debt dilution and results in lower levels of default risk, credit spreads, and leverage. The share of long-term debt is lowered mechanically. The average issuance cost per unit of debt, IC, is higher now because more new debt needs to be issued each period due to the higher repayment rate γ. But this effect is dominated by the reduction in credit spreads. The cost of capital falls and the steady state level of investment increases.

As a second change, we consider a reduction in the corporate income tax τ. The tax cut increases investment, employment, and output. Its effect on credit spreads is driven by two forces. On the one hand, a lower tax decreases the benefit of debt over equity which reduces leverage. On the other hand, the average return on capital falls because of decreasing returns to scale. This increases the default rate and credit spreads. Since spreads are increasing and convex in the stock of existing debt, the marginal cost of long-term debt is higher now. Firms respond by issuing less long-term debt. Long-term debt over total debt increases but the ratio of outstanding long-term debt to capital b/k falls.

Next we consider an increase in firm-level risk σ_ε. Even though firms reduce leverage, the default rate and credit spreads increase as large negative shocks are more frequent.

Table 6: Comparative Statics

Parameters	Results							
	Y	K	L	Leverage	*Spread*	LTD Share	IC	b/k
Benchmark	-	-	-	27.2	2.66	67.4	0.27	18.4
$\gamma = 0.5$	+0.55	+1.28	+0.00	20.6	2.11	48.5	0.41	10.1
$\tau = 0.375$	+1.13	+2.59	+0.33	25.0	2.96	72.3	0.24	17.9
$\sigma_\varepsilon = 0.73$	+0.30	+0.51	+0.07	25.5	3.29	71.2	0.25	18.0
$\eta = 0.002$	+0.95	+2.37	+0.14	25.7	2.54	42.6	0.12	11.1
$\xi = 0.8$	+0.21	+0.26	+0.03	23.4	2.49	79.8	0.17	18.7

Note: All values are in percent. Aggregate output Y, aggregate capital K, and total labor L are expressed as percent changes with respect to the benchmark parametrization from Section 4. *Leverage* is average firm leverage computed as total firm debt over capital. *Spread* is the issuance-weighted credit spread on short-term debt and long-term debt. The *LTD Share* is the average ratio of debt due more than one year from today to total firm debt. IC is the average ratio of debt issuance costs $H(\tilde{b}^S, \tilde{b}^L, b)$ to total firm debt (in percent). b/k denotes the average fraction of outstanding long-term debt to capital.

Aggregate investment, labor, and output increase. This is because firms internalize the full amount of increased upside risk but do not bear the entire increase in downside risk. Very low realizations of ε trigger default but a part of the associated costs is borne by the holders of existing debt and is not internalized by firms. With respect to the benchmark case, outstanding long-term debt over capital falls as higher credit spreads increase the marginal cost of debt dilution and debt overhang.

The fourth change considered is a drop in the debt issuance cost η. This allows firms to lower their share of long-term debt and the average debt issuance cost IC at the same time. The lower amount of long-term debt reduces debt dilution which implies lower default risk, credit spreads, and leverage. Lower credit spreads and lower debt issuance costs increase investment, labor demand, and output.

Finally, we study a higher value of the default cost ξ. Firms optimally respond by choosing lower leverage in order to reduce the default risk. Because of the lower risk of default, debt dilution and debt overhang are less severe. This reduces the marginal cost of long-term debt and allows firms to increase long-term debt over capital. Because of lower credit spreads and lower debt issuance costs, aggregate investment and output increase. In the last counterfactual experiment in Section 6.3, we will come back to the effects of a change in the default cost ξ.

6.2. The Cost of Debt Dilution and Debt Overhang

Future debt dilution and debt overhang lower the price of long-term debt today. Firms would like to promise to maintain low future levels of debt and high levels of investment

Table 7: The Cost of Debt Dilution and Debt Overhang

Experiment	Results								
	Y	K	L	Lev.	*Spread*	$\tilde{b}^S$	$\tilde{b}^L$	IC	W
Benchmark	-	-	-	27.2	2.66	3.7	14.7	0.27	-
No Debt Dilution	+0.72	+0.84	+0.18	20.1	1.95	0.0	13.5	0.12	+0.74
No Debt Overhang	+0.42	+1.74	−0.01	29.4	2.91	3.8	17.5	0.27	+0.14
No DD & No DO	+1.54	+3.54	+0.21	21.8	1.00	0.0	16.0	0.12	+1.20

Note: All values are in percent. Aggregate output Y, aggregate capital K, total labor L, and welfare W are expressed as percent changes with respect to the benchmark parametrization from Section 4. *Lev.* is average firm leverage computed as total firm debt over capital. *Spread* is the issuance-weighted credit spread on short-term debt and long-term debt. $\tilde{b}^S$ and $\tilde{b}^L$ are the average amount of short-term debt and long-term debt, respectively. They are normalized by k^*. IC is the average ratio of debt issuance costs $H(\tilde{b}^S, \tilde{b}^L, b)$ to total firm debt (in percent).

in order to increase the revenue raised on the bond market today. But such a promise is not credible. Once a firm has sold its debt and raised the associated revenue, it has no incentive to take the effects of its actions on the value of existing debt into account. Because creditors have rational expectations, they correctly anticipate and price in future debt dilution and debt overhang. This results in a low price of long-term debt and high costs of capital for firms.

In this section, we disentangle the two distinct forms of this commitment problem. Debt dilution arises if firms are unable to commit to future levels of debt. Debt overhang is present if firms are unable to commit to future amounts of capital. Which of the two distortions is more severe for investment, employment, output, and welfare? Do the two distortions amplify or dampen one another?

To answer these questions, we compare the solution of our benchmark model from Section 4 with three counterfactual experiments in which either debt dilution, debt overhang, or both are eliminated. By allowing firms to choose either the total stock of debt, capital, or both with full commitment, we selectively eliminate debt dilution, debt overhang, or both.

6.2.1. No Debt Dilution

In this experiment we allow firms to commit to a fixed level of total debt $(\tilde{b}^S + \tilde{b}^L)$. All other variables are chosen without commitment just as in the benchmark economy of Section 4. This implies that debt dilution is eliminated but debt overhang is still present.

We allow new entrants to commit to a fixed level of total debt. They choose the amount which maximizes $V(0)$. In general equilibrium, any increase in $V(0)$ leads to additional firm entry and increased labor demand. This drives up the wage and pushes

$V(0)$ back down to zero.

The second row of Table 7 summarizes the results of this experiment. We report equilibrium values for the new steady state. With commitment, firms choose a lower average level of debt. Even though they could benefit ex-post from the opportunity to dilute existing debt, this opportunity would hurt firms ex-ante because creditors would demand high credit spreads.

In the absence of debt dilution, leverage, the default rate, and credit spreads are lower than in the benchmark case. Firms only issue long-term debt. The costs from debt overhang alone are not sufficient in our parametrization to induce firms to issue any short-term debt. Lower credit spreads and lower debt issuance costs imply lower costs of capital which leads to increased investment. Firm value increases as well and more firms enter the economy which drives up wages and labor supply. As a result, aggregate capital, output, and welfare increase.

6.2.2. No Debt Overhang

In the second experiment, we allow new entrants to commit to a fixed level of capital k, that is, capital is chosen to maximize the value of a new firm $V(0)$. As before, all other variables are chosen without commitment. This means that debt overhang is eliminated but debt dilution is still present.

The results are shown in the third row of Table 7. If firms can commit to a fixed value of capital, they choose a higher capital stock than in the benchmark case. Even though firms with existing debt would prefer a lower level of investment ex-post, ex-ante they benefit from the higher stock of capital because this increases the long-term bond price.

The increase in capital is higher than in the previous case without debt dilution. However, the positive effects on GDP and welfare are smaller. Decreasing returns imply that the marginal unit of capital contributes little to firm value. This explains why the large effect of debt overhang on investment does not translate into a large increase in firm value and hence firm entry. In fact, aggregate labor demand falls after the elimination of debt overhang.

This result can be understood by considering firms' maturity choice. In the benchmark model, firms realize that issuing long-term debt instead of short-term debt increases debt overhang and debt dilution in the future. By eliminating debt overhang, the costs of issuing long-term debt are reduced. Firms respond by issuing more long-term debt than in the benchmark case which renders debt dilution more severe. As a result, leverage, default risk, and credit spreads are higher. By eliminating one commitment problem (debt overhang), the size of the second commitment problem (debt dilution) is increased. In other words, debt overhang helps to mitigate debt dilution.

With respect to firm value, the effect of higher credit spreads outweighs the large increase in capital in our parametrization. Credit spreads are inframarginal costs paid on each unit of debt. Changes in spreads therefore have sizeable effects on firm value, firm entry, and labor demand. This explains why debt overhang has a bigger effect on investment, while debt dilution is more important for employment, output, and welfare.

6.2.3. No Debt Dilution & No Debt Overhang

The final experiment allows firms to choose both debt and capital with full commitment. As before, these variables are chosen to maximize the value of a new entrant $V(0)$. This leaves the maturity choice as the only variable chosen without commitment. But in an economy without debt dilution or debt overhang, firms' maturity choice is trivial. In the benchmark model, the only disadvantage of long-term debt is that it gives rise to debt dilution and debt overhang in the future. In the absence of these two distortions, firms optimally issue only long-term debt. Results are shown in the last row of Table 7.

The level of leverage is now higher than when only debt dilution was eliminated. By also eliminating debt overhang, the cost of (long-term) debt is reduced and firms respond by choosing higher leverage.

The interaction between debt dilution and debt overhang is noteworthy. If debt dilution is present, eliminating debt overhang increases credit spreads because it induces firms to build up more long-term debt which renders debt dilution more severe. Without debt dilution, this negative aspect of eliminating debt overhang disappears. Apart from increasing investment, eliminating debt overhang now also reduces credit spreads and raises firm value, entry, and labor demand.

One last observation is that in a dynamic model, debt dilution and debt overhang not only distort firm behavior if the stock of existing debt is high. As the policy functions from the benchmark model in Section 4 show (Figure 2), a new entrant without existing debt chooses a lower level of leverage (about 11%) than the full commitment level of 21.8%. The anticipation of future debt dilution and debt overhang affects a firm's choices even before the firm has sold its first unit of long-term debt.

6.3. A Financial Reform

Standard models of firm financing assume that all debt is short-term. By assumption, debt dilution and debt overhang are absent in that case. This modeling choice is not innocuous, as can be seen by studying the effects of a financial reform in two different models: (1.) a standard short-term debt model, and (2.) the benchmark model from Section 4 with short-term debt and long-term debt.

We consider a financial reform which lowers the default cost ξ. Such a reform could be implemented by speeding up the legal process of bankruptcy, having a more efficient court system, assigning clear control rights to creditors in case of default, etc.

6.3.1. Standard Short-term Debt Model

We first consider a model without long-term debt. Firms finance capital through equity and short-term debt. We parametrize this model to match the empirical targets for leverage and credit spreads that were used in the benchmark model. We obtain $\sigma_\varepsilon = 0.698$, $\eta = 0$, and $\xi = 0.21$. For the remaining parameters, the same values as in the benchmark model of Section 4 are used. This generates a leverage ratio of 27.2% and a credit spread of 2.65%.

Table 8: A Financial Reform

	Y	K	L	Leverage	*Spread*	$\tilde{b}^S$	$\tilde{b}^L$	IC	W
				Only short-term debt					
Pre-Reform	-	-	-	27.2	2.65	20.0	0.0	0.00	-
Post-Reform	$+0.10$	$+0.30$	$+0.04$	29.3	2.79	21.6	0.0	0.00	$+0.02$
				Short- and long-term debt					
Pre-Reform	-	-	-	27.2	2.66	3.7	14.7	0.27	-
Post-Reform	-0.07	-0.06	-0.01	28.7	2.74	5.0	14.4	0.32	-0.08

Note: All values are in percent. Aggregate output Y, aggregate capital K, total labor L, and welfare W are expressed as percent changes with respect to the pre-reform steady state. *Leverage* is average firm leverage (in percent) computed as total firm debt over capital. *Spread* is the issuance-weighted credit spread on short-term debt and long-term debt. $\tilde{b}^S$ and $\tilde{b}^L$ are the average amount of short-term debt and long-term debt, respectively. They are normalized by k^*. IC is the average ratio of debt issuance costs $H(\tilde{b}^S, \tilde{b}^L, b)$ to total firm debt (in percent).

In the short-term debt model, firms roll over the entire stock of debt each period. Through the bond market, they fully internalize all expected costs of default. In order to generate a leverage ratio of 27.2%, the default cost has to be lower than in the benchmark economy with debt dilution and debt overhang: $\xi = 0.21$.

The financial reform lowers the default cost ξ by 1/6. The upper part of Table 8 shows the steady state effects of this reform. Investment, employment, output, and welfare all increase. The same is true for leverage and credit spreads.

In the short-term debt model, leverage is chosen optimally by solving the trade-off between the tax advantage of debt and the full amount of expected default costs. As the default cost ξ is reduced, this trade-off shifts in favor of a higher leverage ratio. In the post-reform steady state, credit spreads are higher but the lower effective tax burden increases investment, employment, output, and welfare.

6.3.2. Benchmark Model with Short-term Debt and Long-term Debt

We now consider the same financial reform in our benchmark model with short-term debt and long-term debt. The results are shown in the lower half of Table 8. As before, the default cost ξ falls by 1/6. The results are strikingly different. The effects of the reform on investment, employment, output, and welfare are the opposite of the short-term debt model.

These differences arise because of debt dilution and debt overhang. Consider the equilibrium before the financial reform. In the benchmark model with debt dilution and debt overhang, firms do not internalize all default costs. In order to generate a leverage ratio of 27.2%, the default cost has to be higher than in the short-term debt model: $\xi = 0.48$. Because of the commitment problem studied above, firms have too much leverage. Credit spreads and debt issuance costs are too high relative to the tax advantage of debt. Any additional increase in leverage pushes firms further away from the optimal leverage ratio.

Now consider the effect of a reduction in ξ. In both models, this reduces the cost of debt financing. This is beneficial. Firms respond by increasing leverage which implies higher default risk and credit spreads. In a model of debt dilution and debt overhang, this has an additional effect. Increased default risk renders debt dilution and debt overhang more severe. The marginal cost of long-term debt rises and firms respond by choosing a lower long-term debt share. But this entails higher debt issuance costs.[21]

In the short-term debt model, the increase in debt dilution and debt overhang is absent. As Table 8 shows, in the benchmark model this effect is strong enough for the financial reform to backfire. Investment, employment, output, and welfare all fall.

This suggests that the standard assumption of short-term debt is not innocuous. A model without long-term debt misses economic effects which can result in misleading policy implications. Since models of firm financing play a key role in various applications (e.g. firm dynamics and misallocation, amplification and propagation of aggregate shocks, transmission of monetary policy), it is important to take the role of long-term debt into account.

7. Conclusion

Firm financing plays a key role in various macroeconomic applications. Economists use models of firm financing to study the amplification and propagation of aggregate shocks, and to assess the role of financial frictions in determining misallocation and total factor productivity. The standard assumption in these applications is that all debt is short-term. Our analysis suggests that this assumption is not without loss of generality. As we show in our final experiment, models which abstract from long-term debt may miss important economic mechanisms. This can result in misleading policy recommendations.

In this paper, we have aimed at studying debt dilution and debt overhang in as clear and transparent a setup as possible. We hope that the results described above will be useful in future work which studies debt dilution and debt overhang in more specific applications. For instance, the present setup could be augmented by adding equity issuance costs and persistent shocks at the firm level. Such an environment could be used to study the role of long-term debt for firm dynamics and misallocation. Adding aggregate shocks would allow examining how debt dilution and debt overhang move over the business cycle and possibly amplify and propagate fundamental shocks. For the case

[21] In the short-term debt model, the ratio of debt issuance costs $H(\tilde{b}^S, \tilde{b}^L, b)$ over total debt is always equal to η. The fact that average debt issuance costs are constant in the short-term debt model is therefore not specific to a parametrization with $\eta = 0$.

of nominal debt, Gomes et al. (2016) have shown that debt dilution and debt overhang have important implications for the transmission mechanism of monetary policy.

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A. Empirical Evidence on Debt Covenants

Myers (1977) argues that covenants which restrict a firm's dividend policy might partially address debt overhang. Debt dilution can be mitigated by leverage limits or minimum interest coverage ratios.

Nash et al. (2003) find that 15.66% of 364 investment grade bond issues in 1989 and in 1996 feature restrictions on additional debt. 8.24% include restrictions of the firm's dividend policy.[22] In a sample of 100 bond issues between 1999-2000, Begley and Freedman (2004), Table 2, p. 24, report that 9% contain additional borrowing restrictions. The percentage for dividend restrictions is identical (9%). Billett et al. (2007), Table III, p. 707, calculate that 22.8% of 15,504 investment grade bond issued between 1960 and 2003 had a covenant which restricts future borrowing of identical (or lower) seniority. 17.1% had a covenant which restricts dividend policy.[23] Reisel (2014), Table 4, p. 259, finds in a sample of 4,267 bond issues from 1989 - 2006 that 5.9% of investment grade bonds feature covenants which restrict additional borrowing or the firm's dividend policy.

These covenants are more common for junk bonds than for investment grade bonds (Billett et al., 2007), and they are more common for bank loans than for corporate bonds (Roberts and Sufi, 2009). Default rates are higher for these two debt classes. Our model shows that debt dilution and debt overhang are more severe if the risk of default is higher (see Section 4.9). This might be a reason for why covenants are more frequent for junk bonds and bank loans.[24]

Covenants which restrict the issuance of secured debt (with effective priority over existing debt) are more frequent than restrictions of unsecured debt (Billett et al., 2007). Secured debt is less than 20% of corporate debt (Compustat), and less than 20% of the number of bond issues (Billett et al., 2007). The majority of U.S. corporate debt is accounted for by senior unsecured corporate bonds (Gomes et al., 2016), and the majority of these bonds is investment grade.

In summary, the empirical corporate finance literature finds that less than 25% of U.S. investment grade corporate bonds include covenants which restrict the issuance of unsecured debt, and less than 20% feature restrictions with the potential to limit debt overhang.

[22] Of the 496 bonds considered in their Compustat sample, 120 feature additional debt restrictions (Table 3, p. 218). Of those, 57 bonds are investment grade (Table 4, p.220). It follows that out of a total of 364 investment grade bonds (Table 2, p.216), 15.66% feature additional debt restrictions. Out of the full sample, 99 bonds include restrictions of the firm's dividend policy (Table 3, p. 218). Of those, 30 bonds are investment grade (Table 4, p.220). It follows that 8.24% of the investment grade bonds in the sample feature dividend restrictions.

[23] Future borrowing of identical (or lower) seniority is restricted by funded debt restrictions (4.5%), subordinate debt restrictions (0.8%), and total leverage tests (17.5%). Dividend policy is restricted by dividend payment restrictions (12.1%) and share repurchase restrictions (5.0%).

[24] Even though financial covenants are frequently included in bank loan agreements, their practical relevance is reduced by the fact that in about two thirds of all covenant violations bank lenders take no action and there are no consequences for the borrowing firm (Roberts and Sufi, 2009).

B. Proofs of Analytical Results

Proof of Proposition 3.1

1. Substituting out $\tilde{b}$ with (11), the first order condition (16) associated to $\bar{\varepsilon}$ becomes:

$$[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c} - \varphi(\bar{\varepsilon})(1+c)\left(\frac{k+(1-\tau)\left[Ak^{\alpha}+\bar{\varepsilon}k-\delta k\right]}{1+(1-\tau)c} - b\right) = 0\,. \tag{36}$$

The marginal benefit of $\bar{\varepsilon}$ is increasing in b. It follows that the interior solution of $\bar{\varepsilon}$ increases in b.

2. Consider the marginal benefit of k as given on the left hand side of first order condition (15). If $\bar{\varepsilon}$ does not respond to the change in b, neither does k. But we know from Proposition 3.1 that $\bar{\varepsilon}$ is increasing in b. A marginal increase of $\bar{\varepsilon}$ affects the benefit of increasing k according to:

$$-\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\frac{1+(1-\tau)[A\alpha k^{\alpha-1}+\bar{\varepsilon}-\delta]}{1+(1-\tau)c} + [1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+r}\frac{\tau c}{1+(1-\tau)c}\,. \tag{37}$$

We consider a change in $\bar{\varepsilon}$ which is caused by an increase in b. Because $\bar{\varepsilon}$ is chosen optimally, the first order condition (16) holds:

$$[1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+r}\frac{\tau c}{1+(1-\tau)c} = \varphi(\bar{\varepsilon})(1+c)\frac{\tilde{b}-b}{k(1+r)}\,. \tag{38}$$

Using (11), it follows that an increase in b raises the benefit of increasing k if and only if:

$$\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\left[\frac{1+(1-\tau)\left[\frac{Ak^{\alpha}}{k}+\bar{\varepsilon}-\delta\right]}{1+(1-\tau)c} - \frac{b}{k} - \frac{1+(1-\tau)[A\alpha k^{\alpha-1}+\bar{\varepsilon}-\delta]}{1+(1-\tau)c}\right] > 0\,. \tag{39}$$

This is the case if and only if:

$$\frac{(1-\tau)\left[\frac{Ak^{\alpha}}{k}-A\alpha k^{\alpha-1}\right]}{1+(1-\tau)c} > \frac{b}{k}\,. \tag{40}$$

Or, equivalently:

$$\frac{(1-\tau)(1-\alpha)Ak^{\alpha-1}}{1+(1-\tau)c} > \frac{b}{k}\,. \tag{41}$$

3. By equation (11), leverage is

$$\frac{\tilde{b}}{k} = \frac{1+(1-\tau)\left[Ak^{\alpha-1}+\bar{\varepsilon}-\delta\right]}{1+(1-\tau)c}\,. \tag{42}$$

We know from Proposition 3.1 that $\bar{\varepsilon}$ is increasing in b. If k is falling in b, $Ak^{\alpha-1}$ is increasing because $\alpha < 1$.

Proof of Proposition 3.2

1. Consider the effect of b on k. The marginal benefit of k responds to an increase in b by $-dp/dk$.

2. Because $\tilde{b}$ is fixed, the effect of b on leverage $\tilde{b}/k$ directly follows from the effect of b on k.

3. It follows from equation (17) that:

$$\frac{d\bar{\varepsilon}}{dk} < 0 \quad \Leftrightarrow \quad 1 + (1-\tau)[A\alpha k^{\alpha-1} + \bar{\varepsilon} - \delta] > 0. \tag{43}$$

C. Derivation of Model Variables

Table 9 in the appendix defines key model variabels. In the following, we derive some of the expressions from Table 9 in more detail.

The total amount of firm debt is the present value of future debt payments:

$$\begin{aligned} D &= \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{1+r}\tilde{b}^L + (1-\gamma)\frac{\gamma+c}{(1+r)^2}\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\tilde{b}^L + ... \\ &= \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{1+r}\tilde{b}^L \sum_{j=0}^{\infty}\left(\frac{1-\gamma}{1+r}\right)^j = \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\tilde{b}^L . \end{aligned} \tag{44}$$

The long-term debt share of a given firm is the present value of debt payments due more than one year from today divided by D:

$$\begin{aligned} &\frac{1}{D}\left((1-\gamma)\frac{\gamma+c}{(1+r)^2}\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\tilde{b}^L + ...\right) \\ &= \frac{1}{D}\frac{\gamma+c}{\gamma+r}\frac{1-\gamma}{1+r}\tilde{b}^L . \end{aligned} \tag{45}$$

The short-term spread compares the gross return (in the absence of default) from buying a short-term bond with the riskless rate:

$$\frac{1+c}{p^S} - (1+r) . \tag{46}$$

The long-term spread compares the gross return (in the absence of default and assuming p^L is constant) from buying a long-term bond with the riskless rate:

$$\frac{\gamma+c+(1-\gamma)p^L}{p^L} - (1+r) = \frac{\gamma+c}{p^L} + 1 - \gamma - (1+r) . \tag{47}$$

Table 9: Key Variables

Capital-output ratio	$\frac{k}{y}$
Labor Share	$\frac{wl}{y}$
Total Debt	$D \equiv \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\,\tilde{b}^L$
Leverage: Debt / Assets	$\frac{D}{k}$
Share of Long-term Debt	$\frac{1}{D}\,\frac{\gamma+c}{\gamma+r}\,\frac{1-\gamma}{1+r}\,\tilde{b}^L$
Short-term Spread	$\frac{1+c}{p^S} - (1+r)$
Long-term Spread	$\frac{\gamma+c}{p^L} + 1 - \gamma - (1+r)$
Macaulay Duration	$\frac{1+r}{\gamma+r}$
Default Rate	$\Phi(\bar{\varepsilon})$
Share of Old Debt	$\frac{b}{D}$

The Macaulay duration is the weighted average term to maturity of the cash flows from a bond divided by the price:

$$\mu = \frac{1}{p_r^L} \sum_{j=1}^{\infty} j\,(1-\gamma)^{j-1}\,\frac{c+\gamma}{(1+r)^j} \;=\; \frac{c+\gamma}{p_r^L}\,\frac{1+r}{(\gamma+r)^2}\,, \tag{48}$$

where p_r^L is the price of a riskless long-term bond:

$$p_r^L = \sum_{j=1}^{\infty}(1-\gamma)^{j-1}\frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{r+\gamma}\,. \tag{49}$$

It follows for the Macaulay duration:

$$\mu = \frac{1+r}{\gamma+r}\,. \tag{50}$$

D. Firm Distribution

Figure 3 shows the stationary firm distribution μ^*. At each point in time, firms differ with respect to the existing stock of debt b. New entrants start without existing debt. Initially, they choose low values of $\tilde{b}^L$ and subsequently build up long-term debt until they default or until they reach the stable value of around $b/k^* = 14\%$. This is where the majority of firms finds themselves. A firm stays at that point until it defaults.

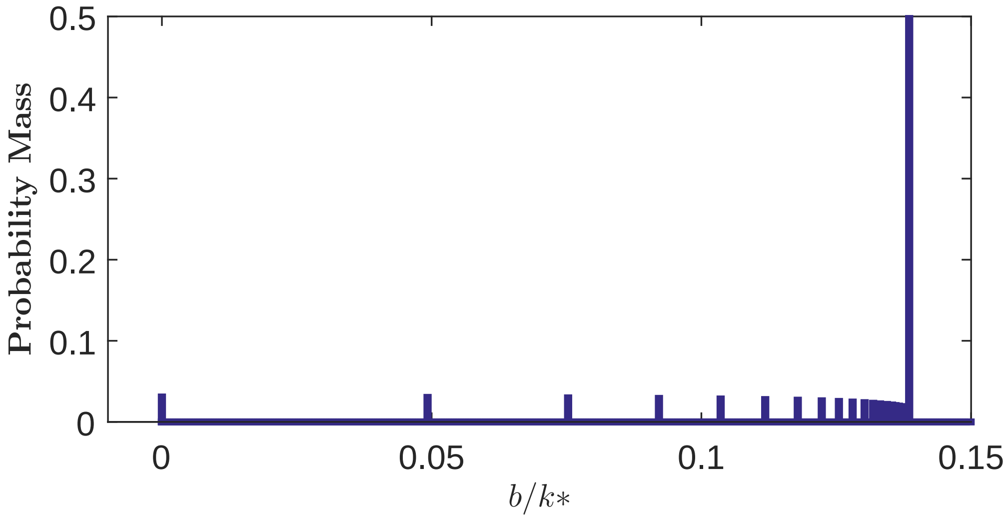

Figure 3: Firm Distribution

E. Data Appendix

In this section, we describe the construction of our data set used in Section 5. We use firm-level balance sheet data from Compustat and information on default events from Moody's Default & Recovery Database.

E.1. Firm Sample

We use annual Compustat data from 1984 to 2015. Moody's default data is available from 1988 onwards. Compustat includes firms listed on three U.S. exchanges: NYSE, AMEX, and Nasdaq. We exclude firms without a U.S. incorporation code and remove financial firms (SIC codes 6000-6999) and utilities (SIC codes 4900-4949). Following Covas and Den Haan (2011), we exclude firms that were part of a large merger or acquisition, delete four big U.S. companies which were strongly affected by an accounting change in 1988 (General Motors, General Electric, Ford, and Chrysler), firm-years with missing `Total Assets`, firm-years which violate the accounting identity (*total assets* $=$ *equity* $+$ *liabilities*) by more than 10 percent, and firm-years with less than \$500'000 in `Total Assets`. We restrict our analysis to firms which have at least three consecutive observations of OLD-Share and Leverage. All balance sheet variables are winsorized at one percent. This leaves us with 10,071 firms and 120,536 firm-years.

From Moody's Default & Recovery Database, we obtain information about default events of debt issues. This information includes a firm identifier, the time of default, and a brief description of the type of default (e.g. 'Missed interest payment', 'Chapter 11', or 'Distressed exchange'). We build a panel of default events and merge it with the balance-sheet data from Compustat. Because naming conventions differ, we employ an algorithm that computes the Levenshtein distance between firm names in Compustat and Moody's to facilitate matching. Out of $1,716$ firm-year observations of default events in Moody's, we can match 820 events involving 687 unique firms to the Compustat database. After cleaning the firm sample in the way described above, in our final sample we are left with 431 different firms with one or more default event during the sample period.

E.2. Variable Definitions

The empirical variables are constructed from the firm panel. We regress all variables on year-fixed effects and keep the residuals. This makes sure that year-fixed effects do not influence results in the pure cross-section of firms. In the panel, a given variable for a given firm is recorded as a time series. For each variable and each firm, we keep the median of this time series and thereby reduce the panel to a cross-section.

The $\texttt{OLD-Share}_t$ of a given firm in year t is defined as

$$\texttt{OLD-Share}_t = \frac{\texttt{debt\_long}_{t-1}}{\texttt{debt\_tot}_t} .$$

Here $\texttt{debt\_long}_{t-1}$ is the stock of firm debt in year $t-1$ with remaining term to maturity above one year. $\texttt{debt\_tot}_t$ is the stock of total firm debt in year t.

Leverage is debt over assets at book value:

$$\texttt{Leverage}_t = \frac{\texttt{debt\_tot}_t}{\texttt{at}_t} ,$$

where $\texttt{at}_t$ is the book value (at historical cost) of total firm assets.

Default events are from Moody's. For a given firm, the dummy variable Default is equal to one if Moody's records at least one default event for this firm during the sample period.

The variable $\texttt{Asset Growth}_t$ is constructed as first differences in log total firm assets:

$$\texttt{Asset Growth}_t = \log(\texttt{at}_t) - \log(\texttt{at}_{t-1})$$

As a robustness check we use the ratio of investment expenditures (i.e. capital expenditures $\texttt{capx}_t$) to total firm assets $\texttt{at}_t$.

As control variables, we use $\texttt{Tobin's q}_t$, $\texttt{Profitability}_t$, $\texttt{Tangibility}_t$, $\texttt{Firm Age}_t$, and $\texttt{Log Sales}_t$. We calculate $\texttt{Tobin's q}_t$ as

$$\texttt{Tobin's q}_t = \frac{\texttt{me}_t + \texttt{liab\_tot}_t}{\texttt{at}_t},$$

where $\texttt{liab\_tot}_t$ is total liabilities and $\texttt{me}_t$ is the market value of equity. We calculate it as $\texttt{me}_t = \texttt{csho}_t \cdot \texttt{prcc\_f}_t + \texttt{pstkl}_t$, where $\texttt{csho}_t$ is common shares outstanding, $\texttt{prcc\_f}_t$ is the firm's stock price at the end of year t, and $\texttt{pstkl}_t$ is the liquidating value of preferred stock. $\texttt{Profitability}_t$ is $\texttt{ib}_t/\texttt{at}_t$, that is, income before extraordinary items over total assets. $\texttt{Tangibility}_t$ is $\texttt{ppent}_t/\texttt{at}_t$, where $\texttt{ppent}_t$ is tangible fixed property (at historical cost) less accumulated depreciation. $\texttt{Firm Age}_t$ is the number of years since the firm's entry into the Compustat sample. We proxy firm size by $\texttt{Log Sales}_t$, i.e. the natural logarithm of net sales.

The Altman Z-score is computed as

$$\text{Z-score} = 1.2 \cdot \frac{\texttt{work}_t}{\texttt{at}_t} + 1.4 \cdot \frac{\texttt{ret}_t}{\texttt{at}_t} + 3.3 \cdot \frac{\texttt{ebit}_t}{\texttt{at}_t} + 0.6 \cdot \frac{\texttt{me}_t}{\texttt{liab\_tot}_t} + 1.0 \cdot \frac{\texttt{sales}_t}{\texttt{at}_t},$$

where $\texttt{work}_t$ is working capital, $\texttt{ret}_t$ is retained earnings, $\texttt{ebit}_t$ is earnings before interest and taxes, and $\texttt{sales}_t$ is net sales.