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Abstract

We develop a rational theory of *liquidity sentiments* in which the market outcome in any given period depends on agents' expectations about market conditions in future periods. Our theory is based on the interaction between adverse selection and resale considerations giving rise to an intertemporal coordination problem that yields multiple self-fulfilling equilibria. We construct "sentiment" equilibria in which sunspots generate fluctuations in prices, volume, and welfare, all of which are positively correlated. The intertemporal nature of the coordination problem disciplines the set of possible sentiment dynamics. In particular, sentiments must be sufficiently persistent and transitions must be stochastic. We consider an extension with production in which asset quality is endogenously determined and provide conditions under which sentiments are a *necessary* feature of any equilibrium. A testable implication of the model is that assets produced in good times are of lower average quality than those produced in bad times. We discuss the predictions of our theory within the context of several applications.

JEL: D82, E32, E44, G12.

Keywords: Sentiment; Liquidity; Asset Prices; Capital Reallocation; Business Cycles.

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1 Introduction

In a frictionless market, all gains from trade are realized and durable assets are held by parties that value them the most. As a result, competitive prices reflect not only the gains from trade today, but also all expected future gains from trade. In the presence of frictions, some gains from trade may remain unrealized, which depresses prices. In such an environment, there is a close connection between *liquidity*—the ease with which assets are reallocated to their most productive use—and asset prices. In this paper, we explore the extent to which rational expectations about future liquidity or *sentiments* can fluctuate over time and influence prices and quantities.

We analyze a dynamic asset market, in which asset owners are privately informed about the common value component or quality of their asset, which is either high or low. Gains from trade arise over time because owners experience idiosyncratic shocks to their private value of ownership. Potential buyers compete for assets, but they face a lemons problem as in Akerlof (1970), since they do not observe the quality of owners' assets. A buyer who purchases an asset in any given period becomes an owner in the next period. The important feature of our environment is that buyers must worry not only about the quality of an assets for which they bid, but also about market liquidity in the future if they want to resell. In order to emphasize our main results, it is useful to mention that when only one of the two considerations (adverse selection or resale considerations) are present, the equilibrium is unique and the economy features no aggregate volatility.

Our main result is that the interaction between adverse selection and resale concerns generates an intertemporal coordination problem, which gives rise to multiple self-fulfilling equilibria and generates endogenous volatility. The reason is that, when buyers anticipate the need to sell assets in the future, their willingness to pay for them today depends on their beliefs about future market conditions. If buyers believe that the market will be liquid and prices will be high in the future, they will bid more aggressively for assets today, and thus be able to attract a better pool of assets today. And conversely, if buyers expect illiquidity in the future, they will offer lower prices today and attract a worse (and smaller) pool of assets.

To illustrate these ideas, we first consider candidate equilibria that we term *non-sentiment equilibria*. A defining property of these equilibria is that market liquidity is constant over time, and thus so too are agents' expectations about future liquidity. This class includes an efficient equilibrium, in which all owners with low private valuation trade their assets immediately and, as a result, the prices, output and welfare are permanently high. It also includes an inefficient equilibrium, in which only low quality asset owners trade and, as a result, prices, output and welfare are permanently low. We show that there exists: (1) a lower bound, $\underline{\pi}$, on the proportion of high quality assets, π, such that the efficient equilibrium exists when $\pi \geq \underline{\pi}$, and (2) an upper bound, $\bar{\pi}$, such that the inefficient equilibrium exists when $\pi \leq \bar{\pi}$. Importantly, $\underline{\pi} < \bar{\pi}$ and, therefore, the two equilibria coexist for intermediate π.

We then consider *sentiment equilibria*, in which agents' (rational) expectations about future market liquidity depend non-trivially on a publicly observable sunspot process that is extrinsic to the economy (i.e., unrelated to fundamentals). Sentiment equilibria can be characterized by sets of "good" and "bad" states as well as a transition matrix for how the sentiment process evolves. In good states, agents have a positive outlook on future market conditions, which results in high prices today and, as a result, all gains from trade being realized. In bad states, agents correctly anticipate that the market is likely to be illiquid in the future, and as a result, the market is illiquid today. We demonstrate that the coexistence of multiple non-sentiment equilibria (i.e., $\pi \in (\bar{\pi}, \underline{\pi})$) is necessary and sufficient for the existence of sentiment equilibria. Moreover, the set of possible sentiment dynamics is disciplined by the primitives of the model. That is, unlike repeated static coordination problems, sentiment equilibria cannot be driven by an arbitrary stochastic process, but rather must exhibit certain properties. Most notably, the sentiment process must (i) be sufficiently persistent and (ii) feature non-deterministic transition dynamics.

We extend our model to incorporate endogenous asset production. This extension allows us to (i) determine the distribution of asset quality endogenously and (ii) provide conditions under which sentiments are a necessary feature of any equilibrium. In each period, a mass of producers supply assets. Each producer exerts unobservable costly effort, which affects the quality of the asset produced. After production, producers can trade their assets in the market. We first show that, in any sentiment equilibrium, the quality of assets produced is counter-cyclical, lower in good times and higher in bad times. Second, we show that when production costs are intermediate, any equilibrium must involve sentiments. That is, prices and liquidity *must be* endogenously volatile. Intuitively, if agents expect liquid markets and high prices to persist indefinitely, the quality of produced assets would be too low, and future buyers would not be willing to offer high prices, which renders the market illiquid in the future and contradicts expectations. Conversely, if agents expect illiquidity and low prices to persist indefinitely, the quality of assets produced would be too high, and future buyers would make aggressive offers thereby inducing a liquid market, which again contradicts expectations. Thus, non-sentiment equilibria are unsustainable.

Though our model abstracts from institutional details of specific markets, we discuss several interpretations of the model and explore the predictions of sentiment equilibria within the context of these applications. The first application is the (re)allocation of capital among firms. Within this context, the model's predictions match the stylized (and still somewhat puzzling) facts in Eisfeldt and Rampini (2006). In particular, that reallocation of capital is pro-cyclical, but the cross-sectional dispersion of productivity is counter-cyclical. In addition, aggregate TFP in our economy is endogenous and fluctuates with market sentiments. Thus, sentiments can be an important source of macroeconomic volatility.

Second, we consider an application in which, due to financial frictions, entrepreneurs must sell their existing projects in order to undertake new ones. In sentiment equilibria, good states involve high growth fueled by liquid secondary markets enabling all new investment opportunities to be pursued. In bad states, growth is lower and some new investments are not pursued because entrepreneurs forego them in favor of managing their existing project. This application of our model is related to work by Eisfeldt (2004), Kurlat (2013), and Bigio (2015). One important difference is that heterogeneity in project quality is short-lived in their models, whereas it is long-lived in ours. Thus, while the existing literature has shown that (short-lived) adverse selection can serve to amplify aggregate shocks, we demonstrate that (long-lived) adverse selection can, in fact, be the source of the aggregate shocks.

The model's predictions also match stylized facts in housing markets which exhibit strong positive correlation between prices and transaction volume and negative correlation between prices and time on the market (Mayer, 2011). Large movements in housing prices are difficult to explain based on fundamentals and are thus often interpreted as "bubbles". The sentiment equilibria in our model exhibit similar time-series behavior: prices and volume rise and fall despite no obvious changes in fundamentals. More generally, our model suggests that sentiments, liquidity, and prices are intrinsically connected even when agents are fully rational and prices are competitive. Thus, sentiments cannot be separated from fundamentals; both are essential for determining asset valuations.

1.1 Related Literature

Our paper naturally relates to the recent and growing literature that embeds adverse selection into dynamic economies.[1] This literature highlights that novel dynamics can emerge because the joint distribution of assets for sale and gains from trade changes over time. In a competitive

[1] See, for example, Eisfeldt (2004), Martin (2005), Kurlat (2013), Guerrieri and Shimer (2014), Bigio (2015), Chari et al. (2010), Gorton and Ordoñez (2014, 2016), Benhabib et al. (2014), Daley and Green (2016) and Fuchs et al. (2016).

framework, Janssen and Karamychev (2002) and Janssen and Roy (2002) show that when the gains from trade are persistent, past liquidity has a negative effect on current liquidity. Intuitively, if more of the gains from trade were realized yesterday, there will be more adverse selection in the market today. This can lead to deterministic liquidity cycles. Daley and Green (2016) and Fuchs et al. (2016) explicitly model re-trade considerations and construct equilibria with time-varying trading volume.[2] Those papers focus on the role of information (either past trading behavior or exogenous news) as a signal of quality about the distribution of assets. In contrast, we intentionally focus on a setting where uninformed agents' beliefs about asset quality are fixed over time. In our model, the novel dynamics emerge as a result of an intertemporal coordination and changes in the expectations of future market liquidity.

Other related work in the area considers markets with search frictions. For example, Chiu and Koeppl (2016) model the interaction between adverse selection and search frictions, and explore policies designed to alleviate the adverse selection problem when the fraction of low quality assets in the market is so large that there would be no trade absent an intervention. Maurin (2016) also considers such a setting and constructs equilibria with cycles where the proportion of high quality assets increases over time until pooling becomes feasible. Unlike our sentiment equilibria, these equilibria are deterministic and are not driven by intertemporal coordination. Finally, Mäkinen and Palazzo (2017) consider a more general search and matching technology that allows for congestion externalities. Their focus is on the additional negative effect (and policies to overcome it) from the fact that unshocked traders with low quality assets remain in the market creating congestion for shocked sellers.

Plantin (2009) and Malherbe (2014) are also related to our work, although the strategic considerations in their papers are contemporaneous rather than dynamic. Malherbe (2014) shows that multiple equilibria can arise due to complementarities in firms' cash-holding decisions. If a firm increases its cash-holdings in the first period, then it is less likely that a liquidity shock will generate an economic reason to trade. As a result, there will be smaller gains from trade and more adverse selection in the market. This, in turn, makes it more attractive for other firms to also hoard cash. Thus, there can be two equilibria, one in which firms expect other firms not to hoard cash and the second period market to work well, and another in which firms expect other firms to hoard cash and, as a result, the second period market dries-up. A similar mechanism is present in Plantin (2009). Although there is no cash-hoarding by firms in his setting, the number of investors who decide to buy the bond in the first period affects the potential market size for the bonds and hence their price in the future. The contemporaneous

[2] The importance of re-trade considerations in asset markets goes back to Harrison and Kreps (1978). See also Lagos and Zhang (2015, 2016) for recent related work within search-theoretic environment.

complementarity can lead to self-fulfilling market failures. It is important to highlight that equilibrium multiplicity in these papers arises due to static coordination failures whereas the multiplicity in our paper arises due to an intertemporal coordination failure.[3]

The intertemporal aspect of the coordination leading to multiplicity of equilibria relates our work with the broad literature on fiat money and rational bubbles.[4] Yet, there is an important difference between our work and most of this literature. In our model, the assets generate real output and the price is always pinned down by fundamentals. That is, we do not rely on a violation of the transversality condition in order to have positive prices. Moreover, the role of the assets in our economy is not to serve as a medium of exchange, as is the case in search-theoretic models of money in the spirit of Kiyotaki and Wright (1989).

Dating back to Shell (1977) and Cass and Shell (1983), there is a rich literature on sunspots in macroeconomics, where an extrinsic random variable can affect economic outcomes.[5] In our model, the nature of the intertemporal coordination problem combined with strategic trade puts discipline on the set of feasible sunspot processes. In particular, the sunspot process needs to be both stochastic and sufficiently persistent in order to generate intertemporal coordination on it; how persistent in turn depends on model parameters. A natural question is what drives sentiments? In the core of the paper we consider them being driven by a stochastic process extrinsic to the economy. However, as we discuss later, sentiments could also be driven by real economic variables, in which case one can interpret sentiments in our model as an amplification mechanism (Manuelli and Peck, 1992).

Recently, there has been a renewed interest among macroeconomists to understand how sentiments—in the form of correlated shocks to agents' information sets—can be drivers of aggregate fluctuations, as in the work of Angeletos and La'O (2013).[6] In this literature, dispersion of information among agents about aggregate states is an essential ingredient. We contribute to this literature by showing that, in the presence of adverse selection, sentiments which coordinate agents' expectations about future market conditions can generate aggregate fluctuations even when information about aggregates is common to all economic agents. Moreover, with endogenous production, these sentiments must be part of any equilibrium when production costs are not too extreme. We are not aware of a similar result in the literature.

[3]Indeed, as in the global games literature, Plantin (2009) obtains a unique equilibrium by introducing a noisy private signal about the probabilities of default of the bonds.

[4]See, for example, the early papers by Samuelson (1958), Tirole (1985), Weil (1987), Santos and Woodford (1997), and the more recent work by Martin and Ventura (2012) and Dong et al. (2017). Barlevy et al. (2015) provide a thorough overview of various theories of asset bubbles.

[5]See Shell (2008) for a recent survey of the sunspot literature and Benhabib and Farmer (1999) for a survey of the literature on indeterminacy.

[6]Other recent work includes papers by Lorenzoni (2009); Hassan and Mertens (2011); Benhabib et al. (2015).

The rest of the paper is organized as follows. In Section 2, we present the model. In Sections 3 and 4, we conduct our main analysis. In Section 5, we discuss several applications and extensions of the model. We conclude in Section 6. All proofs are in the Appendix.

2 The Model

Time is infinite and discrete, indexed by $t \in \{0, 1, ...\}$. There is a unit mass of indivisible assets, indexed by $k \in [0, 1]$, which are identical in every respect except for their common value or "quality," which we denote by $\theta_k \in \{L, H\}$. The probability that any given asset is of high quality is $P(\theta_k = H) = \pi \in (0, 1)$, which is also the fraction of high-quality assets in the economy. Assets are long-lived and qualities are fixed over time though our results are robust provided there is at least some persistence in asset quality (see Section 5.2).

There is a mass $M > 1$ of ex-ante identical agents, indexed by $m \in [0, M]$. Each agent can own at most one asset. We refer to agents who own assets as *owners* and to the rest as potential *buyers*. At each date t, an owner m, has a private value from asset ownership or "productivity," which is either low or high and is denote by $\omega_{mt} \in \{l, h\}$. The flow value that agent m derives from owning asset k at date t depends on both her private value (ω_{mt}) and the common value of the asset (θ_k), and it is given by $u(\theta_k, \omega_{mt})$. All agents are risk neutral and share a common discount factor $\delta \in (0, 1)$.

High-quality assets deliver a higher flow payoff $u(H, \omega) > u(L, \omega)$, and agents with a high private value of ownership derive a higher flow payoff, $v_\theta \equiv u(\theta, h) > c_\theta \equiv u(\theta, l)$. For this reason, when $\omega_{mt} = l$, we say that owner m is *shocked* since there are gains from transferring the asset to an unshocked agent. For simplicity, we assume each owner's status is *i.i.d.*, each period an owner is shocked with probability $P(\omega_{mt} = l) = \lambda \in (0, 1)$, which is also the fraction of shocked owners in the market in each period.[7] In addition, we assume that $c_H > v_L$, which implies that the common value component is sufficiently important that strategic considerations due to adverse selection remain relevant when the owner is shocked.

The market for assets is competitive and anonymous—in each period, at least two unshocked buyers are randomly matched with an owner, and they compete for the owner's asset a la Bertrand.[8] When an owner receives offers, she decides which (if any) offer to accept. If the owner rejects all offers, she continues to be an owner in the next period and is rematched with a new set of buyers. If the owner accepts an offer, then she sells her asset and enters the pool of

[7] That private value shocks are independent over time facilitates tractability, but is inessential for our main results; see Section 5.2.

[8] Perfect competition among buyers is not essential; see Section 5.2.

potential buyers. A buyer whose offer is accepted, acquires the asset and becomes an owner in the next period, whereas a buyer whose offer is rejected remains a buyer in the next period. We will assume throughout that the agents have "deep pockets," so that their budget constraints do not bind when bidding for assets.[9]

Trade in our economy may be hindered by the presence of asymmetric information. In particular, both asset quality and ownership status are privately known by the asset owner and not observable to buyers.[10] In dynamic environments with asymmetrically informed agents, the history of past trades, or lack thereof, can signal information about asset quality (Hörner and Vieille, 2009; Fuchs and Skrzypacz, 2015). For both parsimony and tractability, we will intentionally abstract from this possibility by assuming that the past trading history of individual assets is not observed by buyers. Therefore, when making offers at date t, each asset looks identical to each buyer. Furthermore, because the productivity shocks are *iid*, the joint distribution of (θ, ω) among asset owners at the beginning of any trading period is independent of the history of play and constant over time.[11]

The price of an asset at date t is the maximal bid of the buyers for that asset at date t. Since all assets appear identical to buyers, we restrict attention to equilibria in which the price is also the same across all assets at any date t. We denote this (common) price by P_t.

We refer to an owner with productivity status ω and an asset of quality θ as a (θ, ω)-owner. Given an owner's private information, the current price, and her expectation about future prices, the problem facing an owner at date t is when, if ever, to accept an offer. Let $V_t(\theta, \omega)$ denote the equilibrium payoff to a type (θ, ω)-owner at date t, which solves

$$V_t(\theta, \omega) = \max_{T \geq t} E_t \left\{ \sum_{s=t}^{T-1} \delta^{s-t} u(\theta, \omega_s) + \delta^{T-t} P_T \middle| \theta, \omega \right\}, \tag{1}$$

where E_t denotes the expectation operator conditional on the public history at date t. The Bellman equation is

$$V_t(\theta, \omega) = \max \left\{ P_t, u(\theta, \omega) + \delta E_t\{V_{t+1}(\theta, \omega') \middle| \theta, \omega\} \right\}, \tag{2}$$

where the next period's realization of a random variable is denoted with a prime. Clearly, it is

[9] E.g. in each period, each agent has a sufficiently large endowment of the numeraire good, and the agents' preferences over the numeraire good are linear.

[10] Asymmetric information about the private value component ω is not essential for our results, see Section 5.2.

[11] That is, θ_k and ω_{mt} are independently distributed with $Pr(\theta_k = H) = \pi$ and $Pr(\omega_{mt} = h) = 1 - \lambda$.

optimal for a (θ, ω)-owner to accept a (maximal) offer of p at date t if

$$(\theta, \omega) \in \Gamma_t(p) \equiv \{(\theta, \omega) : u(\theta, \omega) + \delta E_t\{V_{t+1}(\theta, \omega') \big| \theta, \omega\} \leq p\}. \tag{3}$$

Thus, Γ_t characterizes owners' strategy in period t. For convenience and without affecting the set of equilibrium payoffs, we will adopt the convention that a (θ, ω)-owner accepts an offer of p in period t with probability one if she is indifferent.

Our equilibrium notion requires that the buyers' offers must be optimal, which can formally be decomposed into two conditions. First, because buyers are identical, symmetrically informed, and compete in Bertrand fashion, they must earn zero expected profit conditional on their offer being accepted. That is, if $\Gamma_t(P_t) \neq \emptyset$ then

$$P_t = E_t\{v_\theta + \delta V_{t+1}(\theta, \omega') | (\theta, \omega) \in \Gamma_t(P_t)\}. \tag{4}$$

Second, we require that a buyer cannot profitably deviate from the equilibrium price by making a higher or lower price offer. Any offer $p < P_t$ will be rejected with probability one. Therefore, a profitable deviation does not exist provided that for all $p \geq P_t$,

$$p \geq E_t\{v_\theta + \delta V_{t+1}(\theta, \omega') | (\theta, \omega) \in \Gamma_t(p)\}. \tag{5}$$

Agents' expectations about the future affect the seller's willingness to trade at a given price today (through (2)-(3)) and the buyer's willingness to offer a given price today (through (4)-(5)). Of course, in equilibrium, these expectations must be rationalized by future behavior. A primary goal of this paper will be to characterize the extent to which these expectations can (rationally) vary over time and then study the implications for aggregate dynamics. In order to economize on notation and technical detail, we will restrict attention to equilibria in which these expectations are stationary with respect to a *sentiment process* that follow a homogenous Markov chain with a finite state space $\mathcal{Z} = \{z_1, z_2, ..., z_n\}$.

Definition 1 *A stationary rational expectations equilibrium (REE) is a value function V, beliefs E, a price function P, and a Markov chain Z with state space $\mathcal{Z}$ and transition matrix $Q = [q_{ij}]$, such that for each $Z_t \in \mathcal{Z}$:*

(i) $V_t(\theta, \omega) = V(\theta, \omega, Z_t)$ *solves* (1),

(ii) $\Gamma_t(p) = \Gamma(p, Z_t)$ *satisfies* (3),

(iii) *If* $\Gamma(P_t, Z_t) \neq \emptyset$ *then* $P_t = P(Z_t)$ *satisfies* (4),

(iv) For all $p \geq P_t$, (5) holds, and

(v) $E\{V(\theta, \omega, Z_{t+1})|Z_t = z_i\} = \sum_{j=1}^{n} V(\theta, \omega, z_j) q_{ij}$.

It is worth emphasizing several points about our equilibrium definition. First, the sentiment process is purely an extrinsic coordination device that is unrelated to the economic payoffs associated with asset ownership or joint distribution over (θ, ω) among owners. Second, while we have incorporated the sentiment process into our equilibrium definition, an equilibrium need not involve sentiments in any economically meaningful way (e.g., $\mathcal{Z}$ can be a singleton, see Section 3.1).

Henceforth, we will further restrict attention to equilibria in which the Markov chain is *irreducible*, meaning that is possible to get to any state starting from any state (though doing so may involve many transitions).[12] This restriction is not necessary for most of our results. It is also not without loss with respect to the set of possible dynamics. For example, it rules out equilibria with absorbing states, which can exist. However, assuming irreducibility simplifies exposition and formal statements of results without compromising the main economic insights.

2.1 Frictionless Benchmark

Before characterizing equilibria, we briefly remark on a benchmark economy in which asset quality is publicly observable.[13] Observability of asset quality suffices to ensure that allocations are efficient. The following proposition characterizes the unique equilibrium of this benchmark.

Proposition 1 (Observable Quality) *If asset qualities are publicly observable, then the equilibrium is unique, in it all assets are efficiently allocated and, for all t, the price of a θ-quality assets is $p_\theta = (1-\delta)^{-1} v_\theta$.*

For any given (observable) quality, buyers value the assets weakly more than the owners (strictly so if owners are shocked). Thus, in equilibrium, all assets must be reallocated from shocked owners to buyers, i.e., the asset allocation is efficient. Finally, because markets are competitive, a type-θ asset is priced at the present discounted value of v_θ.

[12] Formally, irreducibility requires that for any two states $z_i, z_j \in \mathcal{Z}$, there exists an integer $n < \infty$ such that $\Pr(Z_n = z_i | Z_0 = z_j) > 0$.

[13] Formally, our notion of equilibrium can be modified in two ways to accommodate the benchmark. First, prices are indexed by θ. Second, the condition of the expectation in both (4) and (5) (i.e, $(\theta, \omega) \in \Gamma_t(p)$) is replaced by θ.

3 Equilibrium

In this section, we characterize equilibria of our model. We start by providing a partial characterization of any equilibrium, which narrows the set of possible allocations. We then characterize the set of equilibria in which sentiments do not play a role and show that two such equilibria can arise, which are ranked in terms of both prices and welfare (Theorem 1). We then provide necessary and sufficient conditions under which sentiment equilibria exist, characterize their properties, and discuss the implications for aggregate dynamics (Theorem 2, and Propositions 4 and 5).

Proposition 2 *In any equilibrium and for all $Z_t \in \mathcal{Z}$*

$$V(L, l, Z_t) = V(L, h, Z_t) = p(Z_t) \leq V(H, l, Z_t) < V(H, h, Z_t).$$

An immediate implication is that, in every period and regardless of Z_t, (L, l)-owners sell their assets, whereas (H, h)-owners do not. Furthermore, it is without loss with respect to the set of equilibrium payoffs and prices to restrict attention to equilibria such that (L, h)-owners also trade in every period. Thus, with regard to which assets are traded and therefore how assets are allocated among agents, the only question is whether (H, l)-owners trade. If so, then assets are allocated efficiently in that period and we refer to the market as being *liquid*. If not, then some high quality assets are inefficiently retained by low productivity owners and we refer to the market as being *illiquid*.

3.1 Non-Sentiment Equilibria

We first consider a simple class of equilibria in which sentiments do not play a role: the allocations are the same in every period and prices are constant. These equilibria help illustrate the link between prices and liquidity as well as how an intertemporal coordination problem can lead to multiplicity.

Definition 2 *An equilibrium is a* **non-sentiment equilibrium** *if market liquidity is the same in every period with probability one.*

From Proposition 2, it follows that there can be two types of non-sentiment equilibria, depending on whether (H, l)-owners trade. We adopt the following definition in order to distinguish among them.

Definition 3 *A non-sentiment equilibrium features* **efficient trade** *if* (H, l)*-owners trade. Otherwise, if only low quality assets trade, we say that it features* **inefficient trade***.*

In the efficient trade equilibrium, all shocked owners trade and the assets are efficiently reallocated each period. Instead, in the inefficient trade equilibrium, the allocation is inefficient because the unproductive owners with high quality assets retain ownership. Given a candidate type of non-sentiment equilibrium, the equilibrium price and value functions are uniquely pinned down. Whether such a candidate is in fact an equilibrium then rests on whether conditions (i) and (iv) hold (i.e., whether owners or buyers have a profitable deviation).

The following theorem shows that a non-sentiment equilibrium always exists and provides necessary and sufficient conditions for each type of equilibrium.

Theorem 1 (Non-Sentiment Equilibrium) *There exist thresholds* $\underline{\pi} < \bar{\pi} \in (0, 1)$ *such that:*

1. *The efficient trade equilibrium exists if and only if* $\pi \geq \underline{\pi}$*,*

2. *The inefficient trade equilibrium exists if and only if* $\pi \leq \bar{\pi}$*.*

Notably, the two equilibria coexist when $\pi \in [\underline{\pi}, \bar{\pi}]$*. When they coexist, both the price and welfare are higher in the efficient trade equilibrium (welfare is higher in a Pareto sense).*

The unexpected part of the theorem is that the two equilibria coexist for a generic set of parameters.[14] One intuition for the multiplicity is that a form of coordination problem arises, albeit an *intertemporal* one. An informal explanation goes as follows. If buyers today expect that buyers in the future will offer higher prices, then their unconditional value for an asset is high and thus they are willing to make a high offer today. At this high price, an (H, l)-owner is willing to sell today. That is, the expectation of future market liquidity generates liquidity in the market today. Conversely, if buyers today expect that future buyers will offer low prices then their unconditional value today for the asset is low. Hence the highest (pooling) price they are willing to offer is also low and at this low offer, an (H, l)-owner prefers to retain her asset.

From the discussion above, it should be clear that dynamic considerations are essential for this coordination problem to arise. The next proposition shows that the parameter region where multiple equilibria arise expands when agents care more about the future, but vanishes as they become arbitrarily impatient ($\delta \to 0$).

[14] In a static model (or our model with $\delta = 0$), there exists a single (i.e., non-generic) value of π such that both equilibria exist.

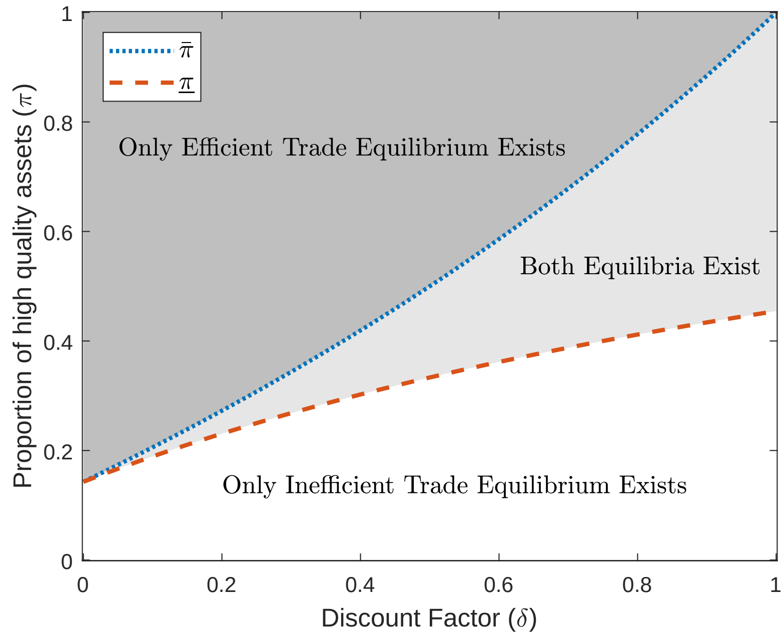


FIGURE 1: **Non-Sentiment Equilibrium Set and Role of Dynamics**. Unless stated otherwise, the parameters used are: $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $v_H = 1$ and $v_L = 0.45$.

Proposition 3 *The wedge $\bar{\pi} - \underline{\pi}$ is increasing in δ, and goes to zero as $\delta \to 0$. Thus, the equilibrium becomes generically unique as resale considerations vanish.*

Figure 1 illustrates this result graphically by plotting the thresholds $\bar{\pi}$ and $\underline{\pi}$ against the discount factor δ. As we can see, the region of multiplicity disappears as δ goes zero. Thus, the possibility of multiple equilibria in our setting hinges on the intertemporal coordination problem that arises because, when trading today, the agents care about the future market conditions.

In what follows, we show explicitly how to construct the non-sentiment equilibria, and then formalize the intuition above for why multiple equilibria arise in our setting. We begin with the construction of the efficient trade equilibrium.

Efficient trade equilibrium

Let p^{ET} and V^{ET} denote the candidate equilibrium price and value function in an efficient trade equilibrium. Recall that in an efficient trade equilibrium, all but the (H,h)-owners trade every period. Therefore, owners values are given by

$$V^{ET}(L,l) = V^{ET}(L,h) = V^{ET}(H,l) = p^{ET}, \tag{6}$$

$$V^{ET}(H,h) = v_H + \delta E\{V^{ET}(H,\omega')\}. \tag{7}$$

Since buyers have $\omega = h$, the zero-profit condition requires the

$$p^{ET} = \hat{\pi} V^{ET}(H,h) + (1-\hat{\pi})(v_L + \delta E\{V^{ET}(L,\omega')\} \tag{8}$$

where $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi+1-\pi}$ is the probability that the asset is of high quality, conditional on being sold. A buyer has the same value as an (H,h)-owner if the asset turns out to be of high quality (w.p. $\hat{\pi}$), but not the same value as an (L,h) owner if the asset turns out to be of low quality (w.p. $1-\hat{\pi}$). That is because (L,h)-owners sell their asset immediately whereas the buyer must consume the flow payoff for one period before then reselling it. Notably, the buyer understands that, conditional on his offer of p^{ET} being accepted, the probability the asset is of high quality is strictly smaller than π.

Combining (6)-(8), we arrive at the following analytical expression for the candidate price in an efficient trade equilibrium

$$p^{ET} = (1-\delta)^{-1}\left(\hat{\pi} v_H + (1-\hat{\pi})v_L + \delta(1-\hat{\pi})(1-\lambda)\frac{\hat{\pi}(v_H - v_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}\right). \tag{9}$$

To verify that such an equilibrium exists, we must rule out profitable deviations. It is clear that there are no deviations for the buyers, since any such deviation would need to attract the (H,h)-owner, which is impossible without the buyers making losses in expectation. For owners, it is sufficient to check that an (H,l)-owner does not benefit from a one-period deviation (i.e., rejecting p^{ET} for one period). That is,

$$V^{ET}(H,l) = p^{ET} \geq c_H + \delta E\{V^{ET}(H,\omega')\} \tag{10}$$

Letting $\kappa(\hat{\pi}) \equiv \hat{\pi} v_H + (1-\hat{\pi})\, v_L - c_H$ and using equations (6)-(8), this condition can be written as:

$$\kappa(\hat{\pi}) \geq \delta(1-\hat{\pi})\underbrace{E\left\{V^{ET}(H,\omega') - V^{ET}(L,\omega')\right\}}_{\Delta^{ET}}. \tag{11}$$

Thus, the efficient trade equilibrium exists when the static gain from selling in this period, captured by $\kappa(\hat{\pi})$, is greater than the future loss she suffers from selling her H-asset at a price that pools both types of assets, captured by Δ^{ET}; we provide closed-form expressions for Δ^{ET} in the Appendix. The threshold $\underline{\pi}$ in Theorem 1 is the value of π at which condition (11) holds with equality, which can be shown to be interior and unique.

Inefficient trade equilibrium

Let p^{IT} and V^{IT} denote the candidate equilibrium price and value function in an inefficient trade equilibrium. In the inefficient trade equilibrium, only owners of low quality assets trade. Therefore, (L, ω)-owner values are given by:

$$V^{IT}(L,l) = V^{IT}(L,h) = p^{IT}, \tag{12}$$

whereas (H, ω)-owners consume the output from their asset today and in the future,

$$V^{IT}(H,\omega) = u(H,\omega) + \delta E\{V^{IT}(H,\omega')\}. \tag{13}$$

Buyers understand that only low quality assets trade, but low quality assets can always be traded. The zero-profit condition requires

$$p^{IT} = \frac{v_L}{1-\delta}. \tag{14}$$

For existence of such an equilibrium, we must again rule out profitable deviations for the owners and the buyers. It is straightforward to see that there are no deviations for the owners, since H-owners strictly prefer to keep (recall that $c_H > v_L$), whereas L-owners prefer to trade. To rule out deviations for the buyers, it suffices to check that the buyers' profits are non-positive if they make an offer that attracts an (H, l)-owner, i.e., that

$$V^{IT}(H,l) \geq \hat{\pi} V^{IT}(H,h) + (1-\hat{\pi})\left(v_L + \delta E\{V^{IT}(L,\omega')\}\right), \tag{15}$$

Using (12)-(14), this condition becomes:

$$\kappa(\hat{\pi}) \leq \delta\,(1-\hat{\pi})\underbrace{E\left\{V^{IT}(H,\omega') - V^{IT}(L,\omega')\right\}}_{\Delta^{IT}}. \tag{16}$$

Thus, in contrast the the efficient trade equilibrium, the inefficient trade equilibrium exists when the static gain to the (H, l)-owner from selling in this period, captured by $\kappa(\hat{\pi})$, is *lower* than the future loss she suffers by selling her H-asset at a price that pools both types of assets, captured by Δ^{IT}; we provide a closed-form expression for Δ^{IT} in the Appendix. The threshold $\bar{\pi}$ in Theorem 1 is the value of π at which condition (16) holds with equality, which can also be shown to be interior and unique.

What is the source of the multiplicity?

The conditions for the existence of each type of equilibrium, i.e., (11) and (16), look remarkably similar except that the inequality is reversed. For the efficient trade equilibrium to exist, the expected difference between the value of a high and low asset in the next period must be sufficiently low, but it must be sufficiently high for existence of an inefficient trade equilibrium. Naively, it then seems that they cannot simultaneously hold except for non-generic cases. Yet, Theorem 1 clearly states that there is a positive (Lesbegue) measure of π such that both equilibria exist. The crucial observation is that the difference between the expected value of owning a high versus low quality asset depends on the structure of the equilibrium. In the efficient trade equilibrium, the difference is relatively small since assets are regularly pooled at a common price. Whereas in the inefficient trade equilibrium, H and L assets are never pooled which magnifies the difference in their expected value. In short, $\Delta^{ET} < \Delta^{IT}$, thus multiple non-sentiment equilibria exist whenever

$$\frac{\kappa(\hat{\pi})}{\delta(1-\hat{\pi})} \in (\Delta^{ET}, \Delta^{IT}). \tag{17}$$

3.2 Sentiment Equilibria

Thus far, we have considered non-sentiment equilibria, in which the agents' expectations about the future do not vary over time. But can there also exist equilibria in which expectations, prices, and allocations change over time? Our first result shows that the economy cannot feature deterministic variation in liquidity.[15] By Proposition 2, we can partition $\mathcal{Z}$ into two disjoint sets, which we will denote as $\mathcal{Z}_1$ and $\mathcal{Z}_0$, which correspond to the set of states in which the market is liquid and illiquid respectively. We will sometimes refer to $\mathcal{Z}_1$ as "good" states and $\mathcal{Z}_0$ as "bad" states.

Definition 4 *An equilibrium is a* **sentiment equilibrium** *if both $\mathcal{Z}_0$ and $\mathcal{Z}_1$ are non-empty.*

Proposition 4 *A sentiment equilibrium with deterministic transitions between good and bad states does not exist. That is, $z_i \in \mathcal{Z}_1$ ($\mathcal{Z}_0$) if and only if there exists z_j with $q_{ij} > 0$ such that $z_j \in \mathcal{Z}_1$ ($\mathcal{Z}_0$).*

Intuitively, suppose that the market is liquid at t, but will be illiquid at $t+1$ with probability one. Then, regardless of play in $t+2$ and beyond, the expected future market conditions are

[15] When productivity shocks are not *i.i.d.*, sentiment equilibria with deterministic transitions may exist (see Section 5.2).

worse starting from t then starting from $t+1$. Hence, the most a buyer is willing to offer in period $t+1$ is strictly higher than at t. But, if (H,l)-owners are not willing to trade in period $t+1$, then they certainly will not be willing to trade in period t. By a similar reasoning, we can rule out equilibria in which an illiquid market is deterministically followed by a liquid one.

Nevertheless, as we will show next, our economy can feature stochastic sentiment equilibria, in which fluctuations in prices, liquidity, and welfare are driven by changes in market sentiments.

Theorem 2 *A sentiment equilibrium exists if and only if* $\pi \in (\underline{\pi}, \bar{\pi})$.

The theorem shows that the conditions for multiplicity of non-sentiment equilibria are exactly the same as the conditions for the existence of sentiment equilibria. However, the theorem does not shed any light on the characteristics of sentiment equilibria, which we turn to next.

Given any candidate sentiment process, which is fully characterized by $(\mathcal{Z}_0, \mathcal{Z}_1, Q)$, it is straightforward to construct the associated candidate value functions and prices (see the proof of Proposition 5). Next, define

$$\Delta(z) \equiv E\left\{V(H,\omega',Z_{t+1}) - V(L,\omega',Z_{t+1})|Z_t = z\right\} \tag{18}$$

which is analogous to Δ^{ET} and Δ^{IT} (see Section 3.1), except that the expected difference in asset values can now depend on the current sentiment.

Proposition 5 *A candidate is a sentiment equilibrium if and only if*

$$\frac{\kappa(\hat{\pi})}{\delta(1-\hat{\pi})} \in \left[\max_{z\in\mathcal{Z}_1} \Delta(z), \min_{z\in\mathcal{Z}_0} \Delta(z)\right]. \tag{19}$$

As alluded to in Proposition 4, a crucial feature of any sentiment equilibrium is that the sentiments be sufficiently persistent. That is, in order for the market to be liquid today given some $z \in \mathcal{Z}_1$, agents must *expect* that the market is sufficiently likely to be liquid in the future, meaning $\Delta(z)$ is relatively small. Conversely, in order for the market to be illiquid today given some $z \in \mathcal{Z}_0$, agents must *expect* that the market is sufficiently likely to be illiquid in the future, meaning $\Delta(z)$ is relatively large. And of course, future market conditions must rationalize these expectations.

To illustrate the implications of Proposition 5, let us begin by considering a simple class of candidate sentiment equilibria, where $\mathcal{Z} = \{b, g\}$ and Z_t follows a symmetric first-order Markov process with persistence parameter $\rho = \mathbb{P}(Z_t = z|Z_{t-1} = z) \in (0,1)$. In the g state, agents coordinate on a liquid market (i.e., $\mathcal{Z}_1 = \{g\}$), whereas in the b state they coordinate on an

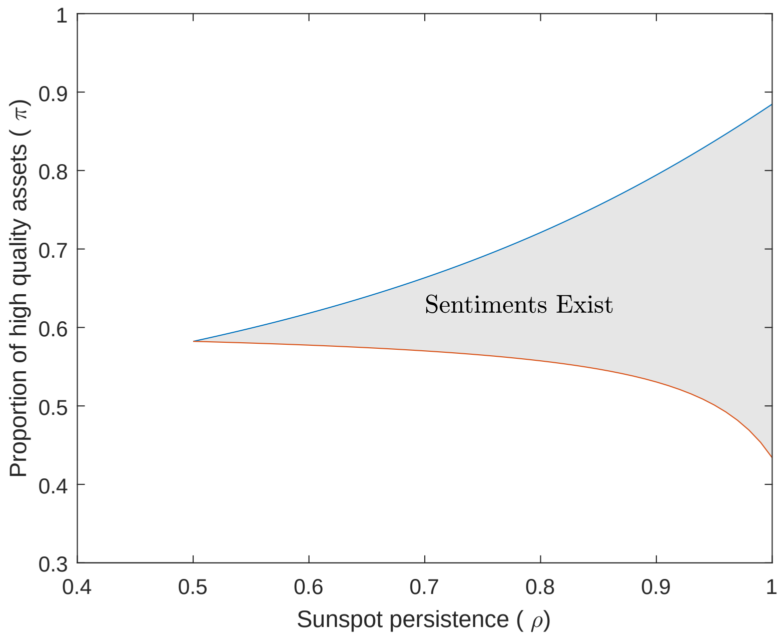


FIGURE 2: **Sentiment Equilibrium Existence Set**. The figure illustrates all the combination of the parameters π and ρ for which the binary-symmetric sentiment equilibrium exists.

illiquid market (i.e., $\mathcal{Z}_0 = \{b\}$). We refer to this class of processes as a binary-symmetric sentiment process with persistence ρ.

Corollary 1 *A sentiment equilibrium with a binary-symmetric sentiment process with persistence ρ exists if and only if $\pi \in (\underline{\pi}, \bar{\pi})$ and $\rho \geq \bar{\rho}$, where $\bar{\rho} \in \left[\frac{1}{2}, 1\right)$ depends on the primitives.*

This result emphasizes the role of intertemporal coordination for the existence of multiple equilibria in our setting. The realization of the sentiment must not only signal to the agents what to play today, but it must also be informative about how the equilibrium play will proceed in the future. These two objectives are accomplished precisely by a sentiment process that is sufficiently persistent. To understand why the persistence is needed, note that the more persistent is the process the larger is the expected difference in asset values in the illiquid state, $\Delta(b)$, and the smaller is the expected difference in asset values in the liquid state, $\Delta(g)$. As $\rho \to 1$, $\Delta(b) \to \Delta^{IT}$ and $\Delta(g) \to \Delta^{ET}$. Thus, for ρ large enough and $\pi \in (\underline{\pi}, \bar{\pi})$,

$$\frac{\kappa(\hat{\pi})}{\delta(1-\hat{\pi})} \in [\Delta(g), \Delta(b)],$$

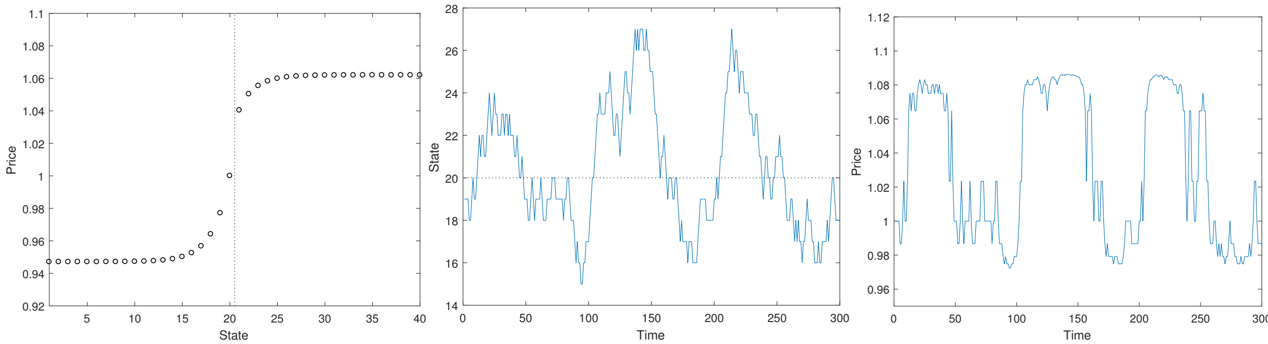
which, by Proposition 5, completes the argument.

The amount of persistence that a sentiment process needs depends on model parameters, as illustrated in Figure 2. Here, the shaded region depicts the combination of the parameters π and ρ for which a binary-symmetric sentiment equilibrium exists. The lower boundary of the region depicts the combinations of π and ρ for which the (H, l)-owner is indifferent between trading and retaining her asset in the good state. It is downward sloping because the pooling bid is higher both when the pool quality is higher and when the good state is expected to last longer. On the other hand, the upper boundary depicts the combinations for which the buyers make exactly zero profits by deviating and attracting the (H, l)-owner in the bad state. It is upward sloping because the buyers' willingness to pay for the asset pool is higher when the pool quality is higher but lower when the bad state is expected to last longer. In the interior, neither the owners nor the buyers want to profitably deviate from equilibrium play. Figure 2 emphasizes that, in contrast to static coordination problems, a sentiment equilibrium cannot be driven by an arbitrary stochastic process, but rather is disciplined by model parameters.

The binary-symmetric sentiment example is perhaps the simplest illustration of how sentiments can drive equilibrium behavior. Yet, the dynamics can be much richer. We illustrate the dynamics of an economy with a richer sentiment process in Figure 3. In this example, $\mathcal{Z} = \{1, ..., N\}$, the transition matrix has the form:

$$Q = \left\{ \begin{array}{ccccc} \rho & 1-\rho & 0 & ... & 0 \\ \frac{1-\rho}{2} & \rho & \frac{1-\rho}{2} & ... & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & ... & \frac{1-\rho}{2} & \rho & \frac{1-\rho}{2} \\ 0 & 0 & ... & 1-\rho & \rho \end{array} \right\}$$

with $\mathcal{Z}_0 = \{1, 2..., n^* - 1\}$ and $\mathcal{Z}_1 = \{n^*, n^* + 1..., N\}$. Thus, the market is liquid when $Z_t \geq n^*$ and it is illiquid otherwise. A feature of this example that gives rise to richer dynamics is that even if the market remains in a liquid state, agents expectations about future liquidity can change. For instance, the market is liquid when $Z_t = N$ and when $z_t = n^*$, but when $Z_t = N$, traders expect that the market will remain liquid for at least the next $N/2$ periods, whereas when $Z_t = n^*$, there is risk of illiquidity in the next period. As in the binary-symmetric example, such a sentiment equilibrium exists provided the parameter ρ is sufficiently high, a property that is reflected in the cyclical dynamics in Figures 3(b) and 3(c).



(a) Price function (b) Sentiment path (c) Price path

FIGURE 3: **Richer Sentiments**. The left-panel illustrates the price as a function of the state. The middle and right panels illustrate a simulated path for the state and price respectively. The parameters used for the sentiment process are $N = 40$, $n^* = 20$, $\rho = 0.4$.

4 Asset Production

In this section, we explore the role of sentiments in determining the distribution of asset quality in the economy. In order to do so, suppose that in each period there is a mass $\mu \in (0, 1)$ of "producers" each of whom can create an asset. Each producer chooses an investment level q at cost $c(q)$, with $c'(q) > 0$, $c''(q) \geq 0$. A producer who chooses an investment level q produces a H-quality asset with probability q and an L-quality asset with probability $1 - q$. Thus, more investment corresponds to a higher likelihood of creating an H-quality asset but also a higher cost. To keep the environment stationary, we assume that each period a fraction μ of assets mature before paying off and their owners exit the market. As a result, the factor with which each agent discounts asset payoffs becomes $\hat{\delta} = \delta(1 - \mu)$.

Each asset takes one period to produce: a producer in period t becomes the owner of the asset in period $t + 1$ and faces the same *i.i.d.* process of productivity shocks as other owners in the economy. We assume that the vintage of the asset is observable, which seems plausible and facilitates a tractable analysis.[16] In other words, in each period there will be a different market for each vintage of asset.

Given a candidate equilibrium and the current sentiment Z_t, the date-t producer chooses q_t to solve

$$\max_{q \in [0,1]} \left\{ \hat{\delta} \left(q E\{V(H, \omega, Z_{t+1} | Z_t)\} + (1 - q) E\{V(L, \omega, Z_{t+1} | Z_t)\} \right) - c(q) \right\}. \tag{20}$$

[16] If vintage is not observable, then the distribution of quality among all assets in the economy can vary over time, which introduces additional non-trivial dynamic considerations.

Thus, the first order condition for investment at date t is

$$c'(q_t) = \hat{\delta} \underbrace{\left(E\{V(H,\omega,Z_{t+1}) - V(L,\omega,Z_{t+1})|Z_t)\}\right)}_{\Delta(Z_t)} \tag{21}$$

Combining the first order condition with Proposition 5 gives us the following immediate implication.

Proposition 6 *If sentiments are part of an equilibrium with endogenous production, then the quality of assets created in good states is lower than the quality of assets created in bad states.*

Intuitively, if markets are more likely to be liquid next period, then producers have less incentive to create high quality assets in the current period. This finding has testable implications that we discuss in more detail in Section 5.1. However, it does not address the question of whether sentiment equilibria exist when asset production is endogenous. We turn to this question next.

Proposition 7 *When asset production is endogenous:*

(i) Efficient trade is an equilibrium $\iff c'(\underline{\pi}) \leq \underline{c} \equiv \Delta^{ET}\big|_{\pi=\underline{\pi}}$.

(ii) Inefficient trade is an equilibrium $\iff c'(\bar{\pi}) \geq \bar{c} \equiv \Delta^{IT}\big|_{\pi=\bar{\pi}}$.

(iii) If $c'(\underline{\pi}) > \underline{c}$ *and* $c'(\bar{\pi}) < \bar{c}$*, then any equilibrium is a sentment equilibrium (and a sentiment equilibrium exists).*

The first two statements are perhaps not very surprising. Because the incentive to invest is lower in the efficient trade equilibrium, it can only be sustained as an equilibrium when the marginal cost of production is sufficiently low. Conversely, because the incentive to invest is highest in the inefficient trade equilibrium, it can only be sustained as part of an equilibrium when the marginal cost of production is sufficiently high. The third statement is more interesting: when the marginal costs are intermediate, non-sentiment equilibria cannot be sustained—endogenous production requires that sentiments be part of any equilibrium.[17] Figure 4 illustrates this finding. If the marginal cost of production lies entirely in the shaded area (i.e., for all $\pi \in (\bar{\pi}, \underline{\pi})$), then any equilibrium must feature sentiments, whereas if the marginal cost curve lies above the upper line or below the lower line for some $\pi \in (\underline{\pi}, \bar{\pi})$, then non-sentiment equilibria can be sustained.[18]

[17] The conditions in part (iii) of Proposition 7, are not necessary for a sentiment equilibrium to exist because

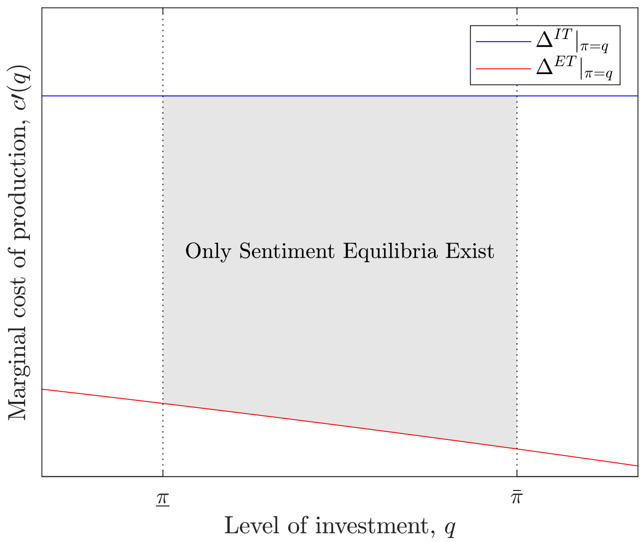


Figure 4: Illustration of Proposition 7.

5 Discussion: Applications and Extensions

5.1 Applications

Our model is intentionally stylized and we abstract from institutional features of specific markets. Therefore, it may be useful to provide several concrete interpretations of the model and discuss the implications of our results. By doing so, we also hope to demonstrate that all sentiment equilibria exhibit certain properties that generate testable implications for specific applications of the model.

Capital Reallocation. Perhaps the most natural application of the model is the re-allocation of existing capital among firms, which by some estimates accounts for a quarter of total investment by firms (Eisfeldt and Rampini, 2006). Within this context, the agents are firms with the technology to operate capital to generate consumption goods and the assets should be interpreted as units of capital. Capital is heterogeneous in quality; all else equal, higher quality capital generates more consumption goods. The idiosyncratic shocks in our model can be interpreted as firm-specific productivity shocks. In the first-best outcome, capital is always immediately reallocated to the most productive firms, in which case aggregate output is constant over time. However, in order to re-allocate capital from one firm to another, the two firms

sentiment equilibria can co-exist with non-sentiment equilibria. A necessary and sufficient condition for the existence of a sentiment equilibrium is that there exists $\tilde{\pi} \in (\underline{\pi}, \bar{\pi})$ such that $c'(\tilde{\pi}) \in (\Delta^{ET}, \Delta^{IT})|_{\pi=\tilde{\pi}}$.

[18] To be more precise, if the marginal cost function lies below the lower line (denoted $\Delta^{ET}|_{\pi=q}$) for some (all) $\pi \in (\bar{\pi}, \underline{\pi})$, then efficient trade is an (the unique) equilibrium. If the marginal cost function lies above the upper line (denoted $\Delta^{IT}|_{\pi=q}$) for some (all) $\pi \in (\bar{\pi}, \underline{\pi})$, then inefficient trade is an (the unique) equilibrium.

must agree to transact. And because capital is heterogeneous—all else equal, higher quality capital generates more of the consumption good—and its quality is privately observed by the firm employing it, such transactions do not necessarily materialize (i.e., if the market is illiquid).

In sentiment equilibria, aggregate output and aggregate productivity will be at (below) the first-best level in good (bad) states. In particular, the aggregate output is given by

$$Y_t = \iint u(\theta_k, \omega_{mt}) \gamma_t(k, m) dk dm,$$

where $\gamma_t(k, m)$ is the indicator for firm m operating capital unit k at date t. Aggregate productivity is just rescaled output since the mass of capital units is fixed. Sentiment equilibria exhibit fluctuations in aggregate output and productivity, despite the absence of aggregate shocks to fundamentals. By Propositions 4 and 5, periods of high (and low) output must be sufficiently persistent and transitions between high and low output states must be stochastic, which gives rise to (endogenous) business-cycle dynamics driven by rational changes in expectations about the future state of the economy.

Moreover, in good states, all firms operating capital have high productivity whereas in bad states some firms with low productivity operate capital. At the same time, capital reallocation is higher in good states than in bad ones. Thus, sentiment equilibria exhibit properties in line with stylized facts documented by Eisfeldt and Rampini (2006); capital reallocation is pro-cyclical while the dispersion in productivity is counter-cyclical.

New Investment with Financial Frictions. Rather than gains from trade arising from re-allocating existing capital among firms, suppose instead that the gains arise from a difference in investment opportunities. To be more specific, interpret the agents in our model as entrepreneurs who can either manage existing projects or start new ones. All entrepreneurs are equally good at managing existing projects. But, in order to start a new project, an entrepreneur must have a new idea, which arrives randomly (the idiosyncratic shock). Due to frictions in financial markets, in order for an entrepreneur to turn her new idea into a project, she must sell her existing project. When they arrive, all new ideas are equally good, however once an entrepreneur invests in a new idea and creates a project, its quality is realized and privately observed by the entrepreneur. Naturally, high quality projects create more of the consumption good. The efficient outcome is for all new ideas to be undertaken. However, an entrepreneur managing a high-quality project may decide not to undertake a new idea due to adverse selection in the market for existing projects.

This interpretation of the model is similar to that in the work by Eisfeldt (2004), Kurlat

(2013), and Bigio (2015). One important difference is that project quality is persistent in our model whereas it is short-lived in theirs. Indeed, as we discuss in Section 5.2, some degree of persistence in project quality is necessary for sentiment equilibria to exist. Thus, whereas the aforementioned literature has shown that adverse selection can amplify aggregate shocks, we show that it can, in fact, be the source of aggregate shocks. More specifically, in sentiment equilibria, both investment and growth will be driven by the market sentiment, which must evolve stochastically (Proposition 4). All new ideas will be undertaken in good states, but some will be foregone in bad states. Due to the (necessary) persistence of sentiments, periods of high or low investment and growth will persist in waves but will eventually end with a shift in the sentiment.

Because sentiments are persistent, not only will existing projects be more liquid in good states, but entrepreneurs will also find investing in new projects more profitable in good states because these projects are expected to trade more efficiently in the future. This has interesting implications when investment opportunities are not identical. For example, suppose the entrepreneur privately observes $q \in [0, 1]$, which corresponds to the probability that the idea will result in a good project if undertaken and suppose the distribution of q is $i.i.d.$ Then, while the quantity of investment will be higher in good states, both the average quality of new investment and the return on new investment will be higher in bad states (similar to Proposition 6).

Real Estate. As a third and final application of the model, consider a local real estate market. The assets are residential homes within a particular area and agents are households. Homes are heterogeneous in quality, which is privately observed by the household who owns and occupies it. The flow payoff corresponds to the utility or consumption value a household experiences from living in the home. All households experience a higher flow value from occupying a high-quality home, but whether the household is a good fit for a home (i.e., whether $\omega = h$) may change over time due to unforseen changes in jobs, preferences, or family composition (the idiosyncratic private value shock). In the efficient outcome, all households who are not a good fit immediately sell their homes to households with a higher flow value. Of course, due to the adverse selection problem, a household owning a high-quality home that is not a good fit may choose to continue to live in the home.

Within this context, the predictions of any sentiment equilibrium are as follows. First, prices and transaction volume are positively correlated, and they are negatively correlated with time-to-sale. In good times, prices and volume are high and owners with a reason to move do so quickly. Conversely, in bad times, prices and volume are low and (H, l)-households delay the sale of their home until market conditions improve. Second, real estate prices exhibit fluctuations even in the absence of aggregate shocks. These predictions are consistent with numerous empirical examinations of real-estate markets (see Mayer (2011) for a survey of this literature). Large movements in housing prices are difficult to explain based on fundamentals and therefore have been interpreted by many as "bubbles" driven by non-rational agents (Scheinkman and Xiong, 2003; Barberis et al., 2016). The time-series of prices in our sentiment equilibria exhibit similar behavior (see Figure 3) but obtain within a rational expectations framework.

Finally, a testable implication of our results in Section 4 is that homes produced in good times will be of lower average quality than those produced in bad times. We are not aware of any existing empirical evidence regarding this prediction.

5.2 Extensions

Here we discuss several assumptions of the model and explore some extensions and alternative specifications. Our discussion of these extensions will proceed at at an informal level. Formal results are available from the authors upon request.

Sentiments as an amplification mechanism. As discussed in Manuelli and Peck (1992), the early sunspot literature was motivated by the idea that small shocks to fundamentals are not very different from sunspots. They show that, in an overlapping generations endowment economy with money, small shocks to fundamentals can serve as the coordination device for different monetary equilibria. Furthermore, in the limit, as the underlying shocks have no direct effect on endowments, for every equilibrium of the pure sunspot economy with no shocks to endowments, there is a sequence of equilibria of the economy with risky endowments that converges to it. Our model can also be extended to allow for aggregate shocks to fundamentals which can then serve as a coordination device for agents' expectations regarding the future market conditions. Of course, as we have highlighted, these shocks will need to be persistent enough in order to coordinate expectations.

To illustrate this point, suppose that the flow payoff of assets is a function of some observable aggregate state $X_t \in \{G, B\}$, which follows a persistent and observable two-state Markov process. Concretely, consider the case where in state $X_t = G$ the flow payoff to a (θ, ω)-owner is $(1 + \varepsilon)u(\theta, \omega)$, whereas in state $X_t = B$ it is $(1 - \varepsilon)u(\theta, \omega)$ for some $\varepsilon \in (0, 1)$. Note that when $\varepsilon = 0$, we are back to our baseline setup without aggregate shocks. It is therefore straightforward to show that, for ε small enough, X_t can serve as the coordination device for a sentiment equilibrium. Such an equilibrium will display an amplification of fundamental shocks: although the shocks have a small effect on payoffs, they change expectations about future and therefore the pool of assets that are traded today, which can have a large impact on equilibrium prices, liquidity, output and welfare.

Persistent private value shocks. We have assumed that the idiosyncratic private value shocks are independent over time. We made this assumption in order to focus on the forward looking nature of the equilibrium and the role of sentiments. If we introduce persistence into the idiosyncratic shocks, then the equilibrium will depend not only on the agents' expectations about the future but also on the history of play. This is due to the fact that with positively correlated shocks, the joint distribution of (θ, ω) among asset owners is not necessarily stationary. When shocks are positively correlated over time, liquidity in the past is bad for liquidity today. A lot of trade yesterday implies most of the gains from trade have been realized and there is little reason to trade today.

However, the agents' concern about future market conditions and the intertemporal coordination problem stemming from it would still be present. Indeed, most of our results can be generalized to an environment with persistence in the idiosyncratic shocks. One notable difference is that, deterministic transitions between good and bad states can be part of an equilibrium when shocks are persistent (i.e., Proposition 4 no longer holds). As Maurin (2016) has shown in an environment with search, it is possible to create deterministic trading cycles. They are characterized by a few periods of illiquidity followed by one period of liquidity, and so on. During the periods of illiquidity, the average pool of potential sellers improves (i.e., the fraction (H, l)-owners increases). Eventually, the pool quality is sufficiently high that buyers are willing to offer a pooling price. Immediately after, there are few (H, l)-owners in the economy and thus the market is again illiquid until enough (H, l)-owners have accumulated.

Quality persistence and durability. We have also assumed that asset quality is (perfectly) persistent and that assets do not depreciate. While both of these assumptions can be relaxed, some degree of each is needed for the existence of sentiment equilibria. To see why some quality persistence is necessary, consider the expected difference in continuation values when asset quality can switch from one period to the next:

$$\Delta(z) = E\{V(\theta', \omega', Z_{t+1})|\theta = H, Z_t = z\} - E\{V(\theta', \omega', Z_{t+1})|\theta = L, Z_t = z\}$$

Fixing the agents' expectations about the future, the less persistent is asset quality, the smaller is $\Delta(z)$. As a result, expectations about future market conditions play a less important role in determining whether the market is liquid today. In the extreme when asset quality is *i.i.d.*,

$\Delta(z) = 0$ regardless of agents' expectations about the future and there is no scope for sentiments.

A simple way to capture asset depreciation is by incorporating a Poisson arrival at which the asset fully depreciates or matures. It is not difficult to show that this extension of the model in which assets depreciate with probability ρ each period is isomorphic to our model without depreciation and with a discount factor $\delta(1-\rho)$. Thus, a higher rate of depreciation has the same effect as a decrease in δ; faster depreciation reduces both $\bar{\pi}$ and $\underline{\pi}$ (see Figure 1) as well as the wedge between them (Proposition 3).

Competition. We have assumed that buyers are competitive and, hence, all rents go to the sellers. This assumption is convenient, but not necessary. Our main results also extend to the setting where buyers have some (or all) of the bargaining power. When bidding for an asset, buyers would still take into account that they may want to resell the asset in the future. Their expectations of future market conditions would continue to play a role in determining how aggressively they bid. As a result, the intertemporal coordination problem we have highlighted, which is key for the existence sentiment equilibria, remains present. The only qualitative change is that, with some bargaining power, the buyers may forego trade with the high-quality asset owners (even when they can break even by doing so), in order to extract rents from the low quality ones. The implication of less than perfect competition is that it becomes more difficult to sustain efficient trade and easier to sustain inefficient trade, therefore, the thresholds $\bar{\pi}$ and $\underline{\pi}$ in Theorem 1 (or Figure 1) would be strictly higher than under perfect competition.

Information Structure. Finally, we have assumed the idiosyncratic shocks are privately observed. Relative to the alternative environment in which these shocks are publicly observable, this assumption implies that the severity of the adverse selection problem is larger. With publicly observable idiosynchratic shocks, (L, h)-owners will be unable to pool with owners of high quality assets and are effectively excluded from the market. This improves the pool of traded assets, but leaves the mechanism underlying sentiment equilibria unchanged. Essentially, all of the results in Section 3 hold if $\hat{\pi}$ is replace by π. Since $\hat{\pi} < \pi$, this implies that the thresholds characterizing the set of equilibria in Theorem 1 are strictly lower than with unobservable private value shocks.

6 Conclusions

We study a dynamic market in which asset owners have private information about their asset quality and experience shocks to the private value of ownership, generating repeated gains from trade. The interaction of adverse selection with resale concerns gives rise to an intertemporal coordination problem that can sustain multiple self-fulfilling equilibria. We construct sentiment equilibria in which agents expectations about future liquidity vary over time and affect equilibrium prices and allocations. In sentiment equilibria, the price is equal to the expected fundamental value, yet prices display large fluctuations due to changes in sentiments resembling behavior that is often interpreted as "bubbles." Unlike static coordinations problems, the dynamics of sentiment equilibria are disciplined by model parameters. Notably, the sentiment process on which agents coordinate must be both stochastic and sufficiently persistent. When asset production is endogenous, our model predicts that the quality of produced assets is lower in good times than in bad times; furthermore, we show that for a wide range of parameters any equilibrium must involve sentiments. Finally, we discuss the predictions of our theory within the context of several applications.

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Appendix

Proof of Proposition 1. See text. ■

Proof of Proposition 2. From the zero profit condition (4), the equilibrium price must satisfy:

$$P_t = E\{v_\theta + \delta V_{t+1}(\theta, \omega')|(\theta,\omega) \in \Gamma_t(P_t)\} \geq E\{v_\theta + \delta V_{t+1}(\theta,\omega')|\theta = L\} \tag{22}$$

where the right-hand side is equal to the value of the (L, h)-owner if she were to keep the asset for a period. Thus, it must be that $(L, h) \in \Gamma_t(P_t)$ and $V_t(L, h) = P_t$. On the other hand, the (L, l)-owner has a weakly lower value than the $(L, 1)$-owner since the quality of her asset is the same, but the payoff she derives while keeping it is lower. Hence, in equilibrium we must also have $(L, l) \in \Gamma_t(P_t)$ and $V_t(L, l) = P_t$. Finally, $V_t(H, \omega) \geq P_t$ holds trivially since the owner always has the option to trade at the equilibrium price, and we have that $(H, h) \notin \Gamma_t(P_t)$ because the low quality assets always trade and thus

$$P_t = E\{v_\theta + \delta V_{t+1}(\theta,\omega')|(\theta,\omega) \in \Gamma_t(P_t)\} < v_H + \delta E_t\{V_{t+1}(\theta,\omega')|\theta = H\} = V_t(H, h), \tag{23}$$

i.e. buyers cannot attract the (H, h)-owner without making losses in expectation. ■

Proof of Theorem 1. That there can at most be two types of non-sentiment equilibria follows from Proposition 2, which shows that there are only two possibilities depending on whether the (H, l)-owner trades or not.

<u>Efficient trade equilibrium.</u> The equations (6), (7), and (8) characterize the equilibrium owner values and asset price in candidate efficient trade equilibria. Since this system is linear, if an efficient trade equilibrium exists, there is only one of its kind. Moreover, this equilibrium exists if and only if inequality (10) is satisfied. Thus, combining (6) through (10), the efficient trade equilibrium exists if and only if:

$$(c_H - \hat{\pi} v_H - (1-\hat{\pi})\, v_L) + \delta\,(1-\hat{\pi}) \underbrace{\frac{(1-\lambda)\,(1-\hat{\pi})\,(v_H - v_L)}{1-\delta\,(1-\hat{\pi})\,(1-\lambda)}}_{\Delta^{ET}} \leq 0, \tag{24}$$

where $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi + 1 - \pi}$. The left-hand side is strictly decreasing in π, positive at $\pi = 0$ and negative at $\pi = 1$. Hence, the threshold $\underline{\pi} \in (0, 1)$ exists, is unique, and the efficient trade equilibrium exists if and only if $\pi \geq \underline{\pi}$.

<u>Inefficient trade equilibrium.</u> The equations (12), (13), and (14) characterize the equilibrium owner values and asset price in candidate inefficient trade equilibria. Since this is a system of

linear equations, if an inefficient trade equilibrium exists, there is only one of its kind. Moreover, this equilibrium exists if and only if inequality (15) is satisfied. Thus, combining (12) through (15), the inefficient trade equilibrium exists if and only if:

$$0 \leq (c_H - \hat{\pi} v_H - (1-\hat{\pi})\, v_L) + \delta\,(1-\hat{\pi}) \underbrace{\frac{(1-\lambda)\,(v_H - v_L) + \lambda\,(c_H - v_L)}{1-\delta}}_{\Delta^{IT}}, \tag{25}$$

where $\hat{\pi} \equiv \frac{\lambda \pi}{\lambda \pi + 1 - \pi}$. The right-hand side is strictly decreasing in π, positive when $\pi = 0$ and negative when $\pi = 1$. Hence, the threshold $\bar{\pi} \in (0,1)$ exists, is unique, and the inefficient trade equilibrium exists if and only if $\pi \leq \bar{\pi}$.

Existence and Multiplicity. Next, we show that $\underline{\pi} < \bar{\pi}$, which will establish that an equilibrium exists and that the two equilibria coexist whenever $\pi \in (\underline{\pi}, \bar{\pi})$. From (24) and (25), we have that $\underline{\pi} < \bar{\pi}$ if and only if:

$$\frac{(1-\lambda)\,(1-\hat{\pi})\,(v_H - v_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}|_{\pi=\underline{\pi}} < \frac{(1-\lambda)\,(v_H - v_L) + \lambda\,(c_H - v_L)}{1-\delta}, \tag{26}$$

but this inequality holds because, for any $\pi < 1$

$$\begin{aligned} \frac{(1-\lambda)\,(1-\hat{\pi})\,(v_H - v_L)}{1-\delta(1-\hat{\pi})(1-\lambda)} &\leq \frac{(1-\lambda)\,(v_H - v_L)}{1-\delta(1-\lambda)} \\ &< \frac{(1-\lambda)\,(v_H - v_L) + \lambda\,(c_H - v_L)}{1-\delta}, \end{aligned}$$

where we used that $c_H > v_L$.

Finally, we have shown in the text that the asset prices are strictly higher in the efficient trade equilibrium. But, since the asset prices are higher, it must be that the (L, ω)-owners are better off, the (H, l)-owner is better off by revealed preference, and the (H, h)-owner is better off since she becomes a (H, l)-owner with positive probability. Thus, the efficient trade equilibrium Pareto dominates the inefficient trade equilibrium. ■

Proof of Proposition 3. Consider the expressions defining the thesholds $\underline{\pi}$ and $\bar{\pi}$:

$$(c_H - \hat{\pi} v_H - (1-\hat{\pi})\, v_L) + \delta\,(1-\hat{\pi}) \frac{(1-\lambda)\,(v_H - v_L) + \lambda\,(c_H - v_L)}{1-\delta}|_{\pi=\bar{\pi}} = 0, \tag{27}$$

and

$$(c_H - \hat{\pi} v_H - (1-\hat{\pi})\, v_L) + \delta\,(1-\hat{\pi}) \frac{(1-\lambda)\,(1-\hat{\pi})\,(v_H - v_L)}{1-\delta\,(1-\hat{\pi})\,(1-\lambda)}|_{\pi=\underline{\pi}} = 0, \tag{28}$$

where in both cases the left-hand side is decreasing in π, since $\hat{\pi}$ is increasing in π.

First, note that

$$\lim_{\delta\to 0}\bar{\pi} = \lim_{\delta\to 0}\underline{\pi} = \frac{\frac{c_H - v_L}{v_H - v_L}}{\frac{c_H - v_L}{v_H - v_L} + \left(1 - \frac{c_H - v_L}{v_H - v_L}\right)\cdot\lambda},$$

and thus the equilibrium becomes generically unique as $\delta \to 0$.

Second, observe that (i) thresholds $\underline{\pi}$ and $\bar{\pi}$ coincide as $\delta \to 0$, (ii) $\frac{(1-\lambda)(1-\hat{\pi})(v_H - v_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}|_{\pi=\underline{\pi}} < \frac{(1-\lambda)(v_H-v_L)+\lambda(c_H-v_L)}{1-\delta}$ (see proof of Theorem 1), and (iii) $\frac{(1-\lambda)(1-\hat{\pi})(v_H-v_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}$ is decreasing in π. Therefore, $\bar{\pi}$ is increasing faster in δ than $\underline{\pi}$, and so the wedge $\bar{\pi} - \underline{\pi}$ is increasing in δ. ■

Proof of Proposition 4. Let $\kappa(\hat{\pi}) \equiv \hat{\pi}v_H + (1-\hat{\pi})v_L - c_H$. If $z_i \in \mathcal{Z}_1$, then by the same logic as in the construction of the efficient trade equilibrium, it must be that

$$\kappa(\hat{\pi}) \geq \delta(1-\hat{\pi})\underbrace{E\{V(H,\omega',Z_{t+1}) - V(L,\omega',Z_{t+1})|Z_t = z_i\}}_{\equiv\Delta(z_i)}.$$

Similarly, if the market is illiquid at date $t+1$ w.p.1. then it must be that for all j such that $q_{ij} > 0$,

$$\kappa(\hat{\pi}) \leq \delta(1-\hat{\pi})\underbrace{E\{V(H,\omega',Z_{t+2}) - V(L,\omega',Z_{t+2})|Z_{t+1} = z_j\}}_{\equiv\Delta(z_j)}.$$

Thus, we must have that $\Delta(z_j) \geq \Delta(z_i)$. Thus, let us assume that this is the case.

We will now show that $\Delta(z_i) > \Delta(z_j)$ for all j such that $q_{ij} > 0$, implying a contradiction. Because trade is inefficient w.p.1. in the next period starting from $Z_t = z_i$, and because $\Delta(z_j) \geq \Delta(z_i)$,

$$\begin{aligned}\Delta(z_i) &= (1-\lambda)v_H + \lambda c_H - v_L + \delta E\{\Delta(Z_{t+1})|Z_t = z_i\}\\ &\geq (1-\lambda)v_H + \lambda c_H - v_L + \delta\Delta(z_i)\\ &= \frac{(1-\lambda)v_H + \lambda c_H - v_L}{1-\delta}.\end{aligned}$$

On the other hand, because there is positive probability of efficient trade at some point in the future starting from $Z_{t+1} = z_j$,

$$\Delta(z_j) < \frac{(1-\lambda)v_H + \lambda c_H - v_L}{1-\delta}.$$

Therefore, $\Delta(z_i) > \Delta(z_j)$ for all j such that $q_{ij} > 0$. ■

Proof of Proposition 5. Let N denote the number of elements in $\mathcal{Z}$. Let $I_{\mathcal{Z}}$ denote the

$N \times N$ identity matrix and $1_{\mathcal{Z}}$ be the $N \times 1$ vector of ones. Next, let $I_{\mathcal{Z}_1}$ ($I_{\mathcal{Z}_0}$) be the matrix which coincides with I_N except that it has zeros on the diagonal entries that correspond to the states $z \in \mathcal{Z}_0$ ($z \in \mathcal{Z}_1$). It is straightforward to construct the candidate sentiment equilibrium prices $p = \{p(z)\}_{z\in\mathcal{Z}}$ and values $V(\theta,\omega) = \{V(\theta,\omega,z)\}_{z\in\mathcal{Z}}$ from equations (2)-(5) as follows:

$$V(H,l) = I_{\mathcal{Z}_1} \cdot p + I_{\mathcal{Z}_0} \cdot (c_H \cdot 1_{\mathcal{Z}} + \delta Q(\lambda V(H,l) + (1-\lambda) V(H,h))), \tag{29}$$

$$V(H,h) = v_H \cdot 1_{\mathcal{Z}} + \delta Q(\lambda V(H,l) + (1-\lambda) V(H,h)), \tag{30}$$

$$p = I_{\mathcal{Z}_1} \cdot (\hat{\pi} V(H,h) + (1-\hat{\pi})(v_L \cdot 1_{\mathcal{Z}} + \delta Q p)) + I_{\mathcal{Z}_0} \cdot (v_L \cdot 1_{\mathcal{Z}} + \delta Q p). \tag{31}$$

Thus, in order to establish the result, we only need to check that there are no profitable deviations in all states $z \in \mathcal{Z}$:

1. There are no profitable deviations for the owners if and only if:

$$I_{\mathcal{Z}_1} \cdot (c_H \cdot 1_{\mathcal{Z}} + \delta Q(\lambda V(H,l) + (1-\lambda) V(H,h))) \leq I_{\mathcal{Z}_1} \cdot p, \tag{32}$$

   i.e. $\forall z \in \mathcal{Z}_1$, the (H,l)-owner prefers to trade than keep her asset.

2. There are no profitable deviations for the buyers if and only if:

$$I_{\mathcal{Z}_0} \cdot (\hat{\pi} V(H,h) + (1-\hat{\pi})(v_L \cdot 1_{\mathcal{Z}} + \delta Q p)) \leq I_{\mathcal{Z}_0} \cdot V(H,l), \tag{33}$$

   i.e. $\forall z \in \mathcal{Z}_0$, the buyers cannot make positive profits by attracting the (H,l)-owner.

Next, as in the text, define:

$$\Delta(z) \equiv E\{V(H,\omega',Z_{t+1}) - V(L,\omega',Z_{t+1}) | Z_t = z\}. \tag{34}$$

Using the equilibrium prices and values above to solve for $\Delta \equiv \{\Delta(z)\}_{z\in\mathcal{Z}}$, we get:

$$\Delta = Q \cdot M \cdot v, \tag{35}$$

where

$$M = [I_{\mathcal{Z}} - (I_{\mathcal{Z}_1} \cdot (1-\lambda) \cdot (1-\hat{\pi}) + I_{\mathcal{Z}_0}) \cdot \delta Q]^{-1}, \tag{36}$$

$$v = I_{\mathcal{Z}_1} \cdot (1-\lambda) \cdot (1-\hat{\pi}) \cdot (v_H - v_L) \cdot 1_{\mathcal{Z}} + I_{\mathcal{Z}_0} \cdot ((1-\lambda) \cdot v_H + \lambda \cdot c_H - v_L) \cdot 1_{\mathcal{Z}}. \tag{37}$$

After some algebra, the conditions for no profitable deviations become:

$$I_{\mathcal{Z}_1} \cdot \delta \left(1-\hat{\pi}\right) \Delta \leq I_{\mathcal{Z}_1} \cdot \left(\hat{\pi} v_H + \left(1-\hat{\pi}\right) v_L - c_H\right) \cdot 1_{\mathcal{Z}}, \tag{38}$$

and

$$I_{\mathcal{Z}_0} \cdot \delta \left(1-\hat{\pi}\right) \Delta \geq I_{\mathcal{Z}_0} \cdot \left(\hat{\pi} v_H + \left(1-\hat{\pi}\right) v_L - c_H\right) \cdot 1_{\mathcal{Z}}, \tag{39}$$

which establishes the result. ■

Proof of Theorem 2. If $\pi \in (\underline{\pi}, \bar{\pi})$, then by construction the binary symmetric sentiment equilibrium exists whenever $\rho \geq \bar{\rho}$ (see Corollary 1). On the other hand, suppose that a sentiment equilibrium exists, and the sentiment process is Z_t that takes values in set $\mathcal{Z}$.

For any given $z \in \mathcal{Z}$, we can express $\Delta(z)$ recursively as follows:

$$\begin{aligned}\Delta\left(z\right) = &\sum_{z' \in \mathcal{Z}_1} \mathbb{P}\left(Z_{t+1} = z' | Z_t = z\right) \cdot \left(\left(1-\lambda\right) \cdot \left(1-\hat{\pi}\right) \cdot \left(v_H - v_L\right) + \delta \cdot \left(1-\lambda\right) \cdot \left(1-\hat{\pi}\right) \cdot \Delta\left(z'\right)\right) \\ &+ \sum_{z' \in \mathcal{Z}_0} \mathbb{P}\left(Z_{t+1} = z' | Z_t = z\right) \cdot \left(\lambda \cdot v_H + \left(1-\lambda\right) \cdot c_H - v_L + \delta \cdot \Delta\left(z'\right)\right).\end{aligned}$$

Since $\lambda \cdot v_H + (1-\lambda) \cdot c_H - v_L > (1-\lambda) \cdot (1-\hat{\pi}) \cdot (v_H - v_L)$, and because the Markov chain is irreducible, $\forall z \in \mathcal{Z}$:

$$\begin{aligned}\Delta\left(z\right) &< \left(1-\lambda\right) \cdot \left(1-\hat{\pi}\right) \cdot \left(v_H - v_L\right) + \delta \cdot \sum_{z' \in \mathcal{Z}} \mathbb{P}\left(Z_{t+1} = z' | Z_t = z\right) \cdot \Delta\left(z'\right) \\ &\leq \frac{\lambda \cdot v_H + \left(1-\lambda\right) \cdot c_H - v_L}{1-\delta} = \Delta^{IT}.\end{aligned}$$

Analogously, $\forall z \in \mathcal{Z}$, we have:

$$\begin{aligned}\Delta\left(z\right) &> \left(1-\lambda\right) \cdot \left(1-\hat{\pi}\right) \cdot \left(v_H - v_L\right) + \delta \cdot \left(1-\lambda\right) \cdot \left(1-\hat{\pi}\right) \cdot \sum_{z' \in \mathcal{Z}} \mathbb{P}\left(Z_{t+1} = z' | Z_t = z\right) \cdot \Delta\left(z'\right) \\ &\geq \frac{\left(1-\lambda\right) \cdot \left(1-\hat{\pi}\right) \cdot \left(v_H - v_L\right)}{1 - \delta \cdot \left(1-\lambda\right) \cdot \left(1-\hat{\pi}\right)} = \Delta^{ET}.\end{aligned}$$

Thus, the requirement that there be no profitable deviations for the owners is:

$$\frac{\kappa\left(\hat{\pi}\right)}{\delta\left(1-\hat{\pi}\right)} \geq \max_{z \in \mathcal{Z}_1} \Delta\left(z\right) \implies \frac{\kappa\left(\hat{\pi}\right)}{\delta\left(1-\hat{\pi}\right)} > \Delta^{ET}, \tag{40}$$

whereas the requirement that there be no profitable deviations for the buyers is:

$$\frac{\kappa\left(\hat{\pi}\right)}{\delta\left(1-\hat{\pi}\right)} \leq \min_{z\in\mathcal{Z}_0} \Delta\left(z\right) \implies \frac{\kappa\left(\hat{\pi}\right)}{\delta\left(1-\hat{\pi}\right)} < \Delta^{IT}. \tag{41}$$

But these are equivalent to requiring that $\pi \in (\underline{\pi}, \bar{\pi})$ (see Section 3.1). ■

Proof of Corollary 1. From Proposition 5, a binary symmetric sentiment equilibrium exists if and only if:

$$\Delta\left(g\right) \leq \frac{\hat{\kappa}\left(\hat{\pi}\right)}{\delta\left(1-\hat{\pi}\right)} \leq \Delta\left(b\right).$$

We show that this is equivalent to $\pi \in (\underline{\pi}, \bar{\pi})$ and $\rho \geq \bar{\rho}$ for some $\bar{\rho} \in \left[\frac{1}{2}, 1\right)$. We will do this in two steps.

(*i*) We show that $\Delta\left(g\right) \lesseqqgtr \Delta\left(b\right)$ if and only if $\rho \gtreqqless \frac{1}{2}$. This immediately implies that a candidate $\bar{\rho}$ must be greater than $\frac{1}{2}$.

(*ii*) We show that $\Delta\left(g\right)$ is decreasing and $\Delta\left(b\right)$ is increasing in ρ for $\rho \geq \frac{1}{2}$, and that $\lim_{\rho\to 1} \Delta\left(g\right) = \Delta^{ET}$ and $\lim_{\rho\to 1} \Delta\left(b\right) = \Delta^{IT}$. The existence of threshold $\bar{\rho}$ then follows from the fact that $\pi \in (\underline{\pi}, \bar{\pi})$ is equivalent to $\Delta^{ET} < \frac{\hat{\kappa}(\hat{\pi})}{\delta(1-\hat{\pi})} < \Delta^{IT}$.

For (*i*), define $\alpha \equiv (1-\lambda)\cdot(1-\hat{\pi})\cdot(v_H - v_L)$ and $\beta \equiv \lambda \cdot v_H + (1-\lambda)\cdot c_H - v_L$, where note that $\alpha < \beta$. We can express $\Delta(g)$ and $\Delta(b)$ as follows:

$$\Delta\left(g\right) = \rho\cdot\left(\alpha + \delta\cdot(1-\lambda)\cdot(1-\hat{\pi})\cdot\Delta\left(g\right)\right) + (1-\rho)\cdot\left(\beta + \delta\cdot\Delta\left(b\right)\right), \tag{42}$$

$$\Delta\left(b\right) = (1-\rho)\cdot\left(\alpha + \delta\cdot(1-\lambda)\cdot(1-\hat{\pi})\cdot\Delta\left(g\right)\right) + \rho\cdot\left(\beta + \delta\cdot\Delta\left(b\right)\right). \tag{43}$$

Combine (42) and (43) to get:

$$\Delta\left(b\right) - \Delta\left(g\right) = (1-2\rho)\cdot\left(\alpha - \beta + \delta\cdot(1-\lambda)\cdot(1-\hat{\pi})\cdot\Delta\left(g\right) - \delta\cdot\Delta\left(b\right)\right).$$

Clearly, $\Delta\left(b\right) = \Delta\left(g\right)$ if $\rho = \frac{1}{2}$. Next, if $\rho < \frac{1}{2}$, then:

$$\Delta\left(b\right) - \Delta\left(g\right) < (1-2\rho)\cdot\left(\alpha - \beta - \delta\cdot\left(\Delta\left(b\right) - \Delta\left(g\right)\right)\right),$$

and thus $\Delta\left(b\right) < \Delta\left(g\right)$. But, if $\rho > \frac{1}{2}$, then:

$$\Delta\left(b\right) - \Delta\left(g\right) > (1-2\rho)\cdot\left(\alpha - \beta - \delta\cdot\left(\Delta\left(b\right) - \Delta\left(g\right)\right)\right),$$

and thus $\Delta\left(b\right) > \Delta\left(g\right)$.

For (ii), differentiate (42) and (43) with respect to ρ to get:

$$0 = (1 - \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi})) \cdot \frac{d\Delta(g)}{d\rho} + (1 - \delta) \cdot \frac{d\Delta(b)}{d\rho},$$

$$\frac{d\Delta(g)}{d\rho} = \frac{(\alpha + \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot \Delta(g)) - (\beta + \delta \cdot \Delta(b))}{1 - \rho \cdot \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) + (1 - \rho) \cdot \delta \cdot \frac{1 - \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi})}{1 - \delta}}.$$

Thus, since from (i) we have $\Delta(g) \leq \Delta(b)$ for $\rho \geq \frac{1}{2}$, it follows that $\frac{d\Delta(g)}{d\rho} < 0 < \frac{d\Delta(b)}{d\rho}$ for $\rho \geq \frac{1}{2}$. Finally, it is clear that as $\rho \to 1$, $\Delta(g) \to \frac{\alpha}{1 - \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi})} = \Delta^{ET}$ and $\Delta(b) \to \frac{\beta}{1 - \delta} = \Delta^{IT}$. ■

Proof of Proposition 6. Follows from (i) the first order condition in (21), (ii) $c'' \geq 0$ and (iii) the fact that $\Delta(z_1) < \Delta(z_0)$ for any $z_0 \in \mathcal{Z}_0$ and $z_1 \in \mathcal{Z}_1$. ■

Proof of Proposition 7. (i) In the efficient trade equilibrium, production optimality requires that:

$$c'(\pi) = \hat{\delta} \cdot \Delta^{ET},$$

where $\hat{\delta} = \delta(1 - \mu)$, and recall that:

$$\Delta^{ET} = \frac{(1 - \hat{\pi})(1 - \lambda)(v_H - v_L)}{1 - \hat{\delta}(1 - \hat{\pi})(1 - \lambda)},$$

which depends on the actual quality of assets π and is decreasing in π. Since $c(\cdot)$ is convex, this defines a unique candidate quality π of assets produced. From Theorem 1, therefore, efficient trade equilibrium exists if and only if $\pi \geq \underline{\pi}$ or equivalently $c'(\underline{\pi}) \leq \hat{\delta}\Delta^{ET}|_{\pi = \underline{\pi}}$.

(ii) In the inefficient trade equilibrium, production optimality requires that:

$$c'(\pi) = \bar{c} \equiv \hat{\delta} \cdot \Delta^{IT},$$

and recall that:

$$\Delta^{IT} = \frac{(1 - \lambda)v_H + \lambda c_H - v_L}{1 - \hat{\delta}},$$

which is independent of π. Again, this defines a unique candidate quality π of assets produced. From Theorem 1, therefore, inefficient trade equilibrium exists if and only if $\pi \leq \bar{\pi}$ or equivalently $c'(\bar{\pi}) \geq \underline{c} \equiv \hat{\delta}\Delta^{IT}$.

(iii) It follows immediately that any equilibrium must feature sentiments if both $c'(\underline{\pi}) > \bar{c}$ and $c'(\bar{\pi}) < \underline{c}$. Next, we prove existence of sentiment equilibrium.

Consider a simple sentiment process Z_t that takes values in $\mathcal{Z} = \{g, b\}$, with $\mathbb{P}_t(Z_t = g) = \gamma \in (0, 1)$ for all t. It is straightforward that in such a candidate equilibrium $\Delta(g) = \Delta(b) = \Delta$,

where:
$$\Delta = \frac{\gamma \cdot (1-\hat{\pi})(1-\lambda)(v_H - v_L) + (1-\gamma)\cdot((1-\lambda)v_H + \lambda c_H - v_L)}{1 - \gamma \cdot \hat{\delta}(1-\hat{\pi})(1-\lambda) - (1-\gamma)\cdot \hat{\delta}},$$
and note that $\Delta \uparrow \Delta^{IT}$ as $\gamma \downarrow 0$ and $\Delta \downarrow \Delta^{ET}$ as $\gamma \uparrow 1$.

Production optimality requires that the quality of assets satisfy:
$$c'(\pi) = \hat{\delta} \cdot \Delta. \tag{44}$$
And, from Proposition 5, this candidate is an equilibrium if and only if
$$\frac{\kappa(\hat{\pi})}{\hat{\delta}(1-\hat{\pi})} = \Delta. \tag{45}$$
Observe that (a) for γ close to 1, by our assumption that $c'(\underline{\pi}) > \bar{c}$ and continuity we have that: $c'(\pi) = \hat{\delta} \cdot \Delta \Longrightarrow \frac{\kappa(\hat{\pi})}{\hat{\delta}(1-\hat{\pi})} < \Delta$, and (b) for γ close to zero, by our assumption that $c'(\bar{\pi}) < \bar{c}$ and continuity we have that: $c'(\pi) = \hat{\delta} \cdot \Delta \Longrightarrow \frac{\kappa(\hat{\pi})}{\hat{\delta}(1-\hat{\pi})} > \Delta$. Thus, by continuity, there exist π and γ such that both (44) and (45) hold and the candidate is an equilibrium. ■