Codes of Best Practice in Competitive Markets for Managers

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Abstract

We study firms’ corporate governance in environments where possibly heterogeneous shareholders compete for possibly heterogeneous managers. A firm, formed by a shareholder and a manager, can sign either an incentive contract or a contract including a Code of Best Practice. A Code allows for a better manager’s control but makes manager’s decisions hard to react when market conditions change. It tends to be adopted in markets with low volatility and in low-competitive environments. The firms with the best projects tend to adopt the Code when managers are not too heterogeneous while the best managers tend to be hired through incentive contracts when the projects are similar. Although the matching between shareholders and managers is often positively assortative, the shareholders with the best projects might be willing to renounce to hire the best managers, signing contracts including Codes with lower-ability managers.

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1 Introduction

The last two decades have witnessed the creation and diffusion of Codes of Best Practice (or Codes of Corporate Governance) all around the world. A Code of Best Practice is a list of rules promoted by a regulator suggesting how a firm should supervise management.\(^1\) Code’s recommendations cover a wide range of Corporate Governance issues: for example, board structure, executive compensation, as well as the role played by institutional investors and capital structure (Bolton \textit{et al}., 2002). Yet, the two most relevant features shared by any Code of Best Practice are, first, its voluntary nature\(^2\) and, second, that they aim to improve manager’s oversight.\(^3\)

In this paper, we analyze the implementation of Codes in markets where possibly heterogeneous shareholders compete for possibly heterogeneous managers. We study which firms use a Code and how this depends on the characteristics of the set of shareholders and managers, as well as each firm’s output market. We model each firm as an agency relationship (Jensen and Meckling, 1976), where each shareholder hires one manager to conduct her project. A shareholder can hire a manager either through an incentive contract or through a contract that includes a Code of Best Practice. The success of each project depends on the decisions taken by the manager and also on product market conditions. Both shareholder and manager know the (ex-ante) distribution of the market conditions parameter; however, only the manager learns the actual realization of the market conditions once the contract has been signed.

The adoption of the Code is a mechanism that allows the shareholder to reduce manager’s discretion (see, for instance, Dahya \textit{et al}., 2002). However, this improvement in Board’s control goes along with a decrease in flexibility at the manager’s decision level. We model this trade-off in a simple way: managers’ payment depends on the final outcome if he is hired through an incentive contract while a Code allows the shareholder to propose

\(^1\)The introduction of Codes of Best Practice was leaded by the Cadbury Report (1992).

\(^2\)In some countries, the legislation asks firms either to comply with each rule or to explain why they do not comply (“comply or explain” principle).

\(^3\)See, for instance, Aguilera \textit{et al}., 2004) and European Commission (2001) for the number and diffusion of Codes of Best Practice. See also IOSCO (2006) for a recent survey of compliance with Board Independence rules proposed by OECD (2004). Nowadays, OECD (2004) can be considered the standard for Codes of Best Practice.
a contract specifying (ex-ante) the manager’s decisions.\textsuperscript{4} Thus, a shareholder chooses between a contract that easily adjusts to the environment but requires paying informational rents (an incentive contract) and a contract that allows to reduce manager’s informational rents but imposes large costs if shareholder’s predictions over the market conditions were erroneous since no adjustment is possible (a Code contract).

In the study of one isolated partnership, we show that a Code is more likely to arise when projects are highly profitable, managers are efficient, have low exogenous outside opportunities and there is low variance in market conditions.

The main purpose of our paper is the analysis of the adoption of Codes in environments where shareholders compete for managers, where the identity of matched partners, and not only the contract, is endogenous rather than exogenous. With this purpose, we follow the approach adopted in Dam and Pérez-Castrillo (2006) and Serfes (forthcoming) and we model a two-sided market where the transaction takes the form of a contract (instead of an object or a monetary transfer).\textsuperscript{5} We take stability as the solution concept for this market. An outcome (i.e., a matching between shareholders and managers and a set of contracts) is stable if it can not be blocked by a shareholder-manager pair who would sign a more profitable contract for both parties.\textsuperscript{6}

The analysis of the shareholder-manager competitive market gives rise to several interesting results. First, in environments with shareholders own projects of different expected return while all managers have the same ability to conduct them, Codes are adopted by those shareholders owning the best projects. Second, also in this type of environment, identical managers end up with quite different ex-post utilities due to two different fac-

\textsuperscript{4}Similar to Alonso-Paulí (2007), we consider that the adoption of the Code allows the shareholder to reduce manager’s discretion. In Alonso-Paulí (2007), adopting the Code prevents the manager from taking certain (bad) actions. In this paper, adopting the Code lets the shareholder choose which actions the manager will take.

\textsuperscript{5}Dam and Pérez-Castrillo (2006) fully characterize a market with homogeneous principals and heterogeneous agents enjoying limited liability, whereas Serfes (forthcoming) analyzes a market with heterogeneous principals and agents with CARA utility functions. See Roth and Sotomayor (1990) for a general presentation of matching markets with and without money.

\textsuperscript{6}Stability and competitive equilibrium are very close concepts (for matching models where the parties decide on money instead of contracts see, for instance, Shapley and Shubik, 1972, Roth and Sotomayor, 1990, and Pérez-Castrillo and Sotomayor, 2002). Any stable outcome is also a competitive equilibrium and viceversa.
tors: a) managers signing contracts including Codes obtain lower utility than managers signing incentive contracts, and b) managers being hired through incentive contracts by shareholders with better projects also obtain higher utility. Third, when the market is composed by homogeneous shareholders and heterogeneous managers, the best managers are hired through incentive contracts in such a way that the Code of Best Practice is not implemented in the relationships involving top managers (contrary to the conclusion obtained in the analysis of an isolated relationship).

Fourth, when both sides are formed by heterogeneous agents, we provide conditions under which the matching is positively assortative, that is, shareholders with better projects hire better managers. This is always the case when either all the contracts (in a stable outcome) are based on incentives, or they all include Codes. We also show cases where the coexistence of incentive contracts and contracts including Codes makes that the matching is not positively assortative. In particular, a shareholder with a good project may opt for a contract including a Code of Best Practice, attracting a less efficient manager when the more efficient is “too expensive” because he is being hired in the market through an incentive contract. Finally, we discuss the welfare effects of introducing Codes of Best Practice. Albeit its voluntary nature, the use of Codes is not always welfare enhancing. We find that, in general, introducing Codes tends to be welfare enhancing if the environment faced by the firms displays a low variance, while it decreases welfares in environment with intermediate variance.

The literature dealing with Codes of Best Practice is still very scarce. With regard to the theoretical literature, only Alonso-Paulí (2007) deals with the analysis of Codes. He concentrates on the effect of its adoption on managers’ incentives and studies the design (by a regulator) of the optimal Code. Empirical literature has been developing along with the creation of Codes of Best Practice. It investigates the relation between Corporate Governance provisions and its effect on firm’s performance. This analysis has not reached a consensus in its conclusions. For instance, while Arcot and Bruno (2006), Gompers et al. (2003) and Fernández and Gómez (2002) find positive effects of the adoption of the Code in US, UK and Spain, respectively, Nowak et al. (2004) and De Jong et al. (2006), find no effect of Code’s recommendations for Germany and The Netherlands, respectively.

Our paper follows a long tradition of studies on the effect of several Corporate Governance mechanisms. Following Shleifer and Vishny (1997), Corporate Governance deals
with the ways in which the suppliers of finance to corporations assure themselves of getting a return on their investment. Berle and Means (1930) were the first to analyze modern corporations highlighting the potential weaknesses generated by the separation between dispersed owners of the corporation and its managers. The literature has extensively analyzed the use of mechanisms such as takeovers, large shareholders, boards of directors and manager’s compensation to solve the agency problem between shareholders and managers. Our paper contributes to this literature by analyzing a different mechanism: the Codes of Best Practice.

Regarding the disciplinary role of the market, the influential papers by Grossman and Hart (1980) and Scharfstein (1988) study the main effects of the threat of takeovers and establish the takeover guidelines for this mechanism to be effective. On the role played by large shareholders, Admati et al. (1994) and Huddart (1993) show that they tend to under-monitor because they balance the benefits from monitoring the manager with the costs of having undiversified portfolios. On the contrary, over-monitoring may arise in presence of specific investments (Aghion and Tirole, 1997, Burkart et al., 1997) or similarly if shareholders may enjoy private benefits of control (La Porta et al., 1998). Hermalin and Weisbach (1998) and Adams and Ferreira (2007) have developed important contributions to understand the functioning of the board of directors. The former analyzes the process by which directors get selected and the influence of the manager on this process. The latter studies the monitoring and the advising tasks developed by any board of directors. Finally, the role played by an appropriately chosen executive compensation scheme has been extensively studied (see, for instance, the pioneer work by Jensen and Meckling, 1976, and the paper by Baker et al., 1988) and has been nicely summarized in Murphy (1999).

The paper is organized as follows. In Section 2, we present the main features of the model and the corresponding solution concept. The properties that the contract displays in stable outcomes are stated in Section 3. Section 4 studies particular manager-shareholder markets and provides characteristics for the most general environment. The welfare effect of introducing Codes is discussed in Section 5. Finally, Section 6 concludes and discusses some extensions of the model. All the proofs are in the Appendix.
2 The Model

2.1 Shareholders and Managers

We consider the market for managers where \( n \) risk neutral shareholders \( S = \{s_1, s_2, s_3, \ldots, s_n\} \) meet \( N \) risk neutral managers \( M = \{m_1, m_2, m_3, \ldots, m_N\} \). We denote shareholders by \( s, s_i, s_i^* \), etc. and, similarly, managers are represented by \( m, m_j, m_j^* \), etc. Each shareholder owns a project but she lacks the skills to develop it. Instead, each manager has the ability to conduct one project. Thus, shareholders and managers have to match in pairs to carry out projects and a contract is signed for each partnership with this objective. Managers enjoy limited liability over income, their wage can not be negative in any contingency.

Both shareholders and managers may be heterogeneous agents. Shareholders may differ in the profitability of the project they own while managers may diverge in their ability to conduct shareholders’ projects. We allow for the possibility that both shareholders and managers can seek for alternative partners and sign new contracts. Hence, the matching between shareholders and managers will be endogenous.

2.2 Projects

Once a shareholder-manager pair is formed, a firm is constituted and the manager is in charge of taking decisions concerning the project. We assume for simplicity that projects are independent in the sense that, once constituted, a firm’s profits only depend on decisions taken in that firm. The project yields a revenue \( R_i > 0 \) for shareholder \( s_i \) if it is successful, whereas the asset has value 0 in case of failure. The value of \( R_i \), for \( i = 1, \ldots, n \), is public information. Without loss of generality, we order projects as \( R_1 \geq R_2 \geq \ldots \geq R_n > 0 \). The probability of success of the project depends on the manager’s decision or effort \( e \) and on some random shock \( h \). In particular, we assume that the probability of success is \( eh \).

Manager \( m_j \)’s effort is his own private information and it has a cost \( c_j(e) = c_j e^2 \), with \( c_j > 0 \). Managers’ ability (the inverse of \( c_j \)) is public information, and we order managers depending on their ability: \( 0 < c_1 \leq c_2 \leq \ldots \leq c_N \); that is, a lower index corresponds to a more efficient manager.

The random variable \( h \) represents the uncertainty in the output market of a project. This industry-specific component can reflect differences among sectors, countries, etc. It is
ex-ante unknown to both parties. It is common knowledge that $h$ is distributed according to $F(h)$ on the interval $[\underline{h}, \overline{h}]$ and it is revealed only to the manager after he accepts the contract and before he decides on the effort.\footnote{An alternative interpretation for the random variable $h$ is that the value of success is industry-specific ($hR$) rather than the probability of success being industry-specific ($he$). This interpretation yields the same project’s expected value.} We denote $\alpha(h) = \int_{\underline{h}}^{\overline{h}} h dF(h)$ the mean of the random shock. Also, we denote $\theta(h) = \int_{\underline{h}}^{\overline{h}} h^2 dF(h)$ so that the variance of the distribution of $h$ is $\text{Var}(h) = \theta(h) - \alpha(h)^2$.

We denote by $\{S, M, R, c\}$ the market for managers and shareholders, where $R \equiv (R_1, R_2, ..., R_n)$ denotes the vector of shareholders’ projects and $c \equiv (c_1, c_2, ..., c_n)$ corresponds to the vector of managers’ abilities.

### 2.3 Contracts and payoffs

When shareholder $s_i$ and manager $m_j$ form a firm, they sign the contract that will govern their relationship. The contract can take two forms. It can be based on an incentive scheme (IS contract) or it can include a Code of Best Practices (CBP contract).

If the firm $(s_i, m_j)$ signs an IS contract, it has the form $W^IS_{s_i,m_j} = (w_R, w_0)$. The first component of the contract $w_R$ is the transfer to the manager in case revenue $R_i$ is obtained, the second part of the contract $w_0$ is the transfer in case of failure, when the result is 0. Under contract $(w_R, w_0)$, the manager will select the effort once he has observed the realization of the market conditions, $h$. He will select the contingent effort $e(w_R, w_0; h)$ that maximizes his utility, i.e.,

$$e(w_R, w_0; h) = \arg\max\{w_0 + h e(w_R - w_0) - c_j e_j^2\},$$

which implies that the level of effort is:

$$e(w_R, w_0; h) = \frac{h}{c_j} (w_R - w_0). \quad (ICC)$$

The previous equation represents the Incentive Compatibility Constraint (ICC). It states that the manager tends to exert a higher level of effort if the bonus is large ($w_R - w_0$), if the market condition is particularly profitable (high $h$), or if he has good skills (low $c_j$).
Alternatively, the firm \((s_i, m_j)\) can sign a CBP contract. A CBP is a monitoring technology that allows the shareholder to gather better information about the manager’s decisions. We model the adoption of a Code in a very simple way. We assume that the board’s control allows to make the manager’s decisions ex-ante contractual, i.e., the shareholder can ask the manager for an *specific* level of effort. Albeit manager’s decisions are ex-ante contractible, shareholders do not know still the realization of the market conditions \(h\). Therefore, the agency problem between the manager and the shareholder is not fully solved. A CBP contract for firm \((s_i, m_j)\) is then a vector \(W_{CBP}^{s_i, m_j} = (w_R, w_0, \varepsilon)\) that specifies the payment to the manager in case of success and failure of the project, as well as the effort he must exert.

Any contract must be *acceptable* for both the manager and the shareholder. Both agents must be better off signing the contract than not signing any contract. This is a necessary condition that requires that they are better under the contract than staying apart from the market. Contract \(W_{s_i, m_j}\) is acceptable for shareholder \(s_i\) if it offers her non-negative profits. It is acceptable for manager \(m_j\) if his expected utility under \(W_{s_i, m_j}\) is not lower than the utility he would obtain by exiting the market. We call this the “outside utility” and denote it \(U\). We write the previous Acceptability constraints as follows:

\[
\pi_{s_i}(m_j, W_{s_i, m_j}) \geq 0, \quad (PC_{s})
\]
\[
V_{m_j}(s_i, W_{s_i, m_j}) \geq U, \quad (PC_{m})
\]
where \(\pi_{s_i}(m_j, W_{s_i, m_j})\) is shareholder \(s_i\)’s expected profits and \(V_{m_j}(s_i, W_{s_i, m_j})\) is manager \(m_j\)’s expected utility when they sign the contract \(W_{s_i, m_j}\).

Furthermore, contracts have to satisfy managers’ limited liability which implies the following constraints:

\[
w_R \geq 0, \quad (LL_{R})
\]
\[
w_0 \geq 0. \quad (LL_{0})
\]
Contracts which are not acceptable for one of the parties or that do not satisfy limited liability constraints will be discarded by either the shareholder or the manager. We say that they are feasible contracts:

**Definition 1** A contract \(W_{s_i, m_j}\) is *feasible* for \((s_i, m_j)\) if it satisfies the acceptability and limited liability constraints \((PC_{s})\), \((PC_{m})\), \((LL_{R})\), and \((LL_{0})\).
2.4 Matching

In our model, the identity of the partners in any firm is endogeneous. A manager will not only chose between signing a contract with a particular shareholder or staying out of the market, but he also has the option to form a firm with any other shareholder. We will represent the identity of the partners forming firms through a matching function that associates shareholders with managers. We now describe a matching in this economy.

**Definition 2** A (one-to-one) matching for the market \( \{S, M, R, c\} \) is a mapping \( \mu: S \cup M \rightarrow S \cup M \) such that (i) \( \mu(s_i) \in M \cup \{s_i\} \) for all \( s_i \in S \), (ii) \( \mu(m_j) \in S \cup \{m_j\} \) for all \( m_j \in M \), and (iii) \( \mu(s_i) = m_j \) if and only if \( \mu(m_j) = s_i \) for all \( (s_i, m_j) \in S \times M \).

A matching is a function that describes which firms are formed. Every pair involving a particular shareholder and a particular manager is a firm. That is, the firm \((s_i, m_j)\) is formed under the matching function \( \mu \) if \( \mu(s_i) = m_j \) (or, analogously, \( \mu(m_j) = s_i \)). The matching function also indicates when an agent (shareholder or manager) is not involved in any firm.

In addition, we need to describe which contract governs any relationship. The only requisite is that contracts within a firm must be feasible.

**Definition 3** A menu of contracts \( W \) compatible with a matching \( \mu \) for the market \( \{S, M, R, c\} \) is a vector of feasible contracts, one for each firm formed under \( \mu \).

A matching and a set of contracts determine a possible organization of our market that we will refer to as an outcome.

**Definition 4** An outcome \((\mu, W)\) for the market \( \{S, M, R, c\} \) is a matching \( \mu \) and a menu of contracts \( W \) compatible with \( \mu \).

The objective of our paper is to characterize the equilibrium outcomes in the shareholders/managers market. That is, we identify the characteristics of both, the contracts emerging and the firms formed. First, contracts signed between shareholders and managers are endogenous. In the traditional analysis of agency problems between managers and shareholders, the literature has thoroughly analyzed the properties of contracts in an isolated principal/agent relationship. Second, we also require that the matching itself is endogenous.
To be an equilibrium, the outcome \((\mu, \mathcal{W})\) shall be immune to potential blocking from any shareholder-manager pair (as it is immune, according to our definition of feasible contracts, to blocking from any individual). This idea corresponds to the concept of stability. It states that it is not sensible to expect \((\mu, \mathcal{W})\) to be an (stable or equilibrium) outcome of the market if there exists any shareholder-manager pair that can form a firm by signing a feasible contract such that both the shareholder and the manager are better-off under the new deal compared to the initial situation \((\mu, \mathcal{W})\).

**Definition 5** An outcome \((\mu, \mathcal{W})\) for the market \(\{S, M, R, c\}\) is stable if there does not exist any pair \((s_i, m_j)\) and any contract \(W'\) feasible for \((s_i, m_j)\) such that \(\pi_{s_i}(m_j, W') > \pi_{s_i}(\mu(s_i), W_{s_i, \mu(s_i)})\) and \(V_{m_j}(s_i, W') > V_{m_j}(\mu(m_j), W_{\mu(m_j), m_j})\).

Stability requires that there does not exist any shareholder-manager pair that can block the current outcome, signing a feasible contract \(W'\) for them. Furthermore, since all contracts in a stable outcome are feasible, a stable outcome is also individually rational.

### 3 Contracts in a stable outcome

In this section, we have a first look at the characteristics of contracts signed in stable outcomes of market \(\{S, M, R, c\}\). The first characteristic implied by stability is that it is not possible for the partners of any existing firm to sign an alternative contract that both find better than the current contract since, otherwise, it would exist a profitable deviation for the partners. That is, contracts in a stable outcome are Pareto optimal among those feasible contracts that satisfy (in case of incentive contracts) incentive constraints. We refer to this notion as (constrained) Pareto Optimality; it is formalized in the following definition.

**Definition 6** A contract \(W_{s_i, m_j}\) for a firm \((s_i, m_j)\) is constrained Pareto optimal if there is no other feasible contract \(W'\) for \((s_i, m_j)\) such that \(\pi_{s_i}(m_j, W') \geq \pi_{s_i}(m_j, W_{s_i, m_j})\) and \(V_{m_j}(s_i, W') \geq V_{m_j}(s_i, W_{s_i, m_j})\), with at least one strict inequality.

Proposition 1 states the optimality property.

**Proposition 1** All the contracts in a stable outcome for the market \(\{S, M, R, c\}\) are constrained Pareto optimal.
The property of constrained Pareto optimality allows to identify any contract in a stable outcome once we know the identity of the partners \((s_i, m_j)\) and the utility obtained by manager \(m_j\). Indeed, the contract is the one that maximizes shareholder \(s_i\)'s expected profits under the constraint that manager \(m_j\) gets this utility level.

In the rest of the section, we will characterize the best contract from shareholder \(s_i\)'s point of view as a function of any possible “reservation utility” level \(U_j\) which has to be achieved by the manager. We denote such a contract as \(W_{s_i,m_j}(U_j)\). Note that the level \(U_j\) will be an equilibrium reservation utility level, determined by the possibility that manager \(s_i\) forms a partnership with other shareholders, when we will perform the complete analysis of stable outcomes in next section.

Remember that the contract \(W_{s_i,m_j}(U_j)\) either takes the form of an Incentive Scheme contract \(W_{IS_{s_i,m_j}}(U_j)\) or a Code of Best Practice contract \(W_{CBP_{s_i,m_j}}(U_j)\). We first identify the contract if \((s_i,m_j)\) sign \(W_{IS_{s_i,m_j}}(U_j)\) then, we calculate the best contract including a CBP \(W_{CBP_{s_i,m_j}}(U_j)\) and finally, as a function of the reservation utility \(U_j\), we state which type of contract is chosen.

Under an IS contract, limited liability constraint \((LL_0)\) makes the incentive condition \((ICC)\) costly for shareholder \(s_i\). However, the impact of limited liability on contracts and on payoffs obtained by the partnership \((s_i,m_j)\) differs depending on the level of manager’s reservation utility \(U_j\). For low values of \(U_j\), the optimal payment scheme depends only on the value of the project \((R_i)\). The shareholder shares half of the value in case of success and the manager ends up with a utility larger than \(U_j\). For large values of \(U_j\), the optimal payment scheme also depends on \(U_j\) as the participation constraint binds. The threshold, denoted by \(\hat{U}_{ij}\), that divides both regions depends on the value of the project and on the distribution of the market specific component as well as on the efficiency of the manager. Formally,

\[
\hat{U}_{ij} = \frac{R_i^2 \theta(h)}{c_j^2}.
\]

Finally, note that the shareholder \(s_i\) will not find acceptable a contract with the manager \(m_j\) if she obtains negative earnings. This situation arises for \(U_j > \hat{U}_{ij} \equiv \frac{R_i^2 \theta(h)}{c_j^2} \). We

\[\text{As we will see, the optimal IS contract when shareholder } s_i \text{ has to ensure manager } m_j \text{ at least } U_j \text{ can give him a level of utility higher than } U_j \text{ (due to the limited liability constraint). That is, } U_j \text{ might not be the actual manager’s utility in a stable outcome. However, we compute the optimal shareholder’s contract for any possible } U_j \text{ as it is the most direct way to develop our analysis.}\]
summarize these findings and identify the contract $W_{s_i,m_j}^{IS}(U_j)$ in Proposition 2.

**Proposition 2** If $(s_i,m_j)$ sign an IS contract $W_{s_i,m_j}^{IS}(U_j)$ in a stable outcome for the market $\{S,M,R,c\}$, then:

(a) $U_j \leq \bar{U}_{ij}$ and the manager’s expected utility is $U_j = \max \left\{ \bar{U}_{ij}, \bar{U}_{ij} \right\}$;
(b) transfers under $W_{s_i,m_j}^{IS}(U_j)$ are $w_0 = 0$ and $w_R = \sqrt{\frac{2c_jU_j}{\theta(h)}}$;
(c) manager’s effort as a function of $h$ is $e^{IS}(h) = h \sqrt{\frac{2U_j}{c_j\theta(h)}}$;
(d) shareholder’s expected profits are $\pi_{s_i}(W_{s_i,m_j}^{IS}(U_j)) = R_i \sqrt{\frac{2\theta(h)U_j}{c_j}} - 2U_j$.

Figure 1 depicts shareholder $s_i$’s profits as a function of manager $m_j$’s reservation utility $U_j$. For any $U_j$ in the interval $[0, \bar{U}_{ij}]$, the optimal contract is the same. It provides the manager an expected utility level of $\bar{U}_{ij}$ (i.e., he obtains “information rents” due to the limited liability constraint). When $U_j \in [\hat{U}_{ij}, \bar{U}_{ij}]$, the efficiency of the (contingent) manager’s effort increases with $U_j$ since the shareholder provides the required higher utility by increasing the salary in case of success (and keeping $w_0 = 0$). Intuitively, profits increase with the value of the project ($R_i$) and with the manager’s efficiency (the inverse of $c_j$). Finally, better market opportunities, summarized by $\theta(h)$, increase profits.

![Figure 1: Shareholder’s profits under Incentive contracts.](image-url)

Under a CBP contract $W_{s_i,m_j}^{CBP}(U_j)$, shareholder and manager not only agree on the payment scheme but also on a pre-specified level of effort. Since the effort is contractual, the only objective of the payment scheme is that the manager accepts to enter into the
relationship. Furthermore, given that both shareholder and manager are risk neutral agents, only the expected wage matters. For the sake of simplicity, and without loss of generality, we stick to fixed wages. Proposition 3 summarizes the characteristics of the contract when the Code is adopted:

**Proposition 3** If $(s_i, m_j)$ sign a CBP contract $W_{s_i,m_j}^{CBP}(U_j)$ in a stable outcome for the market $\{S, M, R, c\}$, then:

(a) manager’s effort under $W_{s_i,m_j}^{CBP}(U_j)$ is $e^{CBP} = \frac{R}{c_j} \alpha(h)$;
(b) transfers are $w_R = w_0 = \frac{R^2 \alpha(h)^2}{2}$;
(c) $U_j \leq \frac{R^2 \alpha(h)^2}{2c_j}$ and the manager obtains a utility of $U_j$;
(d) shareholder’s expected profits are $\pi_{s_i}(W_{s_i,m_j}^{CBP}(U_j)) = \frac{R^2 \alpha(h)^2}{2} - U_j$.

The optimal ex-ante level of effort that maximizes shareholder’s profits depends on the value of the project $R_i$, the manager’s efficiency (the inverse of $c_j$) and also on $\alpha(h)$: the average value of the market conditions. Since manager’s effort must be ex-ante selected and the shareholder does not know the true market conditions, the optimal level of effort is taken as if the true realization was in fact the mean of the distribution (because this is the choice that minimizes the potential losses from an ex-post deviation/mistake). Due to the adoption of the Code, the shareholder does not need to pay informational rents.

Once we have studied the characteristics of $W_{s_i,m_j}^{IS}(U_j)$ and $W_{s_i,m_j}^{CBP}(U_j)$, we proceed to analyze whether shareholder $s_i$ prefers to propose an IS contract or a CBP contract as a function of $U_j$. The main advantage of implementing a Code versus providing incentives is that the former allows a better control of the manager’s actions. Yet, since the market-specific component $h$ is not verifiable, adopting a Code causes a loss in flexibility (ex-post) in the manager’s decision taking. Proposition 4 characterizes the optimal contract.

**Proposition 4** Shareholder $s_i$ obtains higher profits with the contract $W_{s_i,m_j}^{CBP}(U_j)$ than with $W_{s_i,m_j}^{IS}(U_j)$ if and only if $\text{Var}(h) \leq \alpha(h)^2$ and $U_j < U^*_ij$, where

\[
U^*_ij = \frac{R^2}{c_j} \left[\sqrt{\beta(h)^2 + \frac{\beta(h)^2 - \alpha(h)^2}{2}}\right] \quad \text{if } \text{Var}(h) \leq \frac{1}{3} \alpha(h)^2; \text{ and}
\]

\[
U^*_ij = \frac{R^2}{2c_j} \left[\alpha(h)^2 - \frac{\beta(h)^2}{2}\right] \quad \text{if } \text{Var}(h) \in \left(\frac{1}{3} \alpha(h)^2, \alpha(h)^2\right).
\]

\text{We take the convention that the IS contract will be selected in case of indifference between the two contracts.}
Proposition 4 states the conditions for the optimal contract to include a Code of Best Practice. Note that a CBP is never adopted if $Var(h) \geq \alpha(h)^2$. In a nutshell, adopting a CBP requires an environment with low enough variance ($Var(h) \leq \alpha(h)^2$) and a low manager’s utility level ($\bar{U}_j < U^*_j$). Intuitively, a CBP becomes very effective in situations where the environment is not too volatile since the pre-specified level of effort (the optimal corresponding to a situation around the mean of the distribution) does not differ too much from the optimal ex-post effort. On the other hand, the IS contract allows the manager to adapt better to current circumstances, which also benefits the shareholder (although the manager’s decisions will not be optimal). Besides, the Code allows the shareholder to control better the manager which, in turn, implies a lower wage expected cost. This latter effect is less relevant when the manager’s utility is large as, in this case, it is cheaper to give the proper incentives to the manager also ex-post. In fact, manager’s effort increases with the minimum utility and towards the first best level of effort (Proposition 2), while the effort is independent of $\bar{U}_j$ when a CBP is adopted (Proposition 3). Hence, a CBP is more useful when manager’s reservation utility is not too large. Figure 2 illustrates the effect of manager’s utility on the adoption decision when the environment is not too volatile.

![Figure 2: The adoption decision under environments with low variance ($Var(h) < \frac{1}{3} \alpha^2(h)$)](image)

The adoption of a CBP also depends on the ratio $R_i/c_j$, that is, on the value of the project and on manager’s efficiency. The higher this ratio, the higher the effort that will
be asked from the manager; given the more acute agency problem, the shareholder $s_i$ finds the CBP more appealing as it allows better manager’s control. This result is true unless the partnership copes with a very volatile environment.

4 Stable outcomes in the shareholder-manager market

In this Section, we analyze stable outcomes in the market formed by shareholders and managers. The main objectives are to highlight the effect of competition for managers on the contracts signed by firms and to state the composition of the firms: who is matched with whom. We first prove, in Proposition 5, that stable outcomes always exist.

**Proposition 5** The set of stable outcomes in the market \{$S, M, R, c$\} is always non-empty.

We prove this existence result in the Appendix by adapting to our environment the proof developed by Crawford and Knoer (1981) for assignment games.\(^{10}\) In this proof we first show, as they did, that a stable outcome always exists in any “discrete economy” where the possible levels of reservation utility of the managers are discrete, differing by a (possible very small) amount. We then prove that, if the market \{$S, M, R, c$\} has no stable outcome, then also a discrete economy with levels of minimum utility close enough can not have a stable outcome.

An alternative way to prove existence is to propose an outcome for the market and show that it is indeed stable. Since this approach may help to understand how stable outcomes look like, we now develop such a constructive method when there are either two or three partners in each side.

Consider an economy with two shareholders and two managers where forming partnerships is always profitable, that is $\pi_{s_2}(m_2, W_{s_2,m_2}(U)) \geq 0$. Imagine a fictitious auction between the two shareholders to hire the best manager. We denote by $U_{s_i}^{m_1}$, for $i = 1, 2$,

\(^{10}\)See also Gale (1984), Demange and Gale (1985) and Sotomayor (2002) for existence results in assignment games. The main difference between our model and the assignment game is that in ours, matched agents agree on contingent payments and, possibly, on effort, an not only on a fixed monetary transfer.
the maximum utility level for manager \( m_1 \) that makes shareholder \( s_i \) indifferent between hiring \( m_1 \) with a reservation utility of \( U^{s_i,m_1} \) and hiring \( m_2 \) with the outside utility \( U \):

\[
\pi_{s_i}(W_{s_i,m_1}(U^{s_i,m_1})) = \pi_{s_i}(W_{s_i,m_2}(U)).
\]

We can interpret \( U^{s_i,m_1} \) as the "bid" of \( s_i \) for \( m_1 \). It is easy to check that if manager \( s_1 \) is ready to pay more than \( s_2 \) to hire the better manager, i.e., \( U^{s_1,m_1} \geq U^{s_2,m_1} \), then the matching \(((s_1, m_1), (s_2, m_2))\) together with the contracts \((W_{s_1,m_1}(U^{s_i,m_1}), W_{s_2,m_2}(U))\), form a stable outcome. The other case is similar. Remark also that we can easily extend the argument to markets with more than two shareholders or more than two managers.

When there are three shareholders and three managers, we need to proceed in several steps. In the first step, we denote by \( U^{s_i,m_j}_1 \), for \( i = 1, 2, 3 \) and \( j = 1, 2 \), where the subindex refer to step 1, "the bid of \( s_i \) for \( m_j \) against \( m_3 \)" , that is, the maximum utility level such that:

\[
\pi_{s_i}(W_{s_i,m_j}(U^{s_i,m_j}_1)) = \pi_{s_i}(W_{s_i,m_3}(U)),
\]

and we define \( U^{s_i,m_3}_1 = U \), for all \( i \). Also, we denote by \( S(m_j)_1 \) the set of shareholders whose bid for \( m_j \) is the highest, i.e., \( S(m_j)_1 = \{ s_i/ U^{s_i,m_j}_1 \geq U^{s_k,m_j}_1 \text{ for all } k \} \) (note that, by our convention, \( S(m_3)_1 = \{ s_1, s_2, s_3 \} \)). If there exists a matching \( \mu \) such that \( \mu(m_j) \in S(m_j)_1 \) for all \( j \), we take \( \mu \). The matching \( \mu \) with the contracts \( W_{\mu(m_j),m_j}(U^{\mu(m_j),m_j}_1) \) for \( j = 1, 2, 3 \), form a stable outcome. If we can not associate a shareholder to each manager, we need a second step. We take the unique shareholder \( s \) whose bid is the highest for both \( m_1 \) and \( m_2 : s = S(m_1)_1 = S(m_2)_1 \). We decrease simultaneously \( U^{s_1,m_1}_1 \) and \( U^{s_2,m_2}_1 \), keeping the property that \( s \) obtains the same profits with both managers, until one of them reaches the second highest bid. We denote such bids as \( U^{s_1,m_j}_2 \) and \( U^{s_2,m_j}_2 \). We denote all the unchanged bids as \( U^{s_i,m_j}_2 \) instead of \( U^{s_i,m_j}_1 \) and we define \( S(m_j)_2 \) as before, for the new bids. Now, there exists a matching \( \mu \) such that \( \mu(m_j) \in S(m_j)_2 \) for all \( j \) (as \( S(m_3)_2 \) includes three shareholders and another of the sets includes at least two shareholders). We claim that \( \mu \) with the contracts \( W_{\mu(m_j),m_j}(U^{\mu(m_j),m_j}_2) \) for \( j = 1, 2, 3 \), form a stable outcome.

Once we know that stable outcomes always exist in our market, we discuss in the rest of the section their characteristics under several scenarios. First, we deal with the cases where all the agents in one of the two sides are homogeneous, i.e., either all the shareholders hold the same type of project or all the managers are equally efficient. Even in these two simple scenarios, there are interesting results concerning which partnerships
are more prone to introduce CBPs and the utility level the managers achieve. We then provide properties of the stable outcomes in environments where both sides of the market are formed by heterogeneous agents.

4.1 Homogeneous shareholders and heterogeneous managers

Consider the case where the project in hands of all the shareholders offers identical returns, i.e., \( R_i = R \) for all \( i = 1, \ldots, n \) while managers differ in their ability, \( c_1 < c_2 < \ldots < c_N \). Also for simplicity, we are denoting \( s_N \) the less efficient manager with whom a shareholder makes non-negative profits (if there are other managers in the market, we can discard them as they will never be matched in a stable outcome).

Proposition 6 characterizes the stable outcomes in such a market. In the proposition, we denote by \( \tilde{n} = \min \{ n, N \} \) the number of firms that will be formed. Also, to identify the threshold levels that will separate regions in the proposition, we use the following notation: if \( \text{Var}(h) < \alpha(h)^2 \), we denote by \( \pi_j = \pi_s(W_{s,m_j}(U_j^*)) \) the level of profits that the shareholder obtains when manager \( m_j \)'s utility is such that she is indifferent between a CBP contract and an IS contract.\(^{11}\) It is easy to check that \( \pi_j \) increases with the manager’s efficiency, hence it is decreasing in \( j \).

**Proposition 6** When shareholders are homogeneous and managers are heterogeneous, properties (a) - (d) characterize an stable outcome \( (\mu, W) \) for the market \( \{S, M, R, c\} \):

(a) all shareholders have the same profit level \( \pi \); 
(b) \( \pi = 0 \) if \( n > N \); \( \pi \leq \pi_s(W_{s,m_n}(U)) \) if \( n = N \); and \( \pi \in [\pi_s(W_{s,m_{n+1}}(U)), \pi_s(W_{s,m_n}(U))] \) if \( n < N \); 
(c) \( W_{s,m_j} \) is the optimal contract for the manager that gives profits \( \pi \) to the shareholder.

In particular,

(d1) all are IS contracts if \( \text{Var}(h) \geq \alpha(h)^2 \) or if \( \text{Var}(h) < \alpha(h)^2 \) and \( \pi \leq \pi_{\tilde{n}} \); 
(d2) all are CBP contracts if \( \text{Var}(h) < \alpha(h)^2 \) and \( \pi > \pi_1 \);  
(d3) \( W_{s,m_j} \) is a CBP contract if \( j > J \) and \( W_{s,m_j} \) is an IS contract if \( j \leq J \), when \( \text{Var}(h) < \alpha(h)^2 \) and \( J \) is such that \( \pi_J \geq \pi > \pi_{J+1} \).

Since all shareholders hold projects with the same return, the first characteristic highlighted in Proposition 6 is that their profits must be equal. If this was not the case,

\(^{11}\) For notational convenience, we denote \( U_j^* \) instead of \( U_{ij}^* \), since shareholders are homogeneous.
shareholder $s_i$ getting lower profits than $s_{i'}$ could attract manager $\mu(s_{i'})$ by proposing a contract that slightly increases his utility. Second, the level of profits depends on the strength of the competition for managers. Shareholders achieve positive profits when competition for managers is smooth, that is, they are in the short side of the market. Furthermore, their profits are higher when they could hire better managers with lower outside option who are not hired in this market. Third, since CBPs allow better manager’s control, they arise if competition between shareholders is not very strong (so that managers’ level of utility is low), while shareholders are more prone to offer incentive contracts in situations where competition is tough. Also, similar to the conclusion that we obtained in the analysis for an isolated firm (Proposition 4), CBPs tend to be adopted in environments with low variance, while IS contracts are signed under volatile environments.

Finally, Proposition 6 (d3) states who adopts a CBP when a market sustains simultaneously IS and CBP contracts: efficient managers end up being hired through IS contracts while a CBP is used to attract inefficient ones. This contrasts with the conclusion obtained after Proposition 4 suggesting that a CBP would be adopted for efficient managers. Proposition 6 (d3) shows that, when shareholders compete for the best managers, the conclusion is reversed. To attract efficient managers, shareholders offer them a high utility level and this now makes IS contracts more appealing than CBP contracts. This result stresses the relevance of the study of manager-shareholder relationships in a framework where not only contracts but also matching is endogenous.\footnote{Similar to Barros and Macho-Stadler (1998) and Dam and Pérez-Castrillo (2006), the use of incentive contracts has also a positive effect on firm’s efficiency. The better the manager, the closest the effort to its efficient (first-best) level.}

4.2 Heterogeneous shareholders and homogeneous managers

We now consider an economy formed by heterogeneous shareholders ($R_i > R_{i'}$ for all $i < i'$) and equally efficient managers ($c_j = c$ for all $m_j \in \mathcal{M}$). We assume, for simplicity, that there are more managers than shareholders, i.e., $N > n$ and that the partnership between $s_n$ and a manager is feasible. A direct implication of the above is that the reservation utility shareholders will need to provide to managers is the outside utility $U$.\footnote{If $N \geq n$, managers will usually end up with a higher utility than $U$ but the qualitative results will hold.}
Proposition 7 characterizes the unique stable outcome for this market (for notational simplicity, we denote $U_i^*$ instead of $U_{ij}^*$).

**Proposition 7** When shareholders are heterogeneous and managers are homogeneous, properties (a) and (b) characterize the unique stable outcome $(\mu, W)$ for the market \{S, M, R, c\}:

(a) $\mu(s_i) \in M$ for all $s_i \in S$;

(b) all the contracts are $W_{s_i, m} = W_{s_i, m}(U)$.

In particular,

(c1) all are IS contracts if $\text{Var}(h) \geq \alpha(h)^2$ or $U \geq U_1^*$;

(c2) all are CBP contracts if $\text{Var}(h) < \alpha(h)^2$ and $U < U_1^*$;

(c3) $W_{s_i, m}$ is a CBP contract if $i \leq I$ and $W_{s_i, m}$ is an IS contract if $i > I$ if $\text{Var}(h) < \alpha(h)^2$ and $U_{I+1}^* \leq U < U_1^*$.

Proposition 7 establishes three different cases. Case (c1) identifies environments where stable outcomes only involve IS contracts. This situation takes place in volatile markets or when managers’ opportunity cost $(U)$ is very high. Case (c2) considers environments with low variance and low (outside) cost for hiring managers, where all the contracts include a CBP.

Case (c3) characterizes the circumstances under which both types of contracts coexist. The adoption decision in this framework is similar to the case of an isolated firm: shareholders owning good projects prefer to adopt a CBP, saving on incentive costs; their managers receive no informational rents. On the other hand, shareholders with low-income projects prefer to offer incentive contracts.

Interestingly enough, when the environment is volatile $(\text{Var}(h) > \frac{1}{3} \alpha(h)^2)$ but still $\text{Var}(h) < \alpha(h)^2$ the choice of the governance relationship adds an interesting effect on managers’ utility. Although managers are ex-ante homogeneous, their utilities are different due to the two effects: (i) shareholders’ heterogeneity among those offering IS contracts, and (ii) the choice of the type of contract. In this scenario, a shareholder $s_i$ hires a manager through an IS contract by paying him $\hat{U_i}$ (see Proposition 2). Therefore, the better the shareholder (i.e., the higher the value of the project), the larger the actual utility that her manager obtains (since $\hat{U_i}$ is increasing in $R_i$). However, managers hired by the best shareholders (the ones adopting a CBP) are worse off relative to managers
employed by shareholders owning less profitable projects (through IS contracts). Corollary 1 summarizes this finding.

**Corollary 1** Under the conditions set in part (c3) of Proposition 7, when \( \text{Var}(h) > \frac{1}{3} \alpha(h)^2 \), managers’ utility is not monotonic with respect to the value of the project in the unique stable outcome for the market \( \{S, M, R, c\} \): it is increasing as long as the manager signs an IS contract and it sets down to the outside utility \( \underline{U} \) as soon as he signs a CBP contract.

Figure 3 illustrates the utility obtained in the stable outcome when some managers are hired through IS contracts (grey dots) and the rest is hired under CBP contracts (black dots). Note that managers hired under a CBP would be strictly better off under an incentive contract (white dots represents this last effect).

![Figure 3](image)

**Figure 3:** Managers’ utility depending on the value of the project.

### 4.3 Heterogeneous shareholders and heterogeneous managers

We now analyze stable outcomes in markets where both shareholders and managers are heterogeneous. We first state a proposition that provides interesting information about the level of utility obtained by different managers. We have seen that homogeneous managers can end up with different utility levels. However, a general property allows
to rank the level of managers’ utility as a function of their ability: when shareholders compete for managers, better managers always obtain larger utility.

**Proposition 8** The more efficient the (matched) manager, the larger his expected utility in a stable outcome for the market \( \{S, M, R, c\} \).

The main question about the shape of matchings in stable outcomes is whether shareholders with good projects end up hiring efficient managers. If this is a characteristic of the matchings, we say they are positively assortative.

**Definition 7** A matching \( \mu \) for the market \( \{S, M, R, c\} \) is **positively assortative** if shareholders with high-revenue projects are matched with efficient managers, i.e., \( R_i > R_{i'} \) implies that \( c_j \leq c_{j'} \), where \( \mu(s_i) = m_j \) and \( \mu(s_{i'}) = m_{j'} \).

Proposition 9 shows that stable outcomes do indeed always involve positively assortative matchings if all the participants in the market find it optimal to use the same type of contract.

**Proposition 9** If \( (\mu, W) \) is a stable outcome for the market \( \{S, M, R, c\} \), then \( \mu \) is positively assortative if the contracts in \( W \) are all IS contracts, or they are all CBP contracts.

If, for instance, all participants were hiring managers through a CBP contract, then a negative assortative contract could not be stable. The rationale is based on the fact that it is optimal (in terms of total surplus) that the best managers run the best projects, hence any negative assortative matching will be blocked by at least one alternative shareholder-manager partnership creating more value for them.

However, the matching is not necessarily positively assortative when both types of contracts coexist in a stable outcome. In that case, how does a non-positively assortative matching in a stable outcome look like? Proposition 10 provides useful information in that respect.

**Proposition 10** If \( (\mu, W) \) is a stable outcome for the market \( \{S, M, R, c\} \) and if \( \mu(s_i) = m_j \) and \( \mu(s_{i'}) = m_{j'} \) with \( R_i > R_{i'} \) and \( c_j > c_{j'} \), then \( s_i \) and \( m_j \) sign a CBP contract while \( s_{i'} \) and \( m_{j'} \) sign an IS contract.
Therefore, in a non-positively assortative matching in a stable outcome, shareholders with profitable projects hire low-ability managers through CBPs while high-ability managers sign incentive contracts in less profitable firms. This is due to the following trade-off. On the one hand, maximizing total surplus requires a positively assortative matching. Hence, stable outcomes tend to be positively assortative. On the other hand, the better the shareholder, the more likely that she prefers proposing a CBP contract. Therefore, it can be the case that, say, shareholder $s_2$’s best contract to attract manager $m_1$ is an IS contract giving him a rent not lower than $\hat{U}_{21}$ while a shareholder with a more profitable project, $s_1$, would prefer to hire this manager through a CBP. If $s_1$ is forced to pay $m_1$ the rent that $s_2$ is ready to offer, then she would rather attract a less efficient manager through a CBP, as long as the difference in efficiency between the two managers and in the value of the two projects is not too large.

To gain intuition about why a negatively assortative matching can be stable, consider a situation similar to the one displayed in Figure 3, except that now $m_1$ is slightly more efficient than the rest of the managers, $c_1 < c$. The white dots in Figure 4 represent the maximum utility $U_{s_i}$ shareholder $s_i$ for $i \geq I$, is ready to pay to match with $m_1$ (we know that she will do it through an IS contract). If shareholder $s_1$ wants to hire $m_1$, she has to offer him at least $U_{s_1}$. Given the preference of $s_1$ to offer a CBP, she will prefer keeping a “normal” manager at the prize $U$ instead of attracting the slightly better manager at the prize $U_{s_1}$.
5 Introducing Codes: Welfare Considerations

Does the introduction of CBPs yield a welfare improvement? To answer this question, we compare an initial situation where only IS contracts exist (i.e., CBPs are not allowed or they do not exist) with a final situation where both CBP and IS contracts are permitted. As we have stressed, CBPs are voluntary mechanisms; shareholders only adopt them if they earn higher profits, given the managers’ reservation utilities. Therefore, a welfare improvement would be expected as a reasonable outcome. However, this shall not always be the case due to three effects. First, the CBP allows a shareholder to avoid paying informational rents in those cases where an IS contract would ensure the manager a level of utility higher than his reservation utility. Therefore, even if welfare (shareholder’s expected profits plus manager’s expected utility) decreases, the shareholder may be willing to adopt the CBP. Second, the introduction of CBP contracts may have an effect on the managers’ outside utility level. Finally, the introduction of CBPs may have a deep effect on the market structure; a negative assortative matching may arise.

We claim that the introduction of CBPs is likely to enhance welfare in environments with low variance while it has negative consequences for welfare in environments with intermediate variance. We do not need to discuss those environments with high variance.
since the firms that operate in them will never adopt CBPs, hence its introduction will be without consequence in these industries. We briefly make a case for our claim.

First, we show that if the introduction of CBPs does not affect the managers’ level of utility and the composition of the matching, voluntary CBPs enhance welfare in markets with low volatility but they have a negative impact in markets with intermediate volatility. Indeed, the welfare obtained by each profitable partnership depending on the governance mechanism is:

$$W_{ij}^{CBP} = \frac{\alpha(h)^2 R_i^2}{2 \ c_j}$$

and

$$W_{ij}^{IS} = \begin{cases} \frac{3\vartheta(h) R_i^2}{8 \ c_j} & \text{if } U_j < \hat{U}_{ij} \\ \frac{2\vartheta(h)U_j}{c_j} & \text{if } U_j \geq \hat{U}_{ij} \end{cases}$$

We can easily check that, when $U_j < \hat{U}_{ij}$, $W_{ij}^{CBP} \geq W_{ij}^{IS}$ if and only if the variance is low ($\text{Var}(h) \leq \alpha(h)^2$). Also when $U_j \geq \hat{U}_{ij}$, if the optimal contract includes a CBP, i.e., if $U_j < U_{ij}^*$, then this decision decreases welfare if the environment has intermediate variance.\(^{14}\) Therefore, the claim holds, for example, in markets with many managers of similar ability since the introduction of CBPs will modify neither the expected utility of the managers nor the nature of the matching.

Second, we discuss situations where the introduction of voluntary CBPs may affect the managers’ utility. We argue that the previous claim often holds, although some new effects make CBPs less appealing from a welfare point of view. To discuss a situation where the introduction of CBPs does not change the matching (it remains positively assortative) but it modifies managers’ reservation utility, let us focus on a market with one “good” manager and many identical “standard” managers. The effect of the CBPs in the firms involving standard managers is the same as before: good for welfare if low variance and bad if intermediate variance. This is also the effect in the firm where the good manager is hired if it adopts a CBP, as the welfare is in this case independent on the manager’s utility. However, this is not necessarily true if this firm uses an IS contract. The introduction of CBP’s, in this case, relaxes competition for managers between shareholders. Given that the other firms can now adopt CBPs, they may make higher profits with their standard managers. Therefore, hiring the good manager through an IS contract might be less appealing. The (equilibrium) reservation utility of the good manager may decrease if CBPs can be adopted, which would imply a lower welfare in the firm where he is working.

\(^{14}\)With intermediate variance, it is always the case that $U_{ij}^* < \hat{U}_{ij}$. 

A third effect is due to the fact that the matching may not longer be possitively assortative if IS and CBP contracts coexist. This effect is also typically detrimental for welfare, as a possitively assortative matching is optimal from a total surplus point of view.

Therefore, we can conclude that, in our model, the introduction of voluntary CBPs decreases welfare in environments with intermediate volatility. On the other hand, it certainly increases welfare in environments with low volatility as long as it does not induce too many changes in the structure of the market, either through drastic decreases in managers’ level of utility or through changes in the equilibrium allocation of managers to firms.

6 Conclusion and Extensions

This paper has explored the way competition affects shareholders’ willingness to adopt Codes of Best Practice. We have modeled a Code as a monitoring mechanism that allows shareholders to control ex-ante managers’ decisions. Adopting the Code, though, impedes a flexible managers’ reaction to changing market conditions.

Due to ex-post inflexibility, CBPs tend to be adopted in environments where predicting market conditions is not a complex task for shareholders. More mature industries, such as utilities, banking, food and drink sector, should be, according to our predictions, examples of industries where CBPs are more likely to be adopted. On the contrary, when the environment faces high volatility, the best a shareholder can opt for is to leave manager’s hands free (i.e., to offer him an incentive contract). High-tech sectors such as dot-com industries, or pharmaceutical companies should tend to use incentive contracts, letting the manager take the major decisions.

Our findings suggest that the characteristics of the set of shareholders and the set of managers in the market have a deep effect on the decision of adopting a CBP. When shareholders have similar projects, i.e., firms’ technologies are similar, the CBPs do not seem to be the right mechanisms to attract the best managers. Indeed, partnerships will agree on this governance structure only in environments with low level of competition for managers. In addition, when both types of governance structures coexist, the lower the manager’s ability, the more likely that a Code is adopted. Instead, when managers are of similar ability, the shareholders with best projects prefer to adopt the Code since
this governance pattern allows a shareholder to reduce her manager’s rents. In fact, our analysis suggests that the best shareholders might be willing to renounce to hire the best managers, who would be offered incentive contracts, and hire instead lower ability managers through CBPs. Hence, although the matching between shareholders and managers is always positively assortative when only one type of governance structure exists in the market, the property may fail to hold due to the coexistence of both governance structures.

Since the characteristics of the market are linked to the type of industry a firm operates in and to the type of project, it seems natural to ask how the conclusions of our analysis would be affected if shareholders’ heterogeneity was not due to the value of their project but to the distribution function of the market specific characteristics (that is, we would have a distribution function $F_i(h)$ for each shareholder $i$). Similar to the conclusions obtained when heterogeneity is due to the value of the project, also here the matching is “positively assortative” when all the firms end up under the same type of governance structure. However, the meaning of “positively assortative” depends now on the type of governance. If all contracts include a Code, then the best managers are hired by those shareholders whose market-specific component has a higher mean. However, if they are all incentive contracts, the “best” shareholders are those in markets where the combination of mean and variance (expressed in the variable $\theta_i(h)$) is higher; these are the shareholders who end up forming firms with the higher-ability managers. Also, we have some information about how a non-positively assortative matching in a stable outcome looks like. For example, if all shareholders own projects in markets whose shocks have the same average then, in a non-positively assortative matching, shareholders in markets with higher volatility hire low-ability managers through incentive contracts while high-ability managers sign contracts including a Code in firms producing in markets with lower variance.

Finally, we have not considered the possibility that firms, once created, could compete against each other in the product market. In our model, there was competition among shareholders to catch the best managers and among managers to work for the shareholders that offer the best contracts, but there was no competition among firms. A firm’s profits were independent of the composition of the other firms. Extending our model to explicitly allow firms’ market competition seems computationally demanding. However, the analysis
developed so far provides enough elements to be able to anticipate the effects of firms’ competition on the use of Codes of Best Practices.

For our purposes, firms’ market competition should have two main implications. First, it makes the best managers even more appealing than before as shareholders will be ready to offer good salaries to these managers not only because of their value for the firm but also to avoid that they are hired by the market competitors. According to our results, such an increase in managers’ utility should favour the use of incentive contracts. Second, market competition typically allows improving incentive contracts by making use of yardstick contracts, where a manager is paid according not only to his absolute performance, but also as a function of his relative performance with respect to others. Both effects go in the direction of making incentive contracts more appealing. Therefore, we should expect to observe less use of Codes in those markets characterized by tough competition.

7 Appendix

Proof. of Proposition 1. Assume \((\mu, W)\) is stable, but the contract \(W_{s_i,m_j} \in W\) signed by \((s_i, m_j)\), where \(\mu(s_i) = m_j\), is not constrained Pareto optimal. (a) First, suppose there exists a feasible contract \(W'\) for the partnership \((s_i, m_j)\) such that \(\pi_{s_i}(m_j, W') > \pi_{s_i}(\mu(s_i), W_{s_i,\mu(s_i)})\) and \(V_{m_j}(s_i, W') > V(\mu(m_j), W_{m_j,\mu(m_j)})\). In that case, \((s_i, m_j)\) will block \((\mu, W)\) with \(W'\). This contradicts the initial fact that \((\mu, W)\) is stable.

(b) Second, suppose there exists a feasible contract \(W'\) for the partnership \((s_i, m_j)\) such that \(\pi_{s_i}(m_j, W') > \pi_{s_i}(\mu(s_i), W_{s_i,\mu(s_i)})\) and \(V_{m_j}(s_i, W') = V(\mu(m_j), W_{m_j,\mu(m_j)})\). Consider the contract \(W''\) that includes both salaries higher that \(W'\) by \(\epsilon > 0\). Manager’s effort will be the same under \(W''\) and \(W'\) (if these contract include a CBP this will happen by contract; if they are incentive contracts, the ICC does not change). If \(\epsilon\) is small enough, \(W''\) satisfies \(\pi_{s_i}(m_j, W'') > \pi_{s_i}(\mu(s_i), W_{s_i,\mu(s_i)})\) and \(V_{m_j}(s_i, W'') > V(\mu(m_j), W_{m_j,\mu(m_j)})\), which we already know contradicts the stability of \((\mu, W)\).

The analysis of the third possibility requires a more detailed understanding of the (optimal) contracts between shareholder and manager. We will develop such an analysis in the rest of Section 3 using (b), i.e., there can not exist a contract that leaves the manager indifferent by improves shareholder’s profits. We can check afterwards that, among those contracts, it is not possible to improve manager’s expected utility without
Proof. of Proposition 2. The contract \( W^{IS}_{s_i,m_j} (U_j) \) is the solution to the following programme:

\[
\max_{\{w_R,w_0,e\}} \int \{ h e(w_R, w_0; h) [R_i - w_R] - [1 - h e(w_R, w_0; h)] w_0 \} dF(h)
\]

s.t. \((ICC),(LL_R),(LL_0)\), and

\[
\int \{ h e(w_R, w_0; h) w_R + [1 - h e(w_R, w_0; h)] w_0 - c_j \frac{e(w_R, w_0; h)^2}{2} \} dF(h) \geq U_j.
\]

We can rewrite the programme by plugging \((ICC)\) into the objective function and the last constraint. After some calculations we obtain:

\[
\max_{\{w_R,w_0\}} \left\{ -w_0 + \frac{\theta(h)}{c_j} (w_R - w_0)(R_i - (w_R - w_0)) \right\}
\]

s.t. \(w_0 + \frac{\theta(h)}{2c_j} (w_R - w_0)^2 \geq U_j\) \hspace{1cm} (1)

\(w_0 \geq 0\). \hspace{1cm} (2)

where we have omitted \((LL_R)\) since it is implied by \((ICC)\) and (2).

Let \(\lambda\) and \(\rho\) be the Lagrange multipliers corresponding to (1) and (2), respectively. The Kuhn-Tucker (first-order) conditions of the above maximization problem are (1), (2), \(\lambda \geq 0\), \(\rho \geq 0\), and:

\[
\frac{\theta(h)}{c_j} (R_i - 2(w_R - w_0)) + \lambda \frac{\theta(h)}{c_j} (w_R - w_0) = 0 \hspace{1cm} (3)
\]

\[
-1 - \frac{\theta(h)}{c_j} (R_i - 2(w_R - w_0)) + \lambda - \lambda \frac{\theta(h)}{c_j} (w_R - w_0) + \rho = 0 \hspace{1cm} (4)
\]

\[
\lambda \left[ w_0 + \frac{\theta(h)}{2c_j} (w_R - w_0)^2 - U_j \right] = 0 \hspace{1cm} (5)
\]

\[
\rho w_0 = 0. \hspace{1cm} (6)
\]

First, simplifying (3) we get:

\[
\lambda = 2 - \frac{R_i}{(w_R - w_0)} \hspace{1cm} (7)
\]
and plugging (7) into (4) we obtain:

$$\rho = -1 + \frac{R_i}{(w_R - w_0)}.$$  

(8)

We study the different regions where Kuhn-Tucker conditions may be satisfied:

Case 1: $\lambda > 0$, $\rho > 0$ (Both (1) and (2) are binding). Payment in case of failure is $w_0 = 0$ following (1) while in case of success is $w_R = \sqrt{\frac{2R_i c_j}{\theta(h)}}$ from (2). Finally, from (7) and (8) this is possible only if $U_j \in \left( \frac{R_i^2}{c_j} \frac{\theta(h)}{8}, \frac{R_i^2}{c_j} \frac{\theta(h)}{2} \right)$.

Case 2: $\lambda = \rho = 0$. (3) implies $R_i = 2(w_R - w_0)$ and plugging this into (4) we get $-1 = 0$, which is not possible.

Case 3: $\lambda > 0$, $\rho = 0$ ((1) is binding). From (8) we get $w_R - w_0 = R_i$. This implies that $\lambda = 1$. Then, (1) implies $w_0 = U_j - \frac{R_i^2}{c_j} \frac{\theta(h)}{2}$. Since $w_0 \geq 0$, this case is only possible if $U_j \geq \frac{R_i^2}{c_j} \frac{\theta(h)}{2}$.

Case 4: $\lambda = 0$, $\rho > 0$ ((2) is binding). From (2) we obtain $w_0 = 0$, and using (7), we get $w_R = \frac{R_i}{2}$. This implies, by (8), $\rho = 1$. In this case (1) holds only if $U_j \leq \frac{R_i^2}{c_j} \frac{\theta(h)}{8}$.

If $U_j > \frac{R_i^2}{c_j} \frac{\theta(h)}{2}$, the optimum must lie in Case 3 and $\pi_{s_i} = -w_0 < 0$ which is not feasible. If $U_j < \frac{R_i^2}{c_j} \frac{\theta(h)}{8}$, the optimum lies in Case 4, where $U_j = \frac{R_i^2}{c_j} \frac{\theta(h)}{8}$ and shareholder’s profits are the same as if $U_j = \frac{R_i^2}{c_j} \frac{\theta(h)}{8}$. These facts prove part (a) in Proposition 2.

Also, it is easily checked that the solution at the borders of cases 3 and 4 coincide with the solution at the borders of Case 1 (the solution is continuous). Therefore, the optimal contract has the shape found in Case 1, which proves part (b). Finally, parts (c) and (d) follow from the contracts in Case 1.

**Proof. of Proposition 3.** First, since there is no need to give the manager incentives, a fixed wage is optimal, we denote it by $w (= w_R = w_0)$. The contract $W_{s_i,m_j}(U_{j})$ is the solution to the following problem:

$$\max_{\{c,w\}} \int \bar{h} eR_i dF(h) - w = \alpha(h) eR_i - w$$

s.t. $w - c_j \frac{e^2}{2} \geq U_j$

(9)

$$w \geq 0.$$  

(10)

From (9), $w \geq c_j \frac{e^2}{2} + U_j$ which implies that (10) is not binding. Also, since $w$ affects negatively shareholder $s_i$’s expected profits, (9) is binding. This proves part (c) of Proposition 29.
3. Therefore, shareholder’s problem is:

\[ e^{CBP} \in \arg \max_e \{ \alpha(h) e R_i - c_j \frac{e^2}{2} \}; \]

hence, \( e^{CBP} = \frac{R_i}{c_j} \alpha(h) \). This shows part (a). Part (b) is obtained by plugging \( e^{CBP} \) into (9) binding. Finally, to prove part (d), we substitute \( e^{CBP} \) and the optimal wage into shareholder \( s_i \)'s expected profits. ■

**Proof of Proposition 4.** We compare the profit functions of propositions 2 and 3. First, we compare both profit functions at the extreme values \( U_j = 0 \) and \( \tilde{U}_{ij} \):

\[
\begin{align*}
\pi_{s_i}(W^{CBP}_{s_i,m_j}(0)) &\leq \pi_{s_i}(W^{IS}_{s_i,m_j}(0)) \iff \frac{R_i^2 \alpha(h)^2}{c_j} \leq \frac{R_i^2 \theta(h)}{c_j} 4 \iff \\
\alpha(h)^2 &\leq \frac{1}{2} \theta(h) \iff Var(h) \geq \alpha(h)^2.
\end{align*}
\]

\[
\pi_{s_i}(W^{CBP}_{s_i,m_j}(\tilde{U}_{ij})) = \frac{R_i^2 \alpha(h)^2}{c_j} - \frac{R_i^2 \theta(h)}{c_j} \leq 0 = \pi_{s_i}(W^{IS}_{s_i,m_j}(\tilde{U}_{ij}))
\]

where the inequality holds since \( Var(h) = \theta(h) - \alpha(h)^2 > 0 \). Moreover, \( \frac{\partial(\pi_{s_i}(W^{CBP}_{s_i,m_j}(U_j)))}{\partial U_j} = -1, \frac{\partial(\pi_{s_i}(W^{IS}_{s_i,m_j}(U_j)))}{\partial U_j} = 0 \) for \( U_j \leq \tilde{U}_{ij} \), and \( \pi_{s_i}(W^{IS}_{s_i,m_j}(U_j)) \) is a decreasing and concave function of \( U_j \) for \( U_j \in \left[ \tilde{U}_{ij}, \tilde{U}_{ij} \right] \) with \( \frac{\partial(\pi_{s_i}(W^{IS}_{s_i,m_j}(U_j)))}{\partial U_j} = 0 \) for \( U_j = \tilde{U}_{ij} \) and \( \frac{\partial(\pi_{s_i}(W^{IS}_{s_i,m_j}(U_j)))}{\partial U_j} = -1 \) for \( U_j = \tilde{U}_{ij} \). Therefore, the functions \( \pi_{s_i}(W^{CBP}_{s_i,m_j}(U_j)) \) and \( \pi_{s_i}(W^{IS}_{s_i,m_j}(U_j)) \) cross at most once in \( \left[ 0, \tilde{U}_{ij} \right] \).

The previous properties imply first, that \( \pi_{s_i}(W^{CBP}_{s_i,m_j}(U_j)) \leq \pi_{s_i}(W^{IS}_{s_i,m_j}(U_j)) \) for all \( U_j \) if \( Var(h) \geq \alpha(h)^2 \). Second, note that \( \pi_{s_i}(W^{IS}_{s_i,m_j}(U_j)) = \frac{R_i^2 \theta(h)}{c_j} \) for all \( U_j \leq \tilde{U}_{ij} \) and \( \pi_{s_i}(W^{CBP}_{s_i,m_j}(U_j)) \) is strictly decreasing w.r.t. \( U_j \). We evaluate profits at \( \tilde{U}_{ij} \) and we obtain

\[
\begin{align*}
\pi_{s_i}(W^{CBP}_{s_i,m_j}(\tilde{U}_{ij})) &\leq \pi_{s_i}(W^{IS}_{s_i,m_j}(\tilde{U}_{ij})) \iff \frac{R_i^2 \alpha(h)^2}{c_j} - \frac{R_i^2 \theta(h)}{c_j} \leq \frac{R_i^2 \theta(h)}{c_j} 4 \iff \\
\alpha(h)^2 &\leq \frac{3}{4} \theta(h) \iff Var(h) \geq \frac{1}{3} \alpha(h)^2.
\end{align*}
\]

Therefore, if \( Var(h) \in \left( \frac{1}{3} \alpha(h)^2, \alpha(h)^2 \right) \) the Code is adopted if and only if \( U_j < U^*_j \), where \( U^*_j \in \left[ 0, \tilde{U}_{ij} \right] \) is implicitly defined by \( \pi_{s_i}(W^{CBP}_{s_i,m_j}(U^*_j)) = \pi_{s_i}(W^{IS}_{s_i,m_j}(U^*_j)) \), i.e., \( U^*_j = \frac{R_i^2}{2c_j} \left[ \alpha(h)^2 - \frac{\theta(h)}{2} \right] \).

Finally, if \( Var(h) \in \left( 0, \frac{1}{3} \alpha(h)^2 \right) \), we have \( \pi_{s_i}(W^{CBP}_{s_i,m_j}(\tilde{U}_{ij})) > \pi_{s_i}(W^{IS}_{s_i,m_j}(\tilde{U}_{ij})) \) and \( \pi_{s_i}(W^{CBP}_{s_i,m_j}(\tilde{U}_{ij})) < \pi_{s_i}(W^{IS}_{s_i,m_j}(\tilde{U}_{ij})) \). By the properties of the derivatives of the profit functions, there exists a unique \( U^*_j \in (\tilde{U}_{ij}, \tilde{U}_{ij}) \) such that the Code is adopted if \( U_j < U^*_j \).
Level $U_{ij}^*$ is the smaller of the two values for which $\pi_{s_i}(W_{s_i,m_j}^{CBP}(U_{ij}^*)) = \pi_{s_i}(W_{s_i,m_j}^{IS}(U_{ij}^*))$, i.e., $R_i\sqrt{\frac{2\theta(h)U_{ij}}{c_j}} - 2U_{ij}^* = \frac{R_i^2 \alpha(h)}{c_j} - U_{ij}$. It corresponds to the expression stated in part (a) of the Proposition.

**Proof. of Proposition 5.** We are going to adapt to our environment the proof of existence developed by Crawford and Knoer (1981) for assignment games. We first consider a discrete economy, where the possible levels of reservation utility of the managers are $U, U+1, U+2$, and so on. The numbers can be as small as wished, we denote $U+r$ instead of $U+\epsilon r$ for notational simplicity.

Given that the participation constraint in a contract $W_{s_i,m_j}(U_{ij})$ might not be binding when the optimal contract is an IS contract, we denote as $W_{s_i,m_j}(U_{ij})^+$ the optimal contract when $s_i$ is imposed only to select contracts that provide $m_j$ a final utility of $U+r$, where $r$ must be some non-negative natural number.

We apply the following algorithm that defines the possible offers by shareholders to managers and the way they should act at any time $t$:

R1. $U_j(0) = U$ for all $j \in N$.

R2. Each shareholder $s_i$ makes an offer $W_{s_i,m_k}^+(U_k(0))$ to the manager $m_k$ with whom she obtains the largest non-negative profits, given the level of reservation utility $U_j(0)$ that she must guaranty to any manager $m_j$. That is, $k \in \arg \max_{j \in N} \{\pi_{s_i}(W_{s_i,m_j}^+(U_j(0)))\}$.

R3. Each manager who receives one or more offers, rejects all but his favorite, which he tentatively accepts. Ties are broken at any time in any manner.

R4. Offers not rejected in previous periods remain in force. If manager $m_j$ rejected an offer from some shareholder in period $t-1$, then $U_j(t) = U_j(t-1) + 1$; if not, $U_j(t) = U_j(t-1)$.

Rejected shareholders continue to make offers to their favorite managers, taking into account the current permitted reservation utility levels, as long as they make non-negative profits.

R5. The process stops when no rejections are issued in some period. Managers then accept the offers that remain in force from the shareholders they have not rejected.

Claim 1. After a finite number of periods, no rejections are issued, every manager gets at most one offer, and the process stops.

This claim follows the fact that the increments in the minimum utility are discrete and that a shareholder’s profits are negative if the reservation utility she needs to offer to the manager is high enough.
Claim 2. The process converges to a discrete stable allocation in the discrete market previously defined.

By construction, the algorithm always provides individually-rational outcomes. Hence, we prove that the final allocation of the process is indeed discrete stable if we show that it can not be blocked by a shareholder-manager pair. By Claim 1, the process converges to an outcome, denote it by \((\mu, \mathcal{W})\). Suppose that \((\mu, \mathcal{W})\) is not discrete stable. Then, there exists a couple \((s_i, m_j)\) such that

\[
\pi_{s_i}(W_{s_i, m_j}^+ (U_j + 1)) > \pi_i,
\]

where \(U_j\) and \(\pi_i\) are, respectively, the utility and profits currently obtained by \(s_i\) and \(m_j\) under \((\mu, \mathcal{W})\) (remember that \(U_j = U + r\), for some natural number \(r\)). We note that, denoting by \(T\) the period where the process has stopped, \(U_j \geq \overline{U}_j(T)\); in fact, \(U_j = \overline{U}_j(T)\) if the final contract signed by \(m_j\) includes a CBP, but \(U_j\) can be strictly larger than \(\overline{U}_j(T)\) if \(m_j\) signs an IS contract. However, we know that shareholder \(s_i\) has preferred \(\mu(s_i)\) to \(m_j\) when the minimum utility level to offer to \(m_j\) was, at most, \(\overline{U}_j(T) + 1\) (the utility level can have been smaller at the time \(s_i\) made her decision, as she might have been provisionally matched with \(\mu(s_i)\) for some periods before \(T\)). That is, \(\pi_{s_i}(W_{s_i, m_j}^+ (\overline{U}_j(T) + 1)) \leq \pi_i\). Hence, \((s_i, m_j)\) can not block the outcome.

Claim 3. The market \(\{\mathcal{S}, \mathcal{M}, \mathcal{R}, \mathcal{C}\}\) has a stable outcome.

Suppose that \(\{\mathcal{S}, \mathcal{M}, \mathcal{R}, \mathcal{C}\}\) does not have a stable outcome. Take any individually rational outcome \((\mu, \mathcal{W})\) where the contracts are constrained Pareto optimal. We denote any such an outcome as \((\mu, \mathcal{U})\), where \(\mathcal{U} = (U_1, ..., U_N) \in \mathbb{R}^N\) is the vector of managers’ utilities under \((\mu, \mathcal{U})\) (hence, \(\mathcal{W}\) is the optimal vector of contracts for the shareholders given the managers they are assigned to according to \(\mu\) and the managers’ utility levels \(\mathcal{U}\)). We denote by \(\Psi\) the set of all matchings \(\mu\) for which there exists a vector \(\mathcal{U}\) such that \((\mu, \mathcal{U})\) is individually rational. Let:

\[
\beta([s_i, m_j]; (\mu, \mathcal{U})) \equiv \pi_{s_i}(W_{s_i, m_j}(U_j)) - \pi_{s_i}(W_{s_i, \mu(s_i)}(U_k)),
\]

where \(m_k = \mu(s_i)\); that is, \(\beta([s_i, m_j]; (\mu, \mathcal{U}))\) is the (possibly negative) extra benefits that \(s_i\) can obtain by deviating from \((\mu, \mathcal{U})\) with \(m_j\). Also, define

\[
B[(\mu, \mathcal{U})] \equiv \max_{(s_i, m_j)} \beta([s_i, m_j]; (\mu, \mathcal{U})].
\]
Given that \((\mu, \mathcal{U})\) does not have a stable outcome, \(B[(\mu, \mathcal{U})] > 0\) if \((\mu, \mathcal{U})\) is individually rational. Moreover, \(b[(s_i, m_j); (\mu, \mathcal{U})]\) is a continuous function of \(\mathcal{U}\) (profits are continuous in the manager’s utility), hence \(B[(\mu, \mathcal{U})]\) is also a continuous function of \(\mathcal{U}\). For any \(\mu \in \Psi\), let

\[
G(\mu) \equiv \min_{(U_1, \ldots, U_N)} B[(\mu, \mathcal{U})]
\]

\[\text{s.t. } U_j \geq U \text{ for all } j \]

\[
\pi_{\mu(m_j)} (W_{\mu(m_j), m_j}(U_j)) \geq 0 \text{ for all } j,
\]

and

\[
H \equiv \min_{\mu \in \Psi} G(\mu).
\]

The function \(G(\mu)\) is well defined for any \(\mu \in \Psi\): \(B[(\mu, \mathcal{U})]\) is a continuous function and the feasible set is non-empty and compact, hence, \(B[(\mu, \mathcal{U})]\) reaches its minimum at some feasible vector of utilities. Moreover, \(G(\mu) > 0\) for all \(\mu \in \Psi\) since \(B[(\mu, \mathcal{U})] > 0\) for every \((\mu, \mathcal{U})\) in the feasible region. Finally, \(H > 0\) since \(\Psi\) is a finite set.

Therefore, we have proven that there exists \(H > 0\) such that, for any individually rational outcome \((\mu, \mathcal{W})\) we can find a pair \((s_i, m_j)\) whose benefits by deviating are larger than \(H\). Note that we can always split the extra profits \(B\) obtained by the shareholder between her and her manager by increasing both manager’s salaries by \(B/2\), which does not alter his incentives. Hence, if we choose the unit of measurement smaller than \(H/2\), this would imply that any individually rational allocation can be improved upon by a least one partnership \((s_i, m_j)\) in the corresponding discrete market as well, which would contradict Claim 2.

**Proof of Proposition 6.** We prove part (a) is necessary. Suppose, without loss of generality, that shareholder \(s_1\) obtains lower profits than \(s_2\). Then, \(s_1\) could hire the manager who is currently with \(s_2\), offer him a slightly better utility level than before (through, say, the same type of contract than \(s_2\) was offering) and make strictly higher profits. This is not possible in a stable outcome. Part (b) easily follows from the maximum and minimum profits that the shareholder hiring the worst manager (or not hiring at all) can make. Part (c) follows after Proposition 1. Finally, it is immediate that if the contracts satisfy (a) - (c), then the outcome is stable.

To prove part (d1), we note that if \(\text{Var}(h) \geq \alpha(h)^2\), the CBP is never adopted according to Proposition 4. If \(\text{Var}(h) < \alpha(h)^2\) and \(\pi \leq \bar{\pi}_n\), then \(\pi \leq \bar{\pi}_j\) for all managers,
since \( \pi_j \) is decreasing in \( j \). This implies that \( U_j \geq U_j^* \) since the constrained Pareto optimal contract between the shareholder and the manager \( m_j \) that provides the shareholder a profit level smaller or equal than \( \pi_j \) shall give \( m_j \) higher utility level than \( U_j^* \) (which he would obtain in the case the shareholder would get \( \pi_j \)). Therefore (Proposition 4), An incentive contract is optimal for all \( j \). A very similar argument allows to prove part (d2).

When \( \text{Var}(h) < \alpha(h)^2 \) and \( J \) is such that \( \pi_J \geq \pi > \pi_{J+1} \) then \( \pi_j \geq \pi \) if and only if \( j \leq J \). As we have argued above, \( \pi_j \geq \pi \) implies that the constrained Pareto optimal contract between a shareholder and \( m_j \) is an IS contract. In fact, \( W_{s;m_j} \) is a IS contract if and only if \( j > J \), as stated in part (e) of the Proposition.

**Proof of Proposition 7.** Given \( N > n \) and that managers are homogeneous, any contract in any stable outcome should be the best contract for the shareholder when she only needs to offer \( U \) to the manager. It is then immediate that (a) and (b) characterize the (unique) stable outcome.

Parts (c1), (c2), and (c3) easily follow given the characteristics of the optimal contract given in Proposition 4 and the fact that \( U_1^* > \ldots > U_{I-1}^* > U_I^* > \ldots > U_n^* \).

**Proof of Corollary 1.** Under the conditions set in part (c) of Proposition 1, a manager matched with a “bad” shareholder is hired through an IS contract, which implies that he achieves \( \hat{U}_i \), whereas a manager matched with a “good” shareholder achieves \( U < \hat{U}_i \) through a CBP contract. Finally \( \frac{\partial \hat{U}_i}{\partial R_i} = \frac{R_i}{c} \left[ \sum \theta(h) - \sum \theta(h)-\alpha(h)^2 \right] > 0 \).

**Proof of Proposition 8.** Let \( m_j \) and \( m_{j'} \), with \( c_j < c_{j'} \), be two managers that are matched under the stable outcome \((\mu, W)\) and let \( U_j \) and \( U_{j'} \) be the level of utility they obtain. Suppose, by contradiction, that \( U_j \leq U_{j'} \). By inspection of a shareholder’s profits in Propositions 2 and 3, it is easily checked that they are increasing in manager’s efficiency, for a given level of manager’s utility. Therefore, if \( s_{j'} = \mu(m_{j'}) \) offers \( W_{s_{j'},m_j}(U_{j'}) \) to manager \( m_j \), she will obtain higher profits than in the outcome \((\mu, W)\) while \( m_j \) achieves the utility level \( U_{j'} \geq U_j \). It is always possible to modify that contract to make sure that both \( s_{j'} \) and \( m_j \) obtain higher profits than under \((\mu, W)\), that is, they can block the outcome, which contradicts the fact that it is stable.

**Proof of Propositions 9 and 10.** We do the proof by contradiction. Take two matched shareholders \( s_i \) and \( s_{j'} \), with \( m_j = \mu(s_i) \) and \( m_{j'} = \mu(s_{j'}) \), such that \( R_i > R_{j'} \) while \( c_j > c_{j'} \). Denote by \( U_j \) and \( U_{j'} \), with \( U_j < U_{j'} \) according to Lemma 8, the level of utility obtained by managers \( m_j \) and \( m_{j'} \) in the stable outcome \((\mu, W)\). The contracts
signed by shareholders \( s_i \) and \( s_{i'} \) are, respectively, \( W_{s_i, m_j}(U_j) \) and \( W_{s_{i'}, m_j'}(U_{j'}) \).

We are going to prove that unless \( W_{s_i, m_j}(U_j) = W_{s_{i'}, m_j}(U_j) \) and \( W_{s_{i'}, m_j'}(U_{j'}) = W_{s_{i'}, m_{j'}}(U_{j'}), \) the following inequality holds:

\[
\pi_{s_i}(W_{s_i, m_{j'}}(U_{j'})) + \pi_{s_{i'}}(W_{s_{i'}, m_{j}}(U_{j})) > \pi_{s_i}(W_{s_i, m_{j}}(U_j)) + \pi_{s_{i'}}(W_{s_{i'}, m_{j'}}(U_{j'})). \tag{11}
\]

Therefore, either \( \pi_{s_i}(W_{s_i, m_{j}}(U_{j})) > \pi_{s_i}(W_{s_i, m_{j}}(U_j)) \) or \( \pi_{s_{i'}}(W_{s_{i'}, m_{j'}}(U_{j'})) > \pi_{s_{i'}}(W_{s_{i'}, m_{j'}}(U_{j'})). \) However, this cannot happen in a stable outcome. Indeed, suppose for instance that the first inequality was true. Shareholder \( s_i \) could offer to manager \( m_{j'} \) a contract that would guarantee this manager an expected utility slightly larger than \( U_{j'} \) while keeping for herself expected profits larger than \( \pi_{s_i}(W_{s_i, m_{j}}(U_{j})) \). That is, the partnership \((s_i, m_{j'})\) could block the outcome \((\mu, \mathcal{W})\).

We now prove inequality \((11)\).

\((a)\) Consider first that both are IS contracts, i.e., \( W_{s_i, m_j}(U_j) = W_{s_{i}', m_j}(U_j) \) and \( W_{s_{i'}, m_{j'}}(U_{j'}) = W_{s_{i'}, m_{j'}}(U_{j'}). \) From Proposition (2), the optimal payment has a different shape depending on the level of utility. We know that \( U_j \geq \hat{U}_{ij} \) and \( U_{j'} \geq \hat{U}_{i'j'} \). This also implies that \( U_j \geq \hat{U}_{i'j'} \). We consider now two cases:

\((a1) \ U_j \geq \hat{U}_{i'j'} \). In this case, equation \((11)\) is equivalent to:

\[
R_i \sqrt{\frac{2\theta(h)U_j}{c_{j'}}} - 2U_j + R_{i'} \sqrt{\frac{2\theta(h)U_{j'}}{c_j}} - 2U_j > R_i \sqrt{2\theta(h)U_j}{c_j} - 2U_j + R_{i'} \sqrt{\frac{2\theta(h)U_{j'}}{c_{j'}}} - 2U_{j'},
\]

which holds given that:

\[
[R_i - R_{i'}] \left[ \sqrt{\frac{U_{j'}}{c_{j'}}} - \sqrt{\frac{U_j}{c_j}} \right] > 0.
\]

\((a2) \ U_{j'} \in [\hat{U}_{i'j'}, \hat{U}_{ij'}]. \) In this case, \( \pi_{s_i}(W_{s_{i'}, m_{j'}}(\hat{U}_{ij'})) = \frac{R_i^2 \theta(h)}{4c_{j'}} \) and \((11)\) is implied by:

\[
\frac{R_i^2 \theta(h)}{4c_{j'}} + R_{i'} \sqrt{\frac{2\theta(h)U_j}{c_j}} - 2U_j > R_i \sqrt{2\theta(h)U_j}{c_j} - 2U_j + R_{i'} \sqrt{\frac{2\theta(h)U_{j'}}{c_{j'}}} - 2U_{j'},
\]

which is equivalent to:

\[
f(R_i, R_{i'}) = \frac{R_i^2 \theta(h)}{4c_{j'}} - \left( R_{i'} \sqrt{\frac{2\theta(h)U_{j'}}{c_{j'}}} - 2U_{j'} \right) - (R_i - R_{i'}) \sqrt{\frac{2\theta(h)U_{j}}{c_j}} > 0.
\]

We see that \( \frac{\partial f(R_i, R_{i'})}{\partial R_i} > 0 \) if and only if \( U_j < \hat{U}_{ij'} \frac{c_j}{c_{j'}} \), which always holds in this region. Then, \( f(R_i, R_{i'}) > f(R_i = R_{i'}, R_{i'}) \) for any \( R_i > R_{i'} \), hence \((11)\) holds if \( g(U_{j'}) \equiv f(R_i = U_{j'}) \).
that this is the case. Indeed, when 

\[ U_{i'} > \hat{U}_{i'j'}, \] 

having its minimum at 

\[ \hat{U}_{i'j'} = 0 \] 

Therefore, (11) holds.

(b) Suppose that all stable contracts include a CBP. Equation (11) is implied by:

\[
\frac{R_i^2 \alpha(h)}{2c_j} - U_j + \frac{R_j^2 \alpha(h)}{2c_j} - U_j > \frac{R_i^2 \alpha(h)}{2c_j} - U_j + \frac{R_j^2 \alpha(h)}{2c_j} - U_j,
\]

i.e.,

\[
[R_i^2 - R_j^2] \left[ \frac{1}{c_j} - \frac{1}{c_j} \right] > 0,
\]

which always holds.

(c) We now consider that the existing contracts are \( W_{s_i, m_j}^{IS}(U_j) \) and \( W_{s_i, m_j}^{CBP}(U_j) \). We show that they can not be part of a stable outcome if and only if we prove that

\[
\pi_{s_i'}(W_{s_i, m_j}^{IS}(U_j)) + \pi_{s_j}(W_{s_i, m_j}^{CBP}(U_j')) > \pi_{s_i}(W_{s_i, m_j}^{IS}(U_j)) + \pi_{s_j}(W_{s_i, m_j}^{CBP}(U_j'))
\]

since either \((s_i', m_j)\) or \((s_i, m_j')\) can do better than under the original contracts. Given that \( U_j \) is the level of utility obtained by \( m_j \), and \((s_i, m_j)\) sign an IS contract, it is necessarily the case that \( U_j > \hat{U}_{ij'i} \). Also, \( R_i > R_i' \) implies that \( U_j > \hat{U}_{i'j'} \). Therefore, we can rewrite (12) as:

\[
R_i' \sqrt{\frac{2\theta(h)U_j}{c_j}} - 2U_j + \frac{R_i^2 \alpha(h)}{2c_j} - U_j > R_i \sqrt{\frac{2\theta(h)U_j}{c_j}} - 2U_j + \frac{R_i^2 \alpha(h)}{2c_j} - U_j,
\]

i.e.,

\[
[R_i^2 - R_i'^2] \frac{\alpha(h)^2}{2c_j} > [R_i - R_i'] \frac{2\theta(h)U_j}{c_j},
\]

or,

\[
U_j < \left[ \frac{R_i + R_i'}{2} \right]^2 \frac{[\alpha(h)^2]^2 c_j}{2\theta(h) \frac{2c_j}{c_j}}.
\]

(13)

For the contract \( W_{s_i', m_j'}^{CBP}(U_j') \) to be optimal, it is necessarily the case (according to Proposition 4) that \( U_j' \leq U_{i'j'}^* \). Since \( U_j < U_j' \) (see Lemma 8), \( U_j < U_{i'j'}^* \). Therefore, equation (13) certainly holds if \( U_{i'j'}^* \) is lower or equal than the right-hand side of (13). We claim that this is the case. Indeed, when \( Var(h) \leq \frac{1}{3} \alpha(h)^2 \),

\[
U_{i'j'}^* = \frac{R_i'^2}{c_j} \left[ \frac{\sqrt{\theta(h)} - \sqrt{\theta(h) - \alpha(h)}^2}{2} \right]^2 \leq \left[ \frac{R_i + R_i'}{2} \right] \frac{[\alpha(h)^2]^2 c_j}{2\theta(h) \frac{2c_j}{c_j}} \iff \left[ \frac{\theta(h) - \sqrt{\theta(h) \sqrt{\theta(h) - \alpha(h)}^2}}{\alpha(h)^2} \right]^2 \leq \left[ \frac{R_i + R_i'}{2} \right] \frac{[\alpha(h)^2]^2 c_j}{2c_j}.
\]
While the right-hand side of the last equation is always larger than 1, given that \( R_i > R_{i'} \) and \( c_j > c_{j'} \), the left-hand side is smaller or equal than 1 if and only if:

\[
\frac{\theta(h)}{\alpha(h)^2} - \sqrt{\frac{\theta(h)}{\alpha(h)^2} - 1} \leq 1 \iff \frac{\theta(h)}{\alpha(h)^2} - 1 \leq \sqrt{\frac{\theta(h)}{\alpha(h)^2} - 1} \iff \sqrt{\frac{\theta(h)}{\alpha(h)^2} - 1} \leq \sqrt{\frac{\theta(h)}{\alpha(h)^2}}
\]

which always holds. The claim when \( Var(h) \in \left( \frac{1}{3} \alpha(h)^2, \alpha(h)^2 \right) \) is equivalent to:

\[
\frac{R_i^2}{2c_{j'}} \left[ \alpha(h)^2 - \frac{\theta(h)}{2} \right] \leq \left[ \frac{R_i + R_{i'}}{2} \right] \frac{[\alpha(h)^2]^2}{2\theta(h)} \frac{c_j}{c_{j'}} \iff \frac{\theta(h)}{\alpha(h)^2} - \frac{1}{2} \left[ \frac{\theta(h)}{\alpha(h)^2} \right]^2 \leq \left[ \frac{R_i + R_{i'}}{2R_{i'}} \right] \frac{c_j}{c_{j'}}.
\]

The left-hand side of the equation is always lower than 1. Hence, equation (12) holds and the initial contracts can not be part of a stable outcome. ■

References


