Legal Evolution and Contract Evolution under Imperfect Enforcement

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Abstract

We model the joint evolution of contracts and precedents by introducing imperfect enforcement into a standard incomplete contracts setup. We assume that biased trial courts can refuse to verify novel evidence but are bound to respect precedents, namely to verify evidence that other judges verified in past cases. We find that optimal contracts are innovative (contingent on both precedents and novel evidence), but noisy evidence and judicial biases introduce enforcement risk and cause incentives to be low-powered. Litigation of innovative contracts refines the law, making it more informative. This evolution improves enforcement and makes contracts more complete, thereby enabling higher-powered incentives and improving welfare. This beneficial mechanism is hampered by judicial bias, which slows down legal evolution and causes enforcement risk to persist for a long time.

Keywords: Contracts, Imperfect enforcement, Legal evolution, Precedents
JEL codes: D86, K12, K40, K41

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1 Introduction

Market transactions can achieve efficient outcomes under the conditions of the Coase theorem, which include the reliable enforcement of contingent contracts (Coase 1960). If courts are costly to use (Townsend 1979) or unable to verify complex contingencies (Grossman and Hart 1986), contracts are incomplete and efficient arrangements become unattainable. Empirical evidence bears out the economic importance of courts. A large body of work documents that common law and its legal rules positively predict the development of financial, labor and other markets (La Porta, Lopez-de-Silanes and Shleifer 2008), as well as the use of flexible and innovative contracts (Lerner and Schoar 2005; Qian and Strahan 2007). These findings have revived the long-standing view that common law promotes economic efficiency thanks to the efficiency-oriented nature of its judges (Posner [1973] 2014) and the adaptability of precedents (Hayek 1960). As courts resolve disputes and develop case law, they refine legal rules and improve contract enforcement. This development, in turn, allows parties to write better contracts, bringing them closer to the first best.

The link between legal evolution and private contracting cannot be accounted for by existing theories that take contract incompleteness as a given and time-invariant constraint (Grossman and Hart 1986). To address this issue, we need a model in which enforcement frictions change over time as the law develops and contracts endogenously adapt. This paper develops such a model by introducing imperfect enforcement and legal evolution in a standard incomplete contracts setup.

A buyer and a seller engage in the exchange of a widget of uncertain quality. Quality-contingent pay is needed to induce productive effort in this relationship. There are many noisy signals of quality, some more informative than others. Payments are enforced in light of the signals that parties present and judges choose to verify in court. Contracts are incomplete because parties cannot fix ex ante which specific piece of evidence judges should use ex post. Intuitively, when enforcing complex clauses involving the parties’ best efforts, good faith, or intent, courts retain some discretion to fill in their meaning ex post.

We then characterize the legal system as follows. First, we assume that precedents allow parties to contract on some specific pieces of evidence. When a court rules that a certain proxy is material to determine quality, stare decisis binds future judges to also regard it as material. A gap is filled, and parties can contract ex ante on the quality proxy established by precedent. Instead, when litigants present novel evidence in court, judges have discretion in deciding which evidence is material and which is not.

Second, following legal realism (Frank 1930, 1949; Stone 1946, 1964; Posner 2005) and previous work (Gennaioli and Shleifer 2007; Gennaioli 2013), we assume that some judges
are subject to biases. Allowing for judicial bias is important because legal evolution is often seen as a cure for the vagaries of judges (Priest 1977). Judicial biases may reflect attitudes towards specific litigants or a judge’s ideological preferences, whose influence is well documented (e.g., Pinello 1999; Klein 2002; Zywicki 2003; Sunstein, Schkade, and Ellman 2004). In our setup, pro-seller judges dismiss evidence favoring the buyer and pro-buyer judges dismiss evidence favoring the seller. This behavior does not only make adjudication less fact-intensive, but it also influences contracts and legal evolution.

We study optimal contracts by using a mechanism-design approach. Parties can contract on the quality proxies based on precedent. They can also choose to let courts consider novel evidence. A contract based only on precedents is perfectly enforced. A contract based also on new evidence, which we dub an innovative contract, is exposed instead to some judicial discretion. The optimal mechanism, then, need not only induce the seller to provide high quality ex ante. It must also, at the enforcement stage, induce litigants to report to judges the evidence they collect, and judges to verify such evidence correctly. We obtain two main results.

First, if parties choose to contract they write a contract that is both path-dependent and innovative. It conditions the seller’s payment on evidence based on precedents, but also on novel evidence that has not been used in court yet. Contracting on both novel evidence and precedents is optimal because they both contain valuable information about quality. Despite its use of novel evidence, however, the optimal contract remains incomplete because enforcement frictions reduce the verifiability of quality. To begin with, the parties may fail to collect informative evidence. What is more, biased judges suppress information generated by parties in court. These frictions increase the cost of incentivizing the seller. Thus, parties are forced to use low-powered incentives and efficiency declines.

Second, there is a virtuous cycle between the use of innovative contracts and legal evolution. When parties write and litigate innovative contracts, judges are constantly asked to evaluate new evidence. As a result, precedents become more informative, allowing parties to write better contracts. This process reduces enforcement frictions because: i) more informative proxies of quality become reliably available to parties, and ii) binding precedents restrict judicial discretion, which limits the economic cost of judicial biases. Due to both effects, legal evolution narrows down residual uncertainty in the verification of quality. As a result, the cost of incentives falls over time and parties profitably write higher-powered contracts, increasing effort and productive efficiency. When the law is fully developed, contracts reach the full verifiability benchmark and welfare is maximal.

This positive feedback loop is hindered by judicial bias, which slows down legal evolu-
tion. First, biased trial courts suppress new facts to favor particular litigants, reducing the speed at which contracts become more complete. Second, biased courts reduce the incentive for parties to contract in the first place, which hurts future parties by slowing down legal evolution even further.\(^1\) Hence, our analysis shows that case law is more effective at resolving contractual incompleteness when it arises from the novelty and complexity of a transaction than when it is due to a failure of the courts. In the former case, litigation induces a progressive refinement of legal rules. In the latter, not only are parties discouraged from contracting, but the law may also fail to develop even in the presence of new facts. Society may be stuck for a long time with undeveloped law and unpredictable enforcement.

We contribute to the literature on legal evolution.\(^2\) Relative to early papers (Priest 1977; Rubin 1977), recent work focuses on judicial behavior (Gennaioli and Shleifer 2007; Ponzetto and Fernandez 2008; Niblett 2013b; Anderlini, Felli, and Riboni 2014). Our view of legal evolution as a way to enrich the empirical content of the law is closest to Gennaioli and Shleifer’s (2007) model of distinguishing. Our main innovation is to consider the role of legal evolution for private contracting, and in particular the mutual feedback between contractual and legal evolution.\(^3\)

Our analysis highlights a strong link between contracts and the law. In standard contract theory, parties write from scratch optimal contracts tailored to their own needs and constraints. In reality, however, parties do not write contracts ex nihilo, but rather introduce marginal innovations into slowly evolving established contract forms. Choi, Gulati and Posner (2013) document empirically this pattern in the context of sovereign bonds. In our model, this pattern is due to imperfect legal enforcement. Established contract forms embody the legal capital and hence predictable enforcement created by past judicial decisions. Incremental contractual innovations, on the other hand, allow parties to benefit from extra flexibility, while keeping enforcement hazards low enough. Contracts are neither fully flexible nor completely rigid. Their degree of flexibility reflects past decisions and the parties’ trust in the ability of courts to enforce new, hard-to-verify clauses.

As a consequence, the law plays a key role in supporting private contracting. Court systems or transactions in which judicial bias is less prevalent encourage flexible and in-

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\(^1\)Key to this second effect is that private contracting is a public good in our framework. Contracting and litigation bring into contract law new clauses and new evidence that help future parties write better contracts. Individuals fail to internalize these positive dynamic externalities, so contracting may be suboptimally low in equilibrium, particularly if judges are biased.

\(^2\)Gennaioli (2013) offers a static analysis of contracting under judicial bias. Other papers studying the static effects of judicial error when it is due to judicial bias or corruption are Glaeser, Scheinkman and Shleifer (2003), Glaeser and Shleifer (2003), and Bond (2009).

\(^3\)Anderlini, Felli and Riboni (2014) study contracting and precedents with time-inconsistent judges, but they do not consider contract innovation.
novative contracting. This speeds up legal evolution, fostering the use of more complete contracts and stronger incentives. The development of investor rights in common law systems may be a product of this positive feedback loop. Fiduciary duties are an implied term in financial contracts that allows courts to protect shareholders against managerial abuse. Like our open-ended contract terms, fiduciary duties are a residual concept that “can include situations that no one has foreseen or categorized ... and in fact has led to a continuous evolution in corporate law” (Clark 1986, p. 141). Judicial decisions on fiduciary duties do not merely fill gaps in the courts’ policing of managerial discretion. They also allow firms and investors to make their contracts more complete by selectively opting in or out of certain provisions (Butler and Ribstein 1998).

This progressive refinement of laws and contracts arguably fostered the development of flexible contracting and financial markets in common law systems (Johnson et al. 2000, La Porta, Lopez-de-Silanes and Shleifer 2008). Our model shows that the same evolutionary process may not be as virtuous in areas of law in which trial courts are biased and thus more reluctant to properly verify case facts. Product liability, in which successful contractual solutions have not emerged (Rubin 1993), may be a case in point.

The paper is organized as follows. Sections 2 and 3 lay out the contracting setup and enforcement risk. Sections 4 and 5 study the optimal contract and legal evolution when partnerships are homogeneous. Section 6 extends the model to the case in which partnerships are heterogeneous. Section 7 concludes. Proofs are in the Appendix.

2 The Model

2.1 Setup

We build our analysis on the repetition of a transaction that has three conventional ingredients of incomplete contracts models: non-contractible effort, risk neutrality and limited liability (Bolton and Dewatripont 2004). Time is discrete, with an infinite horizon. At the beginning of each period $t = 0, 1, ...$ a penniless entrepreneur (the seller) and a wealthy customer (the buyer) meet and choose whether to form a partnership involving the supply of a relationship-specific widget. Parties live for one period.

During each period $t$, production occurs in two stages. First, the seller exerts effort

\footnote{In the words of Butler and Ribstein (1998, p. 29): “fiduciary duties are not distinct from the contract ...despite the fact that fiduciary duties are imposed on parties who have not drafted around them. ... If the parties can choose the terms by either accepting them or contracting around them, the result of this choice is a contract.”}

\footnote{This assumption matters only for our analysis of legal evolution. We return to it in Section 5.}
$a \in [0, 1]$ at a non-pecuniary cost $C(a)$. Second, with probability $a$ the widget is realized to be “good,” taking value $v > 0$; with probability $1 - a$, the widget is “bad,” taking value zero. We keep time subscripts implicit until Section 3. The widget is an experience good, such as a professional service or the management of a public company, whose value is learned by the buyer only after consuming it. We impose the following restrictions on the seller’s cost function:

$$C(a) > 0, C'(a) > 0, C''(a) > 0, \text{ and } C'''(a) \geq 0 \text{ for all } a \in (0, 1), \tag{1}$$

with limit conditions $C(0) = 0$, $\lim_{a \to 0} C'(a) = 0$, and $\lim_{a \to 1} C''(a) > v$.

If at time $t$ the partnership is formed, the seller’s first-best effort level is:

$$a_{FB} = \arg \max_a \{av - C(a)\} = C^{-1}(v) \in (0, 1), \tag{2}$$

which corresponds to joint surplus

$$\Pi_{FB} = \max_a \{av - C(a)\} = vC^{-1}(v) - C\left(C^{-1}(v)\right) > 0. \tag{3}$$

If the partnership is not formed, the seller obtains 0, while the buyer obtains utility $u_B \geq 0$. We interpret $u_B$ as the surplus obtained by the buyer if he acquires a general-purpose widget in the market. In the case of managing a corporation, $u_B$ can be viewed as the payoff from short-term management, devoid of any firm-specific investment. Forming the partnership is first-best efficient if and only if $\Pi_{FB} \geq u_B$.

2.2 Contracting

In period $t$, the seller and the buyer meet. If they form a partnership, the seller makes a take-it-or-leave-it contract offer to the buyer (so the seller has full bargaining power). Next, the seller exerts effort, which determines the likelihood of producing a valuable widget. The widget is produced, the buyer consumes it, and the contract is enforced.

Under full observability, the first best is implemented by requiring the buyer to pay the seller a price $p = a_{FB}v - u_B$ if he exerted effort $a_{FB}$, and zero otherwise. Unfortunately, effort is unobservable (and non-contractible), so this solution does not work.

If widget quality is observable and perfectly verifiable after consumption, the parties can specify a quality-contingent price $p_q$ for $q \in \{0, v\}$. In the optimal contract, the buyer’s participation constraint is binding. Otherwise, the seller could raise $p_q$ for all $q$ and still ensure participation without affecting effort provision. As a result, the seller
chooses \( p_v \geq 0 \) and \( p_0 \geq 0 \) to maximize joint surplus \( av - C(a) - u_B \) subject to the buyer’s binding participation constraint \( a(v - p_v) - (1 - a)p_0 = u_B \), and to the seller’s incentive-compatibility constraint \( p_v - p_0 = C'(a) \). The problem can be rewritten as:

\[
\max_{a \in [0, 1]} \{ av - C(a) - u_B \} \tag{4}
\]

subject to

\[
av - u_B \geq \min_{p_v, p_0 \geq 0} \{ ap_v + (1 - a)p_0 \} \quad \text{s.t. } p_v - p_0 = C'(a). \tag{5}
\]

In (5), the optimal price \( p_q \) minimizes the cost of inducing any effort \( a \). This minimum cost defines the set of effort levels that can be implemented given the buyer’s participation constraint. The seller chooses the surplus-maximizing effort \( a \) from this set.

**Proposition 1** When quality \( q \) is contractible, the optimal contract sets a positive price only when quality is high (\( p_0 = 0 \) and \( p_v = C'(a_{SB}) > 0 \)).

The first best is attained if and only if the buyer’s outside option is nil (\( u_B = 0 \)). The partnership is formed if and only if the buyer’s outside option is sufficiently low:

\[
u_B \leq U_B = \max_{a \in [0, 1]} \{ a[v - C'(a)] \}. \tag{6}
\]

When the partnership is formed, second-best effort and joint surplus decrease with the buyer’s outside option and increase with the value of a high-quality widget (\( \partial a_{SB}/\partial u_B < 0 < \partial a_{SB}/\partial v \) and \( \partial \Pi_{SB}/\partial u_B < 0 < \partial \Pi_{SB}/\partial v \) for all \( u_B \in (0, U_B) \)).

The optimal contract specifies zero payment to the seller in case of low quality (\( p_0 = 0 \)). With this provision, wasteful payments are minimized, thereby reducing the cost of incentives. A similar property is at work when imperfect enforcement allows parties to contract only on an imperfect signal of quality.

When the buyer’s outside option is zero, the first best is attained by setting \( p_v = v \), which makes the seller the full residual claimant. When the buyer’s outside option is positive, however, the payment must be reduced to \( p_v < v \). As a result, the first best cannot be achieved because of the seller’s wealth constraint. Ideally, the seller would like to pay \( u_B \) to the buyer and “purchase the firm” from him, which would elicit first-best effort. This arrangement is infeasible because the seller is penniless. Hence, when \( u_B > 0 \) second-best effort and joint surplus are below the first best and decrease with the buyer’s outside option. We assume that Condition (6) always holds, so the partnership is feasible when quality is fully contractible.
3 Litigation and Imperfect Verifiability

Contracts are enforced in court, using signals of quality. Contracts are incomplete because parties cannot specify ex ante which precise signals will be used in adjudication and how. We now describe the structure of signals as well as the legal infrastructure that governs adjudication. We refer to the partnership occurring at time $t$ as “partnership $t$.”

3.1 General Evidence and Precedents

The quality of the seller’s job is reflected in a set of signals, or pieces of evidence, that represent the legal arguments that can be used in litigation (e.g., a legal statement of the functionality of the widget or of its timely delivery). There is a continuum $I$ of such pieces of evidence, each of which is uniquely identified by an index $i \in [0, 1]$. The binary value $e_t(i) \in \{-1, 1\}$ of piece of evidence $i$ offers a noisy signal of quality: $e_t(i) = -1$ is a perfect signal of low quality, while $e_t(i) = 1$ is a noisy signal of high quality.

Formally, if quality at $t$ is high ($q_t = v$) all pieces of evidence are positive ($e_t(i) = 1$ for all $i$). If instead quality is low ($q_t = 0$) each piece of evidence $i$ takes value

$$e_t(i) = \begin{cases} 1 & \text{for } i < \xi_t \\ -1 & \text{for } i \geq \xi_t \end{cases},$$

where $\xi_t$ is an i.i.d random variable that captures the noisiness of evidence. It has a cumulative distribution function $F_\xi(x)$ and continuous density $f_\xi(x) > 0$ on $[0, 1]$.

For any distribution of noise $\xi_t$, pieces of evidence carrying a higher index $i$ are more informative for the transaction at hand. In the limit, $e_t(1)$ almost surely takes values 1 if quality is high and $-1$ if quality is low. A piece of evidence $e_t(i)$ is a sufficient statistic for $q_t$ given a lower-indexed piece of evidence $e_t(j)$ for all $j \leq i$. The index $i$ thus measures the informativeness of a piece of evidence.

Consider now the role of legal uncertainty captured by the distribution of $\xi_t$. When its density $f_\xi(x)$ is concentrated around $\xi_t = 0$ all signals are very informative: they take value 1 if and only if quality is high. When instead $f_\xi(x)$ is concentrated around $\xi_t = 1$ all signals are uninformative: they always take value 1. If a random signal is drawn from the unit interval ($i \sim U[0, 1]$), the probability that it detects low quality is

$$\Pr\{i \geq \xi_t\} = \int_0^1 F_\xi(i) \, di.$$  

The probability of diagnosing low quality declines if the distribution of $\xi_t$ shifts up in
the sense of first-order stochastic dominance. The distribution of $\xi_t$ thus captures the complexity of the transaction: in more sophisticated or innovative sectors it is harder to find evidence that is clearly informative of quality.

Contracts are incomplete because parties have only limited ability to regulate ex ante how specific pieces of evidence will be used in court. We assume that contracts can be fully contingent on precedents, that is, on the pieces of evidence that have been used in past judicial decisions. Instead, they cannot be contingent on specific pieces of evidence that have never been used before.

This assumption captures the fact that real-world contracts contain gaps that must inevitably be filled ex post by the interaction between litigants and judges.\(^6\) The unavoidable process of judicial construction, then, generates significant unpredictability and “reveals a degree of indeterminate judicial discretion inherent in the enforcement of express contractual rights and obligations” (Manesh 2013, p. 6). Precedents reduce such discretion by establishing consistent patterns of contract interpretation. They also enable parties to draft express terms that explicitly prevent their contract from being judicially construed in the same manner used in similar past cases.\(^7\) Uncertainty in enforcement then remains, but mostly limited to contingencies outside precedent.

We formalize this idea as follows. At time $t$ the set of pieces of evidence $I$ is partitioned into a countable subset $P_t \subset I$ of precedents and an uncountable subset $I \setminus P_t$ of novel pieces of evidence. Precedents $P_t$ consist of all pieces of evidence $i \in P_t$ that have been used in past cases (before $t$) and cited in the judicial opinions justifying their outcome. As

\[^6\]In particular, U.S. courts have held that a contract contains an implicit provision whenever it is “clear from what was expressly agreed upon that the parties who negotiated the express terms of the contract would have agreed to [the provision] had they thought to negotiate with respect to that matter” (Katz v. Oak Industries, Inc., 508 A.2d 873, 880 (Del. Ch. 1986)). Such implicit contractual obligations emerge from the implied covenant of good faith and fair dealing, an unwaivable duty inherent in every contract (Restatement (Second) of Contracts § 205 (1981); UCC §1-201(19) and §2-103(1)(b)). In the absence of an unambiguous standard for good-faith conduct, courts apply the implied covenant on an ad-hoc case-by-case basis (Summers 1982).

\[^7\]Courts recognize that “the parties may, by express provisions of the contract, grant the right to engage in the very acts and conduct which would otherwise have been forbidden by an implied covenant of good faith and fair dealing” (VTR, Inc. v. Goodyear Tire Rubber Co., 303 F. Supp. 773, 777 (S.D.N.Y. 1969)). For instance, under Delaware law a limited partnership (or limited liability company) agreement may modify, restrict, or even eliminate the default fiduciary duties owed by company managers. The Delaware Court of Chancery has held that when an agreement grants a party discretion without setting forth its scope, such discretion can only be exercised on a reasonable basis. In response to such precedents, Delaware alternative entities have developed and adopted in their agreements explicit “Sole Discretion Language” specifically meant to ensure that courts will not require the party in question to exercise such discretion reasonably (Altman and Raju 2005). The Court has held that such language “removes the burden from the general partner of showing good cause when a genuine dispute arises,” though it can never permit a party to exercise discretion in bad faith (Fitzgerald v. Cantor, C.A. No. 16297-NC (Del. Ch. Mar. 23, 1999)).
in Gennaioli and Shleifer (2007), precedents specify the facts or dimensions material to a transaction. We assume that precedents are fully contractible: judges have been trained to recognize them and parties know in advance their legal formulation. Parties also know the informativeness $i$ of different precedents. Hence, the contract can be a function of the entire vector of signal realizations $\{e_i(i)\}_{i \in P_t}$. Given the signal structure we assumed, however, it is sufficient for parties to contract only on the realization $e_i(t^*_{i})$ of the most informative precedent $t^*_{i} \equiv \max i \in P_t$ available at $t$. As a sufficient statistic for all precedents, $t^*_{i}$ summarizes the state of precedent at time $t$.

Novel evidence in $I \setminus P_t$ consists of legal evidence that courts have not yet used in their decisions. We assume that parties cannot contract on it because it is prohibitively costly to identify, and thus contract upon, signals that have no record of past use. Formally, this impossibility is reflected in the fact that search for novel evidence is imperfect and undirected. During litigation, each party searches for a random piece of novel evidence from $I \setminus P_t$. With probability $\iota \in (0, 1)$ search is successful: the litigant $L \in \{B, S\}$ randomly draws one piece of evidence in $I \setminus P_t$. The informativeness of the evidence collected by litigant $L$, which we denote $t^L \sim U[0, 1]$, is not immediately observed in court.\footnote{A successful search returns a piece of evidence with unknown informativeness $t^L \sim U[0, 1]$ because the set of novel evidence $I \setminus P_t$ has full measure, given that there is only a countable number of precedents in $P_t$. The assumption that parties recognize the informativeness of a signal if and only if it belongs to the set of precedents is not crucial for our findings, but it simplifies the analysis. It is also realistic. Since novel evidence has never been used before, it is hard for parties to assess its precise informativeness ex post, once the widget value is realized and so is noise $\xi_i$, which is unobservable to the parties. By contrast, the informativeness of precedents can be inferred before contracting by observing the realization of the corresponding signals in many partnerships and by talking to industry peers.}

With probability $(1 - \iota)$ search is unsuccessful and the litigant comes up empty-handed.

For both litigants and judges, novel evidence is “hard:” litigants can hide it but cannot falsify it. If a litigant chooses to present novel evidence in court, the judge can likewise choose to hide it (e.g., by declaring it inadmissible or immaterial). Due to the ambiguity of language and the complexity of facts, the verification of novel evidence can be distorted ex post by opportunistic parties and judges have discretion when verifying it.

### 3.2 Judicial Preferences

Court verification in our model is distorted not only by opportunistic parties, but also by biased judges. Since the contract is litigated ex post, after production has taken place, the impact of contract enforcement is purely distributional. It is therefore natural for ex-post judicial decisions to be shaped by a judge’s distributional preferences. There are three types of judges. A fraction $\beta \in [0, 1)$ have a pro-buyer bias, and wish to minimize payment to
the seller. A fraction $\sigma \in [0, 1)$ have a pro-seller bias, and wish to maximize payment to the seller. The remaining $\omega = 1 - \beta - \sigma$ judges are unbiased: they wish to enforce the contract faithfully. This formulation nests both the case in which pro-seller and pro-buyer judges balance out ($\beta = \sigma$), for instance because they stem from personal idiosyncrasies, and the case in which one type of bias is more prevalent, for instance because the transaction is international and judges favor the local party.

4 The Optimal Contract

A contract in our setting consists of a price schedule $p(\ldots) \geq 0$ specifying a payment from the buyer to the seller contingent on the information presented by the litigants and verified by the judge. The contract cannot also specify a state-contingent trading rule because the widget is an experience good such as a service. Thus, information is not generated or the good is not even produced until the buyer consumes it. We rule out the possibility for the contract to specify punishments for the parties as a whole, such as non-pecuniary criminal penalties or incentive payments to judges. These punishments are illegal in the real world and would not be robust to renegotiation.\footnote{Common law prevents parties from stipulating by contract any kind of penalty, even pecuniary penal damages. “A term fixing unreasonably large liquidated damages is unenforceable on grounds of public policy as a penalty” under U.S. law (12 A.L.R. 4th 891, 899).}

Finally, the judge is the ultimate arbiter of the case. The evidence he chooses to verify is the sole determinant of contract enforcement. Payment cannot be made directly contingent on the information the parties have presented in court, bypassing the judge’s decision to verify it. Thus, litigants are powerless to work around judicial biases by impugning the judge’s ruling. This assumption captures a key feature of trial courts: judicial fact discretion (Frank 1949; Gennaioli and Shleifer 2008). In finding the facts of a case, trial courts have substantial discretion to emphasize or disregard pieces of evidence produced by litigants.\footnote{The appeal process determines no more than a marginal constraint. Appellate courts normally confine themselves to reviewing questions of law, and remain deferential to the findings of fact made by trial courts. In the U.S. federal court system, the standard of review for judge-found facts is that they will not be disturbed unless “clearly erroneous.” This is a very lenient standard, and findings of clear error by courts of appeal are extremely rare.}

The contract is written at the beginning of the partnership, before any information on the value of the widget or the preferences of judges is revealed. The figure below summarizes the sequence of events within a generic period $t$. 

\begin{figure}[h!]
\centering
\includegraphics[width=\textwidth]{fig.png}
\caption{The sequence of events within a generic period $t$.}
\end{figure}
<table>
<thead>
<tr>
<th>$s = 0$</th>
<th>$s = 1$</th>
<th>$s = 2$</th>
<th>$s = 3$</th>
<th>$s = 4$</th>
<th>$s = 5$</th>
<th>$s = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parties meet and write a contract ($p$)</td>
<td>The seller exerts unobservable effort ($\alpha$)</td>
<td>The widget is consumed. Parties observe its quality $q \in {0, v}$</td>
<td>Parties go to court. They observe the realization of the judge’s type $\theta$ and of precedent $e_P$</td>
<td>Parties search for and observe hard evidence ($e_B, e_S$). They argue their case before the judge.</td>
<td>The judge observes and selectively verifies the parties’ reports and evidence based on precedent.</td>
<td>The judge enforces contractual payments</td>
</tr>
</tbody>
</table>

Figure 1: Timing

Figure 1 illustrates the information possessed by different agents. By the revelation principle, any contract $p(\ldots) \geq 0$ can be represented by a two-stage direct revelation mechanism in which litigants and judges truthfully reveal their private information. Intuitively, the choice of an optimal price schedule $p(\ldots)$ solves the following trade-off. By conditioning payment solely on precedent $e_t(i_t^P)$, parties make the contract impermeable to enforcement problems. Instead, by writing an open-ended contract that admits novel evidence, parties make payments contingent on information collected ex post too, but they also subject themselves to the conflicting interests of litigants and to the vagaries of courts, which impose additional constraints on the optimal contract. By adopting a mechanism-design perspective, we identify the best outcome that parties can achieve given the well-defined enforcement problems characterizing the contracting environment.

In the first enforcement stage ($s = 4$ in Figure 1), each litigant $L \in \{B, S\}$ must truthfully reveal two pieces of information. The first is the quality $q_t \in \{0, v\}$ of the widget, which is observed by litigants but not by the judge. We denote by $q$ the realization of quality. The second is the novel evidence collected. Each litigant $L$ privately observes a novel piece of evidence $e_L(i_L^t) \in \{-1, 1\} \cup \{0\}$, where $i_L^t$ is its unknown informativeness, and $e_L(i_L^t) = 0$ denotes an unsuccessful search. We denote by $e_L$ the realization of evidence collection by litigant $L$.

In the second enforcement stage ($s = 5$ in Figure 1), the judge must truthfully reveal the litigants’ reports of quality and novel evidence as well as his own type $b_t \in \{b_B, u, b_S\}$, where $b_L$ denotes a bias in favor of litigant $L$ while $u$ denotes an unbiased judge. The realization of the judge’s type, which we denote by $b$, is observed during litigation by the litigants as well as the judge.

The contractual payment can also be contingent on precedent $e_t(i_t^P)$, whose realization is denoted by $e_P$. This piece of information is perfectly verifiable. As a result, it is not subject to any truthful revelation constraints.
Because parties do not communicate directly with the mechanism designer, contract enforcement depends directly on precedent and on the judge’s report; it reflects the litigants’ reports to the judge only indirectly \((s = 6 \text{ in Figure 1})\). As a result, the payment is a function \(p (q_B'^*, q_S'^*; e_P, e_B', e_S'; b')\), where \(q_B'^*\) and \(q_S'^*\) denote the judge’s report of the litigants’ quality announcement, \(e_B'\) and \(e_S'\) stand for his report of the novel evidence presented by the parties, and \(b'\) is the judge’s report of his own type.

The optimal direct revelation mechanism is the contract that maximizes the seller’s expected payoff

\[
\max_p \left\{ a \mathbb{E} (p|q_t = v) + (1 - a) \mathbb{E} (p|q_t = 0) - C (a) \right\}
\]

subject to the buyer’s participation constraint,

\[
a [v - \mathbb{E} (p|q_t = v)] - (1 - a) \mathbb{E} (p|q_t = 0) \geq u_B;
\]

the seller’s incentive compatibility constraint,

\[
C' (a) = \mathbb{E} (p|q_t = v) - \mathbb{E} (p|q_t = 0);
\]

and the seller’s wealth constraint,

\[
p \geq 0
\]

for every possible realization of \((q_B'^*, q_S'^*; e_P, e_B', e_S'; b')\).\(^{11}\)

Relative to Proposition 1, this problem includes truth-telling constraints for the agents’ reports of their information. Denote by \(\Omega^B_t \equiv \{ q_t = q, e_t (i_t^P) = e_P, e_t (i_t^B) = e_B, b_t = b \}\) the buyer’s information set when he reports \((\tilde{q}_B, \tilde{e}_B)\) to the judge, and by \(\Omega^S_t \equiv \{ q_t = q, e_t (i_t^P) = e_P, e_t (i_t^S) = e_S, b_t = b \}\) the corresponding set for the seller, who reports \((\tilde{q}_S, \tilde{e}_S)\). The buyer’s truth-telling constraints are

\[
\mathbb{E} \left[ p (q, q; e_P, e_B, e_S; b) | \Omega^B_t \right] \leq \mathbb{E} \left[ p (\tilde{q}_B, q; e_P, \tilde{e}_B, e_S; b) | \Omega^B_t \right]
\]

for any feasible report \(\tilde{q}_B \in \{0, v\}\) and \(\tilde{e}_B \in \{0, e_B\}\). That is, the buyer cannot lower his expected payment either by misreporting quality (which is cheap talk) or by hiding his

\(^{11}\)In the seller’s objective (9), as well as in the constraints, the expected payments to the seller \(E (p|q_t = v)\) and \(E (p|q_t = 0)\) are computed across realizations of the litigants’ novel evidence collection \((e_B, e_S)\), and the judge’s type \((b)\). When quality is high, \(e_P = 1\) because no negative signals can be realized. When quality is low, expectations are computed also with respect to the realization of precedent. Formally, given truthful reporting the expectation of payment when quality is high can be written out in full as \(E [p (v, 1; e_B, e_S; b)|q_t = v]\) while the expectation of payment when quality is low can be written out in full as \(E [p (0, 0; e_P, e_B, e_S; b)|q_t = 0]\).
piece of evidence. The expectation here is computed with respect to the seller’s search for private evidence, which the buyer anticipates rationally when making his report.

Similarly, the seller’s truth-telling constraints are

\[
\mathbb{E} \left[ p(q, q; e_P, e_B, e_S; b) | \Omega^S_i \right] \geq \mathbb{E} \left[ p(q, q_S; e_P, e_B, e_S; b) | \Omega^S_i \right]
\] (14)

for any feasible report \( q_S \in \{0, v \} \) and \( e_S \in \{0, e_S \} \). The seller cannot raise his expected payment by making an untruthful report conditional on his information \( \Omega^S_i \).

In the final stage, the truth-telling constraints for pro-buyer and pro-seller judges are respectively equal to

\[
p(q_B, q_S; e_P, e_B, e_S; b_B) \leq p(q'_B, q'_S; e_P, e'_B, e'_S; b')
\] (15)

and

\[
p(q_B, q_S; e_P, e_B, e_S; b_S) \geq p(q'_B, q'_S; e_P, e'_B, e'_S; b')
\] (16)

for any feasible ruling \( q'_B, q'_S \in \{0, v \} \), \( e'_B \in \{0, e_B \} \), \( e'_S \in \{0, e_S \} \) and \( b' \in \{b_B, 0, b_S\} \). These constraints do not involve expectations because judges move last, after litigants reveal to the judge all the information he had not directly observed. The first constraint (eq. 15) means that pro-buyer judges cannot lower payment by untruthfully verifying the litigants’ reports or their preferences. The second constraint (eq. 16) means that, analogously, pro-seller judges cannot raise payment through such selective verification.

We can again solve the contracting problem in two steps. In the first step, the seller minimizes the cost of implementing effort subject to the incentive compatibility, non-negativity, and truth telling constraints (eq. 11 to 16). In the second step, the seller chooses optimal effort. In the Appendix we show that this is a linear programming problem whose solution minimizes the ratio of expected payments when quality is low relative to when it is high:

\[
\Lambda = \frac{\mathbb{E}(p|q_t = 0)}{\mathbb{E}(p|q_t = v)}.
\] (17)

As in Proposition 1, the optimal contract minimizes wasteful payments when quality is low and maximizes incentive payments when quality is high. Now, however, quality is not directly contractible and reports of quality are cheap talk, so payments cannot be perfectly targeted to occur if and only if \( q_t = v \). As a result, the second-best contract minimizes the cost of effort provision by loading payment onto the verifiable signal realizations that are most indicative of high quality.

The optimal mechanism is implemented by a very simple contract.
Proposition 2 The optimal contract for partnership t stipulates that the buyer must pay the seller a price \( p^* > 0 \) if and only if the court verifies evidence of high quality both based on precedent and novel, while it verifies no evidence of low quality.

This contract is similar to the full-verifiability contract of Proposition 1. The incentive payment \( p^* \) is enforced only when evidence based on precedent is positive and moreover the seller presents a novel signal indicative of high quality, while the buyer fails to present evidence of low quality.\(^{12}\)

The optimal contract is thus at once path-dependent and open-ended. Parties take an established contract form and add to it an incremental provision that exploits novel evidence. Motivated by the latter feature, we dub this optimal contract an “innovative contract.” By building upon contracts litigated in the past, parties exploit the information embodied in precedents. As a result, they enhance the predictability of enforcement because precedents have binding power on judges. At the same time, the contract is also contingent on a novel clause whose legal enforcement is not settled. That is, parties invite judicial gap-filling despite the enforcement costs it may entail. By optimally setting \( p^* \) parties protect themselves—at least to some extent—from partisan litigants and biased judges. This ability makes it optimal for them to contract on novel evidence, because such evidence allows the use of more information in enforcement. Unlike in a tort case, contracting parties exert some control ex ante over how such information is used, and as a consequence they never wish to discard it fully.

Nonetheless, the parties’ optimal choice of the price \( p^* \) does not suffice to solve the problems caused by biased enforcement. By destroying valuable information, judicial bias reduces the extent to which incentive payments can track the widget’s value, increasing the cost of providing incentives to the seller. With the optimal contract the payment \( p^* \) is enforced under two conditions. In both cases, precedent is consistent with high quality \( (e_t (i^P) = 1) \) and the seller provides novel positive evidence \( (e_t (i^S) = 1) \). In addition, either the judge is unbiased and no negative evidence is presented \((b_t = u \text{ and} \) 

\[ ^{12} \text{Under imperfect verifiability, the optimal contract cannot rely on direct revelation of quality (} q_B, q_S). \text{Such revelation is cheap talk and the litigants’ interests are perfectly opposed because outcomes in which both litigants are punished are impossible. The contract relies instead, as far as possible, on evidence based on precedent, requiring that it should not prove low quality (} e_t (i_t) = 1). \text{The contract also conditions payment on the presence of positive novel evidence in order to reduce the probability of erroneously rewarding low quality. This feature increases the possibility that the incentive payment is erroneously not enforced. This pro-buyer error, however, can be countered by optimally raising } p^*. \text{Of course, if all judges are pro-buyer then avoiding their errors becomes the parties’ fundamental concern. Accordingly, the Appendix shows that when } \beta = 1 \text{ the optimal contract mandates that payment should be enforced if precedent points to high value and the buyer does not present negative novel evidence. Positive novel evidence is no longer required.} \]
e_t(i_t^B) \in \{0, 1\}), or alternatively the judge is pro-seller (b_t = b_S) and enforces payment irrespective of the buyer’s evidence, which he can discard. When both of these conditions are violated, payment is not enforced. For instance, if the judge is pro-buyer payment is never enforced because he can discard the seller’s positive evidence.

In this taxonomy, there are cases in which the bonus is not enforced even though it should (a pro-buyer judge discards positive informative evidence presented by the seller), as well as cases in which the bonus is enforced even though it should not (a pro-seller judge discards negative evidence presented by the buyer). These enforcement distortions reduce verifiability and welfare. To see this, note that in our model verifiability is endogenous to the optimal contract and is fully characterized by the minimized ratio of expected payments \( \Lambda \) (equation [17]). Given the binary structure of the optimal contract, this ratio coincides with the likelihood ratio of low relative to high quality when payment is enforced:

\[
\Lambda = \frac{\Pr (p = p^*|q_t = 0)}{\Pr (p = p^*|q_t = v)}.
\]  

(18)

When \( \Lambda \) is higher, the information that courts can verify is less diagnostic. When the court enforces payment, we cannot confidently reject the hypothesis that \( q_t = 0 \) in favor of the alternative that \( q_t = v \). Then the innovative contract is more incomplete. This measure of incompleteness is a sufficient statistic for the role of judicial biases (\( \beta \) and \( \sigma \)) and precedent (\( i_t^P \)) on efficiency. The legal environment affects \( \Lambda \) as follows.

**Proposition 3** Suppose the optimal innovative contract is signed.

1. Verifiability of quality is higher when there are fewer biased judges (\( \partial \Lambda / \partial \beta \geq 0 \) and \( \partial \Lambda / \partial \sigma \geq 0 \)), when precedents are more informative (\( \partial \Lambda / \partial i_t^P \leq 0 \)), when litigants are better at collecting novel evidence (\( \partial \Lambda / \partial \xi_t \leq 0 \)) and when evidence is less noisy (\( \Lambda \) decreases if \( \xi_t \) shifts down in the sense of first-order stochastic dominance). Full contractibility (\( \Lambda = 0 \)) is achieved if and only if precedent is perfectly diagnostic of quality (\( i_t^P = 1 \)).

2. When precedents are more informative, judicial biases are less detrimental (\( \partial^2 \Lambda / (\partial \beta \partial i_t^P) \leq 0 \) and \( \partial^2 \Lambda / (\partial \sigma \partial i_t^P) \leq 0 \)) and litigants’ ability to collect novel evidence is less important (\( \partial^2 \Lambda / (\partial \xi_t \partial i_t^P) \geq 0 \)).

Point 1 illustrates the first-order effect of the enforcement parameters. Judicial bias reduces verifiability by destroying information embodied in novel evidence. This cost arises
whether bias is idiosyncratic or systematic. Precedents increase verifiability by generating more precise information (higher $i_t^P$) that judges cannot discard. Better collection of novel evidence naturally increases verifiability as well. Accordingly, verifiability rises when pieces of evidence are more informative ($\xi_t$ is systematically lower).

Point 2 establishes that precedents and novel evidence are substitutes. When precedent is more informative, it is less important that the litigants can generate additional information ($\partial^2 \Lambda / (\partial i \partial i_t) \geq 0$). In addition, the more informative precedent is, the more judges are constrained to use informative evidence. Simply put, precedents reduce the impact of judicial biases on verifiability ($\partial^2 \Lambda / (\partial \beta \partial i_t^P) \leq 0$ and $\partial^2 \Lambda / (\partial \sigma \partial i_t^P) \leq 0$).

These results indicate that more developed precedents are associated with a more level contracting field, which depends less on the legal resources at the parties’ disposal ($i$) and on the biases ($\beta, \sigma$) judges have in favor of or against certain parties. This finding is consistent with evidence that common law helps the development of financial markets by ensuring legal protection of outside investors (La Porta et al. 1998) and enabling companies to be widely held by ordinary investors rather than majority owned by the most sophisticated or most connected market participants (La Porta, Lopez-de-Silanes and Shleifer 1999).

Our result also captures a traditional rationale for stare decisis. Binding precedents not only provide judges with information from past cases but also prevent each judge from having undue influence on legal outcomes (Burke [1790] 1999; Gennaioli and Shleifer 2007; Fernandez and Ponzetto 2012). This benefit of precedent is particularly strong for the contractual transactions we consider here. In tort law, when judicial biases distort precedent creation they may cause persistent distortions of justice (Gennaioli and Shleifer 2007). In our model, by contrast, mistaken judicial decisions do not persist because parties can contractually opt out of them. This way, parties further limit the impact of biases and optimally exploit the information contained in past cases.

As a consequence, the enforcement risks created by judicial biases and legal ambiguity are particularly severe for novel and innovative transactions in which precedents are not developed. Legal evolution, then, exerts a direct impact on optimal contracts and welfare, as we show next.

**Corollary 1** Incentives are higher-powered when precedent is more informative ($\partial p^* / \partial i_t^P >$)

---

13This does not mean that all biases are equally costly in our model. We can prove that pro-seller bias is more costly than pro-buyer bias. This asymmetry arises because pro-buyer judges introduce white noise in the adjudication process (holding for the buyer regardless of the true state), while pro-seller judges introduce a systematic distortion. When quality is low, they undesirably make payment more likely. When quality is high, they cannot desirably increase the probability of payment because they cannot fake novel informative evidence that has not been collected by the seller.
0), when evidence is more informative \((p^* \text{ increases if } \xi_t \text{ shifts down in the sense of first-order stochastic dominance})\) and when there are fewer pro-seller judges \((\partial p^*/\partial \sigma < 0)\).

Greater contractibility leads to higher-powered incentives because they are less likely to be wasted on mistakenly rewarding low quality. Thus, the incentive payment \(p^*\) rises when precedent is more informative and evidence less noisy. The payment \(p^*\) also falls if there are more pro-seller judges. These judges discard the negative signals presented by the buyer, a distortion that also wastes incentive payments.\(^\text{14}\)

The effects of the legal environment \((\beta, \sigma, i_t^P)\) on verifiability and on the incentive payment \(p^*\) determine in turn partnership formation and efficiency. When unverifiability \(\Lambda\) is higher than a threshold \(\hat{\Lambda}\), the optimal contract cannot provide incentives for the seller while leaving the buyer a sufficient payoff. As a result, the partnership cannot be formed.\(^\text{15}\)

The buyer must purchase a widget in the market, deriving utility \(u_B\), and any gains from relationship-specific trade are lost.

When verifiability is high enough that \(\Lambda \leq \hat{\Lambda}\), the partnership is formed and some gains from relationship-specific trade are generated. These gains from trade increase as \(\Lambda\) falls. Higher verifiability enables contracts to specify steeper incentives and guarantees these are more accurately enforced. This effect monotonically increases equilibrium effort and joint surplus \((\partial a/\partial \Lambda < 0 \text{ and } \partial \Pi/\partial \Lambda < 0)\), yielding the following predictions.

**Corollary 2** Partnership \(t\) is formed if and only if precedent is sufficiently informative: \(i_t^P \geq i_t^P\). Partnership formation is more likely when there are fewer biased judges \((\partial i_t^P/\partial \beta \geq 0 \text{ and } \partial \xi_t^P/\partial \sigma \geq 0)\), when evidence is less noisy \((\xi_t^P \text{ decreases if } \xi_t \text{ shifts down in the sense of first-order stochastic dominance})\) and when parties are better at collecting novel evidence \((\partial \xi_t^P/\partial t \leq 0)\).

Conditional on a partnership being formed, the seller’s effort and joint surplus are higher when there are fewer biased judges \((\partial a_t/\partial \beta \leq 0 \text{ and } \partial a_t/\partial \sigma \leq 0)\), when evidence is less noisy \((a_t \text{ rises if } \xi_t \text{ shifts down in the sense of first-order stochastic dominance})\), when

\(^\text{14}\)Incentives also become higher-powered when the high-quality widget is more valuable \((\partial p^*/\partial v > 0)\) and when the buyer’s outside option is lower \((\partial p^*/\partial u_B < 0)\). The effect on \(p^*\) of pro-buyer bias \((\beta)\) is instead ambiguous. Suppose that pro-buyer bias rises. Then, it becomes less likely that \(p^*\) is paid, regardless of widget quality. Two countervailing effects ensue. On the one hand, the incentive payment is less effective, which reduces equilibrium effort and also tends to reduce \(p^*\). On the other hand, the payment is less likely to be wasted when \(q_t = 0\), which increases \(p^*\). The net effect can be either a rise or a fall in \(p^*\).

\(^\text{15}\)The threshold for partnership formation is defined as \(\hat{\Lambda}\) such that

\[
\max_{a \in [0,1]} \left\{ av - \left[ \Lambda / (1 - \Lambda) + a \right] C'(a) \right\} = u_B,
\]

reflecting that the minimum cost of inducing any effort \(a\) is increasing in the likelihood ratio \(\Lambda\).
precedents are more informative ($\partial a_t/\partial i_t^P \geq 0$) and when parties are better at collecting novel evidence ($\partial a_t/\partial t \geq 0$).

Figure 2 depicts the regions of the parameter space where the partnership dissolves ($NC$) and where it forms ($Inn$). On the vertical axis, $\vartheta$ denotes a combination of parameters that increases with each determinant of enforcement frictions (judicial bias as well as the noisiness of evidence and the difficulty of collecting it).

$$i_t^P = i_t^Q$$

![Figure 2: Partnership Formation](image)

When precedents are insufficiently informative relative to enforcement frictions ($i_t^P < i_t^Q$), the quality of verifiable information is so poor that it becomes prohibitively costly to harness effort and the parties prefer not to contract (we are in region $NC$). If instead precedent is sufficiently informative, partnership $t$ is formed. In the figure, an increase in $u_B$ or a decline in $v$ induce a downward shift in the locus $i_t^P = i_t^Q$.

Our model formalizes the role of precedents in shaping verifiability and contracting. When the law is undeveloped ($i_t^P = 0$) noisy evidence and judicial bias hinder verification of quality. As a result, parties write low-powered contracts, effort is suboptimal, and gains from trade are dissipated. As precedents develop ($i_t^P$ increases) residual uncertainty in the verification of quality progressively narrows down. First, developed precedents make contract law more complete, offering an informative signal upon which parties can reliably contract. Second, precedent constrains judges, reducing the cost of their biases. Thanks to both effects, parties can use higher-powered incentives that increase effort provision and welfare. When precedents become fully developed ($i_t^P = 1$) there is no residual uncertainty in contract enforcement and the second best outcome of fully verifiable quality is attained.
5 Legal Evolution

Legal development improves verifiability, contains judicial biases, and fosters contracting. To assess whether this mechanism is really effective, we now study legal evolution itself. In our model there is a two-way feedback between private contracting and legal change. As parties write innovative contracts, judges are allowed to consider new quality proxies that are absent from the stock of precedents. In light of new evidence, for instance, the judge may decide not to enforce the incentive payment even though precedents suggest otherwise. As a result, innovative contracts foster a process of distinguishing whereby judges introduce new material dimensions into the law, as in Gennaioli and Shleifer (2007), improving its informativeness. The law’s development, in turn, allows contracts to become more state-contingent and more efficient. As a result, gains from trade increase, leading to accelerated contracting and legal evolution.

We now study formally this two-way feedback loop. In line with many analyses of legal evolution (e.g., Gennaioli and Shleifer 2007; Anderlini, Felli and Riboni 2014) we assume that litigation is costless, so parties always go to court to enforce their contract. We explore the dynamics of the model first by studying the simplest setting in which all buyer-seller pairs are identical. In this case, at any point in time either all parties contract or nobody does. As a result, legal evolution results only from litigation by the representative partnership.

In Section 6, we let partnerships differ in their resourcefulness in litigation, which we proxy by their ability to collect novel evidence \((e)\). In this richer setting, legal evolution also increases the volume of trade over time, progressively enabling less resourceful parties to contract. This extensive margin represents an additional channel for feedback between contracting and precedents.

To see how litigation affects precedents in our model, note that under the innovative contract from Proposition 2, a judge deciding a case may write four different decisions.

1. The seller wins because evidence based on precedent is positive \((e_t (i^P_t) = 1)\) and he presents novel positive evidence \((e_t (i^S_t) = 1)\), while no negative evidence is verified. This decision establishes a new precedent \((P_{t+1} = P_t \cup \{i^S_t\})\).

2. The buyer wins because evidence based on precedent is negative \((e_t (i^P_t) = -1)\).

\(^{16}\)This assumption is made only for simplicity. If litigation were costly, both litigants could prefer to settle out of court. The standard justification for why parties then go to court is that they hold different priors about the probability of winning the trial. We abstract from modeling this feature because none of our results would depend on the specific states leading or not leading to litigation. Reluctance to litigate would simply slow down legal evolution.
This decision is based on existing precedent and thus does not establish a new one \((P_{t+1} = P_t)\).

3. The buyer wins by presenting negative evidence \((e_t (i_t^B) = -1)\). This decision establishes a new precedent \((P_{t+1} = P_t \cup \{i_t^B\})\).

4. The buyer wins because the seller failed to present positive evidence. This decision is based on absence of evidence and does not establish a new precedent \((P_{t+1} = P_t)\).

The stock of precedent is enriched with new evidence when judges write opinion 1 or 3. Yet, changes in the set of precedents do not necessarily improve its informativeness. In case 1, informativeness improves if and only if the seller happens to draw a new piece of evidence that is more informative than precedents \((i_t^S > i_t^P)\). In case 3, by contrast, informativeness necessarily improves. Under the structure of evidence we assumed, a new signal with a negative realization is more informative than a precedent carrying a positive realization \((i_{t+1}^P = i_t^B > i_t^P)\).

This process implies that innovative contracts do not automatically create new precedents, and that changes in the stock of precedents do not automatically translate into uniform changes in the contracts written by private parties. Laws and contracts change only when the new evidence incorporated into precedent is sufficiently informative. Consistent with this property, Choi, Gulati and Posner (2013) document that many contractual innovations are transient. Sometimes, however, novel contract terms prove effective (i.e., informative enough), so they become the reference for future contracting parties.

To characterize the joint evolution of precedents and contracts, assume that the distribution of noise \(\xi_t\) can be parametrized by its mean \(E \xi_t\) and by a dispersion index \(\gamma \in [0, 1]\) such that \(\text{Var} (\xi_t) = \gamma E \xi_t \left(1 - E \xi_t\right)\).\(^{18}\)

---

\(^{17}\)In some cases, a ruling in the buyer’s favor could be justified in several ways. Suppose that both precedent and the buyer produced negative evidence \((e_t (i_t) = e_t (i_t^B) = -1)\) while the seller failed to produce positive evidence \((e_t (i_t^S) \neq 1)\): then each of the three decisions in the buyer’s favor is possible. We assume that judges choose which decision to render on the basis of two principles. First, in accordance with stare decisis, if evidence based on precedent suffices to settle the case, it is summarily decided without considering novel evidence. Second, due to the need to justify their decision, judges always prefer citing novel evidence than grounding their ruling on the insufficiency of available evidence. As a consequence, judges consider the four decisions in the order given above. They proceed down the line only if they cannot (or neither want nor have to) stop at a lower-numbered decision. This assumption does not qualitatively affect our results, but merely influences the speed of legal evolution. Precedents would evolve more rapidly if judges preferred decision 3 to decision 2, or more slowly if they preferred decision 4 to decision 3.

\(^{18}\)Recall that \(E \xi_t (1 - E \xi_t)\) is the maximum variance of a random variable with expectation \(E \xi_t\) and support on \([0, 1]\). E.g., if \(\xi_t\) has a beta distribution with mean \(E \xi_t\) then \(\gamma\) denotes its fixation index.
Proposition 4  Legal evolution can start without a prior history of contract enforcement \((i_0^P = 0)\) if and only if parties are sufficiently capable of generating novel evidence: \(\iota \geq \iota_0\). Legal evolution is more likely to start when there are fewer biased judges \((\partial \iota_0 / \partial \beta > 0\) and \(\partial \iota_0 / \partial \sigma > 0)\), when evidence is less noisy \((\partial \iota_0 / \partial E \xi_t > 0\) and \(\partial \iota_0 / \partial \gamma > 0)\), and when the buyer’s outside option is lower \((\partial \iota_0 / \partial u_B > 0)\).

When \(\iota \geq \iota_0\), the evolution of precedent is described by a time-homogeneous Markov chain. Given any body of precedents \(i_t^P\), any weakly higher informativeness \(j \geq i_t^P\) is accessible, but any strictly lower informativeness \(j < i_t^P\) is inaccessible. The Markov chain is absorbing: its unique absorbing state is perfectly informative precedent \(i^P = 1\), while all imperfectly informative states \(i^P \in [0, 1)\) are transient.

Legal evolution can start from scratch only if it is profitable to form a partnership when there is no prior history of contract enforcement. Formally, this occurs when \(i_0^P = 0\) so we are in the bottom region of Figure 2, where contracting takes place even at \(i_0^P = 0\) because enforcement frictions \(\vartheta\) are low enough.

The parties’ use of innovative contracts induces a monotonic evolution of contract law towards greater informativeness because in our model precedents do not depreciate. The quality of precedent is described by a monotone increasing and ratcheting process. If informativeness \(i_t^P\) has been attained at time \(t\), then less informative states \(j < i_t^P\) are unattainable in the future. Conversely, any higher level of informativeness can be reached from the initial state \(i_0^P\). In fact it can be reached directly through a single ruling. For any threshold \(j \in [i_t^P, 1)\), there is a strictly positive probability \(\Pr (i_{t+1}^P > j | i_t^P) \) that the judicial decision for partnership \(t\) establishes a new precedent whose informativeness is greater than \(j\). The informativeness of precedents is unchanged with complementary probability \(1 - \lim_{j \to i_t^P} \Pr (i_{t+1}^P > j | i_t^P) > 0\). The unique absorbing state of this process is perfectly informative precedent \((i_t^P = 1)\), and the stationary distribution of the Markov chain is fully concentrated on the absorbing state.

As we prove in the Appendix, the probability that precedent improves is

\[
\Pr \left( i_{t+1}^P > i_t^P | i_t^P \right) = 0 \text{ if } i_t^P < i_t^P, \tag{19}
\]

and

\[
\Pr \left( i_{t+1}^P > i_t^P | i_t^P \right) = a_t (1 - \beta) t (1 - i_t^P) + (1 - a_t) t \\
\times \int_{i_t^P}^{1} [(1 - \sigma) (1 - x) + (1 - \beta) (x - i_t^P) - (1 - \beta - \sigma) t (x - i_t^P) (1 - x)] dF_\xi (x) \tag{20}
\]
if \( i_t^P \geq \bar{i}^P \). As Proposition 2 established, no evolution is possible if \( i_t^P < \bar{i}^P \) because enforcement frictions are too large and precedents not informative enough for partnership \( t \) to be formed. If instead \( i_t^P \geq \bar{i}^P \), there is a strictly positive probability that any precedent is improved. The first term in eq. (20) captures the probability of creating a new and more informative precedent when quality is high, which happens with probability \( a_t \) equal to the seller’s effort. The second term captures the probability of improving precedent when quality is low.\(^{19}\) This expression allows us to analyze how the speed of legal evolution varies with enforcement frictions.

These frictions exert a direct and an indirect effect on the probability that precedent improves. The direct effect occurs conditional on widget quality, and consists in the change in the probability that the judge verifies informative new evidence. The indirect effect of enforcement frictions works instead through changes in partnership formation and equilibrium effort. Below we characterize these effects.

**Proposition 5** If judicial biases are symmetric (\( \beta = \sigma = (1 - \omega)/2 \)) then precedent is more likely to improve when judicial bias is less prevalent (\( \partial \Pr (i_{t+1}^P > i_t^P | i_t^P)/\partial \omega > 0 \)) and parties are more capable of collecting novel evidence (\( \partial \Pr (i_{t+1}^P > i_t^P | i_t^P)/\partial v > 0 \)). When precedent is sufficiently developed, it is less likely to improve the better it is already (\( \lim_{i_t^P \to 1} \partial \Pr (i_{t+1}^P > i_t^P | i_t^P)/\partial i_t^P < 0 \)).

Enforcement frictions directly reduce the speed of legal evolution. Lower ability to collect evidence makes novel informative evidence less common. Most important, stronger judicial bias makes it more likely that new evidence is discarded in order to favor either party, preventing the updating of precedent. More numerous pro-seller judges make it more likely that new negative evidence is discarded when widget quality is low. More numerous pro-buyer judges make it more likely that new positive evidence is discarded. The symmetric scenario \( \beta = \sigma \) simplifies our analysis by balancing out these two cases.

These intuitive direct effects are reinforced by indirect effects. If enforcement frictions become too severe, parties do not contract and legal evolution grinds to a halt. Furthermore, stronger enforcement frictions reduce equilibrium effort and hence the probability that the value of the widget is high. This last effect also hinders legal evolution. When

\(^{19}\)The first term reflects the probability, when quality is high, that a pro-seller or unbiased judge verifies an informative signal collected by the seller. At time \( t \), the probability of picking a signal more informative than precedent equals \( 1 - i_t^P \). The second term reflect the probability that when quality is low precedent is nonetheless positive, and either (i) a negative signal (which must be more informative than precedent) is presented by the buyer and verified by a pro-buyer or an unbiased judge; or (ii) a positive signal more informative than precedent is collected by the seller and verified by a pro-seller or an unbiased judge. An unbiased judge verifies such a positive signal only if the buyer did not present negative evidence.
widget quality is low the buyer may win because evidence based on existing precedents is negative. The court then uses no novel evidence. Conversely, when widget value is high novel evidence is always decisive for the outcome of the case. As a result, lower effort reduces the use of new evidence and slows down legal evolution.

When the law is mature, the quality of precedent intuitively reduces the probability of further evolution. When precedent is more informative, i) new evidence is rarely needed to prove low quality, and ii) new evidence of high quality is likely to be less informative than precedent anyway. Both effects slow down evolution. There is, however, a countervailing indirect effect: better precedents enable partnership formation and induce higher equilibrium effort, which increases the speed of evolution. In the beginning, the indirect effect could dominate, so legal evolution could initially accelerate as precedent improves. The positive feedback loop is strongest at this point. When precedent is sufficiently mature \( (i_t^p \to 1) \), however, the direct effect is sure to prevail and legal evolution eventually slows down as it approaches the absorbing state.

Legal ambiguity also exerts countervailing effects on the speed of evolution. Indirectly, it slows down evolution by reducing partnership formation and effort. On the other hand, its direct effect is positive. If a transaction is straightforward (the distribution of \( \xi_t \) is concentrated around zero), low quality is likely to be uncovered by signals based on existing precedent. Then new evidence is less likely to be verified and legal evolution is slower. Conversely, highly ambiguous transactions (with a distribution of \( \xi_t \) concentrated around one) almost always require new evidence, which at least some judges will verify. Thus, conditional on a quality realization, legal ambiguity tends to speed up evolution.

Proposition 5 highlights a key difference between sources of legal uncertainty in commercial disputes. When contract enforcement is unpredictable because a transaction is novel or ambiguous, case law reliably fills in the gaps and attains greater legal certainty and economic efficiency, in the spirit of Posner (1973). However, this beneficial evolution of precedent cannot be counted upon when litigation is distorted by judicial biases. Then material evidence is suppressed, so imperfect rules cannot be quickly weeded out by litigation, as in Priest (1977) and Rubin (1977).

It is worth analyzing further the indirect effect whereby judicial bias slows down legal evolution by hindering private contracting. In particular, judicial bias can cause private contracting to be too low from a social standpoint. The reason is that legal change has a public-good element. As current parties litigate, precedents tend to evolve for the better and enforcement frictions decline for future parties. Just as with other public goods,  

\[ \text{Then, a new precedent is created unless litigants report no novel evidence, or a pro-buyer judge suppresses it.} \]

20
contracting may then be under-provided because current parties do not internalize these positive dynamic externalities. The wedge between the private and the social returns to contracting is a greater problem when judges are more biased.

To formalize this intuition, suppose that a benevolent government aims at maximizing the long-run welfare function

\[ W_t = \sum_{s=0}^{\infty} \delta^s \mathbb{E}_t \Pi_{t+s} \text{ for } \delta \in (0, 1). \]  

(21)

Such a planner may then prefer to remove the buyer’s outside option \( u_B > 0 \), even if doing so entails a static loss, in order to reap the dynamic gains that result from the creation of better precedents and contracts.

**Proposition 6** Suppose precedent is absent \((\iota_0^P = 0)\) and parties do not contract \((\iota < \iota_0)\). Then, there are two thresholds \( \overline{\iota} \) and \( \underline{\iota} \) (with \( 0 < \overline{\iota} < \iota < \iota_0 \)) such that when \( \iota \in (\underline{\iota}, \overline{\iota}) \) parties contract if the outside option \( u_B \) is removed but not if it is available. In this range, removing the outside option entails a static loss \((\Pi_0|_{u_B=0} < u_B)\), yet it is optimal for a sufficiently patient planner \((W_0|_{u_B=0} > u_B/(1 - \delta) \text{ if } \delta > \delta_0)\). The range is wider when there are more biased judges \((\partial (\overline{\iota} - \overline{\iota}) / \partial \beta > 0 \text{ and } \partial (\overline{\iota} - \overline{\iota}) / \partial \sigma > 0)\).

When the probability \( \iota \) of discovering new evidence is intermediate \((\iota \in (\underline{\iota}, \overline{\iota}))\), removing the outside option \( u_B \) is both necessary and sufficient to induce parties to contract when precedent is undeveloped \((\iota_0^P = 0)\). In this range contemporaneous joint surplus falls if contracting is induced. By definition, removing the outside option reduces the buyer’s surplus from \( u_B \) to zero. It also increases the seller’s surplus from zero to \( \Pi \), but this gain fails to compensate the buyer’s loss when evidence collection is insufficient \((\iota < \overline{\iota} \Leftrightarrow \Pi < u_B)\).

Despite this static loss, a sufficiently patient social planner finds it optimal to force parties to contract even if contracting reduces social welfare in the short run. The intuition is that, by doing so, the planner stimulates litigation, jump starting legal development.\(^{21}\) The resulting evolution progressively increases the surplus obtained by contracting parties. Eventually, contracts and contract law converge to the full-verifiability optimum. The benefit of this intervention is not limited to the presence of biased judges. However, Proposition 6 highlights that the scope for this intervention, and hence the severity of the public-good problem, is greater when judges are more biased. Judicial bias makes it

\(^{21}\)If contracting parties were long-lived, they might partly internalize the future benefit of their current contracting. At the same time, though, because other private parties will benefit from future legal evolution, current contracting would continue to be under-provided even with long-lived parties.
costlier for current parties to contract, and at the same time it makes legal development more valuable.

Our analysis therefore shows that there is a positive feedback loop between legal and contractual evolution, but judicial bias can severely undermine it. On the one hand, strong precedents reduce contractual incompleteness and the cost of judicial bias, allowing parties to use more contingent and higher-powered incentives. On the other hand, legal evolution itself is hindered by biased courts. In the first place, judicial biases forestall the creation of informative precedents because biased trial courts discard new evidence that would upset their preferred outcome for the case. Such new evidence does not become available to appellate courts when establishing new precedents and to future partnerships when writing new contracts. Additionally, judicial bias discourages parties from contracting in the first place. Blunted incentives to contract cause legal evolution to be suboptimally low.

As a result, judicial biases are less likely to be self-correcting through legal evolution precisely when they are most detrimental. This result reinforces in a dynamic environment with optimal contracting Glaeser and Shleifer’s (2003) finding that private contracts cannot solve market failures when adjudication in first-instance courts is subverted. In our setup, this deficiency arises not only because judicial biases reduce the quality of contract enforcement, but also because they slow down legal evolution, making enforcement costs highly persistent, as Niblett, Posner and Shleifer (2010) and Niblett (2013a) document empirically. This effect is particularly strong for areas of law or transactions subject to more prevalent judicial biases, perhaps because they are more ideologically charged, or because they involve litigants who can exert undue influence on judges.

6 Heterogeneous Partnerships

In our analysis above we focused on a representative partnership. As a result, either all parties contract at all times, or they never contract at any time. In reality, at any given point in time some parties find it optimal to contract while others do not. Furthermore, legal evolution changes the overall volume of trade, namely the proportion of parties who find it optimal to contract. Accordingly, innovative contracts spread slowly from more sophisticated parties, who act as early adopters, to less sophisticated parties, who imitate after contractual innovations have become established (Choi, Gulati and Posner, 2013).

To consider these additional possibilities, we now introduce heterogeneity in our model. We assume that partnerships vary in their ability to uncover novel evidence, which is a proxy for sophistication or resourcefulness. Certain parties have a greater ability to
produce novel evidence, for instance because they are represented by more skilled law firms. Formally, each partnership $t$ has an independently drawn realization $\tau_t$ of the probability of sampling novel evidence, with cumulative distribution function $F(\cdot)$ on $[0,1]$. Parties observe the realized probability $\tau_t$ right before deciding whether to contract or not. We then characterize optimal contracting and legal evolution as follows.

**Proposition 7** Provided evidence is not noisier than a threshold $\Xi$, namely $\mathbb{E}\xi_t \leq \Xi$, some partnerships are formed by writing an innovative contract for every stock of precedents $i_t^P \geq 0$. Then, legal evolution is described by a Markov chain with the same properties described in Proposition 4. Contracting is more likely to start when there are fewer biased judges ($\partial \Xi / \partial \beta < 0$ and $\partial \Xi / \partial \sigma < 0$).

Partnership $t$ is formed if the parties’ legal ability is sufficiently high: $\tau_t \geq \zeta_t$. More partnerships are formed when there are fewer biased judges ($\partial \zeta_t / \partial \beta > 0$ and $\partial \zeta_t / \partial \sigma > 0$), and evidence is less noisy ($\zeta_t$ increases if $\xi_t$ shifts up in the sense of first-order stochastic dominance). As legal evolution proceeds over time, more partnerships are formed ($\partial \zeta_t / \partial i_t^P < 0$). All partnerships are formed when precedent is sufficiently informative ($i_t^P \geq \zeta_t^P |_{i=0}$).

These dynamics are analogous to those described by Proposition 4. Initially, only parties with a sufficiently high ability to search for novel evidence $\tau_t$ choose to contract. Their contracting and litigation promote legal evolution. As precedent improves, parties characterized by lower $\zeta_t$ find it beneficial to contract. This expansion in the volume of trade speeds up legal evolution. In the limit, precedent is fully informative ($i_t^P = 1$ remains the unique absorbing state) and all parties contract.

The volume of contracting (though not the joint surplus it generates) reaches its maximum where everybody contracts before precedent becomes fully informative. As in Proposition 2, partnership $t$ is formed if and only if precedent is sufficiently informative ($i_t^P \geq \zeta_t^P$). Intuitively, partnership formation is easier for parties that can more easily collect evidence ($\partial \zeta_t^P / \partial \tau \leq 0$) because their innovative contracts are less incomplete ($\partial \Lambda / \partial \tau \leq 0$). A sufficiently high but still imperfect quality of precedent ($\zeta_t^P |_{i=0} < 1$) suffices to allow a partnership to form even if the parties are completely unable to collect novel evidence ($\tau_t = 0$). Then partnership formation is fully assured in spite of heterogeneity. The maximum volume of contracting is attained earlier when judicial biases ($\beta$ and $\sigma$) are rarer.

Our model thus predicts that innovative contract terms undergo a diffusion process. They are introduced by a sophisticated innovator and then, if they manage to be incorporated into informative precedents, they gradually diffuse to less and less sophisticated parties. As a result, legal evolution does not only increase gains from trade by allowing
parties to write higher-powered incentives. It also increases the volume of trade by making it profitable for more and more parties to contract.

7 Conclusions

We build on the idea that contract incompleteness is not a given and time-invariant constraint on the ability of parties to write contracts. Rather, it reflects problems with judicial enforcement of contracts, such as those stemming from judges’ biases, as well as the stock of legal capital embodied in past judicial decisions. Due to this feature, existing contract forms act as a state variable to which current contracting parties make optimal incremental adjustments. This perspective unveils a positive feedback loop between the development of contracts and the evolution of contract law.

As parties litigate, new precedents are introduced into the body of case law. As a result, some contractual contingencies are verified with greater predictability. Parties respond to this development by writing less incomplete contracts, making payment conditional on more informative proxies of performance recognized by judicial precedents. Enforcement frictions decline, incentives become higher powered, and both the volume of contracting and its efficiency increase. This boost in the volume of contracting in turn increases the frequency of litigation, further speeding up legal evolution. This positive feedback loop can be undermined by judicial biases, which slow down legal change both by making litigation less informative and by reducing the parties’ incentive to contract.

Of course, our analysis is only a first step. Theoretical as well as empirical work is needed to assess more fully the implications of the link between the law and private contracting. In closing the paper, we wish to discuss possible directions for future work as well as limitations of the current analysis.

In our model precedents are fully binding. This assumption crudely captures the role of stare decisis for trial courts. In reality, however, judges can work around precedents by arguing that the dispute before the court materially differs from past cases. Such distinguishing is possible even when it exploits only spurious aspects of the case (Fernandez and Ponzetto 2012). The risk that a biased judge may override the intent of the contracting parties is higher if contract language is ambiguous, if the law attaches little binding power to precedents, or if individual precedents tend to depreciate as transactions develop (Llewellyn 1930; Stone 1946, 1964). We have considered a version of our model in which judges can disregard precedents with some probability. Under this alternative assumption, residual uncertainty in enforcement never disappears. Yet, the evolution of laws and contracts
retains the same qualitative features as in our baseline model.

We have barely considered the implications of our analysis for economic policy. Our model suggests that in the presence of judicial bias it may be socially efficient to create legal mechanisms that complement or replace private contracts. In the spirit of Glaeser and Shleifer (2003), one such mechanism might be regulation. But there are also more market-oriented possibilities, such as contract standardization. This intervention consists in introducing a standard-form contract that is impervious to judicial bias. We assess its consequences within our framework in a companion paper (Gennaioli, Ponzetto and Perotti 2015). As we show in that paper, optimal standardization should temper static and dynamic concerns. On the one hand, standard contracts statically improve the welfare of current parties. On the other hand, by crowding out nonstandard contracts, standardization slows down legal evolution. Due to these effects, standardization should optimally occur only after contract law has become sufficiently developed. Generally speaking, our model indicates that dynamic considerations can play a critical role in the design of legal institutions. Insufficiently developed legal capital heightens enforcement risk, acting as a barrier to effective contractual arrangements.
References


A Mathematical Appendix (for Online Publication)

A.1. Proof of Proposition 1

The cost-minimizing way of inducing effort $a$ given the non-negativity constraint is $p_0 = 0$ and $p_v = C'(a)$. Then second best effort solves the surplus-maximization problem

$$\max_{a \in [0,1]} \{av - C(a)\}$$

subject to the participation constraint

$$\pi_B(a) \equiv a [v - C'(a)] \geq u_B.$$  

The buyer’s share of joint surplus $\pi_B(a)$ is a concave function:

$$\pi''_B(a) = -2C''(a) - aC'''(a) < 0$$

because $C''(a) > 0$ and $C'''(a) \geq 0$ for all $a \in (0,1)$. It has limits $\pi_B(0) = \pi_B(a_{FB}) = 0$ and thus a unique maximum

$$U_B \equiv \max_{a \in [0,1]} \pi_B(a)$$

that is reached at

$$a_B \equiv \arg \max_{a \in [0,1]} \pi_B(a) \in (0, a_{FB}).$$

If $u_B > U_B$ the partnership is infeasible. By the envelope theorem, $\partial U_B / \partial v > 0$.

If $u_B \leq U_B$, second-best effort is $a_{SB} \in [a_B, a_{FB})$ such that $\pi_B(a_{SB}) = u_B$ and $\pi'_B(a_{SB}) < 0$ for all $u_B \in (0, U_B)$. By the implicit-function theorem

$$\frac{\partial a_{SB}}{\partial u_B} = \frac{1}{\pi'_B(a_{SB})} < 0$$

and $\frac{\partial a_{SB}}{\partial v} = -\frac{a_{SB}}{\pi'_B(a_{SB})} > 0$. (A6)

Second-best surplus is $\Pi_{SB} = a_{SB}v - C(a_{SB})$ such that

$$\frac{\partial \Pi_{SB}}{\partial u_B} = [v - C'(a_{SB})] \frac{\partial a_{SB}}{\partial u_B} < 0$$

and $\frac{\partial \Pi_{SB}}{\partial v} = a_{SB} + [v - C'(a_{SB})] \frac{\partial a_{SB}}{\partial v} > 0$ (A7)

for all $a_{SB} < a_{FB} \iff u_B > 0$.

A.2. Proof of Proposition 2

The first-step problem of minimizing the cost of eliciting effort $a$ is

$$\min_p \mathbb{E} [p(0,0; e_P, e_B, e_S; b) | q_t = 0]$$

subject to one equality constraints—the incentive-compatibility constraint in equation (11)—and several inequality constraints—the non-negativity and truth-telling constraints
in equations (12) to (16).

Only some of the inequality constraints are binding. A first set of binding constraints reflects the impossibility of making payment contingent on direct revelation of high quality \( q_t = v \) without also paying for low quality \( q_t = 0 \) to induce truthful revelation. When payment is contingent on novel evidence \( (e_t(i_t^B) \) and \( e_t(i_t^S) \)), the likelihood ratio in equation (17) is minimized by lowering payment when more negative evidence is reported and increasing it when more positive evidence is reported. As a consequence, a second set of binding constraints reflects the litigants’ and biased judges’ ability to hide positive or negative evidence. Finally, the non-negativity constraint is binding.

Whenever \( e_t(i_t^P) = -1 \) precedent suffices to establish incontrovertible evidence of low quality and the optimal payment is nil. Since evidence based on precedent cannot be hidden nor distorted,

\[
p(0, 0; -1, e_B, e_S; b) = 0
\]

for all \( e_B, e_S \in \{0, 1\} \) and all \( b \in \{b_B, u, b_S\} \).

**A.2.1. Pro-Buyer Judges**

Pro-buyer judges use cheap talk to minimize payment, so for any report \( (q_B, q_S; e_B, e_S) \) made by the litigants they enforce the same price regardless of cheap talk \( q_B, q_S \in \{0, v\} \):

\[
p(q_B, q_S; e_P, e_B, e_S; b_B) = p(e_P, e_B, e_S; b_B).
\]

(A10)

Since the seller’s payoff and a pro-buyer judge’s are antithetical, revelation of the seller’s informative private signal \( (e_t(i_t^S) \neq 0) \) through a pro-buyer judge requires a payment independent of \( e_S \in \{-1, 0, 1\} \):

\[
p(e_P, e_B, e_S; b_B) = p(e_P, e_B; b_B)
\]

(A11)

for all \( e_P \in \{-1, 1\} \) and \( e_B, e_S \in \{-1, 0, 1\} \). When the buyer reveals a positive signal \( e_t(i_t^B) = 1 \), pro-buyer judges’ ability to hide information implies the binding constraints

\[
p(e_P, 1; b_B) \leq p(e_P, 0; b_B) \text{ for } e_S \in \{0, 1\}.
\]

(A12)

for all \( e_P \in \{-1, 1\} \).

When the buyer presents a negative signal \( e_t(i_t^B) = -1 \) it provides incontrovertible evidence of low quality. Thus the non-negativity constraints binds:

\[
p(e_P, -1; b_B) = 0
\]

(A13)

for all \( e_P \in \{-1, 1\} \). For non-negative realizations of the buyer’s private signals, the binding truth-telling constraints impose a single price

\[
p(e_P, 0; b_B) = p(e_P, 1; b_B) = p(e_P; b_B)
\]

(A14)

for all \( e_P \in \{-1, 1\} \).
If \( e_P = -1 \) the optimal price is nil, so the optimal price schedule for pro-buyer judges consists of a single price
\[
p(1; b_B) \equiv p_B > 0
\]
to be paid when neither precedent nor the buyer reveal negative evidence. Intuitively, the best verification of novel evidence that can be obtained from pro-buyer judges is to distinguish whether the buyer can prove low quality \( (e_t(i_t^B) = -1) \). The contract cannot rely on evidence of low quality presented by the seller against his own interest \( (e_t(i_t^S) = -1) \), nor can it ask the pro-buyer judge to raise payment when the parties have produced positive signals that he can hide \( (e_t(i_t^L) = 1) \).

Thus, pro-buyer judges provide a reward for high quality
\[
\mathbb{E}[p(v, v; 1, e_B, e_S; b_B)|q_t = v] = p_B
\]
and a wasteful payment for low quality
\[
\mathbb{E}[p(0, 0; e_P, e_B, e_S; b_B)|q_t = 0] = p_B \Pr \{ e_t(i_t^P) = 1, e_t(i_t^B) \neq -1|q_t = 0 \}
= p_B \int_{i_t^P}^{1} (1 - \tau + \tau x) dF_{\xi}(x). \tag{A17}
\]
If all judges have a pro-buyer bias \( (\beta = 1) \) the minimand ratio of expected payments is
\[
\Lambda = \int_{i_t^P}^{1} (1 - \tau + \tau x) dF_{\xi}(x). \tag{A18}
\]
If there are both pro-buyer and unbiased judges, the truth-telling constraint (15) imposes
\[
p_B \leq \min_{q \in \{0, v\}, e_B \in \{0, 1\}, e_S \in \{-1, 0, 1\}} p(q; q; 1, e_B, e_S; u) \tag{A19}
\]

A.2.2. Unbiased Judges

Unbiased judges impose no truth-telling constraints of their own because their preferences consist in faithfully applying the contract. On the other hand, unbiased judges introduce additional truth-telling constraints for the litigants, who must honestly report quality to a judge who is willing to make payment depend on their cheap talk if the contract so stipulates.

The buyer must be induced to reveal truthfully \( q_t = v \). Then \( e_t(i_t^P) = 1 \) with certainty, while \( e_t(i_t^B) = 0 \) with probability \( 1 - \tau \) and \( e_t(i_t^S) = 1 \) with probability \( \tau \) independent of all other random variables. Hence, we can simplify his conditional expectation and write the constraints
\[
\mathbb{E}[p(v, v; 1, e_B, e_S; u)) - p(0, v; 1, e_B, e_S; u)|q_t = v] \leq 0 \text{ for } e_B \in \{0, 1\}. \tag{A20}
\]
For ease of notation, define the conditional probability of individual evidence collection
when quality is low,
\[ F_e(e_S|v) \equiv \Pr \{ e_i(i^S_t) = e_S|q_t = v \} \quad (A21) \]
for \( e_S \in \{0, 1\} \). Then the buyer’s truth-telling constraints are
\[ \sum_{e_S \in \{0, 1\}} F_e(e_S|v) p(v; v; 1, e_B, e_S; u) \leq \sum_{e_S \in \{0, 1\}} F_e(e_S|v) p(0, v; 1, e_B, e_S; u) \quad (A22) \]
for \( e_B \in \{0, 1\} \).

The seller must be induced to reveal truthfully \( q_t = 0 \) even if \( e_t(i^P_t) = 1 \):
\[ \mathbb{E} [p(0, 0; 1, e_B, e_S; u) - p(0, v; 1, e_B, 0; u)|q_t = 0, e_t(i^P_t) = 1, e_t(i^S_t) = e_S] \geq 0. \quad (A23) \]

For ease of notation, define the conditional probability of individual evidence collection when quality is low,
\[ F_e(e_B|0, e_P, e_S) \equiv \Pr \{ e_t(i^B_t) = e_B|q_t = 0, e_t(i^P_t) = e_P, e_t(i^S_t) = e_S \} \quad (A24) \]
for \( e_B \in \{-1, 0, 1\} \) given \( e_P \in \{-1, 1\} \) and \( e_S \in \{-1, 0, 1\} \). Then the seller’s truth-telling constraints are
\[
\sum_{e_B \in \{-1, 0, 1\}} F_e(e_B|0, 1, e_S) p(0, 0; 1, e_B, e_S; u) \geq \sum_{e_B \in \{-1, 0, 1\}} F_e(e_B|0, 1, e_S) p(0, v; 1, e_B, e_S; u) \\
\geq \sum_{e_B \in \{0, 1\}} F_e(e_B|0, 1, e_S) p(0, v; 1, e_B, e_S; u) \quad \text{for } e_S \in \{-1, 0, 1\}. \quad (A25)
\]
The second inequality follows by the non-negativity constraint. It reflects the intuitive optimality of punishing the seller when he falsely reports \( q_S = v \) and his lie is exposed by the buyer’s hard evidence \( e_t(i^B_t) = -1 \).

The buyer’s and the seller’s constraints jointly imply that
\[
\sum_{e_B \in \{0, 1\}} \frac{F_e(e_B|0, 1, 1)}{F_e(1|0, 1, 1)} \sum_{e_S \in \{0, 1\}} F_e(e_S|v) p(v; v; 1, e_B, e_S; u) \\
\leq \sum_{e_B \in \{0, 1\}} \frac{F_e(e_B|0, 1, 1)}{F_e(1|0, 1, 1)} \sum_{e_S \in \{0, 1\}} F_e(e_S|v) p(0, v; 1, e_B, e_S; u) \\
\leq \sum_{e_S \in \{0, 1\}} \frac{F_e(e_S|v)}{F_e(1|0, 1, e_S)} \sum_{e_B \in \{0, 1\}} F_e(e_B|0, 1, e_S) p(0, v; 1, e_B, e_S; u) \\
\leq \sum_{e_S \in \{0, 1\}} \frac{F_e(e_S|v)}{F_e(1|0, 1, e_S)} \sum_{e_B \in \{0, 1\}} F_e(e_B|0, 1, e_S) p(0, 0; 1, e_B, e_S; u). \quad (A26)
\]
The first and last inequality are linear combinations of equations (A22) and (A25), respec-
tively. The inner inequality reduces to
\[
\frac{F_e(0|0, 1, 1)}{F_e(1|0, 1, 1)} p(0, v; 1, 0, 0; u) \leq \frac{F_e(0|0, 1, 0)}{F_e(1|0, 1, 0)} p(0, v; 1, 0, 0; u) \quad (A27)
\]
which is true for all \( p(0, v; 1, 0, 0; u) \geq 0 \) because the probability that the buyer’s search is unsuccessful is \( F_e(0|0, 1, 1) = F_e(0|0, 1, 0) = 1 - \epsilon \) independently of the seller’s signal, while the probability that the buyer uncovers a positive signal is increasing in the seller’s signal:
\[
F_e(1|0, 1, 1) = \epsilon \frac{\int_{1}^{1/p} x^2 dF_\xi(x)}{\int_{1}^{1/p} x dF_\xi(x)} > F_e(1|0, 1, 0) = \epsilon \frac{\int_{1}^{1/p} x dF_\xi(x)}{1 - F_\xi(x)}. \quad (A28)
\]
Intuitively, a positive signal given low quality induces inference of high \( \xi_t \) and thus a higher likelihood that another signal is also positive.

We conjecture that the only binding constraint for the litigants’ truthful reporting of quality \( q_t \) is
\[
\sum_{e_B \in \{0, 1\}} \sum_{e_S \in \{0, 1\}} \frac{F_e(e_B|0, 1, 1)}{F_e(1|0, 1, 1)} F_e(e_S|v) p(v, v; 1, e_B, e_S; u) 
\leq \sum_{e_B \in \{0, 1\}} \sum_{e_S \in \{0, 1\}} \frac{F_e(e_B|0, 1, 1)}{F_e(1|0, 1, 1)} F_e(e_S|v) p(0, 0; 1, e_B, e_S; u). \quad (A29)
\]
Then another binding constraint results from the need to induce the buyer to reveal truthfully a positive signal \( (e_t (i^P_t) = 1) \) when quality is high \( (q_t = v \Rightarrow e_t (i^P_t) = 1) \):
\[
\sum_{e_S \in \{0, 1\}} F_e(e_S|v) p(v, v; 1, 1, e_S; u) \leq \sum_{e_S \in \{0, 1\}} F_e(e_S|v) p(v, v; 1, 0, e_S; u). \quad (A30)
\]
Any combination of the four prices \( p(v, v; 1, e_B, e_S; u) \geq p_B \) for \( e_B, e_S \in \{0, 1\} \) such that
\[
\sum_{e_S \in \{0, 1\}} F_e(e_S|v) p(v, v; 1, e_B, e_S; u) = p_B + \epsilon p_U \quad \text{for} \quad e_B \in \{0, 1\} \quad (A31)
\]
for some constant \( p_U \geq 0 \) is optimal given the truth-telling constraints we have considered so far. Only those with \( p(v, v; 1, 0, 1; u) \geq p(v, v; 1, 1, 0; u) \) are feasible, because the litigants must be incentivized to disclose a positive private signal. Then unbiased judges provide a reward for high quality
\[
\mathbb{E}[p(v, v; 1, e_B, e_S; u) | q_t = v] = 
\sum_{e_B \in \{0, 1\}} \sum_{e_S \in \{0, 1\}} F_e(e_B|v) F_e(e_S|v) p(v, v; 1, e_B, e_S; u) = p_B + \epsilon p_U, \quad (A32)
\]
recalling that the success of the two litigants’ searches is independent.

The wasteful payment for low quality is minimized by minimizing payment whenever a
negative signal is obtained. Thus, the non-negativity constraint is binding for \( e_l (i_t^B) = -1 \):

\[
p(0, 0; 1, -1, e_S; u) = 0 \quad \text{for all} \quad e_S \in \{-1, 0, 1\}.
\]  
(A33)

The truth-telling constraint (A19) is binding for \( e_l (i_t^S) = -1 \):

\[
p(0, 0; 1, e_B, -1; u) = p_B \quad \text{for} \quad e_B \in \{0, 1\}.
\]  
(A34)

Intuitively, the seller should be punished when quality is revealed to be low. When the buyer presents a negative signal (\( e_l (i_t^B) = -1 \)) punishment is constrained because the seller is judgment proof. When the seller collects a negative signal (\( e_l (i_t^S) = -1 \)) punishment is further limited by truth-telling constraints—as we are about to show, at the optimum \( p(0, 0; 1, e_B, 0; u) = p_B \) too.

For ease of notation, define the conditional probability of overall evidence generation,

\[
F_{|q} (e_P, e_B, e_S|q) \equiv \Pr \{ e_l (i_t^P) = e_P, e_l (i_t^B) = e_B, e_l (i_t^S) = e_S|q_t = q \}.
\]  
(A35)

Unbiased judges enforce a wasteful payment for low quality

\[
\mathbb{E} [p(0, 0; e_P, e_B, e_S; u)|q_t = 0] = \sum_{e_B \in \{0,1\}} F_{|q} (1, e_B, -1|0) p_B + \sum_{e_B \in \{0,1\}} \sum_{e_S \in \{0,1\}} F_{|q} (1, e_B, e_S|0) p(0, 0; 1, e_B, e_S; u).
\]  
(A36)

The four prices \( p(0, 0; 1, e_B, e_S; u) \) for \( e_B, e_S \in \{0, 1\} \) are optimally set to minimize it given the binding constraint for truthful reporting of \( q_t \):

\[
\sum_{e_B \in \{0,1\}} \sum_{e_S \in \{0,1\}} \frac{F_e (e_B|0, 1, e_S)}{F_e (1|0, 1, e_S)} F_e (e_S|v) p(0, 0; 1, e_B, e_S; u) = \left[ 1 + \frac{F_e (0|0, 1, 1)}{F_e (1|0, 1, 1)} \right] (p_B + t p_U) \]  
(A37)

Thus, all prices should be minimized except those that minimize

\[
L (e_B, e_S) \equiv F_{|q} (1, e_B, e_S|0) \frac{F_e (1|0, 1, e_S)}{F_e (e_B|0, 1, e_S) F_e (e_S|v)},
\]  
(A38)

such that

\[
L (0, 0) = L (1, 0) = t \int_{i_t^P}^1 x dF_{\xi} (x) > L (0, 1) = L (1, 1) = t \int_{i_t^S}^1 x^2 dF_{\xi} (x)
\]  
(A39)

By the binding truth-telling constraint (A19), the optimum is

\[
p(0, 0; 1, e_B, 0; u) = p_B \quad \text{for} \quad e_B \in \{0, 1\},
\]  
(A40)
with any pair \( p(0, 0; e_B, 1; u) \geq p_B \) for \( e_B \in \{0, 1\} \) such that
\[
\sum_{e_B \in \{0, 1\}} F_e(e_B|0, 1, 1) p(0, 0; e_B, 1; u) = [F_e(0|0, 1, 1) + F_e(1|0, 1, 1)] (p_B + p_U), \tag{A41}
\]
recalling that \( F_e(1|v) = i \). Any such pair is optimal given the truth-telling constraints we have considered so far. Only those with \( p(0, 0; 1, 1, 1; u) \leq p(0, 0; 1, 0, 1; u) \) are feasible, because the buyer must be induced to reveal truthfully a positive signal when quality is low.

Then unbiased judges enforce a wasteful payment for low quality
\[
\mathbb{E}[p(0, 0; e_P, e_B, e_S; u)]_{|q_t = 0} = p_B \int_{i_t}^{1} (1 - t + i x) dF_{\xi} (x) + i p_U \int_{i_t}^{1} (1 - t + i x) x dF_{\xi} (x). \tag{A42}
\]
Intuitively, pro-buyer judges can be made to pay \( p_B > 0 \) when the buyer fails to present evidence of low quality only if unbiased judges make the same payment in the same conditions. Moreover, unbiased judges can make an extra payment \( p_U \geq 0 \) when not only the buyer fails to present evidence of low quality, but the seller also manages to present evidence of high quality.

If there are no pro-seller judges (\( \sigma = 0 \)) the minimand ratio of expected payments is
\[
\Lambda = \frac{p_B \int_{i_t}^{1} (1 - t + i x) dF_{\xi} (x) + (1 - \beta) i p_U \int_{i_t}^{1} (1 - t + i x) x dF_{\xi} (x)}{p_B + (1 - \beta) i p_U} \tag{A43}
\]
such that
\[
\frac{\partial \Lambda}{\partial p_B} = \frac{(1 - \beta) i p_U}{[p_B + (1 - \beta) i p_U]^2} \int_{i_t}^{1} (1 - t + i x) (1 - x) dF_{\xi} (x) \geq 0 \tag{A44}
\]
while
\[
\frac{\partial \Lambda}{\partial p_U} = -\frac{(1 - \beta) i p_B}{[p_B + (1 - \beta) i p_U]^2} \int_{i_t}^{1} (1 - t + i x) (1 - x) dF_{\xi} (x) \leq 0. \tag{A45}
\]
Thus, for \( \sigma = 0 \) and \( \beta < 1 \) the optimal contract has \( p_B = 0 < p_U \) and the minimand ratio of expected payments is
\[
\Lambda = \int_{i_t}^{1} (1 - t + i x) x dF_{\xi} (x). \tag{A46}
\]

A.2.3. Pro-Seller Judges

Pro-seller judges use cheap talk to maximize payment, so for any report \((q_B, q_S; e_B, e_S)\) made by the litigants they enforce the same price regardless of cheap talk \( q_B, q_S \in \{0, v\} \):
\[
p(q_B, q_S; e_P, e_B, e_S; b_S) = p(e_P, e_B, e_S; b_S) \tag{A47}
\]
Since the buyer’s payoff and a pro-seller judge’s are antithetical, revelation of the buyer’s
informative private signal \( e_t (i_t^B) \neq 0 \) through a pro-seller judge requires a payment independent of \( e_B \in \{-1, 0, 1\} \):

\[
p(e_P, e_B, e_S; b_S) = p(e_P, e_S; b_S)
\]

(A48)

for all \( e_P \in \{-1, 1\} \) and \( e_B, e_S \in \{-1, 0, 1\} \). When the seller presents a negative signal \( e_t (i_t^S) = -1 \), pro-seller judges’ ability to hide information implies the binding constraints

\[
p(e_P, -1; b_S) \geq p(e_P, 0; b_S)
\]

(A49)

for all \( e_P \in \{-1, 1\} \) and \( e_B \in \{-1, 0, 1\} \).

If \( e_P = -1 \) the optimal price is nil, so the optimal price schedule for pro-seller judges consists of at most two prices \( \tilde{p}_S \geq 0 \) and \( p_S \geq 0 \) such that

\[
\tilde{p}_S \equiv p(1, -1; b_S) = p(1, 0; b_S) \leq p(1, 1; b_S) \equiv \tilde{p}_S + p_S.
\]

(A50)

Intuitively, the best verification that can be obtained from pro-seller judges is to distinguish whether the seller can present evidence of high quality \( e_t (i_t^S) = 1 \). The mechanism cannot rely on evidence of high quality presented by the buyer against his own interest \( e_t (i_t^B) = 1 \), nor can it ask the pro-seller judge to lower payment when the parties have produced negative signals that he can hide \( e_t (i_t^P) = -1 \).

Thus, pro-seller judges provide a reward for high quality

\[
\mathbb{E}[p(v, v; 1, e_B, e_S; b_S) | q_t = v] = \tilde{p}_S + p_S \Pr \{ e_t (i_t^S) = 1 | q_t = v \} = \tilde{p}_S + v p_s
\]

and a wasteful payment for low quality

\[
\mathbb{E}[p(0, 0; e_P, e_B, e_S; b_S) | q_t = 0] = \tilde{p}_S \Pr \{ e_t (i_t^P) = 1 | q_t = 0 \} + p_S \Pr \{ e_t (i_t^P) = 1, e_t (i_t^S) = 1 | q_t = 0 \}
\]

\[
= \tilde{p}_S \left[ 1 - F_\xi (i_t^P) \right] + v p_s \int c f_\xi (x) \, dx.
\]

(A52)

If all judges have a pro-seller bias \( \sigma = 1 \) the minmand ratio of expected payments is

\[
\Lambda = \frac{\tilde{p}_S \left[ 1 - F_\xi (i_t^P) \right] + v p_s \int c f_\xi (x) \, dx}{\tilde{p}_S + v p_s}
\]

(A53)

such that

\[
\frac{\partial \Lambda}{\partial \tilde{p}_S} = \frac{v p_s}{(\tilde{p}_S + v p_s)^2} \int c f_\xi (x) \, dx \geq 0
\]

(A54)

while

\[
\frac{\partial \Lambda}{\partial p_s} = -\frac{v \tilde{p}_S}{(\tilde{p}_S + v p_s)^2} \int c f_\xi (x) \, dx \leq 0
\]

(A55)

Thus, for \( \sigma = 1 \) the optimal contract has \( \tilde{p}_S = 0 < p_s \) and the minmand ratio of expected
payments is
\[ \Lambda = \int_{i_{t}^{P}}^{1} x dF_{x}(x) \quad (A56) \]

If there are both pro-seller and unbiased judges, the truth-telling constraint (16) imposes
\[ \bar{p}_{S} \geq \max_{q \in \{0,v\}, e_{B} \in (-1,0), e_{S} \in (-1,0)} p(q, q; 1, e_{B}, e_{S}; u) \quad (A57) \]
and
\[ \bar{p}_{S} + p_{S} \geq \max_{q \in \{0,v\}, e_{B} \in (-1,0)} p(q, q; 1, e_{B}, 1; u) . \quad (A58) \]

### A.2.4. Optimal Contract

Since the optimal contract for pro-seller judges has \( \bar{p}_{S} = 0 < p_{S} \), the binding truth-telling constraint (15) uniquely pins down the optimal combination of the four prices \( p(v, v; 1, e_{B}, e_{S}; u) \geq p_{B} \) for \( e_{B}, e_{S} \in \{0,1\} \):

\[
p(v, v; 1, 0, 0; u) = p(v, v; 1, 1, 0; u) = p_{B} < p(v, v; 1, 0, 1; u) = p(v, v; 1, 1, 1; u) = p_{B} + p_{U}, \quad (A59)\]

which enables the minimization of
\[ \bar{p}_{S} = p_{B}. \quad (A60) \]

The optimal contracts for the extreme cases in which judges are respectively all pro-seller or all unbiased are ranked by

\[
\Lambda_{\sigma=1} = \int_{i_{t}^{P}}^{1} x dF_{x}(x) > \Lambda_{\beta=\sigma=0} = \int_{i_{t}^{P}}^{1} (1 - \iota + \iota x) x dF_{x}(x). \quad (A61) \]

Intuitively, unbiased judges provide the best verification, even if they cannot achieve perfect revelation of \( q_{t} \) for any \( i_{t}^{P} < 1 \). Thus, it is optimal to minimize \( p_{S} \) for any \( p_{U} \), so the binding truth-telling constraint (15) also uniquely pins down the optimal pair \( p(0, 0; 1, e_{B}, 1; u) \geq p_{B} \) for \( e_{B} \in \{0,1\} \):

\[ p(0, 0; 1, e_{B}, 1; u) = p(0, 0; 1, e_{B}, 1; u) = p_{B} + p_{U}, \quad (A62) \]

which enables the minimization of
\[ p_{S} = p_{U}. \quad (A63) \]

Then, for any \( p_{B} = \bar{p}_{S} = p_{0} \geq 0 \) and \( p_{U} = p_{S} = p_{1} \geq 0 \), the optimal contract provides a reward for high quality
\[ \mathbb{E}(p|q_{t} = v) = p_{0} + (1 - \beta) \iota p_{1} \quad (A64) \]
and a wasteful payment for low quality

$$\mathbb{E}(p|q_t = 0) = \left[ \int_{i_P}^{1} (1 - \tau + \iota x) dF_\xi(x) + \sigma \int_{i_P}^{1} (1 - x) dF_\xi(x) \right] p_0$$

$$+ \iota \left[ (1 - \beta) \int_{i_P}^{1} x dF_\xi(x) - (1 - \beta - \sigma) \iota \int_{i_P}^{1} x (1 - x) dF_\xi(x) \right] p_1. \quad (A65)$$

The minimand ratio of expected payments has derivatives

$$\frac{\partial \Lambda}{\partial p_0} = \frac{(1 - \beta) \iota p_1}{[p_0 + (1 - \beta) \iota p_1]^2} \int_{i_P}^{1} (1 - x) \left[ 1 - \iota + \sigma \iota + \frac{1 - \beta - \sigma}{1 - \beta} \iota x \right] dF_\xi(x) \geq 0 \quad (A66)$$

and

$$\frac{\partial \Lambda}{\partial p_1} = -\frac{(1 - \beta) \iota p_0}{[p_0 + (1 - \beta) \iota p_1]^2} \int_{i_P}^{1} (1 - x) \left[ 1 - \iota + \sigma \iota + \frac{1 - \beta - \sigma}{1 - \beta} \iota x \right] dF_\xi(x) \leq 0 \quad (A67)$$

Thus, for $\beta < 1$ the optimal contract has $p_0 = 0 < p_1 = p^*$ and the minimand ratio of expected payments is

$$\Lambda = \int_{i_P}^{1} x dF_\xi(x) - \frac{1 - \beta - \sigma}{1 - \beta} \iota \int_{i_P}^{1} x (1 - x) dF_\xi(x). \quad (A68)$$

The optimal mechanism stipulates that the price is nil ($p(\ldots) = 0$) except in the following two cases in which the buyer must pay the seller a positive price $p^* > 0$.

1. Evidence based on precedent is positive, the buyer does not present novel negative evidence, the seller presents novel positive evidence, and the judge is unbiased ($p(q_B, q_S; 1, 0, 1; u) = p(q_B, q_S; 1, 1, 1; u) = p^*$ for all $q_B, q_S \in \{0, v\}$).

2. Evidence based on precedent is positive, the seller presents novel positive evidence, and the judge is pro-seller ($p(q_B, q_S; 1, e_B, 1; b_S) = p$ for all $q_B, q_S \in \{0, v\}$ and $e_B \in \{-1, 0, 1\}$).

Under this optimal mechanism, all the truth-telling constraints we conjectured to be non-binding are slack. Pro-buyer judges attain their bliss point because they never enforce payment. Thus, litigants are indifferent about their reports to pro-buyer judges. Pro-seller judges have no avenue to increase payment further: they would need to disregard precedent or to fake positive evidence that the seller failed to present (because $p(q_B, q_S; e_P, e_B, -1; b_S) = p(q_B, q_S; e_P, e_B, 0; b_S) = 0$), both of which are impossible. The buyer is indifferent about his reports to pro-seller judges, who will completely ignore them, while the seller is happy to report truthfully to a pro-seller judge because their goals coincide.

When the judge is unbiased litigants are incentivized to report truthfully quality $q_t$ because the optimal mechanism ignores their cheap talk $q_B, q_S$. They are incentivized to
report truthfully their private signals because they cannot improve their payoffs by hiding them. The buyer may lower payment to zero by presenting \( e_t (i^B_t) = -1 \) but never raises it by presenting \( e_t (i^S_t) = 1 \). The seller may increase it to \( p > 0 \) by presenting \( e_t (i^S_t) = 1 \) but never lowers it by presenting \( e_t (i^S_t) = -1 \).

The correspondence between the optimal direct revelation mechanism and the intuitive contract in Proposition 2 is straightforward. Under the latter, the buyer hides positive evidence \( e_t (i^B_t) = 1 \) to minimize payment and the seller hides negative evidence \( e_t (i^S_t) = -1 \) to maximize it. An unbiased judge reports truthfully all evidence presented in court. Thus, he enforces payment if and only if evidence based on precedent is positive \( (e_t (i^P_t) = 1) \), the seller presented further positive evidence \( (e_t (i^S_t) = 1) \), and the buyer failed to present negative evidence \( (e_t (i^B_t) \neq -1) \). A pro-buyer judge can and does hide any positive evidence presented by the seller. Thus, he never enforces payment. A pro-seller judge can and does hide any negative evidence presented by the buyer. Thus, he enforces payment whenever the seller presents positive evidence \( (e_t (i^P_t) = 1) \), unless evidence based on precedent is negative \( (e_t (i^B_t) = -1) \).

A.3. Proof of Proposition 3 and Corollary 2

Due to the binary nature of the optimal mechanism described by Proposition 2, we can define the probability that the incentive payment \( p^* \) is enforced given that \( q_t = v \),

\[
\eta_v (\ell , \beta) = (1 - \beta) \ell .
\]  

(A69)

and the probability that it is enforced when \( q_t = 0 \),

\[
\eta_0 (i^P_t, \ell , \beta, \sigma) = \ell \left[ (1 - \beta) \int_{i^P_t}^{1} x dF_\xi (x) - (1 - \beta - \sigma) \ell \int_{i^P_t}^{1} x (1 - x) dF_\xi (x) \right].
\]  

(A70)

These probabilities characterize the minimized likelihood ratio

\[
\Lambda (i^P_t, \ell , \beta, \sigma) \equiv \frac{\mathbb{E} (p|q_t = 0)}{\mathbb{E} (p|q_t = v)} = \frac{\eta_0 (i^P_t, \ell , \beta, \sigma)}{\eta_v (\ell , \beta)}
\]  

(A71)

and the solution of the first-stage cost-minimization problem.

Then the seller’s incentive-compatibility constraint implies that effort \( a \) is induced at minimum cost by an incentive payment

\[
p^* (a; i^P_t, \ell , \beta, \sigma) = \frac{C'' (a)}{\eta_v (\ell , \beta) - \eta_0 (i^P_t, \ell , \beta, \sigma)}.
\]  

(A72)

Substituting this solution, the optimal contract induces effort

\[
\hat{a} = \arg \max_{a \in [0,1]} \{ av - C (a) \}
\]  

(A73)
subject to the buyer’s participation constraint

\[ \pi_B (a; \Lambda, v) \equiv av - \left( a + \frac{\Lambda}{1 - \Lambda} \right) C''(a) \geq \underline{u}_B. \quad (A74) \]

The buyer’s share of joint surplus \( \pi_B (a; \Lambda, v) \) is a concave function of effort:

\[ \frac{\partial^2 \pi_B}{\partial a^2} = -2C''(a) - \left( a + \frac{\Lambda}{1 - \Lambda} \right) C'''(a) < 0 \quad (A75) \]

because \( C''(a) > 0 \) and \( C'''(a) \geq 0 \) for all \( a \in (0, 1) \). It has limit \( \pi_B (0; \Lambda, v) = 0 \) and a unique maximum at

\[ a_B (\Lambda, v) \equiv \arg \max_{a \in [0,1]} \pi_B (a; \Lambda, v). \quad (A76) \]

For sufficiently high values of \( \Lambda \left( i_t^P, \iota, \beta, \sigma \right) \), contract enforcement is so poor that \( \pi_B \) is maximized at \( a = 0 \):

\[ a_B (\Lambda, v) = 0 \text{ for all } \Lambda \geq \frac{v}{v + C''(0)} \quad (A77) \]

because

\[ \frac{\partial \pi_B}{\partial a} (0; \Lambda, v) = v - \frac{\Lambda}{1 - \Lambda} C'''(0). \quad (A78) \]

By the envelope theorem,

\[ \frac{\partial \pi_B}{\partial \Lambda} (a_B (\Lambda); \Lambda, v) = - \frac{C' (a_B (\Lambda, v))}{(1 - \Lambda)^2} < 0 \text{ for all } \Lambda < \frac{v}{v + C''(0)} \quad (A79) \]

In the limit as \( \Lambda \to 0 \), quality becomes perfectly contractible and

\[ \lim_{\Lambda \to 0} \pi_B (a_B (\Lambda, v); \Lambda, v) = \max_{a \in [0,1]} \{ a \left[ v - C' (a) \right] \} \quad (A80) \]

as in Proposition 1. Condition (6) ensures that this is greater than \( \underline{u}_B \). Therefore, there is a threshold

\[ \hat{\lambda} (\underline{u}_B, v) \in \left[ 0, \frac{v}{v + C''(0)} \right] \quad (A81) \]

such that partnership \( t \) is formed if and only if \( \Lambda \left( i_t^P, \iota, \beta, \sigma \right) \leq \hat{\lambda} (\underline{u}_B, v) \). By the implicit function theorem, \( \hat{\lambda} (\underline{u}_B, v) \) is decreasing in the buyer’s outside option \( \underline{u}_B \) and increasing in the value of a high-quality widget \( v \).

If the partnership can be formed, optimal effort is \( \hat{a} (\Lambda, \underline{u}_B, v) \) such that

\[ \pi_B (\hat{a}; \Lambda, v) = \underline{u}_B \quad (A82) \]

which implies

\[ a_B (\Lambda, v) \leq \hat{a} (\Lambda, \underline{u}_B, v) < a_{FB} \quad (A83) \]
and
\[ \frac{\partial \pi_B}{\partial \alpha} (\hat{\alpha} (\Lambda, \underline{u}_B, v); \Lambda, v) < 0 \text{ for all } \Lambda < \hat{\Lambda} (\underline{u}_B, v). \]  

(A84)

By the implicit-function theorem,
\[ \frac{\partial \hat{\alpha}}{\partial \Lambda} = \frac{C' (\hat{\alpha})}{(1 - \Lambda)^2} \left[ v - C' (\hat{\alpha}) - \left( \hat{\alpha} + \frac{\Lambda}{1 - \Lambda} \right) C'' (\hat{\alpha}) \right]^{-1} < 0. \]  

(A85)

Welfare is given by joint surplus
\[ \Pi = \hat{\alpha} v - C (\hat{\alpha}), \]  

(A86)

which is monotone increasing in \( \hat{\alpha} \) for all \( \hat{\alpha} < a_{FB} \), namely whenever \( \Lambda > 0 \) or \( \underline{u}_B > 0 \). Quality is directly contractible if and only if \( \Lambda \left( i^P_t, \iota, \beta, \sigma \right) = 0 \). By Proposition 1, the first best is then attainable if and only if, furthermore, \( \underline{u}_B = 0 \).

Under the optimal mechanism described by Proposition 2,
\[ \Lambda \left( i^P_t, \iota, \beta, \sigma \right) = \int_{i^P_t}^{1} xdF_\xi (x) - \frac{1 - \beta - \sigma}{1 - \beta} \iota \int_{i^P_t}^{1} x (1 - x) dF_\xi (x), \]  

(A87)

such that
\[ \Lambda \left( i^P_t, \iota, \beta, \sigma, \alpha \right) = 0 \Leftrightarrow i^P_t = 1 \]  

(A88)

and more generally
\[ \frac{\partial \Lambda}{\partial i^P_t} = - \left[ 1 - \frac{1 - \beta - \sigma}{1 - \beta} \iota (1 - i^P_t) \right] i^P_t f_\xi (i^P_t) \leq 0, \]  

(A89)
\[ \frac{\partial \Lambda}{\partial \iota} = - \frac{1 - \beta - \sigma}{1 - \beta} \int_{i^P_t}^{1} x (1 - x) dF_\xi (x) \leq 0, \]  

(A90)
\[ \frac{\partial \Lambda}{\partial \beta} = \frac{\sigma \iota}{(1 - \beta)^2} \int_{i^P_t}^{1} x (1 - x) dF_\xi (x) \geq 0, \]  

(A91)

and
\[ \frac{\partial \Lambda}{\partial \sigma} = \frac{\iota}{1 - \beta} \int_{i^P_t}^{1} x (1 - x) dF_\xi (x) \geq 0. \]  

(A92)

If we rewrite
\[ \Lambda \left( i^P_t, \iota, \beta, \sigma \right) = \int_{i^P_t}^{1} \left[ \left( 1 - \iota + \frac{\sigma \iota}{1 - \beta} \right) x + \frac{1 - \beta - \sigma}{1 - \beta} \iota \xi^2 \right] dF_\xi (x), \]  

(A93)

it is immediate that \( \Lambda \) increases if \( \xi \) shifts up in the sense of first-order stochastic dominance. Furthermore,
\[ \frac{\partial^2 \Lambda}{\partial \iota \partial i^P_t} = \frac{1 - \beta - \sigma}{1 - \beta} i^P_t (1 - i^P_t) f_\xi (i^P_t) \geq 0, \]  

(A94)
\[ \frac{\partial^2 \Lambda}{\partial \beta \partial i_t^P} = -\frac{\sigma_i}{(1-\beta)^2} (1 - i_t^P) i_t^P f_\xi (i_t^P) \leq 0, \]  
(A95)

and

\[ \frac{\partial^2 \Lambda}{\partial \sigma \partial i_t^P} = -\frac{\nu}{1-\beta} (1 - i_t^P) i_t^P f_\xi (i_t^P) \leq 0. \]  
(A96)

Since \( \Lambda \) is monotone decreasing in \( i_t^P \), ranging from

\[ \Lambda (0, \nu, \beta, \sigma) = \mathbb{E} \xi_t - \frac{1 - \beta - \sigma}{1 - \beta} \nu (\mathbb{E} \xi_t - \mathbb{E} \xi_t^2) \]  
(A97)

to

\[ \Lambda (1, \nu, \beta, \sigma) = 0 \]  
(A98)

for any

\[ \hat{\Lambda} (u_B, \nu) \leq \mathbb{E} \xi_t - \frac{1 - \beta - \sigma}{1 - \beta} \nu (\mathbb{E} \xi_t - \mathbb{E} \xi_t^2) \]  
(A99)

we can define a threshold \( \hat{i}_t^P (\nu, \beta, \sigma, u_B, \nu) \in [0, 1] \) such that

\[ \Lambda (i_t^P, \nu, \beta, \sigma) \leq \hat{\Lambda} (u_B, \nu) \iff i_t^P \geq \hat{i}_t^P (\nu, \beta, \sigma, u_B, \nu). \]  
(A100)

By the implicit-function theorem, for each parameter \( z \in (\nu, \beta, \sigma) \) the derivative \( \partial i_t^P / \partial z \) has the same sign as \( \partial \Lambda / \partial z \). Analogously, \( i_t^P \) increases if \( \xi_t \) shifts up in the sense of first-order stochastic dominance. We can extend the definition to

\[ i_t^P (\nu, \beta, \sigma, u_B, \nu) = 0 \text{ if } \hat{\Lambda} (u_B, \nu) > \mathbb{E} \xi_t - \frac{1 - \beta - \sigma}{1 - \beta} \nu (\mathbb{E} \xi_t - \mathbb{E} \xi_t^2), \]  
(A101)

and the derivatives are then nil.

**A.4. Proof of Corollary 1**

The optimal price is

\[ p^* = \frac{C' (\hat{a})}{\eta_v - \eta_0}, \]  
(A102)

where \( \hat{a} \) is defined as the right-most solution to

\[ \hat{a} \nu - \left( \hat{a} + \frac{\eta_0}{\eta_v - \eta_0} \right) C' (\hat{a}) = u_B, \]  
(A103)

such that

\[ 0 < v - C' (\hat{a}) < \left( \hat{a} + \frac{\eta_0}{\eta_v - \eta_0} \right) C'' (\hat{a}), \]  
(A104)

where the first-inequality (inefficiency) comes from the definition itself,

\[ v - C' (\hat{a}) = \frac{1}{\hat{a}} \left[ u_B + \frac{\eta_0}{\eta_v - \eta_0} C' (\hat{a}) \right], \]  
(A105)

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and the second (binding participation constraint) from selecting the right-most solution. Comparative statics are then

$$\frac{\partial p^*}{\partial z} = \frac{C'(\hat{a})}{(\eta_v - \eta_0)^2} \left( \frac{\partial \eta_v}{\partial z} - \frac{\partial \eta_0}{\partial z} \right) + \frac{C''(\hat{a})}{\eta_v - \eta_0} \frac{\partial \hat{a}}{\partial z}$$  \hspace{1cm} (A106)

for any parameter z. The implicit-function theorem implies

$$\frac{\partial \hat{a}}{\partial z} = \left[ \left( \hat{a} + \frac{\eta_0}{\eta_v - \eta_0} \right) C''(\hat{a}) - v + C'(\hat{a}) \right]^{-1} \times \left[ \frac{\eta_0 C'(\hat{a})}{(\eta_v - \eta_0)^2} \frac{\partial \eta_v}{\partial z} - \frac{\eta_v C''(\hat{a})}{(\eta_v - \eta_0)^2} \frac{\partial \eta_0}{\partial z} + \hat{a} \frac{\partial v}{\partial z} - \frac{\partial u_B}{\partial z} \right].$$  \hspace{1cm} (A107)

Therefore,

$$\frac{\partial p^*}{\partial i_t^P} = - \frac{C'(\hat{a})}{(\eta_v - \eta_0)^2} \left( \hat{a} + \frac{\eta_0}{\eta_v - \eta_0} \right) C''(\hat{a}) - v + C'(\hat{a}) \frac{\partial \eta_0}{\partial i_t^P} > 0$$ \hspace{1cm} (A108)

because

$$\frac{\partial \eta_0}{\partial i_t^P} = - \iota \left[ (1 - \beta) \left( 1 - \iota + \iota i_t^P \right) + \sigma \iota \left( 1 - \iota i_t^P \right) \right] i_t^P f_\xi \left( i_t^P \right) < 0;$$ \hspace{1cm} (A109)

and

$$\frac{\partial p^*}{\partial \sigma} = - \frac{C'(\hat{a})}{(\eta_v - \eta_0)^2} \left( \hat{a} + \frac{\eta_0}{\eta_v - \eta_0} \right) C''(\hat{a}) - v + C'(\hat{a}) \frac{\partial \eta_0}{\partial \sigma} < 0$$ \hspace{1cm} (A110)

because

$$\frac{\partial \eta_0}{\partial \sigma} = \iota^2 \int_{i_t^P}^{1} x (1 - x) dF_\xi (x) > 0.$$ \hspace{1cm} (A111)

By the same token, $p^*$ declines if $\xi_t$ shifts up in the sense of first-order stochastic dominance.

### A.5. Proof of Proposition 4

The conditions under which each decision is written are the following:

1. Evidence based on precedent is positive ($e_t (i_t^P) = 1$), the seller presents positive evidence ($e_t (i_t^S) = 1$) and one of two additional contingencies is realized.

   (a) The judge is unbiased ($b_t = u$) and the buyer does not present negative evidence ($e_t (i_t^B) \in \{0, 1\}$).

   (b) The judge has a pro-seller bias ($b_t = b_s$).

2. Evidence based on precedent is negative ($e_t (i_t^P) = -1$).

3. Evidence based on precedent is positive ($e_t (i_t^P) = 1$), the buyer presents negative evidence ($e_t (i_t^B) = -1$) and the judge does not have a pro-seller bias ($b_t \in \{b_B, u\}$).
4. Evidence based on precedent is positive \( (e_t(i_t^P) = 1) \) and one of three residual cases is realized.

(a) The judge is pro-buyer \( (b_t = b_B) \) and the buyer does not present negative evidence \( (e_t(i_t^B) \in \{0, 1\}) \).

(b) The judge is unbiased \( (b_t = u) \), the seller does not present positive evidence \( (e_t(i_t^S) \in \{-1, 0\}) \) and the buyer does not present negative evidence \( (e_t(i_t^B) \in \{0, 1\}) \).

(c) The judge is pro-seller \( (b_t = b_S) \) and the seller does not present positive evidence \( (e_t(i_t^S) \in \{-1, 0\}) \).

Suppose that given the current state of precedent \( i_t^P \) partnership \( t \) is formed with an innovative contract that induces optimal effort

\[
a_t = \hat{a} \left( \Lambda (i_t^P, \iota, \beta, \sigma), u_B, v \right) > 0.
\]

(A112)

Then the probability that the informativeness of precedent remains unchanged is

\[
\Pr \left( i_{t+1}^P = i_t^P | i_t^P \right) = (1 - \beta - \sigma) a_i i_t^P \left[ a_t + (1 - a_t) \int_{i_t^P}^1 (1 - \iota + x) dF_\xi (x) \right] \\
+ \sigma t \left\{ a_t i_t^P + (1 - a_t) i_t^P \left[ 1 - F_\xi (i_t^P) \right] \right\} \\
+ (1 - a_t) F_\xi (i_t^P) \\
+ \beta \left[ a_t + (1 - a_t) \int_{i_t^P}^1 (1 - \iota + x) dF_\xi (x) \right] \\
+ (1 - \beta - \sigma) \left\{ a_t (1 - \iota) + (1 - a_t) \int_{i_t^P}^1 \left[ 1 - \iota + x^2 (1 - x) \right] dF_\xi (x) \right\} \\
+ \sigma \left[ a_t (1 - \iota) + (1 - a_t) \int_{i_t^P}^1 (1 - \iota x) dF_\xi (x) \right],
\]

(A113)

where the first two lines corresponds to each subcase of decision 1 with \( i_t^S \leq i_t^P \), the third to decision 2, and the last three to each sub-case of decision 4. Simplifying,

\[
\Pr \left( i_{t+1}^P = i_t^P | i_t^P \right) = 1 - a_t (1 - \beta) t (1 - i_t^P) - (1 - a_t) t \\
\times \int_{i_t^P}^1 \left\{ (1 - \sigma) (1 - x) + [\sigma + (1 - \beta - \sigma) (1 - \iota + x)] (x - i_t^P) \right\} dF_\xi (x).
\]

(A114)

This rewriting highlights the cases in which the informativeness of precedent improves \( (i_{t+1}^P > i_t^P) \). If quality is high (with probability \( a_t \)), a valuable new precedent is created if the seller’s search is successful (with probability \( t \)), his evidence happens to be more informative than the best existing precedent (with probability \( 1 - i_t^P \)), and the judge is willing to verify it because he doesn’t have a pro-buyer bias (with probability \( 1 - \beta \)). If
quality is low (with probability $1 - a_t$), a valuable new precedent can be created only if
evidence based on precedent is positive ($\xi_t > i_t^P$). Then, one possibility is that the buyer
finds negative evidence (with probability $\nu (1 - \xi_t)$), and the judge is willing to verify it
because he doesn’t have a pro-seller bias (with probability $1 - \sigma$). The opposite possibility
is that the seller finds evidence that is positive and yet more informative than precedents
($i_t^P < i_t^S < \xi_t$, with probability $\nu (\xi_t - i_t^P)$). A pro-seller judge always reports it to rule
in the seller’s favor (with probability $\sigma$). An unbiased judge (who decides the case with
probability $1 - \beta - \sigma$) does the same if and only if the buyer does not simultaneously report
negative evidence (with probability $1 - \nu + \nu \xi_t$).\footnote{If an unbiased judge reports the buyer’s negative evidence, he may also report the seller’s positive ev-
dence, but the latter is not only irrelevant for the outcome of the case but also necessarily less informative:
$e_t (i_t^B) = -1 < e_t (i_t^S) = 1 \Rightarrow i_t^S < \xi_t \leq i_t^P$.}

The informativeness of precedent improves when decision 3 is made, and also when
decision 1 is made and the seller’s novel evidence happens to be more informative than
existing precedents ($i_{t+1}^P = i_t^S > i_t^P$). For every value $j \in [i_t^P, 1]$, the probability that the
new precedent is more informative equals

$$
\Pr (i_{t+1}^P > j|i_t^P) = (1 - \beta - \sigma) \nu [1 - j] + (1 - a_t) \nu \int_j^1 (1 - \nu + \nu x) (x - j) dF_\xi (x) \\
+ \sigma [1 - j] + (1 - a_t) \nu \int_j^1 (x - j) dF_\xi (x) \\
+ (1 - \sigma) \nu (1 - a_t) \left[ \int_j^1 (1 - j) dF_\xi (x) + \int_j^1 (1 - x) dF_\xi (x) \right], \quad (A115)
$$

where the first two lines corresponds to each subcase of decision 1 with $i_t^S > j$, and the
last one to decision 3. Simplifying,

$$
\Pr (i_{t+1}^P > j|i_t^P) = a_t (1 - \beta) \nu (1 - j) \\
+ (1 - a_t) \nu \int_j^1 [\sigma + (1 - \beta - \sigma) (1 - \nu + \nu x)] (x - j) dF_\xi (x) \\
+ (1 - a_t) \nu (1 - \sigma) \left[ (1 - j) [F_\xi (j) - F_\xi (i_t^P)] + \int_j^1 (1 - x) dF_\xi (x) \right]. \quad (A116)
$$

The first line describes the probability that precedent improves above informativeness $j$
when quality is high (with probability $a_t$). The seller’s search must be successful (with
probability $\nu$), his evidence must happen to be more informative than $j$ (with probability
$1 - j$), and the judge must be willing to verify it because he doesn’t have a pro-buyer bias
(with probability $1 - \beta$). The second line represents the same decision in the seller’s favor
when quality is actually low (with probability $1 - a_t$). Then evidence based on precedent
must be positive ($\xi_t > i_t^P$). The seller’s search must be successful (with probability $\nu$) and
it must yield evidence that is positive and yet more informative than $j$ ($j < i_t^S < \xi_t$, with
probability $\xi_t - j$). Moreover, either the judge must have a pro-seller bias (with probability
\( \sigma \), or else he must be unbiased (with probability \( 1 - \beta - \sigma \)) and have observed no negative evidence produced by buyer. The latter condition obtains when the buyer’s search fails or when it uncovers positive evidence (with probability \( 1 - \tau + \iota \xi \)).

Given any starting point \( i_0 \geq i(t, \beta, \sigma, u_B) \) consistent with partnership formation, the informativeness of precedent \( i_t^P \) evolves as a time-homogeneous Markov chain with transition kernel

\[
P(i, dj) = p(i, j) \, dj + r(i) \, 1_i(dj),
\]

where \( 1_i \) denotes the indicator function \( 1_i(dj) = 1 \) if \( i \in dj \) and 0 otherwise;

\[
r(i) = 1 - (1 - \beta) \tau (1 - i) \, \hat{\Lambda}(\Lambda(i, \tau, \beta, \sigma), u_B, v) - \tau [1 - \hat{\Lambda}((\Lambda(i, \tau, \beta, \sigma), u_B, v)]
\cdot \int \{(1 - \sigma)(1 - x) + [\sigma + (1 - \beta - \sigma)(1 - \tau + \iota \xi)] \, (x - i) \} \, dF_{\xi}(x) \tag{A118}
\]

describes the discrete probability of a transition from \( i_t^P = i \) to \( i_{t+1}^P = i \); and finally

\[
p(i, j) = 0 \text{ for all } j \in [0, i] \tag{A119}
\]

and

\[
p(i, j) = (1 - \beta) \tau \hat{\Lambda}((\Lambda(i, \tau, \beta, \sigma), u_B, v) + \tau [1 - \hat{\Lambda}((\Lambda(i, \tau, \beta, \sigma), u_B, v)]
\cdot \{ \int [\sigma + (1 - \beta - \sigma)(1 - \tau + \iota \xi)] \, dF_{\xi}(x) + (1 - \sigma) \, [F_{\xi}(j) - F_{\xi}(i)] \}
\tag{A120}
\]

for all \( j \in (i, 1] \) jointly describe the continuous probability density of a transition from \( i_t^P = i \) to \( i_{t+1}^P = j \), which is positive if and only if \( j > i \).

It follows that state \( j \) is accessible from state \( i \) if and only if \( j \geq i \). The state \( i = 1 \) is absorbing because it is impossible to leave: \( r(1) = 1 \) and \( p(1, j) = 0 \) for all \( j \in [0, 1] \). The absorbing state is immediately accessible from any other state, so the Markov chain is absorbing.

The Markov chain can start from any \( i_0^P \geq 0 \) if \( i^P(t, \beta, \sigma, \alpha, u_B, v) = 0 \). The definition of \( i^P \), \( \xi_t \) and \( \gamma \) immediately implies that this condition coincides with

\[
\hat{\Lambda}(u_B, v) \geq \xi_t - \frac{1 - \beta - \sigma}{1 - \beta - \sigma} \, (1 - \gamma) \, \xi_t (1 - \xi_t). \tag{A121}
\]

It can be rewritten as

\[
\tau \geq \tau_0 \equiv \frac{1 - \beta - \sigma}{1 - \beta - \sigma} \, (1 - \gamma) \, \xi_t (1 - \xi_t) \tag{A122}
\]

such that

\[
\frac{\partial \tau_0}{\partial \beta} = \frac{\sigma}{(1 - \beta - \sigma)^2 (1 - \gamma) \, \xi_t (1 - \xi_t)} > 0, \tag{A123}
\]

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\[
\frac{\partial \xi}{\partial \sigma} = \frac{1 - \beta}{(1 - \beta - \sigma)^2} \frac{\mathbb{E}\xi_t - \hat{\Lambda}(u_B, v)}{(1 - \beta - \sigma) \mathbb{E}\xi_t (1 - \mathbb{E}\xi_t)} > 0, \quad \text{(A124)}
\]

\[
\frac{\partial \xi}{\partial u_B} = \frac{1 - \beta}{1 - \beta - \sigma} \frac{1}{\mathbb{E}\xi_t (1 - \mathbb{E}\xi_t)} \frac{\partial \hat{\Lambda}}{\partial u_B} > 0, \quad \text{(A125)}
\]

\[
\frac{\partial \xi}{\partial \mathbb{E}\xi_t} = \frac{1 - \beta}{1 - \beta - \sigma} \frac{(\mathbb{E}\xi_t - \hat{\Lambda})^2 + \hat{\Lambda} (1 - \hat{\Lambda})}{(1 - \gamma) [\mathbb{E}\xi_t (1 - \mathbb{E}\xi_t)]^2} > 0 \quad \text{(A126)}
\]

and

\[
\frac{\partial \xi}{\partial \gamma} = \frac{1 - \beta}{1 - \beta - \sigma} \frac{\mathbb{E}\xi_t - \hat{\Lambda}(u_B, v)}{(1 - \gamma) \mathbb{E}\xi_t (1 - \mathbb{E}\xi_t)} > 0. \quad \text{(A127)}
\]

### A.6. Proof of Proposition 5

When judicial biases are symmetric, \( \beta = \sigma = (1 - \omega) / 2 \). The probability that precedent improves is then

\[
\Pr(i_{t+1}^p > i_t^p | i_t^p) = a_t \frac{1 + \omega}{2} \left( 1 - i_t^p \right)
+ (1 - a_t) \int_{i_t^p}^1 \left[ \frac{1 + \omega}{2} (1 - i_t^p) - \omega u (x - i_t^p) (1 - x) \right] dF_i(x), \quad \text{(A128)}
\]

such that

\[
\frac{\partial}{\partial \omega} \Pr(i_{t+1}^p > i_t^p | i_t^p) = \left[ \frac{1 + \omega}{2} \left( 1 - i_t^p \right) F_i(i_t^p) + \omega \int_{i_t^p}^1 (x - i_t^p) (1 - x) dF_i(x) \right] \frac{\partial a_t}{\partial \omega}
+ \frac{1}{2} a_t u (1 - i_t^p) + (1 - a_t) \int_{i_t^p}^1 \left[ \frac{1}{2} (1 - i_t^p) - \omega u (x - i_t^p) (1 - x) \right] dF_i(x) > 0 \quad \text{(A129)}
\]

because \( \partial a_t / \partial \omega > 0 \); and

\[
\frac{\partial}{\partial t} \Pr(i_{t+1}^p > i_t^p | i_t^p) = \left[ \frac{1 + \omega}{2} \left( 1 - i_t^p \right) F_i(i_t^p) + \omega \int_{i_t^p}^1 (x - i_t^p) (1 - x) dF_i(x) \right] \frac{\partial a_t}{\partial t}
+ a_t \frac{1 + \omega}{2} (1 - i_t^p)
+ (1 - a_t) \int_{i_t^p}^1 \left[ \frac{1 + \omega}{2} (1 - i_t^p) - 2 \omega u (x - i_t^p) (1 - x) \right] dF_i(x) > 0 \quad \text{(A130)}
\]

because \( \partial a_t / \partial t > 0 \) while the integrand on the second line can be written

\[
\frac{1 + \omega}{2} (1 - i_t^p) - 2 \omega u (x - i_t^p) (1 - x) + 2 \omega u (x - i_t^p)^2, \quad \text{(A131)}
\]
a quadratic that is always positive because its determinant is

\[-4\omega t \left[ 1 + \omega - \omega t \left( 1 - i_t^P \right) \right] \left( 1 - i_t^P \right) < 0. \quad (A132)\]

Finally,

\[
\frac{\partial}{\partial i_t^P} \Pr \left( i_{t+1}^P > i_t^P | i_t^P \right) = \left[ \frac{1 + \omega}{2} t \left( 1 - i_t^P \right) F_x \left( i_t^P \right) + t^2 \int_{i_t^P}^1 (x - i_t^P) (1 - x) \, dF_x(x) \right] \frac{\partial a_t}{\partial i_t^P} \\
- a_t \frac{1 + \omega}{2} t - (1 - a_t) \frac{1 + \omega}{2} \left( 1 - i_t^P \right) f_x \left( i_t^P \right) \\
- (1 - a_t) t \int_{i_t^P}^1 \left[ \frac{1 + \omega}{2} - \omega t (1 - x) \right] \, dF_x(x) \quad (A133)
\]

is generally ambiguous because \( \partial a_t / \partial i_t^P > 0 \) while the second line is negative. It is unambiguously negative in the limit:

\[
\lim_{i_t^P \to 1} \frac{\partial}{\partial i_t^P} \Pr \left( i_{t+1}^P > i_t^P | i_t^P \right) = -a_t \frac{1 + \omega}{2} t < 0. \quad (A134)
\]

**A.7. Proof of Proposition 6**

If

\[
\hat{\Lambda} \left( u_B, v \right) < \Lambda \left( i_t^P, t, \beta, \sigma \right) < \hat{\Lambda} \left( 0, v \right) = \frac{v}{v + C_u(0)}, \quad (A135)
\]

contracting is not possible with an outside option of \( u_B \) but it occurs with an outside option of zero. Removing the outside option then brings a static payoff \( \Pi \left( \Lambda_t, 0, v \right) - u_B \).

Recall that \( \Pi \left( \Lambda_t, 0, v \right) \) is a continuous and monotone decreasing function of \( \Lambda_t \) such that \( \Pi \left( \hat{\Lambda} \left( 0, v \right), 0, v \right) = 0 \), while \( \Pi \left( 0, 0, v \right) = \Pi_{FB} \geq u_B \) by the assumption that in the first best the transaction is efficient. Thus, there is a value

\[
\tilde{\Lambda} \left( u_B, v \right) \in \left( \hat{\Lambda} \left( u_B, v \right), \hat{\Lambda} \left( 0, v \right) \right) \quad (A136)
\]

such that static surplus is increased by removing the outside option if and only if \( \Lambda_t \leq \tilde{\Lambda} \left( u_B, v \right) \).

For \( i_0^P = 0 \),

\[
\Lambda_0 = \Lambda \left( 0, t, \beta, \sigma \right) = \mathbb{E} \xi_t - \frac{1 - \beta - \sigma}{1 - \beta} t (1 - \gamma) \mathbb{E} \xi_t (1 - \mathbb{E} \xi_t) \quad (A137)
\]
and therefore
\[
\frac{1 - \beta}{1 - \beta - \sigma (1 - \gamma)} \frac{\mathbb{E} \xi_t - \hat{\Lambda}_0 (0, v)}{\mathbb{E} \xi_t (1 - \mathbb{E} \xi_t)} = \frac{1 - \beta}{1 - \beta - \sigma (1 - \gamma)} \frac{\mathbb{E} \xi_t - \hat{\Lambda}_0 (u_B, v)}{\mathbb{E} \xi_t (1 - \mathbb{E} \xi_t)} = t_0 |_{u_B = 0} \equiv \tilde{t} < t
\]
\[
< \tilde{t} \equiv \frac{1 - \beta}{1 - \beta - \sigma (1 - \gamma)} \frac{\mathbb{E} \xi_t - \hat{\Lambda}_0 (u_B, v)}{\mathbb{E} \xi_t (1 - \mathbb{E} \xi_t)} < \Lambda_0 < \hat{\Lambda} (0, v) \quad (A138)
\]

When $t$ lies in this range, contracting at is possible with no precedent ($i^P_0 = 0$) if the outside option is eliminated, but it entails a static loss because $\Pi (\Lambda_0, 0, v) - u_B < 0$. The range has width
\[
\tilde{t} - t = \frac{1 - \beta}{1 - \beta - \sigma (1 - \gamma)} \frac{\hat{\Lambda}_0 (0, v) - \hat{\Lambda}_0 (u_B, v)}{\mathbb{E} \xi_t (1 - \mathbb{E} \xi_t)} > 0 \quad (A139)
\]
such that
\[
\frac{\partial}{\partial \beta} (\tilde{t} - t) = \frac{\sigma (\tilde{t} - t)}{(1 - \beta) (1 - \beta - \sigma)} > 0 \quad (A140)
\]
and
\[
\frac{\partial}{\partial \sigma} (\tilde{t} - t) = \frac{\tilde{t} - t}{1 - \beta - \sigma} > 0. \quad (A141)
\]

From a dynamic perspective, however, welfare without the outside option is
\[
W^0_t (i^P_t) \equiv \Pi (\Lambda (i^P_t, t, \beta, \sigma), 0, v) + \sum_{s=1}^{\infty} \delta^s \mathbb{E} \left[ \Pi (\Lambda (i^P_{t+s}, t, \beta, \sigma), 0, v) | i^P_t \right]. \quad (A142)
\]

There is a threshold quality of precedent above which removal of the outside option brings static gains:
\[
i^P_t \geq i^P (t, \beta, \sigma, u_B, v) \Leftrightarrow \Lambda (i^P_t, t, \beta, \sigma) \leq \hat{\Lambda}_0 (u_B, v) \Leftrightarrow \Pi (\Lambda_0, 0, v) \geq u_B. \quad (A143)
\]

This threshold is below 1 for all $u_B > 0$ because for $i^P_t = 1$ removing the outside option turns the second best into the first best. Then
\[
W^0_t (i^P_t) \equiv u_B + \sum_{s=1}^{\infty} \delta^s \mathbb{E} \left[ \Pi (\Lambda (i^P_{t+s}, t, \beta, \sigma), 0, v) | i^P_t \right] > \frac{1}{1 - \delta} u_B \quad (A144)
\]
because precedent has strictly positive probability of improving further above $i^P_t$ with every transaction.

As a consequence, at $t = 0$ and in the absence of precedent welfare without the outside
option is

\[ W_0^0(0) = \Pi(\Lambda_0, 0, v) + \sum_{s=1}^{\infty} \delta^s \mathbb{E} \left[ \Pi(\Lambda(i_s^P, \iota, \beta, \sigma), 0, v) \right] \]

\[ > \Pi(\Lambda_0, 0, v) + \text{Pr}(i_t^P \geq \bar{i}^P | 0) \frac{\delta}{1 - \delta} W_0^0(\bar{i}^P) \]  
\[ (A145) \]

while welfare with the outside option is

\[ W_0^{uB}(0) = \frac{1}{1 - \delta} u_B \]  
\[ (A146) \]

such that the latter is greater than the former for \( \delta = 0 \) but the former is greater than the latter for sufficiently high values of \( \delta \).

**A.8. Proof of Proposition 7**

There are partnerships that are willing to form if and only if

\[ i_t^P \geq \bar{i}^P(1, \beta, \sigma, u_B, v) \iff \Lambda(i_t^P, 1, \beta, \sigma) \leq \hat{\Lambda}(u_B, v), \]  
\[ (A147) \]

and thus for \( i_t^P = 0 \) if

\[ \bar{i}^P(1, \beta, \sigma, u_B, v) = 0 \iff \Lambda(0, 1, \beta, \sigma) \leq \hat{\Lambda}(u_B, v) \iff \underline{l}(\beta, \sigma, u_B, v) \leq 1, \]  
\[ (A148) \]

namely if

\[ \hat{\Lambda}(u_B, v) \geq \left[ 1 - \frac{1 - \beta - \sigma}{1 - \beta} (1 - \gamma) \right] \mathbb{E} \xi_t + \frac{1 - \beta - \sigma}{1 - \beta} (1 - \gamma) (\mathbb{E} \xi_t)^2. \]  
\[ (A149) \]

The right-hand side is a monotone increasing function of \( \mathbb{E} \xi_t \), so the condition can also be written \( \mathbb{E} \xi_t \leq \Xi \) for a threshold \( \Xi \in (0, 1) \) such that \( \partial \Xi/\partial \beta < 0, \partial \Xi/\partial \sigma < 0, \partial \Xi/\partial \gamma < 0, \partial \Xi/\partial u_B < 0 \) and \( \partial \Xi/\partial v > 0 \).

All partnerships are willing to form if and only if

\[ i_t^P \geq \bar{i}(0, \beta, \sigma, u_B, v) \iff \Lambda(i_t^P, 0, \beta, \sigma) \leq \hat{\Lambda}(u_B, v), \]  
\[ (A150) \]

where

\[ \bar{i}^P(0, \beta, \sigma, u_B, v) > 0 \iff \hat{\Lambda}(u_B, v) < \Lambda(0, 0, \beta, \sigma) = \mathbb{E} \xi_t \]  
\[ (A151) \]

and

\[ \bar{i}^P(0, \beta, \sigma, u_B, v) < 1 \iff \hat{\Lambda}(u_B, v) > \Lambda(1, 0, \beta, \sigma) = 0, \]  
\[ (A152) \]

which is ensured by Condition (6). The comparative statics of \( \bar{i}^P(0, \beta, \sigma, u_B, v) \) are a special case of those found for \( \bar{i} \) in Proposition 2 above.

Since \( \beta + \sigma < 1 \), if condition (A148) holds then for all \( i_t^P \in [0, \bar{i}(0, \beta, \sigma, u_B, v)] \) there is
Such that partnership \( t \) is formed if and only if \( t_t \in [\_t, 1] \). The threshold has comparative statics

\[
\frac{\partial t_t}{\partial i^P_t} = -\frac{1 - \beta}{1 - \beta - \sigma} \frac{\int_{i^P_t}^1 xdF(x) - \hat{\Lambda}(u_B, v)}{\int_{i^P_t}^1 x(1 - x)dF(x)} < 0,
\]

\[
\frac{\partial t_t}{\partial \beta} = \frac{\sigma}{(1 - \beta - \sigma)^2} \frac{\int_{i^P_t}^1 xdF(x) - \hat{\Lambda}(u_B, v)}{\int_{i^P_t}^1 x(1 - x)dF(x)} > 0
\]

and

\[
\frac{\partial t_t}{\partial \sigma} = \frac{1 - \beta}{(1 - \beta - \sigma)^2} \frac{\int_{i^P_t}^1 xdF(x) - \hat{\Lambda}(u_B, v)}{\int_{i^P_t}^1 x(1 - x)dF(x)} > 0.
\]

We extend the definition to \( t_t = 0 \) for all \( i^P_t \geq \hat{i}(0, \beta, \sigma, u_B, v) \). Implicitly, the threshold is defined by \( \Lambda(i^P_t, t_t, \beta, \sigma) = \hat{\Lambda}(u_B, v) \). Since \( \partial \Lambda/\partial t \leq 0 \) while \( \Lambda \) increases if \( \xi_t \) shifts up in the sense of first-order stochastic dominance, \( t_t \) also increases if \( \xi_t \) shifts up in the sense of first-order stochastic dominance.

In period \( t \), if the parties draw an ability to collect novel evidence \( t_t < t_t \) the partnership is not formed. If \( t_t \geq t_t \) the partnership is formed and the seller exerts effort

\[
a_t = \hat{a}(\Lambda(i^P_t, t_t, \beta, \sigma), u_B, v) > 0.
\]

Considering that \( t_t \) is a random draw from the distribution \( F_t(.) \), the evolution of precedent is described by

\[
\Pr(i^P_{t+1} > j|i^P_t) = (1 - \beta) \int_{t_t}^1 \hat{a}(\Lambda(i^P_t, h, \beta, \sigma), u_B, v) dF_t(h)
\]

\[
+ t \left\{ \int_j^1 [\sigma + (1 - \beta - \sigma)(1 - t + \epsilon x)](x - j) dF(x) + (1 - \sigma) \int_{i^P_t}^1 (1 - \max\{j, x\}) dF(x) \right\}
\]

\[
\cdot \int_{t_t}^1 \left[1 - \hat{a}(\Lambda(i^P_t, h, \beta, \sigma), u_B, v)\right] dF_t(h) \text{ for all } j \in [i^P_t, 1].
\]

Thus, it is represented by an absorbing Markov chain with the same qualitative properties described by Proposition 4.