Deciding on what to Decide

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Abstract

We study collective decision-making procedures involving the formation of an agenda of issues and the subsequent vote on the position for each issue on the agenda. Issues that are not on the agenda remain unsettled. We use a protocol-free equilibrium concept introduced by Dutta et al. (2004) and show for two prominent voting procedures that essentially any subset of issues may be excluded from the agenda in equilibrium. What is voted upon and what is not depends on the voters' preferences in a subtle manner, suggesting a high degree of instability. We also discuss further conditions under which our results about the variability of agendas result may be qualified. In particular, we study those cases where all issues will be put in the agenda.

Keywords: Issues, agendas, voting rules, equilibrium collections of continuation agendas

JEL-Classification: D71, D72.

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1 Introduction

We study collective decision-making situations where the same agents who will eventually take a vote on their position regarding certain issues, or a subset of them, are in charge of deciding on what issues to take a stand, and which ones to set aside. For example, the present members of a club or a learned society may propose candidates for election, and then vote on the admission or rejection of those who end up being on the ballot, according to a well established rule. Likewise, legislative bodies in charge of elaborating a legal text on a broad subject may first narrow down the set of issues that will go to the floor for a vote on what stand to take on them, while leaving other potential issues unsettled.

There is evidence that different pieces of legislation on the same subject can differ substantially in length and coverage, even at the highest level. This is true, for example, for countries' constitutions. The scope, which measures the percentage of 70 major topics from the Comparative Constitutions Project survey that are included in any given constitution, ranges from 0.21 in New Zealand to 0.81 in Zimbabwe, and the number of human rights in existing constitutions ranges from 2 in Brunei to 99 in Ecuador.[1] We provide a natural and endogenous explanation for these differences, and show how the variability of equilibrium sizes of the sets of issues that will go to the floor depends crucially on one aspect of the voters' preferences. If legislators distinguish between the decision to set aside an issue, and that of have it regulated in an unsatisfactory manner, then variable sizes of the sort we mention can arise. If they treat these two possibilities as being equivalent, all issues will be debated at equilibrium. Likewise, if all club members treat the fact that a candidate is not in the ballot as being the same that this candidate running and loosing, no variability arises, while if these two outcomes are differentiated, we may expect varying ballot sizes.

While exploring what forces drive the size of the set of issues that are addressed under different conditions, we obtain additional insights regarding the role of preferences and voting rules. In particular, we discover an additional way in which chairs become powerful under sequential voting rules. It is well known

[1]See the constitutional ranking statistics on http://comparativeconstitutionsproject.org/ccp-rankings.

that chairs can influence the collective decisions attained by these rules by altering the order of vote. In our context, we can add an additional dimension to that power: the chair can also, by appropriate choices of the voting order, determine whether some issues will or will not go to the floor.

Our work is part of a vast literature on agenda formation we will briefly review below. We certainly do not cover all the grounds that this literature touches upon, but we add aspects to it that we think are novel and different. Notice, to begin with, that the agents who choose what issues to debate are the same who will eventually decide on a position regarding them.[2] This makes our object of study very different than that of electoral contests, where candidates or parties decide what platforms to propose, but then have to rely on the votes of citizens.

In our context, we specifically study how the number and nature of the issues contemplated by a voting body depend on the voting rules to be used, and on the expectations of voters regarding how others will behave.[3] Specifically, we consider a group of voters and a set of potential issues. The alternatives voters face are vectors that describe, for each issue, one of three possible results of their actions: either the issue is not put to vote, or it is, and in that case, one of two positions is adopted. The distinction between issues and alternatives is crucial in all that follows. Issues are items that may be potentially discussed and on which positions may or may not be adopted. Following Riker we "treat as issues whatever some people think are issues" (Riker, 1993, p. 3). We do not enter on their origin or their characteristics, and concentrate on explaining why some of them go into the agenda, and others do not. An agenda, in our setting, is an ordered set of issues on which a position will be adopted after a vote, while an alternative is a vector indicating, for each issue, whether a vote will be taken or not and, if so, what position will be adopted.

Previous models treat alternatives as the primitives of the problem, and then allow for agents to add an alternative to a previous agenda without changing

[2]Our basic results would also obtain if only a part of the total set of voters was involved in the first stage of selecting issues or candidates, in the form of a nominating committee. But since that would make our conclusions easier to reach, we prefer to study the case where all voters play a role at all stages.

[3]Throughout the paper we use the terms voting rule and voting procedure interchangeably.

the nature of those that were already in. In our model, expanding the agenda by including an additional issue completely changes the set of alternatives faced by agents, since the size of the vector of issues to be voted on is increased. An essential feature of our model is that by adding issues to an existing agenda, individuals generate irreversible changes in the sets of actions that are available to others. This facilitates the analysis, because the set of available agenda proposals shrinks along history, and the one containing all issues becomes a terminal point. We consider this to be a realistic feature of our model, that we think was ignored in previous work. In particular, our results are not based on any cyclical pattern and differ from those obtained in the tradition of McKelvey's chaos results (McKelvey, 1979), and from recent work (see Vartiainen, 2014, and references therein) where appropriate modifications of the Banks set (Banks, 1985), and the introduction of history dependence generate predictions regarding stable agendas.

By contrast, our analysis of agenda formation is based on a protocol-free notion of equilibrium proposed by Dutta et al. (2004), that we formalize in the next section. Its main strength is that, unlike in extensive form games, their concept does not assume any a priori established order of moves for the players, and yet is not based on simultaneous play either, as would be the case in normal form games. Armed with this tool, which still allows for the use of one special type of backward induction, we prove that the exact set of issues that will be put to vote is highly dependent on the specification of the voting rule and the strategic considerations of voters. Specifically, given some natural assumptions regarding preference domains we obtain results of the following type: For a given voting procedure, propose an agenda of (almost) any length, and we'll show you a preference profile within the domain that has this agenda as its unique equilibrium. We prove such results for two prominent classes of voting procedures, the amendment procedure and voting by quota. Our results are specific about the reasons why certain issues may not reach the floor. They allow us to provide several sufficient conditions under which, in particular, the only equilibrium agendas are those where all issues will be discussed. Understanding when full agendas will arise is important because previous work by Dutta et al. (2004) showed that, in a different context, that will always be the case in equilibrium as long as the

voting rule is efficient. By contrast, efficiency alone is by no means sufficient to precipitate this same result in our case, since we treat separately the possibility that a position is not adopted on an issue because there is no vote, or because it was brought to the floor and defeated there.

Different sized agendas can emerge under many circumstances. But it is also true that the specific composition of equilibrium agendas hinges on rather volatile features of the voters' preferences, and therefore reflect a high level of potential instability. Hence, although our analysis allows for a notion of equilibrium agendas, it also indicates some of the reasons why societies may quickly abandon any equilibrium in favor of new objects of debate and conflict. This remark fits well with the warnings of noted political scientists regarding the ever-present potential for instability and change in political situations (Riker, 1982, 1993; Cox and Shepsle, 2007). Our analysis also points at an additional form of manipulation that may be in the hands of a chair, as we already announced. Typically, chairs can manipulate sequential voting procedures by adding new alternatives to the list of possible ones (see Moser et al., 2009, and Moser, Fenn et al., 2016) and/or by changing the order of vote once the set of alternatives is given. What we find is that, in addition, the chair may induce voters to change the set of issues (hence the characteristics of the set of available alternatives) under sequential voting procedures by simply announcing the order in which alternatives will be presented for a vote. This type of manipulation could be an additional instrument in the hands of a chairperson, even of one who cannot directly determine what issues or alternatives should be discussed.

The literature on agenda formation is rich, and the following overview is necessarily incomplete. The literature on political agendas as reviewed in Baumgartner (2001) is mainly descriptive. Previous theoretical work mostly considers a specific protocol for the agenda formation and the subsequent voting stage. Austen-Smith (1987), Banks and Gasmi (1987), Baron and Ferejohn (1989), Miller, Grofman, and Feld (1990), Duggan (2006) and Penn (2008) all assume that voting takes place only after the agenda has been built. By contrast, Bernheim, Rangel, and Rayo (2006) and Anesi and Seidmann (2014) analyse the case of "real-time" agenda setting, where any proposal is put to an immediate vote against the current default. In Eguia and Shepsle (2015) the bargaining protocol is endogenized

and chosen by the members of a legislative assembly before the agenda is formed. While all these papers consider the case of complete information like we do, Godefroy and Perez-Richet (2013) use an incomplete information framework to study how the majority quota used to place alternatives on the agenda affects agents' behavior and hence the likelihood to change the status quo. There are only few papers that do not rely on a specific bargaining protocol. Among the notable exceptions is Dutta et al. (2004) who study equilibrium agendas in a model with farsighted agents. Their main result is that the set of equilibrium outcomes for Pareto efficient voting rules coincides with the outcomes when all full agendas are considered. Unlike in their paper, and as we shall discuss later, we show that in our model equilibrium agendas may not contain all issues even if the voting rule is Pareto efficient.

Vartiainen (2014) completes a list of works including Anesi (2006) and Bernheim and Slavov (2009), that start from McKelvey's (1979) chaos theorem and add features to the model in order to avoid infinite cycles and to identify stable agendas. The spirit of his work is similar to ours in different aspects, but there are substantial differences as well. His analysis, like ours and Dutta et al. (2004), is based on a protocol-free equilibrium concept that is implicitly history dependent. However, by clearly distinguishing between issues and alternatives, we reduce the strategic possibilities of voters in a different manner. In these other papers, including Vartiainen (2014), voters can propose any new alternative that satisfies appropriate conditions, whereas we limit their actions to proposing issues, which in addition must not have been proposed before and cannot be eliminated from the floor after they have been added to the agenda. While this makes our model specific, it also adds an important element of descriptive realism, and suggests a very different technical approach, in the line of Dutta et al. (2004), departing for good reason from the approach based on modifications of the Banks set. Another difference with Vartiainen (2014) is that we assume that individuals can, by themselves, add issues to the floor, whereas in his paper agents can only add alternatives if they reach a majority. While we think our analysis could be modified to require the consensus of more than one agent, the difference still remains that the type of changes our voters can induce on the set of alternatives in the agenda are of a different kind. Hence, although appreciative of the existing literature,

we believe our paper contributes a new dimension to previous work.

Finally, let us also mention that there is also a related literature that focusses on strategic candidacy (Osborne and Slivinsky, 1996; Besley and Coate, 1997; Dutta et al., 2001, 2002). The main difference with models of agenda formation like ours is that in strategic candidacy problems, the agents who take the agenda formation decision, by choosing whether to run or not to run, are different from those who will eventually vote.

The outline of the paper is the following. In Section 2 we introduce our model and equilibrium concept. In Section 3 we present an example with a Pareto efficient voting rule where equilibrium agendas can be of arbitrary length. In Section 4 we first study sufficient conditions for equilibrium agendas to be full agendas and then provide general results about the variability of agendas for two prominent voting procedures. Section 5 concludes. All proofs are in the appendix.

2 The Model

We consider a group of $n \geq 2$ agents facing a given set of issues $\mathcal{K} = \{1, \ldots, K\}$ with $K \geq 2$. The group may decide to keep silent on some issues, thus leaving its position on it undefined, while being ready to take a vote on others, in which case one of two positions will be voted upon and adopted for each one of those. We denote by "$-$" the decision to leave an issue out of the voting floor, and by 0 and 1 the two possible positions on issues that are voted upon. Social alternatives are then K-tuples indicating, for each issue, whether or not it was the object of a vote, and, if so, which stand was adopted on it. Accordingly, the set of alternatives then is given by $X = \{0, 1, -\}^{\mathcal{K}}$ and every agent i is assumed to have a strict preference ordering $\succ_i$ on X. By $\mathcal{P}$ we denote the set of strict preference orderings on X.

Our model is supposed to capture key features of decision-making processes undertaken by voting bodies whose members are able to decide on which issues to adopt a definite position, and on which ones to stay silent. For example, the writers of a legal text decide to take a definite stand on certain issues among many,

while leaving the position on others open. The members of a learned society may choose not to nominate certain worthy candidates, while risking to expose others to a vote whose outcome may be negative. Part of these decisions, which are pervasive in practice, may be based on exogenous considerations. But here we want to emphasize that voters' interests may induce them to act strategically, and that their actions will be determinant in defining what issues make the voting agenda, and which ones are excluded. We identify circumstances under which agents will be ready to vote on all issues. This is a worthy reference point, but we also find that the circumstances where it arises are special ones. Hence, identifying other cases where some issues enter the agenda the agenda and others not is equally informative. Of course, this depends on several factors: the voting rule that is used, the potential preferences of agents and the conflict of interests among them.

We consider a two-stage decision making process. In the first stage agents decide which issues to bring to the floor. The result is an agenda, which not only records the set of issues that have been proposed by the agents, but also the order in which the issues have been proposed. After that, in the second stage, agents vote on what position to take on each of the issues of the agenda. Depending on the voting rule, agents may directly vote on the positions for all issues one after the other or they may vote simultaneously on the positions for all issues using a sequential voting procedure on the set of alternatives. In our examples and main results we will consider voting rules that only depend on the set of issues on the agenda, but not on their specific order. Yet, our model also allows for the case where the order of issues plays a role in the second stage.

Agendas

Let $m \in \{1, \ldots, K\}$. Then $a = (a_1, \ldots, a_m)$ with $a_l \in \mathcal{K}$ for $l = 1, \ldots, m$, and $a_l \neq a_{l'}$ for $l \neq l'$ is called an *agenda* of length m. The empty agenda $\varnothing$ where no issue is put to vote is defined to have length 0. By A^m we denote the set of agendas of length m, where $0 \leq m \leq K$, and by $A = \bigcup_{m=0}^{K} A^m$ we denote the set of all agendas.

Let $a \in A$. If an issue k is not on the agenda a, i.e. $k \notin a$, we call k a *free*

issue at a.[4] For $a \in A^m$, where $0 \leq m \leq K-1$, and $k \in \mathcal{K}, k \notin a$, (a,k) denotes the agenda $a' \in A^{m+1}$ with $a'_l = a_l$ for $l = 1, \dots, m$, and $a'_{m+1} = k$.

Once a given set of issues constitute the agenda, the only alternatives that may be attained after voting are those where society chooses either value 0 or 1 on those issues and remains non-committed on the remaining ones. Accordingly, we define the available set of alternatives at agenda a to be the union of all those alternatives that may be potentially chosen when the agenda is a, depending on the preferences of agents. Thus, for a given agenda $a \in A$ the set of *available alternatives* at a, $X(a)$, is given by

$$X(a) = \{x \in X \,|\, \text{for all } k \in \mathcal{K}, x_k \in \{0,1\} \text{ if and only if } k \in a\}.$$

Observe that $X(\varnothing) = \{(-, \dots, -)\}$.

Voting

A voting procedure specifies what alternative is chosen as a function of the agenda and the preferences of agents over alternatives. Formally, a voting procedure on some domain of preference orderings $\mathcal{D} \subset \mathcal{P}$ is a mapping $V : A \times \mathcal{D}^n \to X$ with $V(a,P) \in X(a)$ for all $a \in A$ and $P \in \mathcal{D}^n$. Notice that a voting procedure, in our definition, associates a single outcome to each preference profile and each agenda a. Also, observe that the voting procedure need not be sensitive to the ordering of issues in a, and may only depend on the set of issues in the agenda.

There are many ways in which one can specify voting rules. One of them is to propose a game form that is dominance solvable for each agenda in the sense of Moulin (1979), and to associate to each preference profile the unique Nash equilibrium outcome in undominated strategies of the game induced by that profile and the game form.[5] Another is to associate each agenda and each profile with the result of sincere voting under a sequential rule. In both cases it is well known that the same tree structure may lead to different outcomes

[4] Here and in what follows, for a given agenda $a = (a_1, \dots, a_m) \in A$ we write, for short, $k \in a$ ($k \notin a$) whenever $k = a_l$ for some $l \in \{1, \dots, m\}$ ($k \neq a_l$ for all $l = 1, \dots, m$).

[5] The Nash equilibrium in undominated strategies is obtained by iterative elimination of weakly dominated strategies, where all weakly dominated strategies of all agents are simultaneously eliminated at each stage. This is also known as sophisticated voting (see Farquharson, 1969).

depending on the order of vote on alternatives (see Barberà and Gerber, 2017, and references therein), and hence this order must be determined when defining the voting rule. Notice that the order of vote on these alternatives may not be related to the order of issues in the agenda. See Section 3 for specific cases of application. The definition of a voting rule may also be more direct, as when we introduce, later on, the class of voting by quota methods.

Agenda Formation

In the first stage, starting from the empty agenda agents can unilaterally add issues to the agenda. This process stops when either a full agenda $a \in A^K$ is reached or no agent wants to add further issues. Instead of modeling the agenda formation as an extensive of normal form game, where equilibrium agendas could potentially be very sensitive to the details of the game form, we follow Dutta et al. (2004) and consider an equilibrium collection of sets of continuation agendas defined as follows.

For $a \in A^m$, where $m \in \{0, 1, \ldots, K\}$, let $A(a)$ be the set of *continuation agendas*, i.e.

$$A(a) = \{a' \in A \,|\, a'_k = a_k \text{ for all } k = 1, \ldots, m\}.^6$$

Equilibrium collections of sets of continuation agendas express expectations about the agendas that will result starting from any given agenda a. Since issues are assumed to be added one after the other, expectations at agenda a have to be such that they either do not involve further additions of issues, or else are equilibrium continuations if one further issue is added to a (see condition (E1) below). Moreover, no further additions of issues to an agenda a are expected if and only if no agent would be interested in adding any additional issue after having reached a, in view of what the expected continuations would be (see condition (E2) below). Formally, for given $P \in \mathcal{D}^n$, an *equilibrium collection of sets of continuation agendas* is a collection $(CE(a, P))_{a \in A}$, where $CE(a, P) \subset A(a)$ for each $a \in A$, that satisfies the following two conditions for all $a \in A$:

(E1) $CE(a, P)$ is a nonempty subset of $\bigcup_{k \notin a} CE((a, k), P) \cup \{a\}$.

[6] Observe that $a \in A(a)$ for all $a \in A$, i.e. any agenda is a continuation agenda for itself.

(E2) $a \in CE(a,P)$ if and only if $V(a,P) \succ_i V(a',P)$ for all $a' \in \bigcup_{k \notin a} CE((a,k),P)$ and for all $i = 1,\ldots,n$.[7]

Observe that (E2) is a rather weak stopping requirement because an agent is assumed to stop adding issues to the agenda only if stopping is better than all equilibrium continuations reached when one further issue is added to the existing agenda. Nevertheless, as we will show, equilibrium continuations are not necessarily full agendas, even for restricted domains of preferences and very well behaved voting procedures.

In what follows, for a given equilibrium collection of sets of continuation agendas $(CE(a,P))_{a \in A}$, we will refer to the continuation agendas in $CE(a,P)$ for $a \in A$ as *equilibrium continuations*.

In order to reduce the potential multiplicity of equilibrium collections of sets of continuation agendas we impose a third condition, which we call *consistency* (cf. Dutta et al., 2004). To this end, for $a \in A$ we define an agenda $a' = (a,k,\ldots) \in A$ to be *rationalizable* (relative to a) if $a' \in CE((a,k),P)$ and there exists an agent i and $a'' \in CE(a,P)$ with either $a'' = (a,l,\ldots)$ with $l \neq k$ or $a'' = a$ such that $V(a',P) \succ_i V(a'',P)$. Hence, the continuation agenda $(a,k,\ldots)$ is rationalizable relative to a if it is an equilibrium continuation at (a,k) and if some agent can gain from reaching it rather than sticking to some other equilibrium continuation at a. An equilibrium collection of sets of continuation agendas then is defined to be *consistent* if it satisfies the following condition:

(E3) If $a' \in \bigcup_{l \notin a} CE((a,l),P)$ is rationalizable, then $a' \in CE(a,P)$. Conversely, if $a' = (a,k,\ldots) \in CE(a,P)$ and either $a \in CE(a,P)$ or $a'' = (a,l,\ldots) \in CE(a,P)$ for some $l \neq k$, then a' is rationalizable.

Thus, consistency requires that an equilibrium collection of sets of continuation agendas contains all rationalizable continuation agendas. Moreover, it only contains rationalizable continuation agendas subject to the following two exceptions:

[7] Notice that $V(a',P) \neq V(a,P)$ for all $a' \in \bigcup_{k \notin a} CE((a,k),P)$ since any such agenda a' contains at least one issue $k \notin a$ which implies that $(V(a,P))_k \neq (V(a',P))_k$.

The first is that the agenda a itself is an equilibrium continuation if all agents prefer to stop at a (condition (E2)). The second exception is when there is a unique equilibrium continuation $a' = (a, k, \ldots)$ at a which is then not required to be rationalizable. Observe, however, that the latter case only obtains if there is an agent who prefers continuing over stopping at a and if agents unanimously prefer a' to adding an issue different from k to agenda a.

We will be mainly interested in the agendas that would result in equilibrium when agenda formation starts from the point where all issues are still free. Hence, it is convenient to introduce the following terminology:

Definition 2.1 *Let $a^* \in A$ and $P \in \mathcal{P}^n$. Then a^* is a* ***(consistent) equilibrium agenda at P*** *if there exists a (consistent) equilibrium collection of sets of continuation agendas $(CE(a, P))_{a \in A}$ with $a^* \in CE(\varnothing, P)$.*

Although rather involved, the use of our proposed equilibrium notion poses no existence problems. Moreover, the characterization of families of equilibrium continuation agendas closely follows the steps of a backward induction argument that is quite analogous to the pruning procedure suggested by Arieli and Aumann (2015) for the case of subgame perfect equilibria. Since any full agenda is its own continuation, we can start by asking whether an agenda a that contains all issues but one satisfies the equilibrium requirements. If it does, its continuation full agenda will be pruned. If it does not, then the expectation that a is its own equilibrium continuation is pruned. That leaves us with a family of potential equilibria regarding agendas where at most all issues but one are considered. Then we can continue a similar pruning process for agendas containing all but two issues, and proceed in a similar manner until we reach the case where the agenda is empty and no issue is put to vote. The family of continuation agendas that survives the pruning process is an equilibrium.

Before we consider an example with two issues we record the following result which is a straightforward implication of condition (E1): If an agenda a^* is an equilibrium continuation at some agenda a, then it is an equilibrium continuation at every agenda along the path from a to a^*.

Lemma 2.1 *Let $V : A \times \mathcal{D}^n \to X$ be a voting procedure and let $(CE(a,P))_{a \in A}$ be an equilibrium collection of sets of continuation agendas for some $P \in \mathcal{D}^n$. If $a = (a_1, \ldots, a_m) \in CE((a_1, \ldots, a_l), P)$ for some $l \leq m \leq K$, then*

$$a \in CE((a_1, \ldots, a_k), P) \text{ for all } k = l, \ldots, m.$$

In particular,

$$a \in CE(a, P).$$

3 An Example

We consider the election of new members to a society. There are two candidates, 1 and 2, i.e. the set of issues is $\mathcal{K} = \{1, 2\}$. In this case "$-$" means that the corresponding candidate is not nominated, "1" means that the candidate is nominated and elected and "0" means that the candidate is nominated and not elected. The set of alternatives then is

$$X = \{(-,-), (0,-), (1,-), (-,0), (-,1), (0,0), (0,1), (1,0), (1,1)\}.$$

Let there be three agents with preference orderings on the set of alternatives given in Table 1, where the alternatives in the table are listed in the order of decreasing preference.

Suppose that for any agenda a voting follows the amendment procedure (Farquharson, 1969; Miller, 1977, 1980) for some exogenously given ordering of the attainable alternatives in $X(a)$. That is, if $(x_1, x_2, \ldots, x_N)$ is the given ordering of the alternatives in $X(a)$, then the first vote is over x_1 and x_2, the second vote is over the winner of the first vote and x_3, and so on until all alternatives in $(x_1, x_2, \ldots, x_N)$ are exhausted. In every pairwise vote the winner is selected according to simple majority voting and the winning alternative is the one that survives until the end. Notice that in our example the ordering of the attainable alternatives is relevant only if both issues are on the agenda.[8]

[8] If there is only one issue on the agenda, e.g. issue 1, then there are only two attainable alternatives, $(1,-)$ and $(0,-)$, and the voting outcome is independent of the ordering of these alternatives.

$\succ_1$	$\succ_2$	$\succ_3$
$(0,1)$	$(1,0)$	$(1,1)$
$(0,-)$	$(-,0)$	$(-,1)$
$(-,1)$	$(1,-)$	$(0,1)$
$(-,-)$	$(-,-)$	$(1,-)$
$(0,0)$	$(0,0)$	$(-,-)$
$(-,0)$	$(0,-)$	$(0,-)$
$(1,1)$	$(1,1)$	$(1,0)$
$(1,-)$	$(-,1)$	$(-,0)$
$(1,0)$	$(0,1)$	$(0,0)$

Table 1: Preference orderings $\mathcal{K} = \{1, 2\}$.

Agents are assumed to be sophisticated (Farquharson, 1969) and thus the voting outcome is given by the alternative chosen in an undominated Nash equilibrium, which we obtain by iterative elimination of weakly dominated strategies, where all weakly dominated strategies of all agents are simultaneously eliminated at each stage. Observe that the voting outcome under iterative elimination of weakly dominated strategies is unique if there is an odd number of agents all with strict preferences (see Moulin, 1979, and Barberà and Gerber, 2017). Moreover, it is well known that the amendment procedure is efficient (Miller, 1977, 1980; Barberà and Gerber, 2017).

In order to solve for the equilibrium agendas we first determine the voting outcome for any agenda that contains at most one alternative. At the empty agenda the outcome is the unique attainable alternative $(-,-)$, i.e.

$$V(\varnothing, P) = (-,-).$$

At agenda $a = (1)$ the outcome is

$$V((1), P) = (1,-),$$

because a majority of agents prefers $(1,-)$ over $(0,-)$, and at agenda $a = (2)$ the outcome is

$$V((2), P) = (-,1),$$

since a majority of agents prefers $(-,1)$ over $(-,0)$. Finally, we determine the voting outcome at the full agendas, $(1,2)$ and $(2,1)$, with attainable sets

$$X(1,2) = X(2,1) = \{(0,0),(0,1),(1,0),(1,1)\}.$$

Figure 1 shows the dominance relation on $\{(0,0),(0,1),(1,0),(1,1)\}$ that results from pairwise simple majority voting.

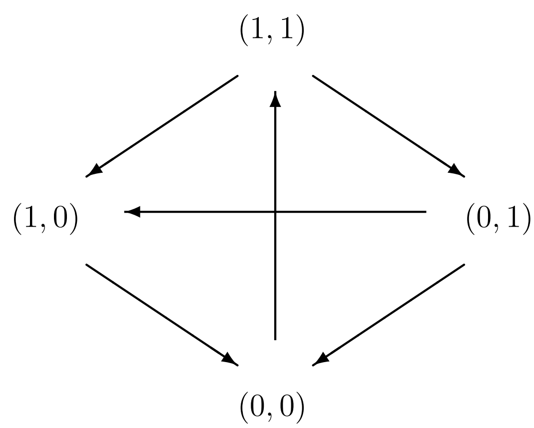

Figure 1: Dominance relation on $\{(0,0),(0,1),(1,0),(1,1)\}$ under pairwise simple majority voting. The arrows point to the alternatives that are beaten under simple majority voting.

It follows from the characterizations in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) that any of the attainable alternatives except for $(1,0)$ is the outcome of sophisticated sequential voting under the amendment procedure for some ordering of the alternatives in $X(1,2) = X(2,1)$.[9]

First consider the case where the ordering of the alternatives in $X(1,2) = X(2,1)$ is such that

$$V((1,2),P) = V((2,1),P) = (0,0).$$

[9]This can also be verified directly: The ordering $((0,0),(0,1),(1,0),(1,1))$ yields outcome $(0,0)$, the ordering $((1,1),(1,0),(0,0),(0,1))$ yields outcome $(1,1)$ and the ordering $((0,1),(1,0),(1,1),(0,0))$ yields outcome $(0,1)$. Finally, no ordering gives outcome $(1,0)$.

Observe that $(0,0)$ is Pareto dominated by $(-,-)$.

We now solve for the equilibrium collection of sets of continuation agendas. To do that we proceed backwards starting from the full agendas $(1,2)$ and $(2,1)$. By (E1) it must be that

$$CE((1,2),P) = \{(1,2)\} \text{ and } CE((2,1),P) = \{(2,1)\}.$$

Now consider agenda (1). By condition (E1), $CE((1),P)$ is a nonempty subset of $\{(1),(1,2)\}$. By condition (E2), $(1) \in CE((1),P)$ is ruled out since agent 1 strictly prefers the voting outcome under the equilibrium continuation $CE((1,2),P) = (1,2)$ over the outcome at agenda (1). Hence,

$$CE((1),P) = \{(1,2)\}.$$

Next consider agenda (2). By condition (E1), $CE((2),P)$ is a nonempty subset of $\{(2),(2,1)\}$. By condition (E2), $(2) \in CE((2),P)$ is ruled out since agent 2 strictly prefers the voting outcome under the equilibrium continuation $CE((2,1),P) = (2,1)$ over the outcome at agenda (2). Hence,

$$CE((2),P) = \{(2,1)\}.$$

Finally, consider the empty agenda. By condition (E1), $CE(\varnothing,P)$ is a nonempty subset of $\{\varnothing\} \cup CE((1),P) \cup CE((2),P) = \{\varnothing,(1,2),(2,1)\}$. Since all agents strictly prefer the voting outcome under the empty agenda over the outcome at any full agenda, (E2) implies that $\varnothing \in CE(\varnothing,P)$. Suppose by way of contradiction that $(1,2) \in CE(\varnothing,P)$. Then, since $\varnothing \in CE(\varnothing,P)$, condition (E3) implies that $(1,2)$ is rationalizable relative to the empty agenda $\varnothing$. However, no agent prefers the voting outcome under agenda $(1,2)$ over the outcome at the empty agenda $\varnothing$ or the outcome at agenda $(2,1)$. Hence, $(1,2)$ is not rationalizable which implies that $(1,2) \notin CE(\varnothing,P)$. Similarly, one proves that $(2,1) \notin CE(\varnothing,P)$. Therefore, we conclude that

$$CE(\varnothing,P) = \{\varnothing\}.$$

Thus, in this case the unique consistent equilibrium agenda is empty and no candidate is nominated and elected.

Now, we develop the same example but under the assumption that the exogenous order of vote under the amendment procedure is such that

$$V((1,2),P) = V((2,1),P) \in \{(0,1),(1,1)\}.$$

Observe that $(0,1)$ is the best alternative for agent 1 and $(1,1)$ is the best alternative for agent 3. Therefore, in this case only full agendas are equilibrium agendas because there is always one agent who is better off by adding an issue to the agenda that was a free issue before. Hence, for all agendas a, $CE(a,P)$ contains full agendas only. Thus, in this case any consistent equilibrium agenda is a full agenda, i.e. both candidates are nominated. However, depending on the order of vote under the amendment procedure, either both candidates or only candidate 2 is elected.

This example illustrates a number of notable points: (1) There are voting procedures and preference profiles for which equilibrium agendas are not full agendas. (2) The equilibrium collection of sets of continuation agendas can be very sensitive to the details of the voting rule, and in particular to the use of a fixed order of vote under sequential voting procedures. (3) As a consequence, if an agent can choose the order of vote under sequential procedures, he can not only influence the outcome for a given agenda, but also the set of issues that a society may choose to leave free.

4 General Results

From now on, we shall qualify our analysis by introducing specific assumptions on the domain of preferences under which voting rules will operate. Our first condition, that we call *betweenness*, is assumed throughout. It is made in order to provide the reader with one consistent interpretation of what it means to avoid sending an issue to the floor. But it will become apparent that our results do not depend on it, and that in fact it just ties our hands and strengthens our conclusions. In fact, the example we considered in the previous section already incorporates the assumption implicitly.

Definition 4.1 *A preference ordering $\succ$ on X satisfies **betweenness** if for all $k \in \mathcal{K}$, and for all $x \in X$, either*

$$(1, x_{\mathcal{K}\setminus\{k\}}) \succ (-, x_{\mathcal{K}\setminus\{k\}}) \succ (0, x_{\mathcal{K}\setminus\{k\}})$$

or

$$(0, x_{\mathcal{K}\setminus\{k\}}) \succ (-, x_{\mathcal{K}\setminus\{k\}}) \succ (1, x_{\mathcal{K}\setminus\{k\}}).[10]$$

Under betweenness, other things being equal, an agent strictly prefers the alternative that takes his preferred position on some issue k over leaving the position open, and he strictly prefers the latter to the alternative where the position is his worse. Observe that this assumption is compatible with the interpretation that agents perceive the resulting indeterminacy as creating a lottery between the competing positions, to be resolved in the future. Then, observe that our assumption will be satisfied whenever the agent's preference ordering can be represented by an expected utility function such that the utility of $(-, x_{\mathcal{K}\setminus\{k\}})$ is the expected utility of a lottery over the set $\{(0, x_{\mathcal{K}\setminus\{k\}}), (1, x_{\mathcal{K}\setminus\{k\}})\}$, where the agent assigns a positive probability to both outcomes, $(0, x_{\mathcal{K}\setminus\{k\}})$ and $(1, x_{\mathcal{K}\setminus\{k\}})$, that is independent of the corresponding probabilities for other open positions, if any.

For later use we also introduce two separability properties of preference orderings.

Definition 4.2

*(1) A preference ordering $\succ$ on $X = \{0, 1, -\}^{\mathcal{K}}$ is **separable** if for all $k \in \mathcal{K}$, $(x_k, x_{-k}) \succ (y_k, x_{-k})$ for some $x_{-k} \in \{0, 1, -\}^{\mathcal{K}\setminus\{k\}}$ implies that $(x_k, x'_{-k}) \succ (y_k, x'_{-k})$ for all $x'_{-k} \in \{0, 1, -\}^{\mathcal{K}\setminus\{k\}}$.*

*(2) A preference ordering $\succ$ on X is **additively separable** on X if there exist scalars $u_k(w) \in \mathbb{R}$ for $k \in \mathcal{K}$, and $w \in \{0, 1, -\}$ such that for $x, y \in X$,*

$$x \succ y \iff \sum_{k=1}^{K} u_k(x_k) > \sum_{k=1}^{K} u_k(y_k).$$

[10] For $x \in \{0, 1, -\}^{\mathcal{K}}$ and $\mathcal{M} \subset \mathcal{K}$ we denote by $x_{\mathcal{M}}$ the projection of x onto $\{0, 1, -\}^{\mathcal{M}}$. Moreover, for $x, y \in \{0, 1, -\}^{\mathcal{K}}$ and $\mathcal{M} \subset \mathcal{K}$, $(x_{\mathcal{M}}, y_{\mathcal{K}\setminus\mathcal{M}})$ is the vector $z \in \{0, 1, -\}^{\mathcal{K}}$ with $z_k = x_k$ for all $k \in \mathcal{M}$ and $z_k = y_k$ for all $k \in \mathcal{K} \setminus \mathcal{M}$.

If $\succ$ satisfies betweenness and additive separability, then for all issues $k \in \mathcal{K}$,

$$\max\{u_k(1), u_k(0)\} > u_k(-) > \min\{u_k(1), u_k(0)\}.$$

4.1 Full Agendas

We start our general analysis by exploring cases where all equilibrium agendas are full agendas (cf. Dutta et al., 2004). One obvious case where all equilibrium agendas are full agendas is when the voting procedure has the property that at any preference profile there is one agent for whom the outcome at any full agenda is this agent's most preferred alternative.[11] This agent will then keep adding issues until a full agenda is reached, i.e. any equilibrium agenda is a full agenda.

Rather than looking at specific voting procedures we may also ask under which conditions on individual preferences there will only be full agendas in equilibrium. To this end let $\mathcal{S}$ be the set of all preference orderings that satisfy separability and betweenness.[12] Let $\succ_i \in \mathcal{S}$. Then for all $k \in \mathcal{K}$ there exists a $w_k^i \in \{0, 1\}$ such that

$$(-, x_{\mathcal{K}\setminus\{k\}}) \succ_i (w_k^i, x_{\mathcal{K}\setminus\{k\}}) \text{ for all } x \in \{0, 1, -\}^K.$$

That is, w_k^i is the worst position on issue k for agent i. We then say that agent i is pessimistic about issue k if no position on that issue, i.e. "$-$", is almost as bad as getting the worst position, i.e. w_k^i, on issue k. Accordingly, in our finite setting agent i is pessimistic about issue k if, whenever i strictly prefers an alternative y over an alternative x, where she receives the worst position on issue k, then either y equals the alternative that is obtained from x by substituting the worst position on issue k by no position "$-$", or y is strictly preferred over that alternative. The formal definition of pessimism is as follows.

[11] Note that this does not imply that the voting procedure is dictatorial because the selected agent may change with the preference profile.

[12] Note that all preference orderings in Table 1 are separable and satisfy betweenness.

Definition 4.3 *Let* $\succ_i \in \mathcal{S}$. *Then* i *is* ***pessimistic*** *about issue* k *if for all* $x, y \in X$ *with* $x_k = w_k^i$,

$$y \succ_i x$$

implies that

$$y \succ_i (-, x_{\mathcal{K}\setminus\{k\}}) \quad \textit{or} \quad y = (-, x_{\mathcal{K}\setminus\{k\}}).$$

The following theorem shows that all equilibrium agendas are full agendas if agents' preferences satisfy separability and betweenness and if all agents are pessimistic about all issues.

Theorem 4.1 *Let* $V : A \times \mathcal{S}^n \to X$ *be a Pareto efficient voting procedure, i.e.* $V(a, P)$ *is Pareto efficient in* $X(a)$ *for all* $a \in A$ *and all* $P \in \mathcal{S}^n$. *If all agents are pessimistic about all issues, then any equilibrium agenda is a full agenda.*

The intuition for Theorem 4.1 is simple: If not deciding on an issue is almost as bad as getting the worst position on that issue, then nothing prevents an agent from adding further issues to an agenda. Hence only full agendas can be equilibrium agendas.

4.2 Variable Agendas

We will now provide a comprehensive analysis of agenda formation under two prominent voting procedures, the amendment procedure, which was already defined in Section 3, and voting by quota, where there is a majority vote on the position for every issue on the agenda. Notice that these procedures have different properties which follows from Barberà et al. (1991): Voting by quota is strategy-proof on the domain of separable preferences, but it is not Pareto efficient. By contrast, the amendment procedure is Pareto efficient, but it is not strategy-proof on any domain containing the set of separable preferences (and on the universal domain in particular).

For both these procedures we will show that apart from some minor qualifications, and unless strong restrictions on preferences are imposed (as we did in Theorem 4.1), for any subset $\mathcal{F}$ of the set of issues $\mathcal{K} = \{1, \ldots, K\}$ there exists a

profile of preference orderings $P = (\succ_1, \ldots, \succ_n) \in \mathcal{P}^n$, such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda at P. Thus, neither of the two procedures imposes any structure on the set of equilibrium agendas.

Amendment Procedure

Suppose voting is according to the amendment procedure for some ordering of the alternatives and simple majority is used throughout. We then have the following result:

Theorem 4.2 *Let $n \geq 3$ be odd and let $\mathcal{F} \subset \mathcal{K} = \{1, \ldots, K\}$ be such that $\#\mathcal{F} \neq 1$ if $K = 2$.*[13] *Then there exists a profile of separable preferences $P \in \mathcal{S}^n$ and some ordering of the alternatives in $X(a)$ for all $a \in A$, such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda a^* at P if voting is according to the amendment procedure for the given orderings of the alternatives at any agenda a.*

We complete our analysis of the amendment procedure by noting that if $K = 2$, then for any profile of separable preferences the set of free issues is either empty or contains all issues. Thus, Theorem 4.2 is tight:

Proposition 4.1 *Let $n \geq 3$ be odd and let $P \in \mathcal{S}^n$ be any profile of separable preferences. Let $K = 2$ and let the voting rule be the amendment procedure for some orderings of the alternatives at any agenda a. Then the set of free issues is either empty or contains all issues.*

Voting by Quota

We now consider another class of voting rules, namely voting by quota. Let $\bar{\mathcal{S}} \subset \mathcal{P}$ denote the set of strict preference orderings that satisfy additive separability and betweenness and let $q \in \{1, \ldots, n\}$. Then *voting by quota q* is the voting procedure $V : A \times \bar{\mathcal{S}}^n \to X$, such that for all $a \in A$, for all $k \in a$, and for all

[13] By "#" we denote the number of elements in a set.

$P \in \bar{\mathcal{S}}^n$,

$$(V(a,P))_k = \begin{cases} 1, & \text{if } \#\{i \,|\, u_k^i(1) > u_k^i(0)\} \geq q \\ 0, & \text{otherwise} \end{cases}, \tag{1}$$

where $(u_k^i(\cdot))_{k \in \mathcal{K}}$ is the collection of scalars in the additively separable utility representation of agent i's preference ordering $\succ_i$.[14] Observe that (1) implies that $V(a,P)$ only depends on the issues in a but not on their specific ordering.

Notably, on the restricted domain of additively separable preferences, for any quota q and for (almost) any set $\mathcal{F}$, there exists a preference profile such that $\mathcal{F}$ is the set of free issues at some equilibrium agenda:

Theorem 4.3 *Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q \in \{1, \ldots, n\}$ and let $\mathcal{F} \subset \mathcal{K}$ be such that $\#\mathcal{F} > 1$ and $\#\mathcal{F} \neq 2$ if n is odd and $q = \frac{n+1}{2}$. Then there exists a preference profile $P \in \bar{\mathcal{S}}^n$ such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda a^* at P.*

We will now argue that the conditions in Theorem 4.3 are tight in the sense that any $\mathcal{F} \subset \mathcal{K}$ that does not satisfy the conditions in the theorem can never be a set of free issues at an equilibrium agenda. The following proposition deals with the case where $\#\mathcal{F} = 1$ and shows that equilibrium agendas never contain all but one issue.

Proposition 4.2 *Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota q and let $P = (\succ_1, \ldots, \succ_n) \in \bar{\mathcal{S}}^n$. If $(CE(a,P))_{a \in A}$ is an equilibrium collection of sets of continuation agendas and if $a^* \in CE(a,P)$ for some $a \in A$, then*

$$a^* \notin A^{K-1}.$$

In particular, no $a^* \in A^{K-1}$ is an equilibrium agenda at P.

We skip the proof of Proposition 4.2 since it is an immediate implication of Lemma 2.1 and the following result:

[14] Voting by quota is a special case of a larger class of voting procedures, called *voting by committees* (Barberà et al., 1991).

Lemma 4.1 *Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q \in \{1, \ldots, n\}$ and let $P = (\succ_1, \ldots, \succ_n) \in \bar{\mathcal{S}}^n$. If $(CE(a,P))_{a \in A}$ is an equilibrium collection of sets of continuation agendas, then*

$$CE(a,P) \subset A^K \text{ for all } a \in A^{K-1}.$$

The intuition for Lemma 4.1 is that if there is only one free issue left, then there is always one agent who is in the winning coalition for that issue. This agent then is better off adding that issue to the agenda since further additions are impossible and hence nothing can deter the agent from her initial move.

The following proposition deals with the other exceptional case of simple majority voting with an odd number of agents, where there can never be only two free issues at an equilibrium agenda.

Proposition 4.3 *Let there be an odd number n of agents and let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q = \frac{n+1}{2}$. Let $P = (\succ_1, \ldots, \succ_n) \in \bar{\mathcal{S}}^n$. If $(CE(a,P))_{a \in A}$ is an equilibrium collection of sets of continuation agendas, then for all $a \in A$,*

$$CE(a,P) \subset A \setminus \left(A^{K-1} \cup A^{K-2}\right).$$

In particular, no $a^* \in A^{K-2}$ is an equilibrium agenda at P, i.e. the set of free issues at an equilibrium agenda never contains two issues only.

5 Conclusion

Agenda formation is an essential part of many decision-making processes. Before we take a decision we have to sort out those issues that we want to settle and those that shall remain unsettled. We have studied the resulting agenda formation process and have demonstrated that essentially any subset of issues may be excluded from an equilibrium agenda. This result holds even under the restrictive assumption that preferences are (additively) separable and satisfy betweenness and that the voting rule is Pareto efficient or strategy-proof. We believe that these results are generic, i.e. except for peculiar rules like the voting procedure that always selects the best alternative for some voter at any full agenda, we

do not expect to find another voting procedure where equilibrium agendas of a particular length do not obtain in general. Yet, our reading of these results is positive, in the sense that the variability of agendas is properly explained in each case, and its causes are pinpointed (the voting rule, the distribution of preferences and agents' chances to have their preferences prevail). In particular, we have shown that any equilibrium agenda is a full agenda whenever all agents are pessimistic about all issues and the voting procedure is Pareto efficient.

It is clear from our results that equilibrium agendas and outcomes are very sensitive to voters' perceptions about the preferences of others regarding not only the potential outcomes of a vote, but also their inclination to postpone the discussion of certain issues. Likewise, equilibrium agendas and outcomes are also sensitive to the choice of voting rules. In particular, when sequential voting rules are to be used, we have shown that agents who can control the order of vote can decisively influence what issues may reach the floor, and which ones will remain unsettled.

Appendix

Proof of Theorem 4.1: Let $V : A \times \mathcal{S}^n \to X$ be a Pareto efficient voting procedure and let all agents be pessimistic about all issues. Let $(CE(a,P))_{a \in A}$ be an equilibrium collection of sets of continuation agendas. We will prove by backwards induction that $CE(a,P) \subset A^K$ for all $a \in A$.

Obviously, the claim is true for any full agenda $a \in A^K$. Suppose the claim is true for all agendas $a \in A$ of length l, where $m+1 \leq l \leq K$ and $1 \leq m \leq K-1$. Let $a \in A^m$. By (E1), $CE(a,P)$ is a nonempty subset of $\bigcup_{k \notin a} CE((a,k),P) \cup \{a\}$. By our induction hypothesis $CE((a,k),P) \subset A^K$ for all $k \notin a$. Suppose by way of contradiction that $a \in CE(a,P)$. Then by (E2), for all agents i,

$$V(a,P) \succ_i V(a',P) \text{ for all } a' \in \bigcup_{k \notin a} CE((a,k),P).$$

Let $y = V(a',P)$ for some $a' \in \bigcup_{k \notin a} CE((a,k),P)$. By the induction hypothesis a' is a full agenda, which implies that $y \in \{0,1\}^K$. Moreover, y is Pareto efficient in $\{0,1\}^K$. Let $x = V(a,P)$. Then $x_k = -$ for all $k \notin a$. Since every agent i is pessimistic about all issues $k \notin a$ it follows that for all i,

$$\big((w_k^i)_{k \notin a}, (x_k)_{k \in a}\big) \succ_i y \quad \text{or} \quad \big((w_k^i)_{k \notin a}, (x_k)_{k \in a}\big) = y.$$

If $((w_k^i)_{k \notin a}, (x_k)_{k \in a}) \succ_i y$ for all i, let $z \in \{0,1\}^K$ be such that $z_k = x_k$ for all $k \in a$. Then, by definition of w_k^i for $k \notin a$ it follows that for all i either

$$z = \big((w_k^i)_{k \notin a}, (x_k)_{k \in a}\big) \succ_i y$$

or

$$z \succ_i \big((w_k^i)_{k \notin a}, (x_k)_{k \in a}\big) \succ_i y.$$

This contradicts our assumption that y is Pareto efficient.

If there exists an i with $((w_k^i)_{k \notin a}, (x_k)_{k \in a}) = y$, let $z \in \{0,1\}^K$ be such that $z_k = 1 - w_k^i$ for all $k \notin a$ and $z_k = x_k$ for all $k \in a$. Notice that $((w_k^i)_{k \notin a}, (x_k)_{k \in a}) = y = \big((w_k^j)_{k \notin a}, (x_k)_{k \in a}\big)$ for some $i \neq j$ implies that $w_k^i = w_k^j$ for all $k \notin a$. Again it follows that $z \succ_i y$ for all i which contradicts our assumption that y is Pareto efficient.

Hence, $a \notin CE(a, P)$ which implies that any agenda in $CE(a, P)$ is a full agenda. This proves the theorem.

$\square$

Proof of Theorem 4.2:

We prove the claim for $n = 3$ agents and note that the extension to an arbitrary odd number of agents $n > 3$ is straightforward: For any preference profile $(\succ_1, \succ_2, \succ_3)$ for three agents we can define a preference profile for $n > 3$ agents, such that every agent's preference ordering is either $\succ_1, \succ_2$ or $\succ_3$ and such that the majority relation on X is preserved.[15]

From now on we assume that $n = 3$. Let $K \geq 2$ and let $\mathcal{F} = \emptyset$. Take any preference ordering $\succ \in \mathcal{S}$ and let $P = (\succ_1, \ldots, \succ_n)$ be such that $\succ_i = \succ$ for all $i = 1, \ldots, n$. Then, for all $a \in A \setminus A^K$ and for all $a' = (a, k, \ldots)$ with $k \notin a$,

$$V(a', P) \succ_i V(a, P). \tag{2}$$

Let $(CE(a, P))_{a \in A}$ be any consistent equilibrium collection of sets of continuation agendas. (2) and (E2) then imply that $a \notin CE(a, P)$ for all $a \in A \setminus A^K$ and we conclude that

$$CE(\varnothing, P) \subset A^K$$

by Lemma 2.1. Hence, there are no free issues at any equilibrium agenda at P.

The remainder of the proof consists of four steps. Steps 1 and 2 deal with the case where there are at least two free issues. In Step 1 we show that for any $K \geq 2$ there exists a preference profile P such that all K issues are free at any consistent equilibrium agenda at P. in Step 2 we use a lexicographic extension of the preferences defined in Step 1 to prove that for $K \geq 3$ and $\#\mathcal{F} \geq 2$ there exists a preference profile P such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda. Steps 3 and 4 consider the case with one free issue. In Step

[15]If $n = 5$ let the preference orderings $\succ'_i$ for agents $i = 1, \ldots, 5$, be given by $\succ'_i = \succ_1$ for $i = 1, 2$, $\succ'_i = \succ_2$ for $i = 3, 4$, and $\succ'_5 = \succ_3$. If $n \geq 7$ let $m \in \mathbb{N}$ and $k \in \{0, 1, 2\}$ be such that $n = 3m + k$, and let the preference orderings $\succ'_i$ for agents $i = 1, \ldots, n$, be given by $\succ'_i = \succ_1$ for $i = 1, \ldots, m$, $\succ'_i = \succ_2$ for $i = m + 1, \ldots, 2m$, and $\succ'_i = \succ_3$ for $i = 2m + 1, \ldots, n$. Let M be the simple majority relation at the preference profile $(\succ_1, \succ_2, \succ_3)$ and let M' be the simple majority relation at the preference profile $(\succ'_1, \ldots, \succ'_n)$. Then it is straightforward to show that for all $x, y \in X$, xMy if and only if $xM'y$.

3 we prove that for $K = 3$ and $\#\mathcal{F} = 1$ there exists a preference profile P such $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda. Finally, in Step 4 we use a lexicographic extension of the preferences defined in Step 2 to extend the case with one free issue from $K = 3$ to an arbitrary number of issues $K \geq 4$.

Step 1: In the following we prove that for any set of issues $\mathcal{K} = \{1, \ldots, K\}$ with $K \geq 2$ there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{S}^3$ such that the set of free issues at any consistent equilibrium agenda is $\mathcal{F} = \mathcal{K}$. Note that this means that $\varnothing$ is the unique consistent equilibrium agenda at P.

Let $\mathcal{K} = \mathcal{F} = \{1, 2\}$. Then Section 3 provides an example where $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda.

Let $\mathcal{K} = \mathcal{F} = \{1, 2, 3\}$ and let the agents' preference orderings be given by Table 2. Note that the agents' preferences are separable and satisfy betweenness. It is immediate to see that there is a Condorcet winner for any agenda a which is the alternative that has position 1 for each issue on the agenda. Thus, for any agenda $a \in A$ the voting outcome under the amendment procedure satisfies

$$(V(a, P))_k = 1 \text{ for all } k \in a.$$

We now solve backwards for the equilibrium collection of sets of continuation agendas. By (E1) it follows that

$$CE(a, P) = \{a\}$$

for all full agendas a.

Now consider an agenda a of length 2. By condition (E1), $CE(a, P)$ is a nonempty subset of $\{a\} \cup CE((a, k), P) = \{a, (a, k)\}$, where $k \notin a$. By condition (E2), $a \in CE(a, P)$ is ruled out since there is always one agent who strictly prefers $(1, 1, 1)$, which is the voting outcome at agenda (a, k), over x with $x_k = -$ and $x_l = 1$ for all $l \neq k$, which is the voting outcome at agenda a. Hence,

$$CE(a, P) = \{(a, k)\}.$$

Next consider an agenda a of length 1, i.e. $a = (k)$ for some $k \in \{1, 2, 3\}$. Let $h, l \notin a, h \neq l$. By condition (E1), $CE((k), P)$ is a nonempty subset of

$\succ_1$	$\succ_2$	$\succ_3$
$(1,1,0)$	$(1,0,1)$	$(0,1,1)$
$(1,1,-)$	$(-,0,1)$	$(-,1,1)$
$(1,-,0)$	$(1,0,-)$	$(0,-,1)$
$(1,-,-)$	$(0,0,1)$	$(0,1,-)$
$(1,0,0)$	$(-,0,-)$	$(-,1,-)$
$(-,1,0)$	$(1,-,1)$	$(0,-,-)$
$(-,1,-)$	$(1,-,-)$	$(-,-,1)$
$(0,1,0)$	$(-,-,1)$	$(-,-,-)$
$(0,1,-)$	$(0,-,1)$	$(1,1,1)$
$(-,-,0)$	$(-,-,-)$	$(1,-,1)$
$(0,-,0)$	$(1,1,1)$	$(1,1,-)$
$(-,0,0)$	$(1,1,-)$	$(1,-,-)$
$(0,0,0)$	$(-,1,1)$	$(0,0,1)$
$(-,-,-)$	$(-,1,-)$	$(0,0,-)$
$(1,1,1)$	$(0,1,1)$	$(-,0,1)$
$(1,0,-)$	$(0,0,-)$	$(-,0,-)$
$(1,-,1)$	$(1,0,0)$	$(1,0,1)$
$(-,1,1)$	$(1,-,0)$	$(1,0,-)$
$(-,-,1)$	$(-,0,0)$	$(0,1,0)$
$(1,0,1)$	$(0,-,-)$	$(-,1,0)$
$(-,0,-)$	$(0,1,-)$	$(0,-,0)$
$(-,0,1)$	$(0,0,0)$	$(-,-,0)$
$(0,1,1)$	$(-,-,0)$	$(1,1,0)$
$(0,-,-)$	$(0,-,0)$	$(1,-,0)$
$(0,0,-)$	$(1,1,0)$	$(0,0,0)$
$(0,-,1)$	$(-,1,0)$	$(-,0,0)$
$(0,0,1)$	$(0,1,0)$	$(1,0,0)$

Table 2: Preference profile for $\mathcal{K} = \mathcal{F} = \{1,2,3\}$.

$\{(k)\} \cup CE((k,h),P) \cup CE((k,l),P) = \{(k),(k,h,l),(k,l,h)\}$. By condition (E2), $(k) \in CE((k),P)$ is ruled out since there is always one agent who strictly prefers $(1,1,1)$, which is the voting outcome at agendas (k,h,l) or (k,l,h), over x with $x_k = 1$ and $x_l = x_h = -$, which is the voting outcome at agenda (k). Hence,

$$CE((k),P) \subset \{(k,h,l),(k,l,h)\}.$$

Finally, consider the empty agenda. By condition (E1), $CE(\varnothing,P)$ is a nonempty subset of $\{\varnothing\} \cup \bigcup_{k=1}^{3} CE((k),P)$. Since all agendas in $CE((k),P)$ for $k = 1,2,3$, are full agendas with voting outcome $(1,1,1)$ and all agents strictly prefer $(-,-,-)$ over $(1,1,1)$, all agents prefer the empty agenda over any full agenda. By (E2) this implies that $\varnothing \in CE(\varnothing,P)$.

It remains to prove that $\varnothing$ is the unique consistent equilibrium agenda. Suppose by way of contradiction that $a \in CE(\varnothing,P)$ for some $a \neq \varnothing$. Then a must be a full agenda and since $\varnothing \in CE(\varnothing,P)$, condition (E3) implies that a is rationalizable relative to the empty agenda $\varnothing$. However, no agent prefers the voting outcome at a full agenda over the voting outcome at the empty agenda $\varnothing$ or any other full agenda a' which could be in $CE(\varnothing,P)$. Hence, a is not rationalizable which implies that $a \notin CE(\varnothing,P)$. Hence, we conclude that

$$CE(\varnothing,P) = \{\varnothing\}.$$

Thus, in this case the unique consistent equilibrium agenda is empty and the set of free issues is given by $\mathcal{K}$.

Let $\mathcal{K} = \mathcal{F} = \{1,2,3,4\}$. We take the preference orderings $\succ_i$ for two issues in Table 1 (Section 3) and extend them in a lexicographic way to preference orderings $\succ_i'$ on $\{0,1,-\}^{\mathcal{K}}$: For $i = 1,2,3$, let $\succ_i'$ be such that

$$(x_1,x_2,x_3,x_4) \succ_i' (y_1,y_2,y_3,y_4)$$

if and only if

$$(x_1,x_2) \succ_i (y_1,y_2)$$

or

$$(x_1,x_2) = (y_1,y_2) \text{ and } (x_3,x_4) \succ_i (y_3,y_4).$$

$\succ'_1$	$\succ'_2$	$\succ'_3$
$(0,1,0,1)$	$(1,0,1,0)$	$(1,1,1,1)$
$(0,1,0,0)$	$(1,0,0,0)$	$(1,1,0,1)$
$(0,1,1,1)$	$(1,0,1,1)$	$(1,1,1,0)$
$(0,1,1,0)$	$(1,0,0,1)$	$(1,1,0,0)$
$(0,0,0,1)$	$(0,0,1,0)$	$(0,1,1,1)$
$(0,0,0,0)$	$(0,0,0,0)$	$(0,1,0,1)$
$(0,0,1,1)$	$(0,0,1,1)$	$(0,1,1,0)$
$(0,0,1,0)$	$(0,0,0,1)$	$(0,1,0,0)$
$(1,1,0,1)$	$(1,1,1,0)$	$(1,0,1,1)$
$(1,1,0,0)$	$(1,1,0,0)$	$(1,0,0,1)$
$(1,1,1,1)$	$(1,1,1,1)$	$(1,0,1,0)$
$(1,1,1,0)$	$(1,1,0,1)$	$(1,0,0,0)$
$(1,0,0,1)$	$(0,1,1,0)$	$(0,0,1,1)$
$(1,0,0,0)$	$(0,1,0,0)$	$(0,0,0,1)$
$(1,0,1,1)$	$(0,1,1,1)$	$(0,0,1,0)$
$(1,0,1,0)$	$(0,1,0,1)$	$(0,0,0,0)$

Table 3: Lexicographic extension of the preference orderings in Table 1 to $\{0,1\}^{\mathcal{K}}$ for $\mathcal{K} = \{1,2,3,4\}$.

For illustration Table 3 gives the agents' preference orderings on $\{0,1\}^{\mathcal{K}}$.

Next we determine the voting outcome at all agendas. At the empty agenda $(-,-,-,-)$ is the unique attainable alternative which implies that

$$V(\varnothing, P) = (-,-,-,-).$$

At agenda (k) for $k \in \{1,2,3,4\}$ there are only two attainable alternatives, x and y with $x_k = 1$, $y_k = 0$ and $x_l = y_l = -$ for $l \neq k$. If $k \in \{1,3\}$, then agents 2 and 3 prefer position 1 over position 0 for issue k which implies that

$$V((1), P) = (1,-,-,-) \text{ and } V((3), P) = (-,-,1,-).$$

If $k \in \{2,4\}$, then agents 1 and 3 prefer position 1 over position 0 for issue k which implies that

$$V((2), P) = (-,1,-,-) \text{ and } V((4), P) = (-,-,-,1).$$

Next we determine the voting outcome at all agendas of length 2. The analysis of case $K = 2$ implies that there exists an ordering of the alternatives in $\{0,1\}^{\{1,2\}}$ such that

$$V((1,2), P) = V((2,1), P) = (0,0,-,-).$$

Similarly, there exists an ordering of the alternatives in $\{0,1\}^{\{3,4\}}$ such that

$$V((3,4), P) = V((3,4), P) = (-,-,0,0).$$

Consider agendas $(2,3)$ and $(3,2)$. Then $(-,1,1,-) \succ_i' (-,0,x_3,-)$ for all $x_3 \in \{0,1\}$ and $i = 1,3$, and $(-,1,1,-) \succ_i' (-,1,0,-)$ for $i = 2,3$. Hence, $(-,1,1,-)$ is the Condorcet winner in $X(2,3)$ which implies that

$$V((2,3), P) = V((3,2), P) = (-,1,1,-).$$

Similarly, we derive

$$V((1,3), P) = V((3,1), P) = (1,-,1,-),$$
$$V((1,4), P) = V((4,1), P) = (1,-,-,1),$$
$$V((2,4), P) = V((4,2), P) = (-,1,-,1).$$

Next consider all agendas of length 3. Let $a \in A^3$ with $4 \notin a$. We will now argue that there exists an ordering of the alternatives in $X(a)$ such that $(0,0,1,-)$ is the voting outcome under the amendment procedure. To see this note that by definition of the preference orderings $\succ_i'$, under simple majority voting $(0,0,1,-)$ is dominated by $(1,0,x_3,-)$ and $(0,1,x_3,-)$ for all $x_3 \in \{0,1\}$ and $(0,0,1,-)$ dominates all remaining alternatives in $X(a)$. Moreover, $(1,0,x_3,-)$ and $(0,1,x_3,-)$ are dominated by $(1,1,1,-)$ for all $x_3 \in \{0,1\}$. It follows from the characterizations in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) that there exists an ordering of the alternatives in $X(a)$ such that $(0,0,1,-)$ is the voting outcome under the amendment procedure. We take this ordering and get

$$V(a,P) = (0,0,1,-) \text{ for all } \ a \in A^3 \text{ with } 4 \notin a.$$

In a similar way one shows that for all $a \in A^3$ with $4 \in a$ there exist orderings of the alternatives in $X(a)$ such that

$$\begin{aligned} V(a,P) &= (-,1,0,0) \text{ for all } \ a \in A^3 \text{ with } 1 \notin a, \\ V(a,P) &= (1,-,0,0) \text{ for all } \ a \in A^3 \text{ with } 2 \notin a, \\ V(a,P) &= (0,0,-,1) \text{ for all } \ a \in A^3 \text{ with } 3 \notin a. \end{aligned}$$

Finally, we determine the voting outcome at all full agendas $a \in A^4$. Note that by definition of the preference orderings $\succ_i'$, under simple majority voting $(0,0,0,0)$ is dominated by $(0,0,1,0), (0,0,0,1), (1,0,x_3,x_4)$ and $(0,1,x_3,x_4)$ for all $x_3, x_4 \in \{0,1\}$, while $(0,0,0,0)$ dominates all remaining alternatives in $X(a)$. Moreover, $(1,1,1,1)$ dominates $(0,0,1,0), (0,0,0,1), (1,0,x_3,x_4)$ and $(0,1,x_3,x_4)$ for all $x_3, x_4 \in \{0,1\}$. Again we use the characterizations in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) to conclude that there exists an ordering of the alternatives in $X(a)$ such that $(0,0,0,0)$ is the voting outcome under the amendment procedure. We take this ordering and get

$$V(a,P) = (0,0,0,0) \text{ for all } \ a \in A^4.$$

We now solve backwards for the equilibrium collection of sets of continuation agendas. By (E1) it follows that

$$CE(a,P) = \{a\} \text{ for all } a \in A^4.$$

Now consider an agenda of length 3. Let $a \in A^3$ and let $1 \notin a$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,1),P) = \{a,(a,1)\}$. By condition (E2), $a \in CE(a,P)$ is ruled out since agent 2 strictly prefers the voting outcome at agenda $(a,1)$, which is $(0,0,0,0)$ over the voting outcome at agenda a which is $(-,1,0,0)$. Hence,

$$CE(a,P) = \{(a,1)\}.$$

In the same way one proves that

$$CE(a,P) = \{(a,k)\} \text{ for all } k \notin a.$$

Next consider agendas of length 2. To begin with, let $a \in \{(1,2),(2,1)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,3),P) \cup CE((a,4),P) = \{a,(a,3,4),(a,4,3)\}$. By condition (E2), $a \in CE(a,P)$ since all agents prefer the outcome under (a), which is $(0,0,-,-)$, over the outcome under $(a,3,4)$ or $(a,4,3)$ which is $(0,0,0,0)$. Moreover, given $a \in CE(a,P)$ none of the agendas $(a,3,4)$ or $(a,4,3)$ is rationalizable relative to a and hence (E3) implies that

$$CE((1,2),P) = \{(1,2)\} \text{ and } CE((2,1),P) = \{(2,1)\}.$$

In a similar way it follows that

$$CE((3,4),P) = \{(3,4)\} \text{ and } CE((4,3),P) = \{(4,3)\}.$$

Let $a \in \{(1,3),(3,1)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,2),P) \cup CE((a,4),P) = \{a,(a,2,4),(a,4,2)\}$. By (E2), $a \in CE(a,P)$ is ruled out since agent 1 strictly prefers the voting outcome at any full agenda, which is $(0,0,0,0)$, over the voting outcome at agenda a, which is $(1,-,1,-)$. Hence,

$$CE((1,3),P), CE((3,1),P) \subset A^4.$$

In a similar way it follows that

$$CE((1,4),P), CE((4,1),P) \subset A^4,$$
$$CE((2,3),P), CE((3,2),P) \subset A^4,$$
$$CE((2,4),P), CE((4,2),P) \subset A^4.$$

Next consider agendas of length 1. To begin with, let $a = (1)$. By condition (E1), $CE((1), P)$ is a nonempty subset of $\{(1)\} \cup \bigcup_{k=2}^{4} CE((1,k), P)$. By (E2), $(1) \in CE((1), P)$ is ruled out since agent 2 strictly prefers the voting outcome at agenda $(1,2) \in CE((1,2), P)$, which is $(0,0,-,-)$, over the voting outcome at agenda (1), which is $(1,-,-,-)$. Moreover, any agenda in $CE((1,3), P)$ or $CE((1,4), P)$ is a full agenda with voting outcome $(0,0,0,0)$. If any such agenda were in $CE((1), P)$, then $(1,2)$ is rationalizable relative to (1) since all agents prefer the voting outcome under $(1,2)$, which is $(0,0,-,-)$, over $(0,0,0,0)$. (E3) then requires that $(1,2) \in CE((1), P)$ which in turn implies that no agenda in $CE((1,3), P)$ or $CE((1,4), P)$ is rationalizable. By (E3) we conclude that no agenda in $CE((1,3), P)$ or $CE((1,4), P)$ belongs to $CE((1), P)$. Hence,

$$CE((1), P) = \{(1,2)\}.$$

In a similar way it follows that

$$\begin{aligned} CE((2), P) &= \{(2,1)\}, \\ CE((3), P) &= \{(3,4)\}, \\ CE((4), P) &= \{(4,3)\}. \end{aligned}$$

Finally, consider the empty agenda. By condition (E1), $CE(\varnothing, P)$ is a nonempty subset of $\{\varnothing\} \cup \bigcup_{k=1}^{4} CE((k), P) = \{\varnothing, (1,2), (2,1), (3,4), (4,3)\}$. Since all agents strictly prefer the voting outcome under the empty agenda, which is $(-,-,-,-)$, over the voting outcome under agendas $(1,2)$ or $(2,1)$, which is $(0,0,-,-)$, and the voting outcome under agendas $(3,4)$ or $(4,3)$ which is $(-,-,0,0)$, it follows that $\varnothing \in CE(\varnothing, P)$ by (E2).

It remains to prove that $\varnothing$ is the unique consistent equilibrium agenda. To this end note that (E3) implies that if any of the agendas $(1,2), (2,1), (3,4), (4,3)$ is in $CE(\varnothing, P)$, then it must be rationalizable. However, the voting outcome under agendas $(3,4)$ and $(4,3)$ is $(-,-,0,0)$ which is strictly worse for all agents than the voting outcome under the empty agenda, which is $(-,-,-,-)$, and the voting outcome under agendas $(1,2)$ and $(2,1)$, which is $(0,0,-,-)$. This implies that neither $(3,4)$ nor $(4,3)$ is rationalizable and by (E3) neither of these agendas is in $CE(\varnothing, P)$. But then neither $(1,2)$ nor $(2,1)$ are rationalizable relative to

$\varnothing$ because all agents strictly prefer $(-,-,-,-)$ over the voting outcome under agendas $(1,2)$ and $(2,1)$, which is $(0,0,-,-)$. Hence, (E3) implies that

$$CE(\varnothing, P) = \{\varnothing\}.$$

Thus, the unique consistent equilibrium agenda is empty and the set of free issues is given by $\mathcal{K} = \{1,2,3,4\}$.

Let $\mathcal{K} = \mathcal{F} = \{1,2,3,4,5\}$. We then construct a preference profile using the preference orderings in Table 1 and Table 2. For $i = 1,2,3$, let $\succ_i^2$ be agent i's preference ordering in Table 1 and let $\succ_i^3$ be agent i's preference ordering in Table 2. We extend these preferences in a lexicographic way to preference orderings $\succ_i'$ on $\{0,1,-\}^{\mathcal{K}}$: For $i = 1,2,3$, let $\succ_i'$ be such that

$$(x_1, x_2, x_3, x_4, x_5) \succ_i' (y_1, y_2, y_3, y_4, y_5)$$

if and only if

$$(x_1, x_2) \succ_i^2 (y_1, y_2)$$

or

$$(x_1, x_2) = (y_1, y_2) \text{ and } (x_3, x_4, x_5) \succ_i^3 (y_3, y_4, y_5).$$

Then, similar to case $\mathcal{K} = \mathcal{F} = \{1,2,3,4\}$ above we can show that the unique consistent equilibrium agenda is empty and hence the set of free issues is given by $\mathcal{K} = \{1,2,3,4,5\}$.

Let $\mathcal{K} = \mathcal{F} = \{1,\ldots,K\}$ with $K \geq 6$. Then either K is even or $K = 2m+3$ for some $m > 1$. In both cases we can proceed as in the previous two cases with $K = 4$ and $K = 5$ to construct an example with three agents and separable preferences such that all issues are free at the unique consistent equilibrium agenda. We simply extend the agents' preference orderings for $K = 2$ and $K = 3$ in a lexicographic way to preference orderings for the given number of issues.

Step 2: Let $\mathcal{K} = \{1,\ldots,K\}$ with $K \geq 3$ and let $\mathcal{F} = \{1,\ldots,F\} \subset \mathcal{K}$ with $F \geq 2$. We will prove that there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{S}^3$ such that the set of free issues at the unique consistent equilibrium agenda is $\mathcal{F}$. To this end take the preference orderings $\succ_i$ used for the case $\mathcal{K} = \mathcal{F}$ in Step 1

and extend them in a lexicographic way to preference orderings $\succ'_i$ on $\{0, 1, -\}^{\mathcal{K}}$: For $i = 1, 2, 3$, let $\succ'_i$ be such that

$$(x_1, \ldots, x_K) \succ'_i (y_1, \ldots, y_K)$$

if and only if one of the following two conditions is satisfied:

(i) There exists some l with $F + 1 \leq l \leq K$, such that $x_k = y_k$ for $k = F + 1, \ldots, l - 1$, and either

$$\begin{aligned} & x_l = 1 \text{ and } y_l \in \{-, 0\} \\ \text{or} \quad & x_l = - \text{ and } y_l = 0. \end{aligned}$$

(ii) $x_k = y_k$ for $k = F + 1, \ldots, K$, and

$$(x_1, \ldots, x_F) \succ_i (y_1, \ldots, y_F).$$

Hence, all agents first consider the positions on issues $F + 1, \ldots, K$ (in that order) and all prefer position 1 over $-$ and $-$ over 0 on these issues. Only if two alternatives have the same positions on all issues $F + 1, \ldots, K$, the positions on the remaining issues are relevant. In that case agent i's preference over the alternatives is determined by the preference $\succ_i$ over the positions on issues $1, \ldots, F$.

Then, by Pareto efficiency of the amendment procedure, $(V(a, P))_k = 1$ for all agendas a with $k \in a$ and $k \in \{F + 1, \ldots, K\}$ independent of the ordering of the alternatives in $X(a)$ under the amendment procedure. Moreover, any consistent equilibrium agenda a must contain all issues in $\{F + 1, \ldots, K\}$. Suppose this were not true, i.e. there exists a consistent equilibrium agenda a with $k \notin a$ for some $k \in \{F + 1, \ldots, K\}$. Lemma 2.1 implies that $a \in CE(a, P)$. Hence, by (E2) it must be true that for all i and for all $a' \in CE((a, k), P))$,

$$V(a, P) \succ_i V(a', P).$$

However, by definition of $\succ_i$ this is impossible since $(V(a', P))_k = 1$ and $(V(a, P))_k = -$. Therefore, we conclude that any consistent equilibrium agenda contains all issues in $\{F + 1, \ldots, K\}$. We will now prove that there are no additional issues on any consistent equilibrium agenda if the order of vote under the amendment procedure is chosen in an appropriate way.

Let $(CE(a,P))_{a\in A}$ be any consistent equilibrium collection of sets of continuation agendas and let a be any agenda that is a permutation of $(F+1,\ldots,K)$. Then, given the definition of agents' preferences, Step 1 implies that there exists an order of vote under the amendment procedure, such that

$$CE(a,P) = \{a\} \tag{3}$$

Moreover, all such agendas a yield the same voting outcome x with $x_k = 1$ for all $k = F+1,\ldots,K$, and $x_k = -$ for all $k = 1,\ldots,K$.

Let a' be any agenda that is either empty or only contains issues in $\{F+1,\ldots,K\}$. We will prove by backwards induction over the number of issues in a', that any agenda in $CE(a',P)$ is a permutation of $(F+1,\ldots,K)$. (3) implies that the claim is true if a' is a permutation of $(F+1,\ldots,K)$. Now suppose the claim is true for all agendas that contain at least $l+1$ issues in $\{F+1,\ldots,K\}$ and no issues in $\{1,\ldots,K\}$, where $0 \leq l < K-F$. Let $a' = (a_1,\ldots,a_l)$ be an agenda with $a_1,\ldots,a_l \in \{F+1,\ldots,K\}$. Since a' does not contain all issues in $\{F+1,\ldots,K\}$, (E1) implies that $CE(a',P)$ is a nonempty subset of $\bigcup_{k\notin a'} CE((a',k),P)$. By the induction hypothesis any agenda in $CE((a',k),P)$ is a permutation of $(F+1,\ldots,K)$ for all $k \notin a'$ with $k \in \{F+1,\ldots,K\}$. By definition of the agents' preferences and the proof in Step 1 it follows that there exists an order of vote under the amendment procedure, such that all agents have the same preferences over voting outcomes $V(a'',P)$ for all $a'' \in CE((a',k),P)$ and for all $k \notin a'$. Moreover, all agents prefer $V(a,P)$, where a is some permutation of $(F+1,\ldots,K)$, over $V(a'',P)$ for any agenda $a'' \in CE((a',k),P)$ for all $k \in \{1,\ldots,F\}, k \notin a'$. Hence, the voting outcomes $V(a'',P)$ for all $a'' \in CE((a',k),P)$ and for all $k \in \{1,\ldots,K\}, k \notin a'$, are Pareto ranked with $V(a'',P)$ being preferred over $V(a''',P)$ for all $a'' \in CE((a',k),P)$ with $k \notin a'$ and $k \in \{F+1,\ldots,K\}$, and for all $a''' \in CE((a',k'),P)$ for all $k' \notin a'$ and $k' \in \{1,\ldots,F\}$. Therefore, consistency (E3) implies that

$$CE(a',P) \subset \bigcup_{k\notin a', k\in\{F+1,\ldots,K\}} CE((a',k),P).$$

Hence, by the induction hypothesis any agenda in $CE(a',P)$ is a permutation of $(F+1,\ldots,K)$. This proves the claim.

We conclude that any agenda in $CE(\varnothing, P)$ is a permutation of $(F+1, \dots, K)$, i.e. the set of free issues at any consistent equilibrium agenda is given by $\mathcal{F}$.

Step 3: Let $\mathcal{K} = \{1,2,3\}$ and $\#\mathcal{F} = 1$. W.l.o.g. let $\mathcal{F} = \{3\}$. We will prove that there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{S}^3$ such that the set of free issues at the unique consistent equilibrium agenda is $\mathcal{F} = \{3\}$. Let preference orderings be given by Table 4. Note that the agents' preferences are separable and satisfy betweenness.

In order to solve for the equilibrium agendas we first determine the voting outcome at all agendas. At the empty agenda $(-,-,-)$ is the unique attainable alternative which implies that

$$V(\varnothing, P) = (-,-,-).$$

At agenda (1) there are only two attainable alternatives, $(1,-,-)$ and $(0,-,-)$. Since agents 2 and 3 prefer $(1,-,-)$ over $(0,-,-)$ it follows that

$$V((1), P) = (1,-,-).$$

At agenda (2) there are only two attainable alternatives, $(-,1,-)$ and $(-,0,-)$. Since agents 1 and 3 prefer $(-,1,-)$ over $(-,0,-)$ it follows that

$$V((2), P) = (-,1,-).$$

At agenda (3) there are only two attainable alternatives, $(-,-,1)$ and $(-,-,0)$. Since agents 2 and 3 prefer $(-,-,1)$ over $(-,-,0)$ it follows that

$$V((3), P) = (-,-,1).$$

Next we consider all agendas of length 2. At agendas, $(1,2)$ and $(2,1)$ the attainable set is

$$X(1,2) = X(2,1) = \{(0,0,-),(0,1,-),(1,0,-),(1,1,-)\}.$$

Note that under simple majority voting $(0,0,-)$ is dominated by $(0,1,-)$ and $(1,0,-)$, where both of the latter agendas are dominated by $(1,1,-)$ which in turn is dominated by $(0,0,-)$. Hence, using the characterization results in Banks

$\succ_1$	$\succ_2$	$\succ_3$
$(0,1,1)$	$(1,0,1)$	$(1,1,0)$
$(0,-,1)$	$(-,0,1)$	$(1,-,0)$
$(0,0,1)$	$(0,0,1)$	$(1,1,-)$
$(0,1,-)$	$(1,0,-)$	$(1,-,-)$
$(0,-,-)$	$(-,0,-)$	$(-,1,0)$
$(0,0,-)$	$(1,-,1)$	$(0,1,0)$
$(-,1,1)$	$(0,0,-)$	$(-,-,0)$
$(1,1,1)$	$(1,-,-)$	$(-,1,-)$
$(-,1,-)$	$(1,1,1)$	$(0,1,-)$
$(1,1,-)$	$(-,-,1)$	$(0,-,0)$
$(-,-,1)$	$(-,1,1)$	$(1,0,0)$
$(-,0,1)$	$(0,-,1)$	$(-,-,-)$
$(1,-,1)$	$(0,1,1)$	$(1,0,-)$
$(-,-,-)$	$(-,-,-)$	$(0,-,-)$
$(-,0,-)$	$(1,1,-)$	$(-,0,0)$
$(1,0,1)$	$(0,-,-)$	$(0,0,0)$
$(1,-,-)$	$(1,0,0)$	$(-,0,-)$
$(1,0,-)$	$(1,-,0)$	$(0,0,-)$
$(0,1,0)$	$(-,0,0)$	$(1,1,1)$
$(-,1,0)$	$(-,-,0)$	$(1,-,1)$
$(0,-,0)$	$(-,1,-)$	$(-,1,1)$
$(0,0,0)$	$(0,0,0)$	$(-,-,1)$
$(-,-,0)$	$(0,-,0)$	$(0,1,1)$
$(-,0,0)$	$(0,1,-)$	$(0,-,1)$
$(1,1,0)$	$(1,1,0)$	$(1,0,1)$
$(1,-,0)$	$(-,1,0)$	$(-,0,1)$
$(1,0,0)$	$(0,1,0)$	$(0,0,1)$

Table 4: Preference orderings for $\mathcal{K} = \{1,2,3\}$ and $\mathcal{F} = \{3\}$.

(1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) we conclude that $(0, 0, -)$ is the outcome under the amendment procedure for some ordering of the alternatives.[16] If we take this ordering of vote under the amendment procedure we obtain

$$V((1, 2), P) = V((2, 1), P) = (0, 0, -).$$

At agendas $(1, 3)$ and $(3, 1)$ the attainable set is

$$X(1, 3) = X(3, 1) = \{(0, -, 0), (0, -, 1), (1, -, 0), (1, -, 1)\}.$$

Since $(1, -, 1)$ dominates any other attainable alternative in pairwise simple majority voting it is the unique outcome under the amendment procedure for any ordering of the alternatives. Hence, we have

$$V((1, 3), P) = V((3, 1), P) = (1, -, 1).$$

Similarly, at agendas $(2, 3)$ and $(3, 2)$ the attainable set is

$$X(2, 3) = X(3, 2) = \{(-, 0, 0), (-0, 1), (-, 1, 0), (-, 1, 1)\}$$

and $(-, 1, 1)$ dominates any other attainable alternative in pairwise simple majority voting. It is therefore the unique outcome under the amendment procedure for any ordering of the alternatives and we get

$$V((2, 3), P) = V((3, 2), P) = (-, 1, 1).$$

Finally, consider all full agendas. The attainable set at any full agenda $a \in A^3$ is

$$X(a) = \{(1, 1, 1), (1, 1, 0), (1, 0, 1), (1, 0, 0), (0, 1, 1), (0, 1, 0), (0, 0, 1), (0, 0, 0)\}.$$

Note that $(0, 0, 1)$ is the unique alternative that dominates $(1, 1, 1)$ under pairwise simple-majority voting. Since $(0, 0, 1)$ is dominated by $(1, 0, 1)$ which in turn is dominated by $(1, 1, 1)$, the characterizations in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) imply that there exists an ordering of

[16] The reader may also verify directly that $(0, 0, -)$ is the outcome if voting takes place in the ordering $((0, 0, -), (0, 1, -), (1, 0, -), (1, 1, -))$ or $((0, 0, -), (1, 0, -), (0, 1, -), (1, 1, -))$.

the alternatives in $X(a)$ such that $(1,1,1)$ is the outcome under the amendment procedure. Hence, for this ordering

$$V(a,P) = (1,1,1) \text{ for all } a \in A^3.$$

We now solve backwards for the equilibrium collection of sets of continuation agendas. By (E1) it follows that

$$CE(a,P) = \{a\} \text{ for all } a \in A^3.$$

Now consider an agenda $a \in \{(1,2),(2,1)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,3),P) = \{a,(a,3)\}$. Since all agents prefer $(0,0,-)$ over $(1,1,1)$ condition (E2) implies that $a \in CE(a,P)$. Moreover, in this case $(a,3)$ is not rationalizable. Hence, by (E3) we get

$$CE((1,2),P) = \{(1,2)\} \text{ and } CE((2,1),P) = \{(2,1)\}.$$

Next consider an agenda $a \in \{(1,3),(3,1)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,2),P) = \{a,(a,2)\}$. By condition (E2), $a \in CE(a,P)$ is ruled out since agent 1 strictly prefers the voting outcome at agenda $(a,2)$ over the voting outcome at agenda a. Hence,

$$CE((1,3),P) = \{(1,3,2)\} \text{ and } CE((3,1),P) = \{(3,1,2)\}.$$

Next consider an agenda $a \in \{(2,3),(3,2)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,1),P) = \{a,(a,1)\}$. By condition (E2), $a \in CE(a,P)$ is ruled out since agent 2 strictly prefers the voting outcome at agenda $(a,1)$ over the voting outcome at agenda a. Hence,

$$CE((2,3),P) = \{(2,3,1)\} \text{ and } CE((3,2),P) = \{(3,2,1)\}.$$

We then move to agendas of length 1. By condition (E1), $CE((1),P)$ is a nonempty subset of $\{(1)\} \cup CE((1,2),P) \cup CE((1,3),P) = \{(1),(1,2),(1,3,2)\}$. By condition (E2), $(1) \in CE((1),P)$ is ruled out since agent 1 strictly prefers the voting outcome at agenda $(1,2)$ over the voting outcome at agenda (1). Suppose by way of contradiction that $(1,3,2) \in CE((1),P)$. Then $(1,2)$ is rationalizable

and (E3) implies that $(1,2) \in CE((1),P)$ and that $(1,3,2)$ must be rationalizable. However, the latter is not true since all agents prefer the voting outcome at agenda $(1,2)$ over the outcome at agenda $(1,3,2)$. Contradiction. Hence, $(1,3,2) \notin CE((1),P)$ and we conclude that

$$CE((1),P) = \{(1,2)\}.$$

By condition (E1), $CE((2),P)$ is a nonempty subset of $\{(2)\} \cup CE((2,1),P) \cup CE((2,3),P) = \{(2),(2,1),(2,3,1)\}$. By condition (E2), $(2) \in CE((2),P)$ is ruled out since agent 2 strictly prefers the voting outcome at the full agenda $(2,3,1)$ over the voting outcome at agenda (2). We conclude that

$$CE((2),P) \subset \{(2,1),(2,3,1)\}.$$

Suppose by way of contradiction that $(2,3,1) \in CE((2),P)$. Since all agents prefer the voting outcome $(0,0,-)$ at agenda $(2,1)$ over the voting outcome $(1,1,1,)$ at agenda $(2,3,1)$ it follows that $(2,1)$ is rationalizable. (E3) then implies that $(2,1) \in CE((2),P)$ and that $(2,3,1)$ is rationalizable. However, the latter is not true since no agent prefers $(1,1,1)$ over $(0,0,-)$. Contradiction. Therefore, we conclude that

$$CE((2),P) = \{(2,1)\}.$$

By condition (E1), $CE((3),P)$ is a nonempty subset of $\{(3)\} \cup CE((3,1),P) \cup CE((3,2),P) = \{(3),(3,1,2),(3,2,1)\}$. By condition (E2), $(3) \in CE((3),P)$ is ruled out since agent 1 strictly prefers the voting outcome at any full agenda over the voting outcome at agenda (3). We conclude that

$$CE((3),P) \subset \{(3,1,2),(3,2,1)\},$$

where all these agendas give the same outcome $(1,1,1)$.

Finally, consider the empty agenda $\varnothing$. By condition (E1), $CE(\varnothing,P)$ is a nonempty subset of $\{\varnothing\} \cup \bigcup_{k=1}^{3} CE((k),P)$, where $CE((1),P) = \{(1,2)\}$ and $CE((2),P) = \{(2,1)\}$ both give voting outcome $(0,0,-)$, and $CE((3),P)$ contains full agendas only, which give outcome $(1,1,1)$. Since agent 1 strictly prefers $(0,0,-)$ over $(-,-,-)$, (E2) implies that $\varnothing \notin CE(\varnothing,P)$. Suppose by way of contradiction that $CE(\varnothing,P)$ contains a full agenda. Since all agents strictly prefer $(0,0,-)$ over $(1,1,1)$, agenda $(1,2)$ is rationalizable and hence is an element

of $CE(\varnothing, P)$ by (E3). Moreover, in this case no full agenda is rationalizable and hence $CE(\varnothing, P)$ must not contain any full agenda. From this contradiction we conclude that

$$CE(\varnothing, P) \subset \{(1,2),(2,1)\}.$$

Thus, in this case the set of free issues at any consistent equilibrium agenda is $\mathcal{F} = \{3\}$.

Step 4: Let $\mathcal{K} = \{1, \dots, K\}$ with $K \geq 4$ and let $\#\mathcal{F} = 1$. W.l.o.g. let $\mathcal{F} = \{3\}$. We will prove that there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{S}^3$ such that the set of free issues at any consistent equilibrium agenda is $\mathcal{F} = \{3\}$. To this end take the preference orderings $\succ_i$ in Table 4 and extend them in a lexicographic way to preference orderings $\succ_i'$ on $\{0, 1, -\}^{\mathcal{K}}$: For $i = 1, 2, 3$, let $\succ_i'$ be such that

$$(x_1, \dots, x_K) \succ_i' (y_1, \dots, y_K)$$

if and only if one of the following two conditions is satisfied:

(i) There exists some l with $4 \leq l \leq K$, such that $x_k = y_k$ for $k = 4, \dots, l-1$, and either

$$\begin{aligned} & x_l = 1 \text{ and } y_l \in \{-, 0\} \\ \text{or} \quad & x_l = - \text{ and } y_l = 0. \end{aligned}$$

(ii) $x_k = y_k$ for $k = 4, \dots, K$, and

$$(x_1, x_2, x_3) \succ_i (y_1, y_2, y_3).$$

Hence, all agents first consider the positions on issues $4, \dots, K$ (in that order) and all prefer position 1 over $-$ over 0 on these issues. Only if two alternatives have the same positions on all issues $4, \dots, K$, the positions on the remaining issues $1, 2, 3$, are relevant. In that case agent i's preference over the alternatives is determined by the preference $\succ_i$ over the positions on issues $1, 2, 3$.

Then, by Pareto efficiency of the amendment procedure, $(V(a, P))_k = 1$ for all agendas a with $k \in a$ and $k \in \{4, \dots, K\}$ independent of the ordering of the alternatives in $X(a)$ under the amendment procedure. Then, analogously

to Step 2, we can use the findings from Step 3 for $\mathcal{K} = \{1, 2, 3\}$ to prove that for any agenda $a \in A$ there exists an order of vote over the alternatives in $X(a)$ under the amendment procedure, such that the set of free issues at any consistent equilibrium agenda is $\mathcal{F} = \{3\}$.

$\square$

Proof of Proposition 4.1: Let $n \geq 3$ be odd and let $P \in \mathcal{S}^n$ be any profile of separable preferences. Let $K = 2$ and let the voting rule be the amendment procedure for some orderings of the alternatives at any agenda a. W.l.o.g. let $V((1), P) = (1, -)$ and $V((2), P) = (-, 1)$. Then there exist sets of voters M_1, M_2 with $\#M_k \geq \frac{n+1}{2}$ for $k = 1, 2$, such that

$$M_1 = \{i \,|\, (1, -) \succ_i (0, -)\} \text{ and } M_2 = \{i \,|\, (-, 1) \succ_i (-, 0)\}.$$

Separability then implies that

$$(1, 1) \succ_i (0, 1) \text{ and } (1, 0) \succ_i (0, 0) \text{ for all } i \in M_1,$$

and

$$(1, 1) \succ_i (1, 0) \text{ and } (0, 1) \succ_i (0, 0) \text{ for all } i \in M_2.$$

Suppose by way of contradiction that the set of of free issues $\mathcal{F}$ contains one issue. W.l.o.g. let $\mathcal{F} = \{2\}$. Then $CE((1), P) = \{(1)\}$ and (E2) implies that

$$(1, -) \succ_i V((1, 2), P) \text{ for all } i. \tag{4}$$

Then there are four cases.
Case 1: $V((1, 2), P) = (1, 1)$. Since betweenness implies that $(1, 1) \succ_i (1, -) \succ_i (1, 0)$ for all $i \in M_2$, this contradicts (4).

Case 2: $V((1, 2), P) = (0, 0)$. Since $(0, 0)$ is dominated by $(1, 0)$ and $(0, 1)$ under pairwise simple majority voting, a necessary condition for $(0, 0)$ to be the outcome under the amendment procedure for some ordering of the set of available alternatives at a full agenda is that $(0, 0)$ dominates $(1, 1)$ (see Barberà and Gerber (2017), Theorem 3.1). Thus, there exists a set of voters M_3 with $\#M_3 \geq \frac{n+1}{2}$, such that $(0, 0) \succ_i (1, 1)$ for all $i \in M_3$. Since M_2 and M_3 both contain at least $\frac{n+1}{2}$ voters, there exists some $i \in M_2 \cap M_3$. Betweenness implies

that $(0,0) \succ_i (1,1) \succ_i (1,-) \succ_i (1,0)$ for all $i \in M_2 \cap M_3$, which contradicts (4).

Case 3: $V((1,2),P) = (1,0)$. Since $(1,0)$ dominates $(0,0)$, but is dominated by $(1,1)$ which in turn dominates $(0,1)$ under pairwise simple majority voting, a necessary condition for $(1,0)$ to be the outcome under the amendment procedure for some ordering of the set of available alternatives at a full agenda is that $(0,0)$ dominates $(1,1)$ (see Barberà and Gerber (2017), Theorem 3.1). Thus, there exists a set of voters M_3 with $\#M_3 \geq \frac{n+1}{2}$, such that $(0,0) \succ_i (1,1)$ for all $i \in M_3$. Since M_1 and M_3 both contain at least $\frac{n+1}{2}$ voters, there exists some $i \in M_1 \cap M_3$. For all $i \in M_1 \cap M_3$, $(1,0) \succ_i (0,0) \succ_i (1,1)$. Betweenness then implies that $(1,0) \succ_i (1,-)$ for all $i \in M_1 \cap M_3$, which contradicts (4).

Case 4: $V((1,2),P) = (0,1)$. Since $(0,1)$ dominates $(0,0)$, but is dominated by $(1,1)$ which in turn dominates $(1,0)$ under pairwise simple majority voting, a necessary condition for $(0,1)$ to be the outcome under the amendment procedure for some ordering of the set of available alternatives at a full agenda is that $(0,0)$ dominates $(1,1)$ (see Barberà and Gerber (2017), Theorem 3.1). Thus, there exists a set of voters M_3 with $\#M_3 \geq \frac{n+1}{2}$, such that $(0,0) \succ_i (1,1)$ for all $i \in M_3$. Since M_2 and M_3 both contain at least $\frac{n+1}{2}$ voters, there exists some $i \in M_2 \cap M_3$. For all $i \in M_2 \cap M_3$, $(0,1) \succ_i (0,0) \succ_i (1,1) \succ_i (1,0)$. Betweenness then implies that $(0,1) \succ_i (1,-)$ for all $i \in M_2 \cap M_3$, which contradicts (4).

We conclude that it is impossible that there is only one free issue if $K = 2$.

$\square$

Proof of Theorem 4.3: Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q \in \{1, \ldots, n\}$ and let $\mathcal{F} = \emptyset$. Take any preference ordering $\succ \in \bar{\mathcal{S}}$ and let $P = (\succ_1, \ldots, \succ_n)$ be such that $\succ_i = \succ$ for all $i = 1, \ldots, n$. Then, for all $a \in A \setminus A^K$ and for all $a' = (a, k, \ldots)$ with $k \notin a$,

$$V(a', P) \succ_i V(a, P). \tag{5}$$

Let $(CE(a,P))_{a \in A}$ be any consistent equilibrium collection of sets of continuation agendas. (5) and (E2) then imply that $a \notin CE(a,P)$ for all $a \in A \setminus A^K$ and we

conclude that

$$CE(\varnothing, P) \subset A^K$$

by Lemma 2.1. Hence, there are no free issues at any equilibrium agenda at P.

Now let $\emptyset \neq \mathcal{F} \subset \mathcal{K}$ be such that $\#\mathcal{F} \neq 1$ and $\#\mathcal{F} \neq 2$ if n is odd and $q = \frac{n+1}{2}$. Let $r = \#\mathcal{F}$ and w.l.o.g. let $\mathcal{F} = \{1, \ldots, r\}$.

We first consider the case where $r \leq n$. In this case, for all i, choose $(u_k^i(\cdot))_{k \in \mathcal{K}}$ such that

$$u_k^i(1) > u_k^i(-) > u_k^i(0) \text{ for all } k \notin \mathcal{F}. \tag{6}$$

If $q \geq \frac{n+1}{2}$, let $\{W_1 \ldots, W_r\}$ be a partition of the set of agents $\{1, \ldots, n\}$ into nonempty subsets W_h, $h = 1, \ldots, r$, such that $\#W_h < q$ for all $h = 1, \ldots, r$. Observe that such a partition exists for any n and any $q \geq \frac{n+1}{2}$ if $r \geq 3$. It also exists for $r = 2$ if either n is even or n is odd and $q > \frac{n+1}{2}$. Then define utility scalars $(u_h^i(\cdot))_{h=1,\ldots,r}$ as follows: For $h = 1, \ldots, r$, and for all $i \in W_h$ let

$$u_h^i(1) \;>\; u_h^i(-) \;>\; u_h^i(0), \tag{7}$$

$$u_{h'}^i(0) \;>\; u_{h'}^i(-) \;>\; u_{h'}^i(1) \text{ for all } h' \in \{1, \ldots, r\} \setminus \{h\}, \tag{8}$$

$$\sum_{h'=1}^{r} u_{h'}^i(-) \;>\; \sum_{h'=1}^{r} u_{h'}^i(0). \tag{9}$$

Observe that $(u_k^i(\cdot))_{k \in \mathcal{K}}$ can always be chosen such that conditions (6), (7), (8) and (9) are satisfied and such that the corresponding preference ordering is strict. With this specification less than q agents prefer position 1 over 0 for any $h \in \mathcal{F}$. This implies that for all $h \in \mathcal{F}$ and for all agendas $a \in A$ with $h \in a$,

$$(V(a, P))_h = 0. \tag{10}$$

If $q < \frac{n+1}{2}$, then it is straightforward to show that there are two cases: Either there exists a partition $\{W_1 \ldots, W_r\}$ of the set of agents $\{1, \ldots, n\}$ into nonempty subsets W_h, $h = 1, \ldots, r$, such that $\#W_h < q$ for all $h = 1, \ldots, r$, and we can use the same utility specification as in the case where $q \geq \frac{n+1}{2}$. Or there exists a partition $\{W_1 \ldots, W_r\}$ of $\{1, \ldots, n\}$ into nonempty subsets W_h, $h = 1, \ldots, r$, such that $\#W_h \geq q$ and $n - \#W_h \geq q$ for at least one $h \in \{1, \ldots, r\}$. In the

latter case, choose the utility scalars $(u_h^i(\cdot))_{h=1,\ldots,r}$ as follows: For $h = 1, \ldots, r$, and for all $i \in W_h$ let

$$\begin{aligned} u_h^i(0) &> u_h^i(-) > u_h^i(1), & (11)\\ u_{h'}^i(1) &> u_{h'}^i(-) > u_{h'}^i(0) \text{ for all } h' \in \{1, \ldots, r\} \setminus \{h\}, & (12)\\ \sum_{h'=1}^{r} u_{h'}^i(-) &> \sum_{h'=1}^{r} u_{h'}^i(1). & (13) \end{aligned}$$

Again observe that $(u_k^i(\cdot))_{k\in\mathcal{K}}$ can always be chosen such that conditions (6), (11), (12) and (13) are satisfied and such that the corresponding preference ordering is strict. With this specification at least q agents prefer position 1 over 0 for any $h \in \mathcal{F}$. This implies that for all $h \in \mathcal{F}$ and for all agendas $a \in A$ with $h \in a$,

$$(V(a, P))_h = 1. \tag{14}$$

(7), (8) and (10) (resp. (11), (12)and (14)) imply that if an agenda contains at most $m \leq r-1$ issues from $\mathcal{F}$, then there exists at least one agent who gets his most preferred position on all m issues, and if an agenda contains all issues from $\mathcal{F}$, then every agent i gets his most preferred position on exactly $r-1$ issues in $\mathcal{F}$. Moreover, given (9) (resp. (13)) every agent prefers an agenda that contains all but the issues in $\mathcal{F}$ over any full agenda: For all agendas a that contain all but the issues in $\mathcal{F}$, and for all full agendas $a' \in A^K$,

$$V(a, P) \succ_i V(a', P) \text{ for all } i. \tag{15}$$

Let $(CE(a, P))_{a\in A}$ be any consistent equilibrium collection of sets of continuation agendas and let $a \in A \setminus A^K$ with $k \in a$ for some $k \in \mathcal{F}$. We will prove that $a \notin CE(a, P)$. By definition of the agents' utility functions there exists an agent i who gets his most preferred position on all issues $l \notin a$. To see this, note that all agents agree on the position for issues not in $\mathcal{F}$ and there are at most $r-1$ issues from $\mathcal{F}$ which are not on agenda a. Hence,

$$V(a', P) \succ_i V(a, P) \text{ for all } a' \in \bigcup_{l\notin a} CE((a, l), P).$$

By (E2) this implies that $a \notin CE(a,P)$. Lemma 2.1 then implies that

$$CE(a,P) \subset A^K \text{ for all } a \in A \text{ with } k \in a \text{ for some } k \in \mathcal{F}. \tag{16}$$

Moreover, if $a \in A$ contains all issues but those in $\mathcal{F}$, then $CE(a,P) = \{a\}$. To see this, observe that any $a' \in CE((a,k),P)$ with $k \notin a$ must be a full agenda by (16). By definition of the agents' utility function it follows that

$$V(a,P) \succ_i V(a',P) \text{ for all } i. \tag{17}$$

(E2) then implies that $a \in CE(a,P)$. Moreover, by (17) and the fact that any $a'' \in CE(a,l)$ for some $l \notin a$ is a full agenda by (16), we conclude that no $a' \in CE((a,k),P)$ with $k \notin a$ is rationalizable relative to a. (E3) then implies that $CE(a,P) = \{a\}$.

Let $0 \leq m \leq K - r$ and let $a \in A^m$ with $h \notin a$ for all $h \in \mathcal{F}$. We will now show inductively over m that $a' \in CE(a,P)$ implies that a' contains all issues but those in $\mathcal{F}$. We have shown above that this is true for $m = K - r$. Suppose that the claim has been proven for all $\overline{m}$ with $\overline{m} \leq m \leq K-r$, where $1 \leq \overline{m} \leq K-r$, and let $a \in A^{\overline{m}-1}$ with $h \notin a$ for all $h \in \mathcal{F}$. By (E1), if $a' \in CE(a,P)$, then either $a' = a$ or $a' \in \bigcup_{k \notin a} CE((a,k),P)$. Let $a' \in CE((a,k),P)$ for some $k \notin a$. If $k \in \mathcal{F}$, then $a' = (a,k,\ldots) \in A^K$ by (16). If $k \notin \mathcal{F}$, then by our induction hypothesis a' contains all issues but those in $\mathcal{F}$ which implies that

$$V(a',P) \succ_i V(a,P) \text{ for all } i,$$

since all agents agree on the position for all issues not in $\mathcal{F}$. (E2) then implies that $a \notin CE(a,P)$. Hence, $CE(a,P) \subset \bigcup_{k \notin a} CE((a,k),P)$ and any $a' \in CE(a,P)$ is either a full agenda or contains all issues but those in $\mathcal{F}$.

Suppose by way of contradiction that there exists $a' \in CE(a,P)$ with $a' \in CE((a,k),P)$ for some $k \in \mathcal{F}$. Then $a' = (a,k,\ldots) \in A^K$ by (16). Let $l \notin \mathcal{F}$ and $l \notin a$ and let $a'' \in CE((a,l),P)$. Then by the induction hypothesis $a'' = (a,l,\ldots)$ contains all issues but those in $\mathcal{F}$ which implies that

$$V(a'',P) \succ_i V(a',P) \text{ for all } i.$$

Hence, a'' is rationalizable relative to a and (E3) implies that $a'' \in CE(a,P)$. Moreover, if both $a' = (a,k,\ldots) \in CE(a,P)$ and $a'' = (a,l,\ldots) \in CE(a,P)$ with

$k \neq l$, then (E3) implies that a' is rationalizable. Therefore, there exists an agent i and some $\hat{a} \in CE(a, P)$ with $\hat{a} = (a, h, \ldots)$ for some $h \neq k$ such that

$$V(a', P) \succ_i V(\hat{a}, P). \tag{18}$$

However, by what we have shown above, any $\hat{a} \in CE(a, P)$ is either a full agenda or contains all issues but those in $\mathcal{F}$. Since a' is a full agenda, this contradicts (18). Hence, if $a' \in CE(a, P)$, then $a' \in CE((a, k), P)$ for some $k \notin \mathcal{F}$ and by the induction hypothesis we conclude that a' contains all issues but those in $\mathcal{F}$. This proves the claim for all m with $0 \leq m \leq K - r$. In particular, any consistent equilibrium agenda a^* at P has the property that $\mathcal{F}$ is the set of free issues at a^*.

It remains to consider the case, where $\mathcal{F} = \{1, \ldots, r\}$ with $r > n$. If $1 \leq q \leq n - 1$, choose $(u_k^i(\cdot))_{k \in \mathcal{K}}$ as follows. For $i = 1, \ldots, n-1$, let

$$\begin{aligned} u_i^i(0) &> u_i^i(-) > u_i^i(1), && (19)\\ u_k^i(1) &> u_k^i(-) > u_k^i(0) \quad \text{for all } k \in \mathcal{K}, k \neq i, && (20)\\ \sum_{k=1}^{r} u_k^i(-) &> \sum_{k=1}^{r} u_k^i(1), && (21) \end{aligned}$$

and for $i = n$ let

$$\begin{aligned} u_k^n(0) &> u_k^n(-) > u_k^n(1) \quad \text{for all } k = n, \ldots, r, && (22)\\ u_k^n(1) &> u_k^n(-) > u_k^n(0) \quad \text{for all } k \in \mathcal{K}, k \notin \{n, \ldots, r\}, && (23)\\ \sum_{\substack{l=1\\ l\neq k}}^{r} u_l^n(-) &< \sum_{\substack{l=1\\ l\neq k}}^{r} u_l^n(1) \quad \text{for all } k = n, \ldots, r, && (24)\\ \sum_{k=1}^{r} u_k^n(-) &> \sum_{k=1}^{r} u_k^n(1), && (25) \end{aligned}$$

Observe that $(u_k^i(\cdot))_{k \in \mathcal{K}}$ can always be chosen such that conditions (19)-(25) are satisfied and such that the corresponding preference ordering is strict. Moreover, note that with this specification, for every issue k there are at least $n - 1$ agents who prefer position 1 over 0, which implies that

$$(V(a, P))_k = 1 \quad \text{for all } a \in A \text{ and for all } k \in a.$$

If $q = n$, choose $(u_k^i(\cdot))_{k\in\mathcal{K}}$ as follows. For $i = 1, \ldots, n-1$, let

$$\begin{aligned} u_i^i(1) &> u_i^i(-) > u_i^i(0), && (26)\\ u_k^i(0) &> u_k^i(-) > u_k^i(1) \quad \text{for all } k \in \mathcal{K}, k \neq i, && (27)\\ \sum_{k=1}^{r} u_k^i(-) &> \sum_{k=1}^{r} u_k^i(0), && (28)\end{aligned}$$

and for $i = n$ let

$$\begin{aligned} u_k^n(1) &> u_k^n(-) > u_k^n(0) \quad \text{for all } k = n, \ldots, r, && (29)\\ u_k^n(0) &> u_k^n(-) > u_k^n(1) \quad \text{for all } k \in \mathcal{K}, k \notin \{n, \ldots, r\}, && (30)\\ \sum_{\substack{l=1\\ l\neq k}}^{r} u_l^n(-) &< \sum_{\substack{l=1\\ l\neq k}}^{r} u_l^n(0) \quad \text{for all } k = n, \ldots, r, && (31)\\ \sum_{k=1}^{r} u_k^n(-) &> \sum_{k=1}^{r} u_k^n(0), && (32)\end{aligned}$$

Again observe that $(u_k^i(\cdot))_{k\in\mathcal{K}}$ can always be chosen such that conditions (26)-(32) are satisfied and such that the corresponding preference ordering is strict. Also, note that with this specification, for every issue k there is at most one agent who prefers position 1 over 0, which implies that

$$(V(a, P))_k = 0 \quad \text{for all } a \in A \text{ and for all } k \in a.$$

For all quotas q the preferences we have specified above have the following properties: Every agent $i \in \{1, \ldots, n-1\}$ gets his most preferred position on all issues but issue i, and agent n gets his most preferred position on all issues but issues $n, \ldots, r$. Moreover, all agents prefer to stop at an agenda that contains all but the issues in $\mathcal{F}$ rather than adding all issues in $\mathcal{F}$ to the given agenda. Finally, agent n prefers to add all remaining issues in $\mathcal{F}$ to any agenda that already contains some issue in $\{n, \ldots, r\}$ and all issues not in $\mathcal{F}$. We will use these properties to prove that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda.

Let $(CE(a, P))_{a\in A}$ be any consistent equilibrium collection of sets of continuation agendas and let $a \in A \setminus A^K$ with $k \in a$ for some $k \in \mathcal{F} = \{1, \ldots, r\}$. We

will prove that $CE(a,P) \subset A^K$. If $a \in A^K$ there is nothing to prove. Hence, let $a \in A^m$ for some $m < K$. The proof is by induction over m. Let $m = K-1$ and let $l \notin a$. If $l \notin \mathcal{F}$, then $V((a,l),P) \succ_i V(a,P)$ for all i and (E2) implies that $a \notin CE(a,P)$. It follows that $CE(a,P) = \{(a,l)\} \subset A^K$. If $l \in \mathcal{F}$, then there are $n-1$ agents who prefer $V((a,l),P)$ over $V(a,P)$ and again we conclude that $CE(a,P) = \{(a,l)\} \subset A^K$.

Now suppose that for all $\bar{m} \leq K-1$ it is true that $CE(a,P) \subset A^K$ for all $a \in A^m$ with $\bar{m} \leq m \leq K-1$ and $k \in a$ for some $k \in \mathcal{F}$. Let $a \in A^{\bar{m}-1}$. If $k \in \{1,\dots,n-1\}$, then agent k gets his most preferred position on all issues not in a, which implies that for all $l \notin a$,

$$V(a',P) \succ_k V(a,P) \text{ for all } a' \in CE((a,l),P).$$

Hence, (E2) implies that $a \notin CE(a,P)$ and

$$CE(a,P) \subset \bigcup_{l \notin a} CE((a,l),P).$$

From the induction hypothesis we conclude that $CE(a,P) \subset A^K$.

If $l \notin a$ for all $l \in \{1,\dots,n-1\}$ and $k \in a$ for some $k \in \{n,\dots,r\}$, let $a' \in CE((a,l),P)$ for some $l \in \{1,\dots,n-1\}$. Then by the induction hypothesis it follows that $a' \in A^K$ and (24), respectively (31) imply that

$$V(a',P) \succ_n V(a,P).$$

Again, (E2) implies that $a \notin CE(a,P)$ and from the induction hypothesis we conclude that $CE(a,P) \subset A^K$.

Now let $a \in A$ contain all issues but those in $\mathcal{F}$. Then using the same argument as in the first part of the proof, where we considered the case $r \leq n$, (21), (25), (28), (32) imply that $CE(a,P) = \{a\}$. Moreover, as in the first part of the proof this implies that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda. This proves the theorem.

$\square$

Proof of Lemma 4.1: Let $V : A \times \mathcal{S}^n \to X$ be voting by quota $q \in \{1,\dots,n\}$ and let $P = (\succ_1,\dots,\succ_n) \in \mathcal{S}^n$. Let $(CE(a,P))_{a\in A}$ be an equilibrium collection

of sets of continuation agendas. Then, for $a \in A^{K-1}$ and $k \notin a$ (E1) implies that

$$CE((a,k),P) = \{(a,k)\}.$$

Moreover, $(V(a,P))_k = -$ and $(V((a,k),P))_k \in \{0,1\}$. If $(V((a,k),P))_k = 1$ ($(V((a,k),P))_k = 0$) then there exists at least one agent i with $u^i_k(1) > u^i_k(0)$ ($u^i_k(0) > u^i_k(1)$). In either case the fact that $\max\{u^i_k(1), u^i_k(0)\} > u^i_k(-)$ implies that

$$V((a,k),P) \succ_i V(a,P)$$

for at least one agent i. Using (E2) we conclude that $a \notin CE(a,P)$ and hence $CE(a,P) = \{(a,k)\} \in A^K$ by (E1).

□

Proof of Proposition 4.3: Let n be odd and let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q = \frac{n+1}{2}$. Let $P = (\succ_1, \ldots, \succ_n) \in \bar{\mathcal{S}}^n$ and let $(CE(a,P))_{a \in A}$ be an equilibrium collection of sets of continuation agendas. Obviously, for any full agenda $a \in A^K$,

$$CE(a,P) = \{a\}.$$

Now consider any agenda $a \in A^{K-1}$ and let $k \notin a$. By condition (E1) and Lemma 4.1, we conclude that

$$CE(a,P) = \{(a,k)\}.$$

Now consider any agenda $a \in A^{K-2}$ and let k and l be the free issues at a. By condition (E1) and the previous reasoning, $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE(a,k) \cup CE(a,l) = \{a, (a,k,l), (a,l,k)\}$. Observe that agendas (a,k,l) and (a,l,k) are outcome equivalent since the voting procedure does not depend on the ordering of the issues in the agenda. Let $x = V(a,P)$ and $y = V((a,k,l),P) = V((a,l,k),P)$. Then $y_m = x_m$ for all $m \in a$ and $y_k, y_l \in \{0,1\}$. By (E2) $a \in CE(a,P)$ if and only if for all agents i,

$$u^i_k(-) + u^i_l(-) > u^i_k(y_k) + u^i_l(y_l). \tag{33}$$

A necessary condition for (33) to hold for all agents is that no agent i is in the winning majority for both issues, k and l. However, if n is odd and $q = \frac{n+1}{2}$,

there always exists at least one agent who belongs to the winning majority for both issues. This implies that (33) is violated for at least one agent i and hence,

$$CE(a, P) \subset A^K \text{ for all } a \in A^{K-2}.$$

Lemma 2.1 then implies that

$$CE(a, P) \subset A \setminus \left(A^{K-1} \cup A^{K-2}\right) \text{ for all } a \in A.$$

$\square$

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