



The Bright Side of the Doom Loop: Banks Exposure and Default Incentives

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Abstract

We revisit the doom-loop debate emphasizing the commitment device that the exposure of the financial sector to sovereign debt provides to the sovereign. If this mechanism is strong then lower exposure or a commitment not to bailout banks, two policy prescriptions that have emerged in this literature, can backfire. We present a simple 3-period model with strategic sovereign default where debt is held by local banks or foreign investors and show that: i) Reducing exposure reduces commitment and hence increases the probability of default, without avoiding the “doom loop”. Furthermore, that allowing banks to buy additional sovereign debt in times of sovereign distress can rule out the doom loop. ii) A no bailout commitment is not sufficient to rule out self-fulfilling expectations.

Keywords: Sovereign Default; Bailout; Doom Loop; Self-fulfilling Crises

JEL Classification E44, E6, F34.

1 Introduction

The “doom loop” or “sovereign-bank nexus” has been identified as a key driver of the European debt crisis by both academics and policy makers.¹ According to this view, problems of sovereign debt sustainability and of financial stability reinforce each other due to mutual exposures between the public and the financial sectors. If public debt loses value due to deteriorating creditworthiness of the public sector, this hurts financial sectors’ balance sheet, since the financial sector is a major holder of public debt. Weakened balance sheets in turn force the government to bailout banks. This implies an expense for the government and hence a

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¹E.g. Brunnermeier (2015), Benassy-Quere et al. (2018)

further deterioration of its fiscal capacity.² This vicious circle can amplify fundamental shocks (Farhi and Tirole (2016)) or even give rise to crises that are purely generated by self-fulfilling pessimistic expectations (Brunnermeier et al. (2016), Cooper and Nikolov (2013)). It can hence explain why the sovereign crises can develop so suddenly and easily spiral out of control.

Based on this analysis a number of proposals to prevent such crises have been made. Cooper and Nikolov (2013) propose that a commitment not to bailout banks would reduce the exposure of the government to the financial sector and hence rule out self-fulfilling crises. Brunnermeier et al. (2016) suggest to reduce the exposure of the financial sector to the government by reducing the amount of public debt relative to their equity to the same end.

However, the doom loop theory largely ignores how the health of the financial system interacts with the incentives of a government to repay. Indeed, *ceteris paribus*, a government would be less inclined to default on its debt if this were to cause major costs to the financial sector. In this paper we revisit the doom loop in a model of self fulfilling equilibria, which takes these interactions into account. Our theory puts the desirability of the above-mentioned policies to address the doom loop into question.

First, we find that it is not desirable to constrain the exposure of the domestic financial sector (banks for short) to the government. This is so because this exposure serves as a commitment device for the government. Reducing the exposure in general reduces commitment and hence increases the probability of default, without avoiding the “doom loop”. Even more strikingly, we show that constraining banks’ ability to increase their exposure in times of sovereign distress only makes things worse. If banks could buy bonds in distressed times, they would support the bond price by delivering additional commitment, thus resolving the distress and ruling out the negative expectations equilibrium. Second, we find that a no bailout commitment is not sufficient to rule out self-fulfilling expectations. Even if a no bailout rule frees the government from the burden to bailout banks, it reduces GDP after a financial crisis, thus reducing its commitment to repay debt after a financial crisis. The bad equilibrium is hence sustained and becomes even worse.

We develop these arguments based on a simple 3 period model of sovereign debt and banks similar to the above cited papers. At period 0 governments issue debt, which is bought by foreign investors and banks. Banks furthermore make loans. They finance their investments by deposits and equity. In period 1 a sunspot shock may hit the economy. If it does, the bond price may drop to a lower value, causing banks to fail. Bank failure entails a cost, because a fraction of the bank’s investments (loans) get destroyed in that event. Thus the government has an incentive to intervene and bail them out. To do so it needs to issue more debt. If this debt is bought by foreign investors, the default incentives of the government increase, making default more likely and hence sustaining the negative expectations. In period 2 all debt matures. The government defaults strategically. If it defaults this again bankrupts banks, which again entails a cost in terms of forgone investment. The model features two equilibria, one where the materialization of a bad sunspot triggers a drop in the bond price causing a bank bailout

²As Brunnermeier et al. (2016) argue, weaker balance sheets also affect the public sector indirectly by causing a credit crunch, which leads to lower output and hence a reduction in the tax base.

and one where it doesn't. Ruling out bailouts in period 1 eliminates the sunspot equilibrium with bailout but just by replacing it by an even worse sunspot equilibrium. However, allowing banks to purchase the debt, which the government issues to bail them out, does rule out the bailout equilibrium because, unlike foreign held debt, domestically held debt does not lead to an increase of the default incentives.

Our model is highly stylized. Yet it highlights that debt re-nationalization in bad times may be just what is necessary to avoid a market turmoil to develop into a full blown crisis. We thus provide an argument against policies that restrict the financial sectors exposure to domestic debt, which was prominently advocated by a group of German and French economists (Benassy-Quere et al. (2018)). This proposal was soon criticized by Messori and Micossi (2018). Indeed, our model provides a formalization of their critique.

Related literature Our paper provides a bridge between two strands of literature. The first is the literature on the doom loop. Brunnermeier et al. (2016) and Cooper and Nikolov (2013) propose 3 period models that are very similar to ours. In their models multiplicity arises through the exact same mechanisms. Leonello (2017) shows that the doom loop can exist even if banks hold no explicit claims to the government on their balance sheets (bonds or debt) but enjoy government guarantees (deposit insurance, bailouts) and resolves the multiplicity of equilibria through global games. Acharay et al. (2014) and Farhi and Tirole (2016) provide a slightly different notion of the doom loop. Instead of generating multiple equilibria, the doom loop serves as an amplifier, so that small fundamental changes can lead to large changes in the equilibrium. What distinguishes our model is that, in these models the default incentives increase in total sovereign debt, while in our model the default incentives increase only in foreign held debt but not in bank held debt, which rather serves as a commitment device. This leads us to arrive to contrary policy conclusions.

The second strand regards the commitment role of domestic exposure to sovereign debt. This idea has been developed both in 3 period models (Balloch (2016), Basu (2010), Bolton and Jeanne (2011), Brutti (2011), Erce (2012), Gennaioli et al. (2014) and Mayer (2011)) as well as in quantitative dynamic models (Sosa-Padilla (2018), Boz et al. (2014), Balke (2018), Engler and Grosse Steffen (2016), Mallucci (2014), Perez (2015), Thaler (2019)). It is furthermore backed up by empirical evidence. E.g. Gennaioli et al. (2014) show that sovereign default premia decrease in domestic bank's exposure to their sovereign. Relative to this literature we contribute by adding a notion of the doom loop.

2 Model

2.1 Setup

We consider a three period economy $t = 0, 1, 2$. The domestic economy is populated by a representative bank, a representative household and a benevolent government. The sovereign issues bonds at $t = 0$, which are held by the local bank and international creditors. At $t = 1$

a sunspot is realized and the sovereign decides if to bailout the bank, at $t = 2$ productivity is revealed, the government decides whether to repay or default, assets pay off and the household consumes.

Initial conditions The period 0 balance sheet of the bank and the government are exogenously given. The government has an initial level of outstanding bonds B_0 that are held by the local bank B_0^h or foreign creditors B_0^f , such that

$$B_0 = B_0^h + B_0^f$$

The initial assets of the bank are bonds B_0^h and loans L_0 . Besides, it has deposit liabilities D_0 . All assets are promises to repay 1 unit in period $t = 2$. Each loan is backed by an investment project.

Period 1 At $t = 1$ the sunspot variable s is revealed, where $s \in \{n, p\}$ refers to normal times n or panic p . Other than that no fundamentals change, but debt markets open again and hence the price of debt may change. This affects the bank's equity E_1

$$E_1 = q_1 B_0^h + L_0 - D_0$$

If the price drops sufficiently, the bank's equity may become negative. In that case the bank is insolvent and a fraction ϕ of the loans and the associated projects are destroyed. This captures the disruptions caused by a bank's insolvency. Let L_1 denote the level of loans that remain at the end of period 1, if the banks are insolvent at the end of the period we have $L_1 = (1 - \phi)L_0$.

However, if banks get in trouble, the government can bailout the bank to avoid loan destruction. We assume that the bailout payment restores banks equity to 0.³ To finance the bailout it issues additional debt to international creditors ΔB_1^f , which also matures at $t = 2$. Denote the price of debt in period 1 by q^{sb} , where s is the sunspot variable and b is the indicator for the bailout decision. The transfer to banks is then given by $q^{s1} \Delta B_1^f$. The size of the transfer is

$$q^{s1} \Delta B_1^f = \max \left\{ D_0 - q^{s1} (B_0^h) - L_0, 0 \right\}$$

where the max operator captures the fact that if the bank is solvent the required bailout is zero. Banks use these funds to invest in a safe asset with return 1; for example a foreign bond. The total investment in the safe asset is given by $S_1 == q^{s1} \Delta B_1^f$.

With a bailout the new level of outstanding debt held by foreign creditors corresponds to $B_1^f = B_0^f + \Delta B_1^f$. Foreign creditors are deep pocketed, competitive and risk neutral. Given a world interest rate of 1, their demand schedule requires the bond price to be equal to one minus the expected default probability at $t = 1$:

³Since the government has no other use for funds at this period and since a bailout smaller than this is not avoiding loan destruction, this is the optimal size of the bailout.

$$q^{sb} = 1 - \text{prob}\{\text{default} \mid s, b\}$$

Period 2 At $t = 2$ productivity ω is revealed, then the government decides upon repayment ($d = 0$) or default ($d = 1$), production takes place, all assets pay off and the household consumes.

Bank equity in period 2, after the government announces its default decision, is:

$$E_2 = L_1 + S_1 - D_0 + B_0^h(1 - d)$$

No bailout is possible at this point. If banks are insolvent in period 2, again a fraction θ of the outstanding loans and associated projects gets destroyed and $L_2 = (1 - \theta)L_1$. Otherwise $L_2 = L_1$.⁴

Production then is given by

$$Y_2 = \omega L_2$$

where ω is productivity and L_2 is the number of surviving loans. ω is a random variable with c.d.f $F(\omega)$ and support $[1, \bar{\omega}]$ where $\bar{\omega}$ is an arbitrary number > 1 . After production all assets pay off: The government taxes the household lump sum to repay its debt (if $d = 0$), the projects pay L_2 to the banks and $(\omega - 1)$ to the household, the bank pays back its deposits D_0 and distributes its equity to the household, who consumes everything. Consumption is hence equal to domestic net income:

$$C_2 = \omega L_2 - (1 - d)(B_1^f)$$

Since the households are risk neutral, the benevolent government's objective is to maximize the expected value of C_2 . However the government has no commitment and hence solves a two stage problem. We summarize the optimization problem in Appendix A.

This concludes the setup of the model. We are now ready to define an equilibrium.

Definition. A subgame perfect equilibrium is given by bailout decisions $\{b^s\}_{s \in \{n, p\}}$, default policy functions $\{d^{sb}(\omega)\}_{s \in \{n, p\}, b \in \{0, 1\}}$ and a price vector $\mathbf{q} = (q^{n0}, q^{n1}, q^{p0}, q^{p1})$ such that

1. The bailout decisions b^s and the default policy functions $d^{sb}(\omega)$ maximize expected consumption, taking as given the pricing vector \mathbf{q} .
2. The price q^{sb} corresponds to the expected repayment probability implied by the default policy function $d^{sb}(\omega)$ for each of the possible scenarios $\{n0, n1, p0, p1\}$.

To focus on the case where banks are exposed to changes in bond prices, we place a few restrictions on parameters.

Assumption 1. *The bank's deposit liabilities exceed its loans $D_0 > L_0$.*

⁴This loss may be different than the loss in period 1, for example because the projects are already almost completed.

This assumption guarantees that the bank becomes insolvent if the bonds become worthless, ensuring that it is costly for the government to default and hence that the exposure of banks acts as a commitment device. In the absence of this assumption, no foreign debt would be sustainable.

Assumption 2. *The banks holdings of sovereign debt (B_0^h) are greater than $\kappa(D_0 - L_0)$ where $\kappa = \left(1 - F\left(\frac{1}{\theta} \frac{B_0^f}{L_0}\right)\right)^{-1} > 0$.*

This assumption ensures that banks hold enough bonds, such that an equilibrium exists where they are solvent, despite having less loans than deposits. It is hence the natural counterpart to the previous assumption. In other words, this assumption ensures that an equilibrium exists where banks are solvent in the non-modeled period 0.

Assumption 3. *If a bank gets insolvent in period 1 it is allowed to continue operating despite negative equity. The fraction of loans that get destroyed in period 1 is at most $\phi L_0 < L_0 + B_0^h - D_0$.*

This assumption guarantees that banks are solvent in period 2 in case of repayment, even if in period 1 they lose a fraction ϕ of the loans. In this case the government has an incentive for repayment, despite a panic in period 1. We will also refer to the case when A3 is violated and the appendix extends our analysis to that scenario. Basically if A3 does not hold, then after a no bailout decision the government has no incentives to repay the debt and consequently the price of sovereign debt is zero in a non-bailout scenario.

Figure 1 illustrates the model as a game in extensive form under these set of assumptions.

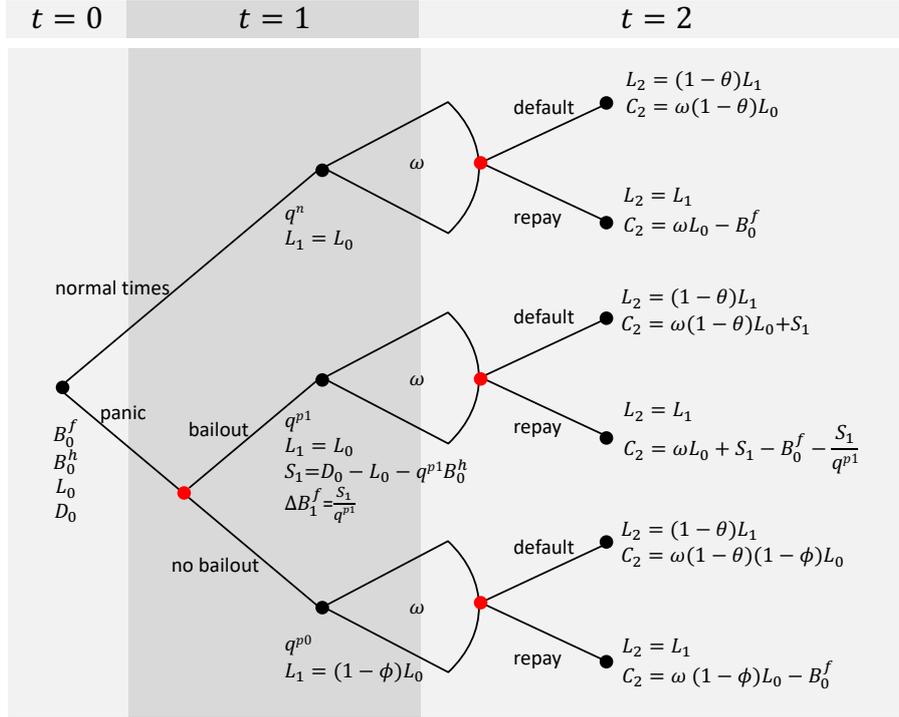
3 The doom loop

This model allows for several equilibria. We start by describing an equilibrium in which the bond price is unaffected by the sunspot and banks are solvent in period 1. Then we describe an equilibrium where banks are solvent in normal times ($s = n$), but insolvent in panic times ($s = p$). In the second equilibrium, the allocation in normal times coincides with the allocation of the first equilibrium.

Proposition 1. *An equilibrium where $q^{s,b}$ is the same for all s and b and banks are solvent in $t = 1$ (and hence require no bailout) exists and is unique. The equilibrium price and policy functions are given by*

$$\begin{aligned}
 q &= 1 - F(\omega^n) \\
 d &= \begin{cases} 1 & \omega < \omega^n \\ 0 & \omega \geq \omega^n \end{cases} \\
 \text{where } \omega^n &= \frac{1}{\theta} \frac{B_0^f}{L_0}
 \end{aligned}$$

Figure 1: Timeline. This figure illustrates the timing of the model. Decision nodes are marked red. The quantities at the terminal nodes hold under Assumptions 1 to 3.



The proof (Appendix B.1) is simple: It starts by solving the optimal repayment choice of the government in period 2 as a function of TFP ω , assuming that L_1, B_1^f, B_1^h are equal to their initial values. It then continues to show that this repayment policy implies a debt price q which is high enough to make banks solvent in period 1.

The optimal default policy is intuitive and in line with much of the literature on sovereign default: The government defaults for TFP below a certain threshold ω^n . The larger the foreign debt burden B_0^f , the larger the incentives to default. Conversely, the higher TFP ω and the greater the number of productive assets L_0 available at the last period, the lower the incentives of default. After all, the “punishment” for default is banks insolvency, thus causing a proportional reduction of bank loans, and hence of the output produced by the productive asset. The equilibrium bond price q follows directly from the default decision. It is equal to the probability of default by the foreign lenders’ asset pricing condition.

Note how in this equilibrium sovereign debt can be sustained purely due to the banks exposure to the government similar to the models cited in the second part of the literature review.⁵

Next we show that besides the fundamental equilibrium described above a sunspot equilibrium exists, where $q^{s,b}$ depends on s and banks are bailed out by the government in case of panic.

⁵By “being sustained” we mean that debt has a positive price at $t=1$ and would have a positive price in $t=0$ as well, although we do not model period 0.

Equilibrium multiplicity and the doom loop.

Proposition 2. *For ϕ sufficiently large, a sunspot equilibrium exists and is characterized by the following:*

-For $s = n$ the policy and price functions are identical to before.⁶

$$b^n = 0 \quad (1)$$

$$q^n = 1 - F(\omega^n) \quad (2)$$

$$d^n(\omega) = \begin{cases} 1 & \omega < \omega^n \\ 0 & \omega \geq \omega^n \end{cases} \quad (3)$$

$$\text{where } \omega^n = \frac{1}{\theta} \frac{B_0^f}{L_0} \quad (4)$$

-For $s = p$, the government decides to bailout banks $b^p = 1$. The price of debt $q^{p,1}$ and the default threshold $\omega^{p,1}$ are the solution to the system

$$q = 1 - F(\omega) \quad (5)$$

$$\omega = \omega^n + \frac{1}{\theta} \frac{\left(\frac{D_0 - L_0}{q} - B_0^h\right)}{L_0} \quad (6)$$

and the default policy is given by

$$d^{p,1}(\omega) = \begin{cases} 1 & \omega < \omega^{p,1} \\ 0 & \omega \geq \omega^{p,1} \end{cases} \quad (7)$$

-The former panic price and policy are supported by the following price and policy for $s = p$, conditional on no bailout,

$$q^{p,0} = 1 - F(\omega^{p,0}) \quad (8)$$

$$d^{p,0}(\omega) = \begin{cases} 1 & \omega < \omega^{p,0} \\ 0 & \omega \geq \omega^{p,0} \end{cases} \quad (9)$$

$$\text{where } \omega^{p,0} = \frac{\omega^n}{1 - \phi} \quad (10)$$

this price and policy are off the equilibrium path.

The proof (Appendix B.2) formalizes a simple idea that we illustrate in Figure 2. Any equilibrium needs to be consistent with the lenders' asset pricing condition (equations (5) and (2)). The blue line in the figure shows the schedule defined by this condition. Since $F(\omega)$ is a c.d.f, $1 - F(\omega)$ has to be non-increasing—the higher the default threshold, the higher the default probability the lower the price.

⁶Where we omit the superscript b as when $s = n$ we have that the banks are solvent and require no bailout.

Furthermore, any equilibrium has to be consistent with the government’s default policy. If banks are solvent in period 1, this policy can conveniently be characterized by the default threshold ω^n defined in equation (4). This threshold is independent of the debt price. In the figure it is shown as the vertical green line. The shaded area shows the levels of q for which the bank would be insolvent ($E_1 < 0$). If the green and the blue line intersect in the non-shaded solvency region the part of the equilibrium associated to $s = n$ exists.⁷

In the case that the bank is insolvent and needs to be bailed out in period 1, the relationship between the bond price and the default threshold given by the government’s default policy is a little more complicated. It is defined by equation (6) and represented by the red schedule in the figure. In this case, the lower the bond price in period 1, the larger the equity shortfall and hence the larger the bailout. Furthermore, the lower the bond price the larger the amount of foreign debt that needs to be issued by the government to finance a bailout of a given size. A larger debt burden in turn implies a higher default probability in period 2. Hence the red schedule is downward sloping. If the red and the blue lines intersect in the shaded (insolvency) region, the sunspot equilibrium exists.⁸

Note that this is the infamous doom loop at play. Just as in Brunnermeier et al. (2016) and Cooper and Nikolov (2013), pessimistic expectations become self-fulfilling. If agents happen to coordinate on the lower bond price, banks become insolvent, forcing the government to increase its debt to finance a bailout, which makes it more likely that the government defaults later on (red curve). A higher default probability in turn justifies a lower bond price (blue curve). The pessimistic expectations are hence validated.

In this discussion of the sunspot equilibrium we focused on the case where bailout is the government’s optimal choice. This is guaranteed by our assumption that the loans losses in period 1 (ϕ) are a large enough (see Appendix B.2). This assumption is natural—if it were not satisfied, banks would never be bailed out and hence the doom loop would not exist. In section 4.3 we turn our attention to the bailout decision and the possibility of committing not to bailout banks.

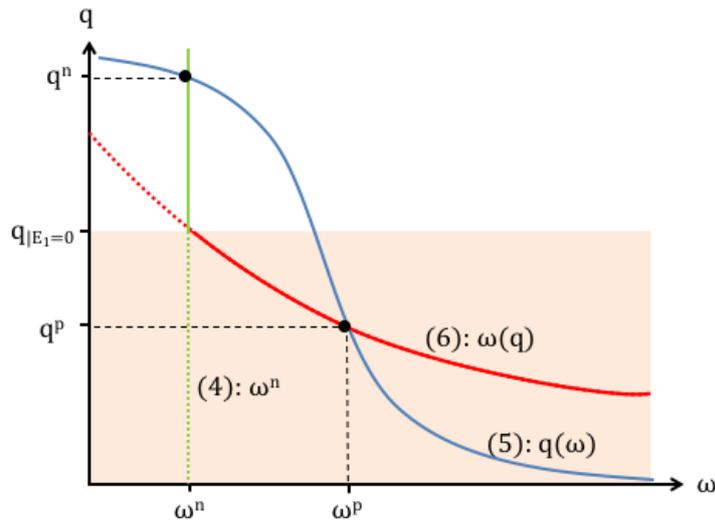
4 Breaking the doom loop

Mutual exposure between the sovereign and the bank generates the doom loop. Thus breaking the doom loop may require reducing the banks exposure to sovereign debt or reducing the sovereign exposure to banks by no bailout rules. Indeed, as Brunnermeier et al. (2016) and Cooper and Nikolov (2013) show, in models where banks’ exposure does not generate commitment to repay, both strategies can be successful at ruling out the bad equilibrium. This section

⁷Assumption 1 ensures that this intersection indeed lies in the solvency region.

⁸The existence of the intersection is guaranteed to exist and lie within the shaded region since (i) $F(\omega)$ has bounded support such that the blue schedule eventually reaches 0 (ii) the red schedule takes the value $q|_{E_1=0} < F(\omega^n)$ for ω^n , converges to 0 from above and is concave. For a generic cumulative distribution function F there may be more than one crossing. In that case we shall from now on focus on the crossing that delivers the highest value of q^p .

Figure 2: Equilibrium



revisits these policies. Taking into account the effect of exposure on repayment incentives turns out to overturn these results.

4.1 The role of the ex ante exposure

We start by considering a reduction in the exposure of banks ex ante. As discussed above, the bank's exposure is the only incentive for the government to repay its debt. Hence it is straightforward to see that a reduction of the banks exposure to such an extent that the bank would be solvent in case of default (violating Assumption 1) would imply that the government would always default. Sovereign debt would hence have no value in equilibrium. While this stark reduction in exposure would rule out the sunspot, it does so at the cost of reducing sovereign debt sustainability. We did not model an explicit reason for why sovereign debt is desirable, but to the extent it is, this strong policy would clearly be undesirable.⁹ Yet it may be desirable to reduce the exposure of banks at the margin. Such a marginal change would not violate Assumption 1 and hence the commitment value of bank exposure would not be reduced. Could such a change improve the conditions in case the panic state materializes?

To think about a marginal reduction in exposure, we first need to formally define exposure. We define banks exposure as the value of their bond holdings in period 1 in normal times (n) relative to their equity. This measure is known as equity multiplier and is given by

$$EM = \frac{q^n B_0^h}{E_1^n}$$

⁹It is straightforward to extend the model in this direction. For example, assume that in period 0 the government must finance some public investment, and the costs of not doing so are prohibitively high.

The larger the equity multiplier, the larger the exposure.¹⁰ Exposure can hence be reduced by increasing bank equity E_1^n or reducing bank bond holdings $q^n B_0^h$. We will focus on the latter. That is we will consider a reduction of exposure through a reduction in $q^n B_0^h$ keeping E_1^n constant.¹¹ This requires a simultaneous adjustment of both the quantity of bonds held by the bank B_0^h and the deposits owed by the bank D_0 .¹² As is evident from Propositions 1 and 2, this change in exposure does not have an effect on the no-sunspot equilibrium or the sunspot equilibrium in normal time.

Formally, we assess the effect on prices of a marginal change in B_0^h , keeping equity in normal times constant. The following proposition summarizes the main result:

Proposition 3. *Lower exposure generates a lower price of sovereign debt and higher default probability in the panic state: $\frac{\partial q}{\partial B_0^h} \Big|_{E_1^n = \bar{E}} > 0$.*

The proof is in Appendix B.3. The intuition is simple: with banks less exposed, it requires a larger reduction in q^n to make them go bust. This result is illustrated in figure 3. The lower exposure shifts downwards the schedule $\omega(q)$ and the threshold for default increases and consequently the equilibrium price decreases.

As the default probability rises, the expected consumption and hence welfare also drops, as now in case of bailout the required issuance of debt to international creditors is larger. Lowering the exposure does not necessarily eliminates the panic event, and if it happens the costs are larger.

4.2 The role of the ex post exposure

In the previous section we saw that reducing the banks ex ante exposure does not help to rule out multiple equilibria. But what about the banks exposure in period 1 – does reducing banks exposure in crisis times help? To analyze this question we first allow the government to bailout the banks by providing them with additional domestic debt. Then we show how this allocation can be decentralized when banks are allowed to re-optimize their portfolio in period 1.

4.2.1 Bailout by domestic bonds

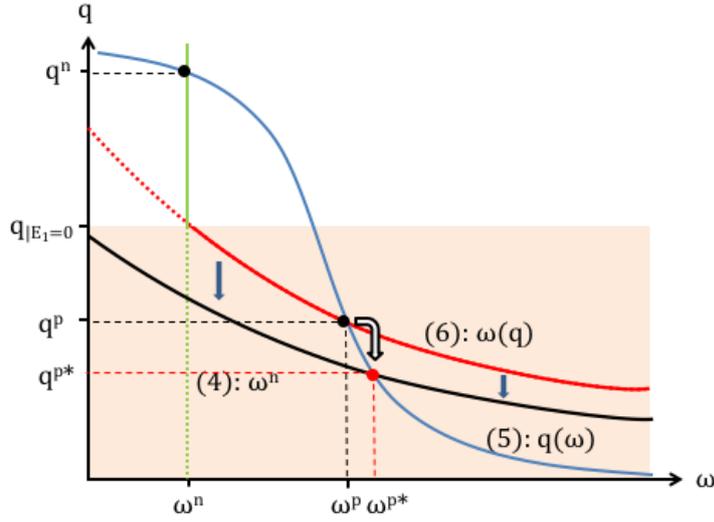
Consider the model as in section 2.1 but now assume that in a bailout the government directly provides the bank with additional sovereign additional debt, instead of borrowing abroad to finance the safe investment. In this case a bailout no longer increases the foreign debt burden, which remains at its initial value B_0^f . Hence the benefit of default no longer increases in the size of the bailout and thus the bond price $q^{p,1}$. The benefit of default does not change with the

¹⁰In particular if $EM > 1$ (which follows from Assumption 1) a fall in the price of sovereign debt by $(EM - 1)\%$ makes banks insolvent.

¹¹Note that E_1 coincides with E_0 under the additional assumption that the panic state is a 0 probability event.

¹²Note that if we were to change B_0^h without adjusting for deposits, given assumption 1, equity would fall more than the sovereign debt holdings value. Generating a higher level of exposure.

Figure 3: Equilibrium lowering B_0^h



bailout or the bond price either, because banks are bankrupt in case of default and solvent in case of repayment no matter how many bonds they got in the course of the bailout in period 1. That means that the doom loop, that leads to multiplicity of equilibria in the baseline model, is no longer active and we can rule out the sunspot equilibrium.

Proposition 4. *When the bank is bailed out with domestic bonds, the sunspot equilibrium ceases to exist.*

That means that increasing the exposure of banks by issuing additional debt in times of self-fulfilling expectations driven crises is benign and, in this simple model, in fact rules out such crises altogether. This is so because such bailout does not interact with the default incentives like a foreign debt financed bailout does. The result in Proposition 4 could also be achieved by issuing the bailout debt to international creditors and letting the newly issued debt to be traded in secondary markets as we describe next.

4.2.2 Portfolio re-optimization and secondary markets

So far our bank was extremely passive; it had no decision to take. Now we extend the model to allow the bank to re-optimize its portfolio in period 1. Loans and deposits are assumed to be illiquid, but the bank can now choose whether to invest the bailout funds it receives (S_1) in domestic debt, subject to a regulatory maximal exposure constraint $q^{p,1}B_1^h \leq \bar{B}$.

Just as in the basic setup, the value of the bailout S_1 is given by

$$S_1 = \min \left\{ D_0 - L_0 - q^{p,1}B_0^h, 0 \right\}$$

and it is financed with the issuance of new debt such that

$$\Delta B_1 = \frac{1}{q^{p,1}} S_1$$

where the new debt is allocated to foreign investors or local banks through the secondary markets satisfying $\Delta B_1 = \Delta B_1^f + \Delta B_1^h$.

The bank operates under limited liability. Its objective hence is to maximize the expected non-negative part of equity in period 2 i.e. $\mathbb{E}[\max\{E_2, 0\}]$, subject to the budget constraint $S_1 + qB_0^h \leq S_2 + qB_1^h$, where S_2 denotes the bailout funds that the bank keeps in the safe asset. The bank is atomistic and thus takes all prices and the governments actions as given. Due to limited liability the bank always has an incentive to buy as much debt as possible, whenever the default probability is positive.¹³¹⁴ It thus invests all the bailout funds into sovereign debt, if the exposure limit permits, or up to the limit \bar{B} otherwise, and invests the rest of the bailout funds in the safe asset. Then we have that the change in sovereign debt holdings and the safe asset holdings by the bank are given by:

$$\begin{aligned} \Delta B_1^h &= \min\{\bar{B}/q^{p,1} - B_0^h, \Delta B_1\} \\ S_2 &= S_1 - q^{p,1} \Delta B_1^h \end{aligned}$$

It is immediately evident that for $\bar{B} \rightarrow \infty$ the equilibrium of this economy coincides with that of the economy analyzed before in Proposition 4, where the government bails out the bank with sovereign debt but no trading is possible after the bailout. On the other hand, for finite \bar{B} the economy could feature multiple equilibria, the following proposition characterizes for which values of \bar{B} this is the case.

Proposition 5. If $\bar{B} \geq D_0 - L_0$ there is no sunspot equilibrium and the equilibrium corresponds to the equilibrium described in Proposition 1. If $\bar{B} < D_0 - L_0$ there is a sunspot equilibrium.

This result establishes that to rule out the Doom Loop is necessary that the regulatory maximal exposure in the panic state has to be greater than the equity shortfall in case of default ($D_0 - L_0$). From Assumption 2 we have that in normal times the exposure is already higher than this threshold as $q^{1,n} B_0^h > D_0 - L_0$. Consequently, the doom loop arises if banks face constraints that forces them to lower bond holdings sufficiently during panic. In such scenario, the panic is self-fulfilling as the lower exposure of banks weakens the incentives of the sovereign to repay. On the contrary, if banks in panic times are allowed to hold bonds up to a value not too much lower than in normal times, the doom loop ceases to exist.

¹³By Assumption 2 a bailout is only necessary if the price is lower than 1 and hence if the default probability is positive.

¹⁴There are other reasons why banks may have a higher valuation of government bonds than foreign investors, especially in times of crisis, such as: Regulatory reasons, financial repression, or non-atomistic behavior of banks.

4.3 Commitment not to bailout

If the necessity for a bailout creates the additional debt that raises the default probability and hence makes the bailout necessary in the first place, it may seem like a useful policy to rule out bailouts and thus rule out the sunspot equilibrium. However this need not necessarily be the case, since ruling out the bailout affects default incentives.

To show this, we analyze the model assuming that the government has credibly committed to never bailout the bank. Starting from Proposition 2, it is straightforward to analyze the possibility of a sunspot equilibrium under the no bailout commitment: Since our equilibrium definition is subgame perfection, it is sufficient to trim the branch of the decision tree related to the bailout (see also figure 1).

Indeed, it turns out that a sunspot equilibrium continues to exist. The following proposition states that and characterizes the sunspot equilibrium

Proposition 6. *Under a no bailout commitment and maintaining the assumption of a high enough ϕ from Proposition 2, a unique sunspot equilibrium exists. For $s = n$ the policy and price functions are as in proposition 2. For $s = p$, default is given by a threshold strategy*

$$d^p(\omega) = \begin{cases} 1 & \text{if } \omega \leq \frac{\omega^n}{1-\phi} \\ 0 & \text{else} \end{cases}$$

and the bond price is $q^p = 1 - F\left(\frac{\omega^n}{1-\phi}\right)$

How does this sunspot equilibrium compare to the equilibrium with a bailout? Under the assumption of large enough loan losses upon insolvency in period 1 (ϕ),¹⁵ a no bailout commitment entails an ex ante welfare loss: Expected consumption conditional on $s = n$ remains unchanged, but expected consumption conditional on $s = p$ drops due to the losses associated to the unresolved banking crisis in period 1. Furthermore, if Assumption 3 is violated (i.e. ϕ is even larger), the panic gets as bad as possible: For $s = p$ the bond price is 0 and default is certain. This is so because in this case the loan losses in period 1 are so high that banks will be insolvent in period 2 even if the government repays its debt. The government hence has no incentive to repay (see Appendix C). In that case ex ante welfare is even lower.

In sum, whenever the doom loop exists, the no bailout commitment is thus both time inconsistent – the government prefers a bailout in period 1 – and ex-ante suboptimal – the expected welfare associated to the sunspot equilibrium is strictly lower with a no bailout commitment than without.

Note that for this result the endogenous commitment to repay provided by the bank's balance sheet is crucial. A banking crisis in period 1, if left unaddressed by the government, increases the default incentives because it weakens the economy in the repayment period. Hence even with a commitment not to bailout banks, which eliminates the traditional doom loop, an alternative form of the doom loop exists. This rationale is absent in Cooper and Nikolov

¹⁵Recall that this assumption is natural, since it is necessary to generate the doom loop

(2013). There the default incentives depend only on the level of public debt but not on the bank's health. Hence a no bailout commitment can rule out the sunspot equilibrium and is thus ex ante optimal, even if ex post suboptimal.¹⁶ Our result calls this conclusion into question.

5 Conclusion

Banks' exposure to sovereign debt give rise to the doom loop: A fall in the price of debt can require a bailout, which raises debt and hence the default probability, justifying the fall in the price of debt. However, the same exposure also provides commitment to the government, thus sustaining sovereign debt. This paper combines these two views to challenge two conclusions that can be derived from looking at the doom loop in isolation: (i) that banks exposure to their government should be reduced (ii) that it is desirable to commit not to bailout banks.

In the context of our model the former policy makes the doom loop worse when applied to bank's ex-ante exposure. Conversely, when banks are allowed to increase their exposure in times of crisis, the doom loop can be avoided. Furthermore, the second policy fails to avoid the doom loop and leads to an even worse negative self-fulfilling loop. These results are no doubt stylized. But they nevertheless may serve as a warning to the policy makers, some of which seem to belief especially in the first conclusion, that is the need to limit banks' exposure.

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¹⁶In their baseline model, default is non-strategic and driven directly by an exogenous “tax capacity” process. In an extension they consider strategic default, but the default incentives are modeled as independent of the bank's balance sheet.

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Appendix

A Problem of the government

The problem at $t = 2$ is

$$W_2^{sb}(\omega) = \max_{d \in \{0,1\}} \left[d \left(C_2^{sb,d} \right) + (1-d) C_2^{sb,r} \right]$$

where the $C_2^{sb,r}$ is the level of consumption for state s and bailout decision b if the government decides to repay. The levels of consumption for each eventuality are given by

$$C_2^{s0,r} = \begin{cases} \omega L_0 - B_0^f & \text{if } q^{s0} B_0^h + L_0 - D_0 \geq 0 \\ \omega(1-\phi)L_0 - B_0^f & \text{if } q^{s0} B_0^h + L_0 - D_0 < 0 \text{ and } B_0^h + (1-\phi)L_0 - D_0 \geq 0 \\ \omega(1-\phi)(1-\theta)L_0 - B_0^f & \text{else} \end{cases}$$

$$C_2^{s0,d} = \begin{cases} \omega L_0 & \text{if } q^{s0} B_0^h + L_0 - D_0 \geq 0 \text{ and } L_0 - D_0 \geq 0 \\ \omega(1-\theta)L_0 & \text{if } q^{s0} B_0^h + L_0 - D_0 \geq 0 \text{ and } L_0 - D_0 < 0 \\ \omega(1-\phi)L_0 & \text{if } q^{s0} B_0^h + L_0 - D_0 < 0 \text{ and } L_0 - D_0 \geq 0 \\ \omega(1-\phi)(1-\theta)L_0 & \text{else} \end{cases}$$

$$C_2^{s1,r} = \begin{cases} \omega L_0 - B_0^f - \left(\frac{1}{q^{s1}} - 1 \right) \left(\max \{ D_0 - L_0 - q^{s1} B_1^f, 0 \} \right) & \text{if } B_0^h + L_0 - D_0 \geq 0 \\ \omega(1-\phi)L_0 - B_0^f - \left(\frac{1}{q^{s1}} - 1 \right) \left(\max \{ D_0 - L_0 - q^{s1} B_1^f, 0 \} \right) & \text{else} \end{cases}$$

$$C_2^{s1,d} = \begin{cases} \omega L_0 - B_0^f + \left(\max \{ D_0 - L_0 - q^{s1} B_1^f, 0 \} \right) & \text{if } L_0 - D_0 \geq 0 \\ \omega(1-\theta)L_0 - B_0^f + \left(\max \{ D_0 - L_0 - q^{s1} B_1^f, 0 \} \right) & \text{if } L_0 - D_0 < 0 \end{cases}$$

and at $t=1$

$$W_1(s) = \max_{b \in \{0,1\}} \mathbb{E} \left[b W_2^{s1}(\omega) + (1-b) W_2^{s0}(\omega) \right]$$

B Proofs of propositions

B.1 Proposition 1

Proof. First we verify that the proposed equilibrium is indeed an equilibrium and then we show that no other equilibrium can exist.

Banks equity in $t = 1$ is given by

$$E_1 = B_0^h q + L_0 - D_0$$

$$\implies E_1 = B_0^h \left(1 - F \left(\frac{1}{\theta} \frac{B_0^f}{L_0} \right) \right) + L_0 - D_0$$

that from Assumption (2) is non negative. Therefore there is no bailout required and no loans are destroyed $L_1 = L_0$.

The default decision in $t = 2$ is the solution to

$$\max \left\{ \omega L_0 - B_0^f, \omega L_0(1 - \theta) \right\} \quad (11)$$

where the first term is the consumption in case of repayment and the second is consumption in case of default. By Assumption 1, default makes the bank insolvent and consequently θ loans are destroyed and thus $L_2 = (1 - \theta)L_1$.

The government defaults whenever

$$\begin{aligned} \omega L_0 - B_0^f &< \omega L_0(1 - \theta) \\ \implies \omega &< \frac{1}{\theta} \frac{B_0^f}{L_0} \end{aligned}$$

and consequently the optimal policy is characterized by a threshold $\omega^n = \frac{1}{\theta} \frac{B_0^f}{L_0}$. Given this default policy the price of sovereign debt in $t = 1$ is given by

$$\begin{aligned} q &= 1 - \text{prob}(\text{Default}) \\ \implies q &= 1 - F(\omega^n) \end{aligned}$$

so the proposed equilibrium is indeed an equilibrium.

Now suppose there is another $\tilde{q} \neq 1 - F(\omega^n)$ that supports another equilibrium where q does not depend on s and banks are solvent in $t = 1$. Since banks are solvent in $t = 1$ there is no bailout and the default decision is exactly the same as before. Note (11) does not depend on q . Therefore the threshold is also the same as before and given by ω^n and the equilibrium price has to satisfy

$$\tilde{q} = 1 - F(\omega^n)$$

a contradiction. □

B.2 Proposition 2

Proof. We have to verify that the price vector \mathbf{q} is indeed an equilibrium price vector. For that it has to be that the default decisions taking as given \mathbf{q} indeed imply a default probability for each possible entry that is equal to $1 - q^{sb}$, for each $s \in \{n, p\}$ and $b \in \{0, 1\}$.

Step 1: Normal times

Let's start with the case $s = n$. Analogous to the proof of proposition 1, if

$$q^n = 1 - F\left(\frac{1}{\theta} \frac{B_0^f}{L_0}\right)$$

then banks are solvent, and there is no bailout in period 1. The government chooses whether

or not to default maximizing

$$\max \left\{ \omega L_0 - B_0^f, \omega L_0(1 - \theta) \right\}.$$

As above, the optimal policy is to default whenever the productivity draw is below $\omega^n = \frac{1}{\theta} \frac{B_0^f}{L_0}$. This policy is hence consistent with the price q^n we started from.

Step 2: Panic with bailout

Next consider the case $s = p$. We first characterize the default decision in case there is a bailout. We guess and later verify that the price $q^{p,1}$ is positive. This condition is necessary to make a bailout feasible. In that case, if the government defaults the bank is insolvent, since it had 0 equity in period 1 when the bond was still having a positive value. The level of consumption in case of a default is

$$C_2^d = (1 - \theta)\omega L_0 + S_1$$

Where we have that θ of the loans are destroyed and S_1 is the bailout transfer. On the other hand, in case of repayment, banks equity E_2 is positive by Assumption 2 and the level of consumption is

$$C_2^r = \omega L_0 + S_1 - \left(B_0^f - \frac{1}{q^{p,1}} S_1 \right)$$

The government decides to default if

$$\begin{aligned} C_2^d &> C_2^r \\ \implies (1 - \theta)\omega L_0 + S_1 &> \omega L_0 + S_1 - \left(B_0^f - \frac{1}{q^{p,1}} S_1 \right) \\ \implies \omega &< \omega^n + \frac{1}{\theta} \frac{\left(\frac{D_0 - L_0}{q^{p,1}} - B_0^h \right)}{L_0} \end{aligned}$$

where we have just rearranged terms and using the definition of S_1 and ω^n . Therefore the optimal default policy is a threshold policy for a given bond price where the threshold is given by

$$\omega^{p,1} = \omega^n + \frac{1}{\theta} \frac{\left(\frac{D_0 - L_0}{q^{p,1}} - B_0^h \right)}{L_0} \quad (12)$$

for international creditors to break even ex-ante it has to be that

$$q^{p,1} = 1 - F(\omega^{p,1}) \quad (13)$$

so the values of q and ω that solve the system (12)-(13) are the equilibrium values. In the last step we show that this solution exists and features $q^{p,1} > 0$.

Step 3: Panic without bailout

Now consider the default decision if there was no bailout. In this case banks are insolvent

for sure if the government doesn't repay. Consumption in case of default hence is

$$C^d = (1 - \theta)(1 - \phi)\omega L_0$$

and in case of repayment

$$C^r = (1 - \phi)\omega L_0 - B_0^f$$

where from Assumption 3 we have banks are solvent if the government fully repays, even in the case of no bailout. Formally this requires that

$$(1 - \phi)L_0 + B_0^h < D_0 \tag{14}$$

in Appendix C we show the case if this assumption is violated. Default is preferred to repayment if

$$\begin{aligned} (1 - \theta)(1 - \phi)\omega L_0 &> (1 - \phi)\omega L_0 - B_0^f \\ \phi &> 1 - \frac{B_0^f}{\theta\omega L_0} \end{aligned}$$

where $1 - \frac{B_0^f}{\theta\omega L_0} < 1$, so there is a threshold $\omega^{p,0}$ for the default strategy in case of no bailout that is given by

$$\omega^{p,0} = \frac{B_0^f}{\theta L_0} \frac{1}{1 - \phi} = \frac{\omega^n}{1 - \phi}$$

from this it follows that the value of sovereign debt in case of no bailout is given by

$$q^{p,0} = 1 - F\left(\frac{\omega^n}{1 - \phi}\right)$$

Step 4: Bailout decision

The welfare in case of no bailout corresponds to

$$\begin{aligned} W^{nb} &= \int_1^{\omega^{p'}} ((1 - \theta)(1 - \phi)\omega L_0) \partial F(\omega) + \int_{\omega^{p'}}^{\bar{\omega}} ((1 - \phi)\omega L_0 - B_0^f) \partial F(\omega) \\ \implies W^{nb} &= ((1 - \phi)L_0 E\{\omega\} - B_0^f) - F(\omega^{p'}) \left((\theta(1 - \phi)L_0) (E\{\omega \mid \omega \leq \omega^{p'}\}) - B_0^f \right) \end{aligned}$$

using the envelope theorem we have that

$$\frac{\partial W^{nb}}{\partial \phi} = L_0 \left(F(\omega^{p,0})\theta\phi E\{\omega \mid \omega \leq \omega^{p'}\} - E\{\omega\} \right)$$

that is negative since $E\{\omega\} \geq E\{\omega \mid \omega \leq \omega^{p,0}\}$ and $F(\omega^{p,0})\theta\phi < 1$. Furthermore, at the limit when $\phi = 1$ we have $W^{nb} = 0$. On the other hand, the welfare in case of a bailout is always positive and does not depend on ϕ . Therefore, there is a threshold for ϕ such that the bailout decision is optimal if ϕ is larger than the threshold.

Step 5: Existence of bailout solution

What remains to be shown is that the system (12)-(13) has a solution for $\omega^{p,1}, q^{p,1}$. From equation (13) we can write ω as a function of q as follows

$$\omega = G_1(q) = F^{-1}(1 - q)$$

where $F^{-1}(\cdot)$ is the inverse function of F . Also define

$$G_2(q) = \omega^n + \frac{1}{\theta} \frac{\left(\frac{D_0 - L_0}{q} - B_0^h\right)}{L_0}$$

so we are left to show that there is a value of $q < \frac{D_0 - L_0}{B_0^h}$ (banks are insolvent) for which

$$G_1(q) = G_2(q)$$

First note that

$$G_2\left(\frac{D_0 - L_0}{B_0^h}\right) = \omega^n < G_1\left(\frac{D_0 - L_0}{B_0^h}\right)$$

where the inequality comes from the fact that $G_1\left(\frac{D_0 - L_0}{q^n B_0^h}\right) = \omega^n$ and G_1 is strictly decreasing. So at $q = \frac{D_0 - L_0}{B_0^h}$ we have that $G_2 < G_1$.

Also as q tends to zero we have that $G_2(q)$ tends to ∞ , but given that ω is bounded, we have that $\min(G_1(0))$ is finite.

So summing up, i) G_1 and G_2 are continuous, ii) $\min(G_1(0))$ is finite and the limit $G_2(0) = \infty$; iii) at $q = \frac{D_0 - L_0}{B_0^h} > 0$ we have that $G_2(q) < G_1(q)$

i) ii) and iii) imply that G_1 and G_2 have to cross at least once in the interval $(0, \frac{D_0 - L_0}{B_0^h})$ and that at this intersection q is positive. Furthermore, the first time $G_1(q)$ and $G_2(q)$ cross, since $G_1(q)$ crosses from above we have that

$$\frac{\partial G_1(q)}{\partial q} < \frac{\partial G_2(q)}{\partial q} \tag{15}$$

□

B.3 Proposition 3

Proof. First we show that the price and default threshold in the normal state are unaffected by a marginal change in the exposure of the banks.

normal state

The change of exposure we consider is a simultaneous variation in B_0^h and D_0 such that E_1^n remains constant. Since E_1^n remains constant, banks are solvent in $t=1$ in normal times. Hence the default threshold and the debt price in period 1 are those described in Proposition 2 and are given by

$$\begin{aligned}\omega^n &= \frac{1}{\theta} \frac{B_0^f}{L_0} \\ q^n &= 1 - F(\omega^n)\end{aligned}$$

So the price and the default threshold in normal times (q^n, ω^n) do not depend on B_0^h or D_0 and a reduction of the exposure of banks is irrelevant if $s = n$.

panic state

Next we show that the price is lower and the default threshold higher in the panic state.

By Proposition 2, in the panic state the price of debt and the default threshold are given by the system

$$q = 1 - F(\omega) \tag{16}$$

$$\omega(q) = \frac{1}{\theta} \frac{B_0^f}{L_0} + \frac{1}{\theta} \frac{\left(\frac{D_0 - L_0}{q} - B_0^h\right)}{L_0} \tag{17}$$

Consequently the equilibrium price satisfies

$$q^p = 1 - F\left(\frac{1}{\theta} \frac{B_0^f}{L_0} + \frac{1}{\theta} \frac{\left(\frac{D_0 - L_0}{q^p} - B_0^h\right)}{L_0}\right) \tag{18}$$

Since we consider simultaneous variations in B_0^h and D_0 such that E_1^n is unchanged, we can replace the parameter D_0 in the above equation by the function $D_0(B_0^h)$, which describes the variation in D_0 necessary to keep E_1^n fixed.

$$q^p = 1 - F\left(\frac{1}{\theta} \frac{B_0^f}{L_0} + \frac{1}{\theta} \frac{\left(\frac{D_0(B_0^h) - L_0}{q^p} - B_0^h\right)}{L_0}\right) \tag{19}$$

and the change in the equilibrium price is

$$\begin{aligned}\frac{\partial q^p}{\partial B_0^h} &= -F'(\omega^p) \left(-\frac{1}{\theta} \frac{1}{L_0} + \left(\frac{1}{\theta} \frac{1}{q^p L_0} \right) \frac{\partial D_0}{\partial B_0^h} + \frac{1}{\theta} \frac{\left(-\frac{D_0 - L_0}{(q^p)^2}\right)}{L_0} \frac{\partial q^p}{\partial B_0^h} \right) \\ \frac{\partial q^p}{\partial B_0^h} \left(1 - F'(\omega^p) \left(\frac{1}{\theta} \frac{\left(\frac{D_0 - L_0}{(q^p)^2}\right)}{L_0} \right) \right) &= -F'(\omega^p) \left(-\frac{1}{\theta} \frac{1}{L_0} + \left(\frac{1}{\theta} \frac{1}{q^p L_0} \right) \frac{\partial D_0}{\partial B_0^h} \right) \\ \frac{\partial q^p}{\partial B_0^h} &= \frac{\left(\frac{1}{\theta} \frac{1}{q^p L_0} \right) \frac{\partial D_0}{\partial B_0^h} - \frac{1}{\theta} \frac{1}{L_0}}{\frac{1}{\theta} \frac{D_0 - L_0}{(q^p)^2} - \frac{1}{F'(\omega^p)}} \\ \frac{\partial q^p}{\partial B_0^h} &= \frac{\left(\frac{1}{\theta} \frac{1}{L_0} \right) \left(\frac{q^n}{q^p} - 1 \right)}{\frac{1}{\theta} \frac{D_0 - L_0}{(q^p)^2} - \frac{1}{F'(\omega^p)}}\end{aligned}$$

where we have used the fact that the corresponding adjustment of D_0 to keep equity constant

is given by $\frac{\partial D_0}{\partial B_0^h} = q^n$ (since $E_1 = L_0 + q^n B_0^h - D_0$ and since q^n is unaffected by changes in B_0^h and D_0).

Given that the price of debt in panic is below the price in normal times $q^p < q^n$ the numerator of this expression must be positive. Since we focus on the crossing that delivers the highest value of q^p , the denominator is positive in the system (16)-(17). This follows from condition (15), that implies

$$\begin{aligned} F'(\omega^p) &> - \left(\frac{\partial \omega^p(q)}{\partial q} \right)^{-1} \\ \implies F'(\omega^p) &> - \left(\frac{\partial \left(\frac{1}{\theta} \frac{B_0^f}{L_0} + \frac{1}{\theta} \frac{(D_0 - L_0 - B_0^h)}{L_0} \right)}{\partial q} \right)^{-1} \\ \implies \frac{1}{F'(\omega^p)} &< \frac{\left(\frac{D_0 - L_0}{q^2} \right)}{\theta L_0} \end{aligned}$$

so we can conclude that

$$\frac{\partial q^p}{\partial B_0^h} > 0$$

and therefore lower exposure implies a lower equilibrium price of debt in the panic state. Consequently the corresponding default threshold is higher. \square

B.4 Proposition 4

Proof. If in the panic equilibrium the price falls and banks become insolvent

$$E_1 = q_1^p B_0^h + L_0 - D_0 < 0$$

the government bailout banks with the transfer of sovereign debt

$$\begin{aligned} q^p \Delta B_1^h &= -E_1 \\ \Delta B_1^h &= -\frac{1}{q^p} E_1 \end{aligned}$$

and there is no change in the debt holdings of foreign creditors $\Delta B_1^f = 0$.

The consumption in case of default is

$$c^d = (1 - \theta)\omega L_0$$

and the consumption in the case of repayment is

$$c^r = \omega L_0 - B_0^f$$

so the government defaults if

$$\begin{aligned} (1 - \theta)\omega L_0 &> \omega L_0 - (B_0^f) \\ \omega &< \frac{B_0^f}{\theta L_0} \end{aligned} \quad (20)$$

and we have that the default threshold is the same as in normal times $\frac{B_0^f}{\theta L_0}$ and consequently the price of debt in panic and normal times is the same. The equilibrium corresponds to the one presented in Proposition 1. \square

B.5 Proposition 5

Proof. If $s = p$ and the government decides to bailout banks, we have that equity before the bailout is

$$E_1 = q_1^p B_0^h + L_0 - D_0$$

and consequently the bailout transfer is given by

$$S_1 = \max \left\{ D_0 - L_0 - q^{p,1} B_0^h, 0 \right\}$$

Hence it has to do a debt issuance of

$$\Delta B_1 = \frac{1}{q_1^p} S_1 \quad (21)$$

Secondary markets open

Now the banks can bid for the newly issued bonds, using their risk-free bond. Banks can at most hold \bar{B} of their assets at market value in bonds. First consider the case in which the constraint binds. In this case the debt holding becomes $B_1^h = \frac{1}{q_1^p} \bar{B}$. The new debt that becomes foreign debt is then

$$\begin{aligned} \Delta B_1^f &= \frac{1}{q_1^p} S_1 - \left(\frac{1}{q^{p,1}} \bar{B} - B_0^h \right) \\ \Delta B_1^f &= \frac{1}{q_1^p} \left(D_0 - L_0 - \bar{B} \right) \end{aligned} \quad (22)$$

With these new levels of local and foreign debt, the consumption if the government defaults is

$$\begin{aligned} c^d &= (1 - \theta)\omega L_0 + S_1 - q^{p,1} \left(\frac{1}{q^{p,1}} \bar{B} - B_0^h \right) \\ &= (1 - \theta)\omega L_0 - (L_0 - D_0) - (\bar{B}) \end{aligned}$$

and under repayment is

$$\begin{aligned} c^r &= \omega L_0 + S_1 - q^{p,1} \left(\frac{1}{q^{p,1}} \bar{B} - B_0^h \right) - B_1^f \\ &= \omega L_0 + S_1 - q^{p,1} \left(\frac{1}{q^{p,1}} \bar{B} - B_0^h \right) - \left(B_0^f + \frac{1}{q^{p,1}} (D_0 - L_0 - \bar{B}) \right) \end{aligned}$$

so the government defaults if $c^d > c^r$, that corresponds to

$$\begin{aligned} (1 - \theta)\omega L_0 + S_1 - q^{p,1}(\bar{B} - B_0^h) &> \omega L_0 + S_1 - q^{p,1}(\bar{B} - B_0^h) - \left(B_0^f + \frac{1}{q_1^p} (D_0 - L_0 - \bar{B}) \right) \\ \omega &< \frac{B_0^f}{\theta L_0} + \frac{\frac{1}{q_1^p} (D_0 - L_0 - \bar{B})}{\theta L_0} \end{aligned}$$

and consequently the default decision follows a threshold strategy and the system of equation which solution correspond to $q^{p,1}$ and $\omega^{p,1}$ corresponds to

$$\begin{aligned} q^{p,1} &= 1 - F(\omega^{p,1}) \\ \omega^{p,1} &= \frac{B_0^f}{\theta L_0} + \frac{\frac{1}{q^{p,1}} (D_0 - L_0 - \bar{B})}{\theta L_0} \end{aligned}$$

Now consider the case where \bar{B} is not binding. In this case all the newly issued debt is acquired by the local banks and consequently $\Delta B_1^f = 0$. We see from equation (22) that this coincides with the previous case when $\bar{B} = D_0 - L_0$. Any larger value of \bar{B} is not binding as what restricts the purchase of sovereign debt by the bank is the constraint that the maximum amount of debt it can buy cannot have a value larger than the bailout received (S_1)

$$\Delta B_1^h = \frac{1}{q_1^p} S_1$$

and using equation (21) this implies that the maximum level of debt purchases are equal to the new debt issued ΔB_1 . This situation is then equivalent to and the price and default threshold are the solution to

$$\begin{aligned} q^{p,1} &= 1 - F(\omega^{p,1}) \\ \omega^{p,1} &= \frac{B_0^f}{\theta L_0} \end{aligned}$$

The two cases can be summarized then for any \bar{B} by an equilibrium price $q^{p,1}$ and default threshold $\omega^{p,1}$ that solve the system

$$\begin{aligned} q^{p,1} &= 1 - F(\omega^{p,1}) \\ \omega^{p,1} &= \frac{B_0^f}{\theta L_0} + \frac{\max \left\{ \frac{1}{q^{p,1}} (D_0 - L_0 - \bar{B}), 0 \right\}}{\theta L_0} \end{aligned}$$

Hence, if $\bar{B} > D_0 - L_0$ the panic price is equal to the normal price and the sunspot equilibrium ceases to exist. \square

C Alternative sunspot equilibrium with zero price of debt in case of no bailout

In Proposition 2 we required that ϕ has to be large enough and at the same time that the banks are solvent after repayment even if there is no bailout. The required value of ϕ might not exist for every possible configuration of the initial allocation that satisfies Assumption 1 and 2. In case the ϕ that is large enough to make the bailout optimal is so large that makes banks insolvent after no bailout, even in case of repayment, then the price of debt in case of no repayment is zero. The next proposition formalizes this case.

Proposition 7. *A sunspot equilibrium exists that is characterized by the following:*

-For $s = n$ the policy and price functions are identical to before:¹⁷

$$b^n = 0 \quad (23)$$

$$q^n = 1 - F(\omega^n) \quad (24)$$

$$d^n(\omega) = \begin{cases} 1 & \omega < \omega^n \\ 0 & \omega \geq \omega^n \end{cases} \quad (25)$$

$$\text{where } \omega^n = \frac{1}{\theta} \frac{B_0^f}{L_0} \quad (26)$$

-For $s = p$, the government decides to bailout banks $b^p = 1$ and the price of debt $q^{p,1}$ and the default threshold $\omega^{p,1}$ are the solution to the system

$$q = 1 - F(\omega) \quad (27)$$

$$\omega = \omega^n + \frac{1}{\theta} \frac{\left(\frac{D_0 - L_0}{q^{sB}} - B_0^h\right)}{L_0} \quad (28)$$

-The price of debt outside the equilibrium path is given by $q^{p,0} = 0$

Proof. We have to verify that the pricing function $q(s, b)$ is indeed an equilibrium pricing function. For that it has to be that the bailout decision and the default decisions taking as given $q^{s,b}$ indeed imply a default probability that is equal to $1 - q^{s,b}$.

Step 1: Normal times

This is analogous to B.2.

Step 2: Panic with bailout

This is analogous to B.2.

Step 3: Panic without bailout

¹⁷Where we omit the superscript b as when $s = n$ we have that the banks are solvent are require no bailout.

Now consider the default decision if there was no bailout. In this case banks are insolvent for sure if the government doesn't repay. Consumption in case of default hence is

$$C^d = (1 - \theta)(1 - \phi)\omega L_0$$

and in case of repayment

$$C^r = (1 - \theta)(1 - \phi)\omega L_0 - B_0^f$$

In this case default is preferred to repayment as $C^d = C^r + B_0^f$.

Step 4: Bailout decision

Furthermore, given the pricing function condition ?? we have that the government always decides to bailout banks as the level of consumption in case of a bailout and default is always greater than the level of consumption in case of no bailout.

$$(1 - \theta)\omega L_0 + S_1 > (1 - \theta)(1 - \phi)\omega L_0$$

Now we have shown that given the price function is consistent with the default decision. Also, that the government decides to bailout the banks given the pricing function.

Step 5: Existence of bailout solution

This is analogous to B.2. □

C.1 Bailout commitment if banks are not solvent after repayment

Assumption 4. *If a bank gets insolvent in period 1 it is allowed to continue operating despite negative equity. The fraction of loans that get destroyed in period 1 is large $(1 - \phi) < \frac{D_0 - B_0^h}{L_0}$.*

This assumption makes sure that a banking crisis in period 1 is bad enough, such that in period 2 banks are not viable even if the government repays its debt. The crisis in period 1 is so bad that there is no avoiding a crisis in period 2.

Once banks are not viable in period 2 there is also no reason for the government to repay any longer. After all it's only motive to repay is to keep the bank solvent; if repayment does not ensure solvency, then default is clearly preferred by the government, since it avoids repaying foreign investors. Hence the government will default regardless of TFP. Anticipating this, the bond price in period 1 in case of a panic must be 0.

Proposition 8. *Under a no bailout commitment and given assumption 4, a unique sunspot equilibrium exists. For $s = n$ the policy and price functions are as in proposition 2. For $s = p$, default is certain $d^p = 0$ and the bond price is zero $q^p = 0$.*

So a no bailout commitment can not rule out the sunspot equilibrium. Furthermore, relative to the case with the bailout, the panic gets as worse as possible: For $s = p$ the bond price is 0 and default not just more probable than in the normal times, but certain. As a result of this, consumption in period 2 is always lower than under the sunspot equilibrium if the panic has realized in period 2. The no bailout commitment is thus both time inconsistent – the

government prefers a bailout in period 1 – and ex-ante suboptimal – the expected welfare associated to the sunspot equilibrium is strictly lower with a no bailout commitment than without.

Proposition 9. *Given assumption 4, in case the panic materializes a bailout in period 1 is preferable for the government. If the probability of $s = p$ is positive, a no bailout commitment is suboptimal ex ante.*

We have thus rationalized that the government always bails out banks in period 1.

Note that for these two results the commitment to repay, which the bank’s exposure provides is crucial. A banking crisis in period 1, if left unaddressed by the government, eliminates the commitment to repay entirely because by assumption 3 it either (i) weakens the banking sector so much that the even repayment can’t avoid a crisis in period 2 or (ii) the banking sector disappears altogether. This rationale is absent in the argument of Cooper and Nikolov (2013), where the default incentives do not on the bailout and where a no bailout commitment hence can rule out the sunspot equilibrium.¹⁸

In fact our model also encompasses this possibility too: If ϕ is small enough (violating assumption A3i) and insolvent banks are allowed to continue operating (violation assumption A3ii) a no bailout commitment rules out the sunspot equilibrium.

¹⁸In their baseline model, default is non strategic and driven directly by an exogenous “tax capacity” process. In an extension they consider strategic default, but the default incentives are modeled as independent of the bank’s balance sheet.