Redistributive Taxation, Public Expenditure, and Size of Government

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Redistributive Taxation, Public Expenditure, and Size of Government

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Abstract

We introduce a model of redistributive income taxation and public expenditure. This joint treatment permits analyzing the interdependencies between the two policies: one cannot be chosen independently of the other. Empirical evidence reveals that partisan confrontation essentially falls on expenditure policies rather than on income taxation. We examine the case in which the expenditure policy (or the size of government) is chosen by majority voting and income taxation is consistently adjusted. This adjustment consists of designing the income tax schedule that, given the expenditure policy, achieves consensus among the population. The model determines the consensus income tax schedule, the composition of public expenditure and the size of government. The main results are that inequality is negatively related to the size of government and to the pro-rich bias in public expenditure, and positively or negatively related to the marginal income tax, depending on substitutability between government supplied and market goods. These implications are validated using OECD data.

JEL-Classification: H23, H50, 050

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1 Introduction

In this paper we study the interdependence between the income redistribution and the composition of public expenditure. Much of Public Finance has treated taxation and expenditure separately. The literature on income redistribution through taxes and cash transfers treats net tax revenue as a given target and [implicitly] disregards how this revenue is spent. Likewise, the public expenditure literature deals with the allocation of the public budget independently of taxation. This paper takes one step towards an integrated analysis of public taxation and expenditure. This joint treatment permits analyzing the interdependencies between the two policies: one cannot be chosen independently of the other. Empirical evidence strongly suggests that, among the two policies, partisan confrontation essentially focusses on public spending and size of government. We examine the case in which the expenditure policy is chosen by majority voting. Given this choice, the income tax schedule is designed so that it achieves consensus among the population, thus avoiding an additional front of political confrontation. The model determines the consensus income tax schedule, the composition of public expenditure and the size of government. The main results are that inequality is negatively related to the size of government and to the pro-rich bias in public expenditure, and positively or negatively related to the marginal income tax, depending on substitutability between government supplied and market goods. These implications are validated using OECD data.

In a recent issue of The Telegraph (18/10/2007), in an article entitled “Time for a new consensus on tax”, former advisor to the UK prime minister Gordon Brown, Chris Wales, describes the politics of taxation with the following words: “The UK tax system has evolved over many generations (...) Yet in all this time, real opportunities for informed debate about taxation choices have been limited. In principle, the people consent to a given level of taxation at the time of the General Election. In practice, there is little constructive debate even then about the level of taxation and even less about the way in which it is levied. (...) The result is that [in the Parliament] there is almost no examination of the design of the tax system as a whole. (...) The search for fairness may ultimately be futile. But the search for a broad consensus is certainly not."

The above view of taxes playing a minor role in political competition and being a matter of consensus instead goes beyond mere anecdotal observation. A similar impression emerges from the data on policy preferences by political parties of all democratic countries since 1945 collected by Budge et

\[\text{2With the exception of public goods, to which we will soon turn.}\]
al (2001, 2006). This work consists of the mapping of the content of electoral party manifestos of each country, party and in each election, into a list of over one hundred key topics. The frequency with which the different topics are raised allows one to quantify the importance attached to each issue and hence provides a rich snapshot of the position of each party in each election campaign. It is striking that “income taxation" or ‘income redistribution" is not included as a distinct entry. This reveals that, in spite of its obvious political relevance in a few countries in recent elections, this topic has not been sufficiently prominent to deserve a specific category. This is not to mean that economic issues have been generally disregarded. Over the entire sample, economic issues occupy fifty percent of the “average" party manifesto. In contrast, there are various categories of public expenditure that have an entry of their own. This is the case of expenditures on environment, culture, social services, social security and health, and education. In all, these lines of public spending represent one third of the economic topics addressed. Clearly, parties appear to systematically choose the expenditure side as their “punch line”.

Consistently with this evidence, Wagshal (2001) finds that the partisan composition of governments has no explanatory power on tax reforms. This finding coincides with the analysis of Cusack and Baramendi (2006) who highlight the puzzle that the social-democratic countries governed by left-wing parties tend to tax labour rather than capital, while the more liberal, market-economy countries governed by conservative parties tax capital relatively more.

Political science has provided two kinds of explanations for this lack of strong partisan competition on taxation. One makes the point that political parties do not see much room for alternative taxation policies, either because all share a similar “paradigm" or because of the role of ‘veto players". The second explanation argues that most of redistribution is performed through public spending and not by means of taxes and that hence it is there where we should look for party competition.

On the first argument, Swank and Steinmo (2002), based on the study of 14 OECD countries, find that for the “empirical record of tax policy change, (...) although statutory tax rates, brackets, and investment incentives have been reduced almost everywhere, we find a remarkable stability in the levels and distribution of tax burdens." They impute the registered tax reforms to

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3The entry that bears the closest relationship to income taxation is entry 503 Social Justice, described as “Concept of equality; need for fair treatment of all people; special protection of underprivileged; need for fair distribution of resources; removal of class barriers; end of discrimination such as racial or sexual discrimination." Clearly, its relationship with income taxation is rather remote.
a "change in paradigm" shared by all governments and not to party competition within countries. Adam and Bevan (2004) also underscore that, “during recent decades, a powerful consensus has developed [which] has included not only the structure of taxes, but also the level of tax rates. This conventional wisdom is probably pretty soundly based, and so to refuse to subscribe to it would be imprudent as well as incurring disapproval from IFIs." (p.60).

The second explanation for the modest political competition over taxation stresses that most redistribution takes place via the benefits provided by the different types of public expenditure rather than directly through income taxes. Consequently, one should expect partisan opposition to be far more lively on the front of the public spending. In an influential paper, Przeworski (1999) makes the point that political struggles over spending levels may partly be fought as struggles over tax structure (p. 43). This is so because “large parts of welfare spending do not come in the form of transfers but rather as services and goods provided by the state, such as the provision of health care, child care, education, and so on” (Cusack and Fuchs, 2002, p. 17). This widespread view is also shared by Ganghof (2005) who insists on that “governments that wish to redistribute through budgetary policy do so mostly on the spending and not the taxing side of the budget” (p. 2) and by Howard (1997) and Ervik (2000) who discusses the relevance of the “hidden” welfare provision. Indeed, data suggests that political confrontation is substantially stronger on public spending. Bruninger (2005), using data on 19 OECD countries from 1971 to 1999, finds support for the general partisan hypothesis to the effect that actual preferences of parties over the structure and size of public spending do matter for policy decisions. Tsebelis and Chang (2004) also find that “significant changes in government composition between one year to the next lead to significant changes in the composition of the budget” (p.473).

In sum, the evidence shows very modest party competition on taxation

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4For the UK 2004/2005 the average yearly non-contributory social cash and near-cash benefits were 40 percent of the benefits in kind received from the public provision of education and health (Jones 2006). Hansen and Weisbrod (1969) and Pechman (1970) developed a controversy in the period 1969-1971 at the Journal of Human Resources over the distributional impact of higher education subsidies. More recently, Le Grand (1982) and Evandrou et al. (1993) have also studied the distributional impact of the public spending. Health and education is strongly redistributive. There are other lines in the public budget, such as foreign service, culture or law-and-order, that accrue benefits increasing with income (or income taxes). In Adam Smith’s words: "The rich, in particular, are necessarily interested to support that order of things, which can alone secure them in the possession of their own advantages. (…) Civil government, so far as it is instituted for the security of property, is, in reality, instituted for the defence of the rich against the poor, or those who have some property against those who have none at all." (Book V. Chap. 1, Part II)
and significant opposition on expenditure because most of effective redistribution is done via public spending rather than directly through redistributive taxes. Our paper explains this behavior as follows. Effectiveness in partisan competition induces the choice of only few dimensions for political confrontation. Because of the interlinkages between taxation and expenditure policies, political parties focus on public spending as their punch line in the political debate. Majority voting thus establishes the expenditure policy/size of government. Given this choice, the political actors—political parties, civil servants, parliamentary committees, lobbies, expert advisors—look for a reshuffling of the distribution of the tax burden that minimizes controversy, by balancing the benefits and the burden experienced by the citizens at the different levels of the income ladder. We show that such consensual reshuffling of income taxation is always achievable, so that partisan competition can focus on the public spending side only.

We model individual preferences to depend upon personal disposable income and on the set of goods and services—both public and/or private in nature—that individuals obtain from the government and that are financed with the net tax revenue. Therefore, income taxation reduces private consumption, but finances the supply of the goods provided by the government. Consequently, the individual valuation of alternative tax and expenditure policies depends upon the balance between private consumption and the public supply of goods. As also stressed in the public goods literature, an appropriate mix of tax and public supply of goods may be found to be individually acceptable.

The largest part of government expenditure consists of the supply of goods and services such as general administration, education, health, law-and-order, infrastructures, culture, or defence, which are in turn financed by the net revenue of taxation (net of the social transfers).

The structure of public spending depends upon two types of issues. The first type concerns the specification of the set of commodities supplied by the government and those that are produced by the private sector. This includes regulations on whether the government reserves monopoly of supply (also

\footnote{The redistributive task of the government has been the object of extensive studies by Moene and Wallerstein (2001a), (2001b) and (2003) and Alesina and Glaeser (2004). Their concern is the relationship between the pre-tax and disposable income inequality: the progressivity of the income tax and the size of the cash and near-cash social transfers. They do not address the public provision of goods and services.}

\footnote{See Warr (1983), Bernheim (1986) and Bergstrom (1986) for the voluntary contributions to public goods. Taylor-Gooby (2007) makes the more general point that attitudes towards income taxation are more sympathetic with people being more aware of the benefits they obtain from public spending.}
that of close substitutes) or permits various degrees of concurrence through
the private market and determines both the composition public expenditure
and substitutability between public and private bundles. For instance, secu-
rity or mail services were the monopoly of the government until a few decades
ago. The stricter the monopoly of the state, and the larger the number of
commodities included, the smaller is the substitutability between the public
and private bundles of commodities. Therefore, cross-country variation in the
substitutability between the two bundles is largely imputable to government
policies rather than to variations in preferences.

The second aspect of expenditure policy refers to the quantities supplied
of each commodity. Different compositions of government spending can be
interpreted as different ways of distributing its benefits over the income dis-
tribution. A pro-rich expenditure policy may have a positive effect on those
who bear the heavier part of the tax burden: the rich can enjoy a significant
fraction of their taxes return to them in the form of goods and services they
value most and this may make them more amenable towards income taxation.

For the sake of tractability we have aggregated all these goods and services
into one single commodity whose valuation varies across individuals accord-
ingly with their income. This variation in the valuation wants to capture how
pro-rich or pro-poor the expenditure policy is. We take two extreme cases as
benchmarks. In the pro-rich case, the structure of government expenditure is
such that it simply gives back to each tax payer its tax contribution. In the
second —egalitarian— case the government expenditure is designed to give
all individuals the same value. We allow for intermediate policies modeled as
convex linear combinations of these two extreme policies. The weight given
to the first expenditure policy indicates the degree of pro-rich bias of the
intermediate policies.

We focus on the tax schedules that for a given public expenditure policy
have the property of achieving consensus. That is, given the distributive
bias of the expenditure policy, we are interested in the tax rate with a degree
of progressivity that is found acceptable by everyone. We show that there
is a unique tax schedule that satisfies this acceptability criterion and we
examine its properties. The income tax schedule is shown to depend on the
composition of the public expenditure and on the substitutability between the
goods and services supplied by the government and the consumption goods
privately obtained through the market. As for the choice of the distributive
bias in the expenditure policy, consensus is no longer possible. We show
that the redistributive bias chosen by majority voting is increasing in income
inequality [the relative gap between mean and median income].

One could argue that in some relevant countries (certainly in the US)
changes in income taxes occupy a significant role in the political debate. As
we shall show in Section 7, our analysis can be rewritten so that the political debate and majoritarian voting is on the size of government. Then, because of the inter-dependence between taxation and expenditure, given the size of government, there is a unique distribution of the benefits and of the burden such that redistribution via income taxation and spending achieves consensus among the actors (political parties, parliamentary committees, lobbies, and civil servants).

What is the net value added of this shift of partisan competition from taxes to public expenditure? Besides bringing the model closer to what appears to be the actual political process, this shift yields three interesting novel insights: (i) it brings into stage the interdependence between taxation and distributive expenditure; (ii) it unveils the impact of public decisions modifying the substitutability between publicly supplied goods and market goods, and (iii) it explains why inequality has a non-monotonic relationship with income tax progressivity.

Our analysis implies that the main determinants of redistribution are: (a) the pro-rich bias in the composition of the public spending (the size of government), and (b) the substitutability between the commodities privately obtained through the market and the commodities publicly provided. The degree of substitutability between the two types of commodities plays a crucial role in determining the attitudes towards taxation. We then examine how the changes in substitutability translate into the progressiveness of income taxation.

For constant-elasticity preferences we obtain explicit, testable results. We focus on three implications: (1) the bias in public spending is negatively related to inequality (the relative gap mean/median incomes); (2) the size of government is negatively related to inequality; and (3) a negative relationship between the marginal tax rate and inequality in countries with high substitutability and a positive relationship in countries with low substitutability. We empirically test these propositions with OECD country data. Due to the poor availability of estimates for the substitutability between public and private spending at the individual level and for the pro-rich bias in public expenditure, we test our propositions with proxies of these entities. The empirical support of the above propositions obtained using the proxies is highly significant.

To test the result of the relationship between the marginal tax rate and inequality, we use threshold regressions and kernel regressions (the latter for...
robustness, so as to not impose any functional form on the data). Threshold regressions allow the researcher to identify exact values of the elasticity of substitution at which the relationship under investigation (the marginal tax rate and inequality) switches from positive to negative. Threshold regressions confirm that for lower elasticities of substitution between public and private goods, there is a positive relationship between marginal tax rates and inequality, and for higher elasticities, there is a negative relationship.

The structure of the paper is as follows. In Section 2 we develop the model. Section 3 defines the notion of consensus taxation, proves the existence and uniqueness of a consensual income tax and shows that this tax is welfare efficient. Section 4 discusses its properties. Section 5 is devoted to the relationship between income tax progressiveness, the pro tax-payer bias of public spending, and the degree of substitutability between the private and public bundles of commodities. Section 6 proves that the expenditure policy chosen by majority voting will be the one preferred by the median voter and shows that the pro-rich bias is negatively related to inequality. In Section 7 we show that for CES preferences the above results can equivalently be expressed in terms of “size of government” instead of “expenditure bias”. Section 8 discusses the strategy of the empirical tests. Section 8.3 tests the relationship between inequality and the bias in the expenditure policy and Section 8.5 with the marginal income tax rate. Finally, section 9 concludes.

2 The Model

2.1 Individuals

Let us assume a continuum of individuals. Individual income is denoted by $y$; it is assumed to be exogenous, and distributed over the population with cdf $F$ on support $[a, \infty)$. We shall denote the average per capita income by $\mu$.

The set of commodities is divided into two bundles, private (denoted $x$) and public (denoted $g$). Individual demand for the private commodities is satisfied through the markets: in view of market prices individuals choose how best to allocate their disposable income. The individual consumption of the publicly supplied commodities is fixed by the government through its expenditure policy.

We assume all commodity prices are constant. This allows us to consider the aggregate expenditure on the two bundles of commodities.

We assume that individual preferences are defined on private and public goods only and are represented by $u(x, g)$. Labour is thus assumed to be
rigidly supplied.

On individual preferences we make the following standard assumptions:

**Assumption 1** $u_x > 0$, $u_g > 0$, $u_{xx} < 0$, $u_{gg} < 0$ and $u_{xg} > 0$. Further, we assume that for $g > 0$, $\lim_{x \to 0} u_x = \infty$ and $\lim_{x \to \infty} u_x = 0$, and for $x > 0$, $\lim_{g \to 0} u_g = \infty$ and $\lim_{g \to \infty} u_g = 0$.

The elasticity of substitution between the two commodity bundles plays a key role in our analysis. A higher consumption of the commodities supplied by the government can be achieved only by accepting higher taxation. This is equivalent to substituting private for publicly provided consumption goods. How much individuals will be willing to give up on private consumption to increase the level of the public bundle depends upon their substitutability. Therefore, the individual attitudes towards taxation will be critically influenced by the elasticity of substitution between the privately and publicly supplied commodities.

### 2.2 Income Taxation

The government raises taxes/transfers in order to redistribute income across individuals. The net public revenue after performing the redistribution of incomes is spent for the provision of the public commodity bundle. Individuals spend their disposable income to purchase private commodities. To save on notation we denote disposable income by $x$.

$t(y)$ denotes the tax (if positive) or subsidy (if negative) allocated to each individual with income $y$. Therefore,

$$x(y) = y - t(y).$$  \hfill (1)

Note that disposable income will exceed pre-tax factor income when $t(y) < 0$.

$t$ denotes the per capita aggregate net surplus/deficit after income redistribution, i.e.

$$\bar{t} = \int t(y) dF(y).$$  \hfill (2)

### 2.3 Public Expenditure

In this section, we analyse the effect of redistribution on the size of government: the share of the public supply of commodities over aggregate factor income. Net tax revenue is endogenously determined together with the income tax schedule.
Analysing the structure of expenditure is important because it establishes the distribution of its benefits. Transferring resources from primary education to the support of arts renders the benefits of public spending to be biased towards the rich. Therefore, the structure of the public expenditure implicitly defines a distribution of the benefits over the taxpayers. We represent the benefit from public expenditure to an individual with income $y$ as equivalent to receiving $g(y)$ worth of public expenditure.

The government's budget is balanced, and hence

$$
\tilde{g} \equiv \int g(y) dF(y) = \tilde{t}. \tag{3}
$$

The “size of government”, $\zeta$ is the weight of the goods and services provided by the government relative to GDP, that is,

$$
\zeta = \frac{\tilde{g}}{\mu}. \tag{4}
$$

To make the "public spending bias towards the rich" (hereafter, the "pro-rich bias") concept operational, let us define two benchmarks: the two extreme cases of the distribution of benefits from public expenditure. In the first case, public spending is fully biased towards the rich and returns the exact amount of taxes paid as benefits: $g(y) = t(y)$. The second benchmark case is the egalitarian expenditure policy: $g(y) = \tilde{g} = \tilde{t}$. The family of “intermediate” expenditure policies are defined as convex linear combinations of these two extreme benchmarks. That is, an expenditure policy with a pro-rich bias $\gamma$ is defined as

$$
g(y, \tilde{g}, \gamma) = \gamma t(y) + (1 - \gamma)\tilde{g} = \gamma t(y) + (1 - \gamma)\tilde{t}, \text{ with } \gamma \in (0, 1). \tag{5}
$$

A budget balanced fiscal policy is thus fully characterized by $\gamma$ and the tax function $t(\cdot)$. As we have argued earlier, the expenditure policy, $\gamma$, is chosen by majority voting. In the next section we shall show that for each such choice of $\gamma$ there is a unique tax function that is acceptable to each tax payer and is welfare efficient.

### 3 Consensus Income Taxation

In the Introduction we have reported on the evidence that partisan competition appears to be strong on expenditure policy and very mild on income taxation. In Section 7 we show that one can as well rewrite the model with voting over the size of government $\zeta$. 

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8In Section 7 we show that one can as well rewrite the model with voting over the size of government $\zeta$. 

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taxation. We have argued that the joint treatment of taxation and expenditure allows one to highlight the fact that an increase in taxes will also bring with it an increase in expenditure. Once the bias in public expenditure has been chosen, all individuals can compare the loss (increase) in private consumption due to higher taxes (subsidies) with the increase (loss) of the benefits from increased public spending. For some income levels the net outcome will be positive and will be favorable to the increase in taxation and for some it will have the opposite effect. We assume that, given the voted $\gamma$, the public administration is interested in reshuffling the income tax—the distribution of the burden—such that the opposition to the tax reform is minimized so that no additional front of party confrontation is initiated.\footnote{See Spector (2000) for an argument as to why rational debate takes place on one dimension only.} Is there a tax function such that the benefits of a marginal change just equal the losses at each income level? As it turns out, for each expenditure policy there is a unique tax function that no one objects and it is welfare efficient. It is for this reason we call this income tax “consensual”.

Surprisingly, the requirement of consensus is neither too stringent nor too loose. For any given distribution of income there is always one and only one tax function that satisfies this property.

3.1 Definition

A tax function $t(\cdot)$ is acceptable to an individual with income $y$ if she does not wish to vary its progressivity. A tax function is consensual if it is unanimously acceptable.

Consider a particular $t(\cdot)$ with net tax revenue $\bar{t}$, as defined in (2). In order to operationalize the notion of “variation of the progressivity” of $t(\cdot)$ we focus on affine transformations $\tilde{t}(\cdot)$ such that

$$\tilde{t}(y) = \alpha + \beta t(y). \quad (6)$$

with

$$\int \tilde{t}(y) dF(y) = \int [\alpha + \beta t(y)] dF(y) = \int t(y) dF(y). \quad (7)$$

Because of (7) we obtain

$$\tilde{t}(y) = \bar{t} + \beta [t(y) - \bar{t}]. \quad (8)$$

The parameter $\beta$ defines the degree of progressiveness of $\tilde{t}(\cdot)$ relative to $t(\cdot)$. $\beta > 1$ implies that all the individuals contributing below average will
see their contribution diminished while the ones with incomes above will contribute more. The opposite holds for $\beta < 1$. Therefore, $\beta > 1$ increases [and $\beta < 1$ decreases] the progressiveness of $t(.)$ relative to $\tilde{t}(.)$. Note that the sign of $\beta$ has no restrictions. We can as well consider $\beta < 1$ so as to invert the direction of transfers between rich and poor.

We place very weak restrictions on the tax functions. We shall work with the set $\Theta$ of all functions from $\mathbb{R}$ to $\mathbb{R}$ that are strictly increasing. The set $\Theta$ is not conditioned to a particular aggregate net tax revenue, it contains all the strictly increasing functions.

Consider any arbitrary $t(\cdot) \in \Theta$ and any given $\gamma$. The valuation of a change in progressiveness by $\beta$ will be

$$u\left( y - [\tilde{t} + \beta(t(y) - \tilde{t})], \gamma[\tilde{t} + \beta(t(y) - \tilde{t})] + (1 - \gamma)\tilde{t} \right). \quad (9)$$

Given a tax function $t(\cdot)$ we denote by $\beta(t(\cdot), y)$ the change that would be preferred by an individual with income $y$.

**Definition 2** A tax function $t(\cdot)$ is individually acceptable to a person with income $y$ if $\beta(t(\cdot), y) = 1$.

We denote by $\mathcal{S}(y)$ the set of all tax functions $t(\cdot) \in \Theta$ that are individually acceptable to earners of income $y$.

We assume that the government chooses the tax function that is acceptable to the largest share possible of the population. We now explore the most demanding acceptability requirement: consensus.

**Definition 3** A tax function $t(\cdot)$ is consensual, $t(\cdot) \in \mathcal{S}$, if it is unanimously acceptable; that is, if $t(\cdot) \in \bigcap_y \mathcal{S}(y)$.

We will show in the next section that such a stringent requirement on income taxation does not yield an empty set.

### 3.2 Existence of Consensus Income Taxation

In this section, we show that the requirement of a consensus income tax schedule yields determinate results: for any given distribution of income there is always one and only one tax function in the set $\Theta$ that satisfies this property.

**Theorem 4** The set $\mathcal{S}$ is non-empty and contains one single element only.
Proof. Consider any arbitrary $t(\cdot)$, $\tilde{t}$ and $\gamma$. The valuation of a $\beta$ affine transformation, as in (9), is

$$u \left( y - (\tilde{t} + \beta (t(y) - \tilde{t})) , \gamma (\tilde{t} + \beta (t(y) - \tilde{t})) + (1 - \gamma)\tilde{t} \right).$$

(10)

It can be readily verified that the utility valuation is concave in $\beta$. Hence, the first order condition fully characterizes the preferred $\beta$.

Differentiating with respect to $\beta$ we obtain

$$\frac{\partial u}{\partial \beta} = (t(y) - \tilde{t}) \left[ -u_x \left( y - \tilde{t}(y), \gamma(y, \tilde{g}) \right) + \gamma u_g \left( y - \tilde{t}(y), \gamma(y, \tilde{g}) \right) \right].$$

Note that for all $t(\cdot) \in \Theta$, $(t(y) - t) \neq 0$ except for at most one value of $y$. Hence, $\beta(y, t(y))$ is implicitly characterized by the condition

$$\frac{u_x \left( y - \tilde{t} + \beta (t(y) - \tilde{t}) \right), \gamma \left( \tilde{t} + \beta (t(y) - \tilde{t}) \right) + (1 - \gamma)\tilde{t}}{u_g \left( y - \tilde{t} + \beta (t(y) - \tilde{t}) \right), \gamma \left( \tilde{t} + \beta (t(y) - \tilde{t}) \right) + (1 - \gamma)\tilde{t}} = \gamma.$$  

(11)

If $t^*(y)$ is universally acceptable, then $\beta(y, t^*(y)) = 1$ for all $y$.

We start with an arbitrary acceptable, then $\beta(y, t^*(y)) = 1$ for all $y$.

We start with an arbitrary parameter $t$ and with the implicit definition of $t(y)$ by

$$\frac{u_x \left( y - t(y), \gamma t(y) + (1 - \gamma)t \right)}{u_g \left( y - t(y), \gamma t(y) + (1 - \gamma)t \right)} = \gamma.$$  

(12)

Because of Assumption 1, the left-hand-side of (12) is strictly increasing in $t(y)$, it goes to infinity as $t(y) \rightarrow y$ and to zero as $t(y) \rightarrow -\frac{1 - \gamma}{\gamma} t$. Hence, for each $t$ and $y$ there exists a unique $t(y)$ satisfying (12). We can thus write

$$t(y) = \psi(y, t, \gamma).$$  

(13)

It can be readily verified that $\psi$ is continuous and strictly increasing in $y$ and continuous and strictly decreasing in $t$.

For an arbitrary $t$, the average tax collection $\bar{t}$ is

$$\bar{t} = \int \psi(y, t, \gamma) dF(y) = \phi(t, \gamma).$$

The socially acceptable tax-transfer policy $t^*(\cdot)$ is given by (13) evaluated at $t^*$, where $t^*$ satisfies $t^* = \phi(t^*, \gamma)$.

We are now required to show that $\phi$ has a fixed point. Since $\psi$ is continuous and strictly decreasing in $t$, so is $\phi$. From (11) we can easily obtain that for $t = 0$, $\psi(y, 0, \gamma) > 0$ for all $y$. Therefore, we have that for $t = 0$, $\phi(0, \gamma) > 0$. Since $\phi$ is continuous and strictly decreasing in $t$, there exists a unique $t^*$ such that $t^* = \phi(t^*, \gamma)$. This completes the proof. ■
A clarifying note on the restriction on the tax function: individual acceptability as defined earlier only considers changes in the steepness of the tax function that leave the aggregate tax revenue unchanged. This restriction may raise concerns of a "hidden restriction" on the tax functions truly under consideration. However, observe that the set $\mathcal{S}(y)$ is obtained after having tested the acceptability of all possible strictly increasing functions with any arbitrary aggregate tax revenue. Therefore, the set $\mathcal{S}(y)$ contains tax functions yielding very different aggregate tax revenues. The restriction that a tax function is consensual, and hence acceptable to all, selects not only the consensual steepness of the tax function, but its net revenue as well. The aggregate tax revenue—and hence the size of government $\varsigma$—is determined together with the shape of the tax function.

We shall show in Section 7 that for CES preferences one can take the size of government, $\varsigma$, as chosen by the voters. Then, the consensual tax function is uniquely chosen together with the bias in government spending. In other words, given the size of government, there is a unique distribution of the benefits of public spending and of the burden of income taxation such that all individuals find it acceptable.

### 3.3 The Shape of the Income Tax Schedule

In this section, we examine the shape of the consensus income tax schedule. We start by showing that the marginal tax rate is positive and does not exceed unity.

**Proposition 5** The marginal tax rate of the consensus tax function satisfies $0 \leq t'(.) \leq 1$.

**Proof.** Totally differentiating (11) with respect to $t(.)$ and $y$ and rearranging we obtain

$$
\frac{dt(.)}{dy} = \frac{u_{xg}u_x - u_{xx}u_g}{(u_{xg}u_x - u_{xx}u_g) + \gamma(u_{yg}u_x - u_{yy}u_x)}.
$$

Observe that the numerator and the first term in the denominator are identical and they are positive because of Assumption 1. Because of Assumption 1, again, the second term in the denominator is also positive. Therefore, the marginal tax rate is positive and less than unity. $\blacksquare$

Given that we have assumed that individual incomes are exogenous, one can question what prevents poor individuals from accepting a full income equalization. Government policy allocates the publicly supplied commodity as an increasing function of one’s contribution in taxes. If a low-income
earner demands more progressiveness, and therefore a larger transfer, she will be trading-off greater private consumption for less public consumption. Likewise, high-income earners may be willing to sacrifice private consumption in order to obtain higher levels of public consumption. At the consensus tax schedule, the marginal rate of substitution between the private and publicly supplied commodities will be equal across the population. It is worth noting that it is precisely this property that makes consensus tax functions welfare efficient.

4 Properties of the Consensus Income Taxation

4.1 Efficiency

We have uniquely characterized an income tax function based on the notion of individual acceptability, combined with a government seeking consensus. The consensus income tax has interesting efficiency properties: the consensus income tax maximizes Social Welfare among all the tax functions in Θ that yield the same net tax revenue.

We define the (Utilitarian) Social Welfare as the sum of the individual utilities, that is,

\[ W(t(.)) = \int u(y - t(y), \gamma t(y) + (1 - \gamma)\tilde{t})dF(y). \]  

(14)

We now show that \( W(t^*(.)) \geq W(t(.)) \) for all \( t(.) \in \Theta \) with net tax revenue \( \tilde{t}^* \).

**Proposition 6** Let \( t^*(.\) be a consensus income tax function with net tax revenue \( \tilde{t}^* \). Then, \( t^*(.\) maximizes the Utilitarian Social Welfare over all tax functions \( t(.) \in \Theta \) with net tax revenue \( \tilde{t}^* \).

**Proof.** We can write

\[ W(t^*(.)) - W(t(.)) = \]

\[ = \int [u(y - t^*(y), \gamma t^*(y) + (1 - \gamma)\tilde{t}^*) - u(y - t(y), \gamma t(y) + (1 - \gamma)\tilde{t})]dF(y). \]

Since \( u(.,.) \) is concave in the tax function, and using (11), we can write

\[ W(t^*(.)) - W(t(.)) \geq \]
\[
\int \left[ t^*(y) - t(y) \right] \left[ -u_x(y - t^*(y), \gamma t^*(y) + (1 - \gamma) \tilde{t}^*) + \gamma u_x(y - t^*(y), \gamma t^*(y) + (1 - \gamma) \tilde{t}^*) \right] dF(y) = 0.
\]

The intuition for this result is as follows. Any local perturbation of a tax function will simultaneously modify the private and public consumptions of the individuals in this income interval and will thus vary the marginal rate of substitution between the two. A necessary condition for a tax function be welfare maximizing is that the marginal rate of substitution between the two consumptions has to be equal across the population. But, this precisely is the condition that characterizes the unique consensus income tax function.

Let us emphasize that a consensus income tax maximizes welfare subject to a net revenue constraint. The implication is that had we fixed an arbitrary exogenous net tax revenue (different from \( \tilde{t}^* \)) we would have found no consensual tax function yielding this arbitrary revenue.

### 4.2 Nash Equilibrium Taxation

Consensus income taxation can also be interpreted as a Nash equilibrium of a voluntary tax-contribution game.

A tax function \( t(\cdot) \) with net tax revenue \( \tilde{t} \) is the tax paid by an individual with income \( y \). Let \( \delta(y) \) denote a deviation from the proposed tax payment. Each individual selects the extent of deviation that suits her best.

A tax function \( t(\cdot) \) is a Nash equilibrium if no individual player deviates from the proposed voluntary tax payment.

We can now state the following Proposition.

**Proposition 7** The consensus income tax \( t^*(\cdot) \) is a Nash equilibrium of the tax-contribution game.

**Proof.** Note first that no individual deviation can modify the net tax revenue \( \tilde{t} \). Thus, for an individual with income \( y \) the payoff of deviating by \( \delta(y) \) is

\[
u \left( y - (t(y) + \delta(y)), \gamma (t(y) + \delta(y)) + (1 - \gamma) \tilde{t} \right).
\]

The first order condition for a maximum is

\[
u_x \left( y - (t(y) + \delta(y)), \gamma (t(y) + \delta(y)) + (1 - \gamma) \tilde{t} \right) = \gamma u_y \left( y - (t(y) + \delta(y)), \gamma (t(y) + \delta(y)) + (1 - \gamma) \tilde{t} \right).
\]
In view of Theorem 1, it is straightforward that if \( t(\cdot) = t^*(\cdot) \) the optimal deviation satisfies \( \delta(y) = 0 \), and that this holds for all \( y \).

This property of consensus income taxation is the analog of the equilibrium in the case of voluntary contributions to public goods.

5 Income Taxation and Public Expenditure: the CES case

In this section, we restrict individual preferences to be of the CES type. This will permit us to examine the effects of income inequality, of expenditure bias and of the elasticity of substitution on the tax schedule and on the size of government.

The family of CES utility functions is given by:

\[
 u(x, g) = \left[ x^{\frac{\sigma-1}{\sigma}} + g^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}, 
\]

with the elasticity of substitution \( \sigma > 0 \).

The marginal utilities to the two types of consumption are

\[
 u_x = x^{-\frac{1}{\sigma}} \left[ x^{\frac{\sigma-1}{\sigma}} + g^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}, \quad \text{and} \\
 u_g = g^{-\frac{1}{\sigma}} \left[ x^{\frac{\sigma-1}{\sigma}} + g^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}.
\]

Therefore,

\[
 \frac{u_x(x, g)}{u_g(x, g)} = \frac{y - t(y)}{\gamma t(y) + (1 - \gamma) t} = \gamma.
\]

We can thus easily obtain that

\[
 t(y) = \frac{y - (1 - \gamma) y^{-\sigma} \bar{t}}{1 + \gamma^{1-\sigma}}. 
\]

Integrating over the incomes \( y \) we can obtain

\[
 \bar{t} = \frac{\mu - (1 - \gamma) \gamma^{-\sigma} \bar{t}}{1 + \gamma^{1-\sigma}}.
\]

Hence,

\[
 \bar{t} = \frac{\mu}{1 + \gamma^{-\sigma}} = \bar{g}. 
\]
Therefore, we obtain that the consensus income tax schedule is linear

\[ t(y) = \tau y - T, \quad (19) \]

where

\[ \tau \equiv \frac{1}{1 + \gamma^{1-\sigma}} \text{ and } T \equiv \frac{1 - \gamma}{1 + \gamma^{1-\sigma}} \frac{\gamma^{-\sigma} \mu}{1 + \gamma^{-\sigma}}. \quad (20) \]

From (18) we obtain the size of the public sector \( \varsigma \) to be

\[ \varsigma = \frac{g}{\mu} = \frac{1}{1 + \gamma^{-\sigma}}. \quad (21) \]

We can now state our results on income taxation and the size of government. First, we discuss the effect of \( \gamma \) and \( \sigma \) on the marginal tax rate \( t'(\cdot) \).

**Proposition 8** Let preferences be CES. Then: (i) the unique consensus income tax is linear; (ii) it is independent of the distribution of income; (iii) the (constant) marginal tax rate, \( t'(\cdot) \equiv \tau \), increases (decreases) with the bias parameter \( \gamma \) if the elasticity of substitution is high (low), \( \sigma > 1 \) (\( \sigma < 1 \)); and (iv) an increase in the elasticity of substitution reduces the marginal tax rate.

**Proof.** Statements (i) and (ii) follow immediately from (19).

Differentiating the marginal tax rate in (19) with respect to \( \gamma \) we obtain

\[ \frac{d\tau}{d\gamma} = (\sigma - 1) \frac{\gamma^{-\sigma}}{(1 + \gamma^{1-\sigma})^2}. \]

This proves statement (iii). As for statement (iv) we differentiate with respect to \( \sigma \) and after simple manipulation we obtain

\[ \frac{d\tau}{d\sigma} = \tau(1 - \tau) \ln \gamma. \]

Noting that \( \ln \gamma < 0 \), statement (iv) results.

With no assumptions on the tax-transfer function, and with CES preferences, we obtain that the unique \( t(\cdot) \) to be consensual is a linear tax function. Therefore, any departure from linearity in taxes requires significant variations in the substitutability of the two bundles of commodities as income varies.

By the same argument, inequality in the distribution of pre-tax income does not play a major role in determining the degree of income redistribution, \( \tau \), unless individual preferences display a significant variation in the degree of substitution as real income changes.
The marginal tax rate depends upon the bias of government spending. The effect of a more egalitarian expenditure policy, $\gamma \to 0$, on the marginal tax rate $\tau$ critically depends upon the degree of substitutability between the two bundles of goods. For low substitutability, $\sigma < 1$, the marginal tax rate tends to unity and for high substitutability, $\sigma > 1$, it tends to zero.

Finally, in economies with a moderate share of government an increase in the substitutability between the two bundles of goods will decrease the marginal tax rate, $\tau$.

Let us now turn to the effects of $\gamma$ and $\sigma$ on the size of government $\varsigma = \frac{a}{\mu}$. From (21) we can easily obtain the following result.

**Proposition 9** Let preferences be CES. Then, the size of government $\varsigma$: (i) increases with the bias parameter $\gamma$; and (ii) decreases with the elasticity of substitution $\sigma$.

The results of Proposition 9 are not surprising. The first result implies that the higher the pro-rich bias in the public spending, the larger will be the size of the government that the population will consider acceptable. The second result implies that increasing the substitutability between the market and the publicly supplied goods will induce a demand for a smaller size of the government.\(^{10}\)

It is worth discussing the case of the tax schedule being linear. Consider the effect of an increase by $\Delta$ of an income $y$. Due to the biased expenditure policy, a linear income tax implies that private and public consumption will also increase at the same rate. If preferences have a falling elasticity of substitution of private for public consumption, individuals with an income increased by $\Delta$ would prefer a more than proportional increase in the supply of the public good and hence would rather favor an increasing marginal tax rate. If the elasticity of substitution were to rise individuals would have a preference for declining marginal tax rates. Clearly, whether individuals unanimously support a tax function with increasing or decreasing marginal tax rates critically depends upon the change in the elasticity of substitution.

\(^{10}\)This result seems in contradiction with Karras’s (1994) argument that the larger is the public sector the more the goods and services supplied will be substitutes for the goods provided through the market. Two points are in order. First, Karras does not take into account that the substitutability between public and private goods critically depends on the political decision of allowing or not the private supply of substitutes (e.g. security, mail service, prisons,...). Second, Karras’s argument does not consider whether such an increase in the size of government would be considered acceptable. This precisely is our point: if the government allows for higher substitutability the policy that will be found consensual will consist of a smaller size of the government sector.
as the consumption levels rise. A similar argument holds for why the tax rate is shown to be independent of the distribution of income.

In our model, the supply of a subset of commodities is the monopoly of the government. This monopoly provides the government with the coercive power to make individuals accept taxation on incomes. How effective this power is critically depends upon the substitutability between this bundle of commodities and the commodities individuals can purchase in the market. Hence, our approach suggests that the rich will lobby more strongly for increasing the substitutability between public and private goods by privatizing as many as possible rather than about the shape of the income tax schedule.

6 Voting over Public Expenditure: the CES case

We have already argued that political scientists have underscored that political confrontation takes place more in the domain of public expenditure than in the setting of income taxation. In the previous section we have characterized the consensus tax function —and the corresponding net tax revenue— as a function of the expenditure policy parameter \( \gamma \). We shall now examine the choice over expenditure policies, \( \tilde{\gamma} \) by majority voting. On this respect we shall simply transpose the analysis of majority voting on income taxation by Romer (1975), Roberts (1977) and Meltzer and Richard (1981) and apply it to the individual preferences over expenditure policy \( \gamma \) and show that a majority voting equilibrium exists. In our case too, the most preferred expenditure policy is monotonic in the individual income and hence the policy that obtains a majoritarian support is the one preferred by the individual with the median income, \( m \). Furthermore, we also obtain that higher inequality —the relative gap between mean and median income— brings a more distributive public spending, i.e. lower \( \gamma \).

**Proposition 10** The majoritarian expenditure policy \( \tilde{\gamma} \) is the one most preferred by the median income voter and is implicitly determined by the unique solution to

\[
\frac{\sigma(1 - \tilde{\gamma})(1 + \tilde{\gamma}^{\sigma-1})}{(1 + \tilde{\gamma}^\sigma)^2} = \frac{\mu - m}{\mu},
\]

where \( m \) is the median income, that is, \( F(m) = 1/2 \). Moreover, \( \tilde{\gamma} \) is strictly decreasing in \( \frac{\mu - m}{\mu} \equiv M \), a measure of income inequality.

**Proof.** Differentiating with respect to \( \gamma \) the utility of an individual with income \( y \) we have that
\[
\frac{du(x(y), g(y))}{d\gamma} = u_x \frac{dx(y)}{d\gamma} + u_g \frac{dg(y)}{d\gamma}.
\]

Since the tax is consensual we can use (11) to obtain
\[
\frac{du(x(y), g(y))}{d\gamma} = u_g \left[ \gamma \frac{dx(y)}{d\gamma} + \frac{dg(y)}{d\gamma} \right].
\]

We know that
\[
\frac{dx(y)}{d\gamma} = -\frac{dt(y)}{d\gamma},
\]
and that
\[
\frac{dg(y)}{d\gamma} = t(y) - \frac{\mu}{1 + \gamma^{-\sigma}} + \gamma \frac{dt(y)}{d\gamma} + \frac{\sigma(1 - \gamma)\gamma^{-(1+\sigma)}\mu}{(1 + \gamma^{-\sigma})^2}.
\]

Therefore,
\[
\frac{du(x(y), g(y))}{d\gamma} = u_g \left[ t(y) - \frac{\mu}{1 + \gamma^{-\sigma}} + \frac{\sigma(1 - \gamma)\gamma^{-(1+\sigma)}\mu}{(1 + \gamma^{-\sigma})^2} \right].
\]

Rearranging we finally obtain that the first order condition for a maximum is
\[
\frac{du(x(y), g(y))}{d\gamma} = \frac{\mu u_g}{1 + \gamma^{1-\sigma}} \left[ \frac{\sigma(1 - \gamma)(1 + \gamma^{\sigma-1}) - \mu - y}{\mu} \right] = 0.
\] (22)

The sign of \(\frac{du(x(y), g(y))}{d\gamma}\) depends on the term in square brackets. The first fraction within the brackets is positive. Hence, for \(y \geq \mu\) the derivative is always positive and these individuals prefer \(\gamma = 1\).

For \(y < \mu\), the sign depends on the first fraction within the brackets, that we denote by \(\Gamma(\gamma)\). That is,
\[
\Gamma(\gamma) \equiv \frac{\sigma(1 - \gamma)(1 + \gamma^{\sigma-1})}{(1 + \gamma^{\sigma})^2}.
\] (23)

It is immediate that \(\Gamma(1) = 0\). When \(\gamma \to 0\) we can easily obtain that \(\lim_{\gamma \to 1} \Gamma(\gamma) \to 1\) if \(\sigma > 1\) and \(\lim_{\gamma \to 0} \Gamma(\gamma) \to \infty\) if \(\sigma < 1\). Therefore, for \(y < \mu\), it follows that
\[
\lim_{\gamma \to 1} \frac{du(x(y), g(y))}{d\gamma} < 0, \quad \text{and} \quad \lim_{\gamma \to 0} \frac{du(x(y), g(y))}{d\gamma} > 0.
\]
Hence, for all individuals with $y < \mu$ there exists a utility maximizing $\gamma$, $\gamma \in (0, 1)$.

We shall now show that for $y < \mu$ there is a unique $\gamma_y$ satisfying (22). To this effect we shall show that if (22) is satisfied for some $\gamma_y$, then $\frac{d\Gamma}{d\gamma}|_{\gamma_y} < 0$.

For notational simplicity we shall drop the superscript $y$.

Since $\frac{\mu - y}{\mu} < 1$ we have from FOC (22) that

$$\Gamma(\gamma) \equiv \frac{\sigma(1 - \gamma)(1 + \gamma^{\sigma - 1})}{(1 + \gamma^\sigma)^2} < 1. \quad (24)$$

Differentiating $\Gamma$ with respect to $\gamma$, using (23) and rearranging we have

$$\Gamma'(\gamma) = -\Gamma(\gamma)\left[\frac{1}{1 - \gamma} + \frac{2\sigma\gamma^{\sigma - 1} - (\sigma - 1)\gamma^{\sigma - 2}}{1 + \gamma^\sigma}ight].$$

Rearranging terms we have

$$\Gamma'(\gamma) = -\Gamma(\gamma)\left[\frac{1 + \gamma^{\sigma - 2}}{(1 + \gamma^{\sigma})^2} + \frac{2\gamma^{\sigma - 1} + \gamma^{2\sigma - 2} - \gamma^{\sigma - 2}}{(1 + \gamma^\sigma)(1 + \gamma^{\sigma - 1})}\right].$$

Using (23) we have

$$\Gamma'(\gamma) = -\sigma\Gamma(\gamma)\left[\frac{1 + \gamma^{\sigma - 2}}{(1 - \gamma)(1 + \gamma^{\sigma - 1})} + \frac{2\gamma^{\sigma - 1} + \gamma^{2\sigma - 2} - \gamma^{\sigma - 2}}{(1 + \gamma^\sigma)(1 + \gamma^{\sigma - 1})}\right].$$

Because of (24) we can finally write

$$\Gamma'(\gamma) < -\sigma\Gamma(\gamma)\left[\frac{1 + \gamma^{\sigma - 2}}{(1 - \gamma)(1 + \gamma^{\sigma - 1})} + \frac{2\gamma^{\sigma - 1} + \gamma^{2\sigma - 2} - \gamma^{\sigma - 2}}{(1 + \gamma^\sigma)(1 + \gamma^{\sigma - 1})}\right] =$$

$$= -\frac{\sigma\Gamma(\gamma)}{(1 - \gamma)(1 + \gamma^{\sigma - 1})(1 + \gamma\gamma^{\sigma - 1})}\left[1 + (1 - \gamma)\gamma^{\sigma - 1} + \gamma^{2\sigma - 2}\right] < 0.$$

This proves that there is a unique utility maximizing $\gamma(y)$ and that individual preferences over $\gamma$ are single peaked.

Since the unique preferred $\gamma$ for each income $y$, $\gamma(y)$, is strictly decreasing in $y$, the expenditure policy that will earn a majoritarian support, $\hat{\gamma}$, is the one preferred by the median voter [with the median income, $m$], $\hat{\gamma} = \gamma(m)$.

Concerning the relationship between $\hat{\gamma}$ and $M(= \frac{\mu - m}{\mu})$, it directly follows from the fact that $\Gamma$ is strictly decreasing in $\gamma$. \(\blacksquare\)
Note that the choice over $\gamma$ is equivalent to choosing affine transformations of the expenditure function. While consensus over tax functions was possible, there is no consensus possible when it comes to expenditure policy and the choice has to be made via majority voting. Paralleling the results in the literature on voting over linear tax functions mentioned before, we also obtain that the median voter will be pivotal for a majority and hence will impose its most preferred policy. Furthermore, as inequality increases the chosen expenditure policy becomes more redistributive —lower $\gamma$.

However, taking into account our results of the previous section, higher inequality does not necessarily bring steeper tax schedules. It depends on the substitutability between the market and publicly supplied goods. Specifically, higher inequality and higher progressivity of the public spending will be associated with lower marginal tax rates if the substitutability is high. This implies that more unequal countries in which the government has allowed for high substitutability between public and private will have lower marginal tax rates than more egalitarian countries with low substitutability between the two. To our knowledge this sheds new light on the puzzle posed by the empirical observation that income inequality appears to be a poor predictor of the level of income taxation. In the next section, we shall test the implication of the model that the sign of the relationship between inequality and the marginal tax rate depends on the substitutability parameter, $\sigma$.

7 Voting over the Size of Government: the CES case

We have mentioned before that in some countries the size of government, more than the distribution of the benefits of public expenditure, is the issue that truly is at the core of the political debate. We shall now show that for the CES case our previous analysis can be equivalently be rewritten with the size of government being the key variable over which the population votes.

Let us start by consensus taxation. Now the question is: given a size of government, $\varsigma$, is there a consistent tax function $t(\cdot)$ and a bias in spending $\gamma$ such that there is consensus over the tax function?

Notice first that, when checking for the acceptability of a tax function, the affine transformation considered had to yield the same net revenue. Then, a tax function was consensual if no one would prefer to change its steepness. To check whether such tax function exists we used the first order condition and obtained the net revenue —the size of government— that satisfies the budget constraint. What we do now is to keep the size of government fix $\varsigma$. 
—hence, \( \bar{t} \) fix—and, using the first order condition, verify whether there is a \( \gamma \) for which the government budget balances.

In (17) we derived that the first order condition requires that

\[
t(y) = \frac{y - (1 - \gamma)\gamma^{-\sigma} \bar{t}}{1 + \gamma^{1-\sigma}}.
\]

Integrating over the incomes we obtain the net revenue that depends on \( \gamma \),

\[
\bar{t}(\gamma) = \frac{\mu - (1 - \gamma)\gamma^{-\sigma} \bar{t}}{1 + \gamma^{1-\sigma}}.
\]

The point now is whether there exists \( \gamma^o \) such that \( \bar{t}(\gamma^o) = \bar{t} \), and by budget balance \( \bar{t}(\gamma^o) = \bar{g} \). It is immediate that, for all \( \varsigma = \bar{t}/\mu \leq 1/2 \), such \( \gamma^o \in [0, 1] \) exists, it is unique, and it is given by

\[
\gamma^o = \left( \frac{\varsigma}{1 - \varsigma} \right)^{\frac{1}{\sigma}}. \tag{25}
\]

The corresponding consensus marginal tax rate is

\[
\tau = \frac{1}{1 + \left( \frac{\varsigma}{1 - \varsigma} \right)^{\frac{1}{\sigma}}}. \tag{26}
\]

From these two expressions it directly follows that: (i) the bias in spending is strictly increasing with government size; and (ii) the marginal tax rate decreases or increases with government size as the elasticity of substitution is smaller or larger than 1.

Finally, observe that since \( \varsigma \) is strictly increasing in \( \gamma \), we can exactly rephrase our results on voting obtained in the previous Section: the majoritarian size of government is the one preferred by the median voter. It is also immediate that the size of government chosen by majority voting is strictly decreasing with inequality, \( M \).

8 Empirical Exercise

8.1 Testable Implications

For the case of constant-elasticity preferences, our model has well-defined, testable implications. Specifically, we shall focus on the following two fundamental implications:
1. Majority voting leads to a choice of bias in public expenditure $\gamma$ that is negatively related to the median to income relative gap, $M$. Alternatively, $\varsigma$ is negatively related to the median to income relative gap, $M$.

2. There is a positive relationship between the marginal tax rate $\tau$ and $M$ when the substitutability $\sigma$ is low, and a negative relationship when the substitutability is high.

We find all these implications to be validated by our empirical analyses. We test for the robustness of the results using two proxies for $\gamma$, $\gamma_{health}$ and $\gamma_{edu}$, and two proxies for $\sigma$, $\sigma_{health}$ and $\sigma_{edu}$. We also use the Gini index, $G$, as a proxy for the mean to median relative gap, $M$. These proxies are discussed in the next subsection.

### 8.2 Empirical Strategy and Data

We shall now test the empirical validity of the relationships described above. We do not have direct data on any of these three independent variables and hence we have had to work with reasonable proxies. Furthermore, our choice of proxies has been severely conditioned by the need of a consistent set of basic information available for a sufficiently large number of countries. We have therefore tested our empirical implications using the OECD database\(^\text{11}\) that includes fifteen countries, listed in the Appendix.

We first discuss our proxy for $\tau$. This is the slope of the affine tax function that is acceptable by all with CES preferences. In our paper the tax function merges the income tax schedule and the several money transfers. In other words, $t(\cdot)$ is the difference between factor income (plus retirement payments and minus retirement contributions) and disposable income.

In purity, we would have had to test whether the difference between the two individual incomes can be represented by an affine function. The only data base available with such individual information is the Luxembourg Income Study database. We performed this exercise but discarded the estimates for two reasons. First, the estimated parameters were unreasonably unstable from year to year, suggesting some possible deficiencies in the raw data. Second, $\gamma$ and $\sigma$ would have to be estimated from completely different sources.

We have therefore used the maximum marginal tax rate in each country reported in the OECD database as a proxy for $\tau$. This implicitly assumes that there is a stable relationship between the maximum marginal tax rate

\(^{11}\)We use the OECD Statistical database obtainable at www.oecd.org/statistics.
and the slope of the affine function that would approximate the difference between factor and disposable individual incomes.

Let us now turn to our estimates for $\gamma$. As defined in Section 2, $\gamma$ captures the pro-rich distributional bias in the public provision of goods and services, which we denote by $G$. This bias depends upon the share in the government budget of the expenditures that mostly benefit the low incomes versus those that benefit the rich taxpayers. For some countries, discussed earlier, there are estimates of the distribution of the benefits of specific lines of government expenditure (essentially, education and health).\footnote{There is a large literature which discusses and identifies the redistributiveness of public expenditures.\cite{Le Grand, 1982} for the redistributive effects of health and education in the UK - he concludes that of the two education is more redistributive). A large debate in the 1980s \cite{Hansen and Weisman, 1969, Pechman, 1970} contest the redistributive effects of higher education in particular, concluding that the redistributive effects of higher education were debatable, and that existing measurement methodologies were not successful in effectively measuring their effects. We do not perform any statistical analyses to test for the relative redistributiveness of the different types of public expenditures in the OECD countries studied; this would entail a separate econometric exercise beyond the purview of this paper.} We are however interested in the distribution of the benefits of the entire government supply of goods and services (including general administration and law-and-order, among others). Therefore, we have had to estimate our own proxies for $\gamma$.

We first compute $G$, the total government expenditure in the provision of goods and services, from OECD data sources. $G$ is obtained by subtracting the expenditures on money transfers from the total amount of government expenditure, as detailed in Appendix B. All estimates are in constant 2000 US dollars. The size of government is then computed as

$$\varsigma_i = \frac{G_i}{GDP_i}.$$ 

As for estimating a proxy for $\gamma$, we focus on two redistributive, pro-poor public expenditures: health and education, $G_{\text{health}}$ and $G_{\text{edu}}$. Bearing in mind that $\gamma$ is the pro-rich bias, we estimate two proxies:

$$\gamma_i = 1 - \frac{G_i}{G},$$

where $i = \text{health, education}$.

We also require an estimate for the elasticity of substitution between the two bundles of commodities, private and publicly provided, $\sigma$. The substitutability between public and private expenditure has been a recurrent topic in macroeconomics. Since the work of Barro (1981), there have been
numerous attempts at estimating the elasticity of substitution. Aschauer (1985) finds a significant degree of substitutability between the two variables for the United States. Karras (1994) finds that they are complementary or unrelated, using data for 30 countries. Evans and Karras (1996) provide additional evidence supporting the complementarity using data for 54 countries. More recently, Amano and Wirjanto (1998) for the US show that the two variables are unrelated or have very weak complementarity. For Japan, Hamori and Asako (1999) find a significant degree of substitutability, while for Okubo (2003) the two bundles are complementary or unrelated. Finally, Bouakez and Rebei (2006) with the same specification of preferences as ours - but with habit formation - estimate $\sigma = 0.332$.

Unfortunately, we are unable to use these [quite contradictory] estimates for the following reasons. First, in most of the macroeconomics literature substitutability is defined by the sign of the cross derivative and by the corresponding elasticity, $\sigma$. Second, the models are all inter-temporal and this aspect proves to be critical for the estimates. Ni (1995) empirically finds that when the two expenditures add linearly in the preferences, the estimates indicate substitutability, while if the two expenditures enter the utility function non-separably one obtains complementarity. Third, the estimates are perplexingly contradictory. Finally, much of the literature estimates a world elasticity using panel data, whereas we require country estimates, to obtain a ranking across countries. Kwan (2006), for example, using co-integration methods, has found that for nine East Asia countries while the two bundles are substitutes on the average, they are complements in others. In sum, we cannot base our empirical work on these estimates.

Our approach to the estimation of $\sigma$ is therefore as follows. The substitutability between the two bundles of commodities depends upon the nature of individual preferences and on the degree of monopoly that the government keeps for itself for some subset of commodities, as discussed earlier. For many OECD countries the postal system or security has been a public monopoly until fairly recently. Today, however, the rich can supplement the public supply of police force, for instance, by purchasing additional private security. Similarly, in many countries education and health have high degrees of "publicness" while in others, a good share of the demand is satisfied through the private market. The larger the share of the expenditure channelled through the market, the higher is the substitutability between the public and the private provision of these goods. For our purposes, therefore, we use a metric $\lambda$, which equals the ratio of private over the total of public and private expenditures in health and in education, as a proxy for the elasticity of substitution.

Using this ratio, we proxy the elasticity of substitution by making $\sigma =$
If all is private, and $\lambda = 1$, then the elasticity is infinity. We compute $\lambda$ for education and for health using comparative data of public and private expenses available at the OECD.\textsuperscript{13} Using the two $\lambda$s, we obtain $\sigma_{\text{edn}}$ and $\sigma_{\text{health}}$.

Indeed, these are very rough proxies for the “true” elasticities of substitution. Our empirical exercise, however, rests on the “ranking” of the countries by their degree of substitutability and not on the absolute value of the elasticity of substitution.

Since both education and health expenditures have similar pro-poor incidence, we could have used data on education only for our proxies for $\gamma$ and $\sigma$. However, there is a criticism often discussed in the literature against the use of education: i.e. that of potential reverse causality. Indeed, education is directly beneficial to the poor, but it also enhances their earning potential in future. Consequently, countries with a large share of expenditure in education will exhibit lower levels of inequality. To account for this possible endogeneity, we complement our analysis with expenditures on health. While not as strongly redistributive as primary education,\textsuperscript{14} the eventual effect of health expenditures on the earning potential of the poor —and therefore on inequality— is of a smaller order of magnitude.

Finally, we also compute the relative mean-median gap, $M = (\mu - m)/\mu$. Unfortunately, complete data is not available for all years. We therefore supplement our analysis by using $G$ as a proxy for $M$ in our empirical estimations. For this, we use the country Gini estimates\textsuperscript{15}

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\textsuperscript{13}Estimates of $\lambda$ are available at http://darp.lse.ac.uk/expenditures/. The data source for our estimates is the OECD Social Expenditure database. Data has been available only for different spreads of years - the earliest being 1980 till 2008. The spread of years has varied from country to country, with the least spread being 1997 to 2007, and the maximum spread being 1980 to 2008.

\textsuperscript{14}Accordingly with the estimates for the UK reported in the Table below, the benefits from education are more tilted towards the poor than the benefits from the National Health Service.

\textbf{UK: All Households, 2004-2005}

<table>
<thead>
<tr>
<th>Quintile</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-tax income</td>
<td>4.280</td>
<td>11.200</td>
<td>21.580</td>
<td>34.460</td>
<td>66.330</td>
</tr>
<tr>
<td>Benefits in kind</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- education</td>
<td>2.585</td>
<td>2.015</td>
<td>2.083</td>
<td>1.608</td>
<td>1.221</td>
</tr>
<tr>
<td>- health</td>
<td>3.629</td>
<td>3.577</td>
<td>3.204</td>
<td>2.754</td>
<td>2.446</td>
</tr>
<tr>
<td>Source: Jones (2006)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{15}The database used is V2.0c at http://www.wider.unu.edu/research/Database/en_GB/database/.
Table 1: OLS Regression $M$ and $G$ on $\gamma$

<table>
<thead>
<tr>
<th></th>
<th>$M$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{\text{health}}$</td>
<td>-2.74*</td>
<td>-.476*</td>
</tr>
<tr>
<td>$\gamma_{\text{edn}}$</td>
<td>-0.02</td>
<td>.034</td>
</tr>
</tbody>
</table>

Notes: *: Significant at the 1% level
†: Significant at the 5% level
‡: Significant at the 10% level

### 8.3 Relationship between $\gamma$ and $M$

In Section 6 we derive that the relationship between the distributional bias of public expenditure $\gamma$ and $M$ resulting from majority voting is negative. We now estimate the relationship between these two entities. We first estimate OLS regressions of the relationship given by

$$\gamma_{it} = \kappa_0 + \kappa_1 M_{it} + \epsilon_{it}$$

where $M_{it}$ is the value of $M$ in country $i$ in time $t$ and $\gamma_{it}$ is the distributional bias in public expenditure for country $i$ in time $t$. The results are presented in Table 1 below.

We observe that the relationship is negative and significant for $\gamma_{\text{health}}$. We also observe that the relationship is not significant when using $\gamma_{\text{edn}}$. This is, however, not very surprising. We have already discussed that the expenditure in education may have some serious endogeneity problems. We obtain similar results when we use the Gini index $G$ instead of $M$.

For robustness, we estimate kernel regressions to observe the nature of the relationship between $\gamma$ and $M$.$^{16}$ The model estimated is given by

$$\gamma_{it} = g(M_{it}) + \nu_{it},$$

where, as before, $\gamma_{it}$ is the distributional bias in public expenditure for country $i$ in time $t$, $M_{it}$ is the value of $M$ for country $i$ in time $t$, $g(\cdot)$ is a generic function and $\nu_{it}$ is an error term. The estimated kernel regression is displayed in Figure 1

$^{16}$We use the Epanechnikov kernel estimator (Silverman 1986) for the kernel regressions, which is standard in the literature.
Figure 1: Kernel regression of $\gamma_{health}$ on $M$
The OLS results above are supported by the kernel regressions in Figure 1 which clearly shows the relationship to be monotonically decreasing for $\gamma_{health}$.

To summarize, Section 6 establishes that there is a negative relationship between $\gamma$ and $M$. Our empirical exercise validates this result with both OLS and kernel regressions.

### 8.4 Relationship between $\zeta$ and $M$

In this section we investigate the relationship between the size of government, $\zeta_{it}$ and $M_{it}$. Section 7 establishes the relationship between $\zeta$ and $M$ to be negative. We use the same dataset used in the previous sub-section to estimate this relationship. There are 325 observations for the same countries that we have used in the previous analysis.

We estimate the model below using OLS:

$$
\zeta_{it} = \psi_0 + \psi_1 M_{it} + \epsilon_{it},
$$

(30)

where $\zeta_{it}$ is the ratio of government expenditure to GDP for country $i$ in time $t$, $M_{it}$ is the $(mean - median)/mean$ of country $i$ in time $t$, $\epsilon_{it}$ is a random error term. The estimates of the OLS model are presented in Table 2.

<table>
<thead>
<tr>
<th>coefficients</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_1$</td>
<td>-.484$^*$</td>
</tr>
<tr>
<td>cons</td>
<td>.461$^*$</td>
</tr>
</tbody>
</table>

Notes: $^*$: Significant at the 1% level  
†: Significant at the 5% level  
‡: Significant at the 10% level

Table 2: OLS Regression of size of government on M

We obtain a significant negative relationship between $\zeta$ and $M$, as established in Section 7.

For robustness, we estimate a kernel regression, to observe the nature of the relationship between $\zeta$ and $M$ without imposing a functional specification. We estimate the model:

$$
\zeta_{it} = H(M_{it}) + \nu_{it},
$$

(31)
where, $\zeta_{it}$ is the size of the government for country $i$ in time $t$, $M_{it}$ is the value of $M$ for country $i$ in time $t$, $H(.)$ is a generic function and $v_{it}$ is an error term.

The estimated kernel regression is displayed in Figure 2.

The kernel regression clearly shows a strong negative relationship between size of government and inequality as measured by $M$.

8.5 Relationship between $\tau$ and $M$

In this section we investigate the relationship between $\tau$ and $M$, conditional on the value of $\sigma$, the elasticity of substitution. We use both $\sigma_{health}$ and $\sigma_{edn}$ as measures of the elasticity of substitution. Our model implies (Section 5) that for low values of $\sigma$, there exists a positive relationship between the marginal tax rate and $M$, while for higher values of $\sigma$, this relationship turns negative.

We use threshold regressions to estimate the relationship between $\tau$ and $M$. Threshold regressions estimate the threshold value $\sigma, \sigma^*$, for which there exist two different regimes of the relationship between $\tau$ and $M$, above and below the threshold value $\sigma^*$.

We estimate the following model:
\[ \tau_{it} = \beta_0 + \beta_1 M_{it} I(\sigma_{it} \leq \sigma^*) + \beta_2 M_{it} I(\sigma_{it} > \sigma^*) + \nu_{it}, \]  \hspace{1cm} (32)  

where \( \tau_{it} \) is the dependent variable (the marginal tax rate corresponding to country \( i \) in time \( t \)), \( M_{it} \) is the value of \( M \) for country \( i \) in time \( t \), \( \sigma_{it} \) is the elasticity of substitution for country \( i \) in time \( t \), and the threshold variable, assumed to be strictly exogenous, \( \sigma^* \) is the threshold parameter. Also, \( \beta_1 \) and \( \beta_2 \) are the slope parameters for each of the two regimes, and \( \nu_{it} \) is a random disturbance term. The function \( I(\sigma_{it} \leq \sigma^*) \) is an indicator variable that takes the value 1 if \( \sigma_{it} \leq \sigma^* \) and 0 otherwise. The threshold value of \( \sigma_{it} \) is the estimate at which the likelihood function achieves a local minimum.\(^{17}\)

In Table 3 we present the threshold regression estimates for the model above, using the computed values of \( M \), and \( \sigma_{health} \) and \( \sigma_{edn} \) as the threshold variables. For the regressions with \( \sigma_{health} \) as the threshold variable, we observe that for values of \( \sigma_{health} \leq 0.63 \), there is a significant positive relationship, and for values greater there is a significant negative relationship between \( \tau \) and \( M \). Likewise, for \( \sigma_{edn} \leq 0.83 \), we observe a significant positive relationship and for greater values, a negative relationship (not significant for our sample). The likelihood function for the threshold regression with \( \sigma_{health} \) as the threshold variable is presented in Figure ??.

<table>
<thead>
<tr>
<th>Regimes</th>
<th>( \sigma_{health} )</th>
<th>( \beta )</th>
<th>( \sigma_{edn} )</th>
<th>( \beta )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>( \sigma_{health} \leq 0.63 )</td>
<td>2.26*</td>
<td>( \sigma_{edn} \leq 0.83 )</td>
<td>1.86†</td>
<td>320</td>
</tr>
<tr>
<td>Regime 2</td>
<td>( \sigma_{health} &gt; 0.63 )</td>
<td>-0.23*</td>
<td>( \sigma_{edn} &gt; 0.83 )</td>
<td>-0.04</td>
<td>46</td>
</tr>
</tbody>
</table>

Notes:  
*: Significant at the 1% level  
†: Significant at the 5% level  
‡: Significant at the 10% level

Table 3: Threshold Regression of \( \tau \) on \( M \) for \( \sigma_{health} \) and \( \sigma_{edn} \) as threshold variables

From the threshold regressions we obtain a robust relationship between the marginal tax rate \( \tau \) and \( M \) with the elasticity of substitution based

\(^{17}\)There is no asymptotic theory to obtain p-values corresponding to the threshold value obtained, thus our reported threshold values of \( \sigma \) depends on the value obtained by minimising the likelihood function. Estimations were performed using Bruce Hansen’s Gauss programmes obtained from http://www.ssc.wisc.edu/~bhansen/progs/progs_threshold.html

\(^{18}\)We compute the country specific measures of \( M \) using data on the mean and median values of household income available in the WIDER dataset.
Figure 3: Likelihood function of the threshold regression with substitutability of health, achieving a minimum at value 0.63
on health expenditures: for low values of $\sigma_{health} \leq 0.63$ the relationship is positive and significant, and is negative and equally significant for high values of elasticity of substitution.

When using the Gini measure as a proxy for $M$, we obtain the following results.

<table>
<thead>
<tr>
<th>Regimes</th>
<th>$\sigma_{health}$</th>
<th>$\beta$</th>
<th>$N$</th>
<th>$\sigma_{edn}$</th>
<th>$\beta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>$\sigma_{health} \leq 0.92$</td>
<td>-1.465*</td>
<td>332</td>
<td>$\sigma_{edn} \leq 0.18$</td>
<td>0.431‡</td>
<td>190</td>
</tr>
<tr>
<td>Regime 2</td>
<td>$\sigma_{health} &gt; 0.92$</td>
<td>-0.277*</td>
<td>34</td>
<td>$\sigma_{edn} &gt; 0.18$</td>
<td>-1.1*</td>
<td>176</td>
</tr>
</tbody>
</table>

Notes: *: Significant at the 1% level
‡: Significant at the 5% level
‡‡: Significant at the 10% level

Table 4: Threshold Regression of $\tau$ on $G$ for $\sigma_{health}$ and $\sigma_{edn}$ as threshold variables

Note that the results are more robust using the elasticity of substitution based on education expenditures. We would like to mention here that in an earlier version of the paper based on the previous OECD data set the results were strongly significant for the regimes above and below the threshold values. We have however opted to use the latest data made available by the OECD.

Combining the results in Section 6 on voting over the expenditure policy with the results in Section 3 on consensus taxation, we have derived that the relationship between income taxation and inequality is conditioned by the substitutability parameter $\sigma$. Indeed, we have obtained that for low levels of substitutability more inequality leads to a higher tax rate, while for higher substitutability the relationship is the opposite. Our empirical exercise validates this result.

It is worth noting that our results therefore provide an explanation for why direct linear regressions between taxation and inequality have obtained such poor results.

9 Conclusion

In this paper we have jointly treated public taxation and spending. This joint treatment has allowed us to address that the two policies cannot be chosen independent of each other. Based on recent empirical evidence, that
partisan confrontation is much stronger on expenditure policies rather than on income taxation, we examine the case in which expenditure policy is chosen by majority voting and income taxation is consistently adjusted. This model allows us to address novel issues such as the interdependence between income taxation, the composition of public spending and the substitutability between public and private goods.

The main results are that inequality is negatively related to the pro-rich bias in public expenditure and to the size of government, and that inequality is positively or negatively related to the marginal income tax, depending upon the elasticity of substitution between public and private goods being low or high. Both implications are validated by our empirical exercise using kernel regressions and threshold regressions, respectively.

The paper has substantial room for improvement on both counts: theoretical and empirical. While redistributive activity of the government through taxes and transfers has attracted the interest of researchers, the role of public spending has been comparatively neglected. We know too little about the redistributive impact of the different components of the government budget $G$. Even for countries where redistributive effects are regularly estimated, such as the UK, they focus on five budget lines only: education, health, housing subsidies, travel subsidies, and school meals. But the impact of the rest of budget lines remain unexplored. For most countries such estimates simply do not exist. This lack of information is paralleled by a similar lack of modelling on how the change in the structure of government spending affects the consumer behavior and well-being.

The analysis of the substitutability between the private and publicly provided goods and services deserves further attention. We are aware of no empirical work estimating this degree of substitutability nor of any formal modeling of the effect of the regulation of the private substitutive supply of goods and services that are also being provided by the state.

There is much to be gained by the joint analysis of public taxation and expenditure. Our work is but a first step in this direction.

References


A Countries used in the study - OECD database

The countries which are used for our analysis are as follows. Data has been obtained from the OECD database, at www.oecd.org/statistics

Austria
Belgium
Canada
Czech Republic
Denmark
Finland
France
Germany
Greece
Hungary
Iceland
Ireland
Italy
Japan
Korea
Netherlands
New Zealand
Norway
Poland
Portugal
Slovak Republic
Spain
Sweden
United Kingdom
United States

The range of years for which the data is used varies from 1980s to 2005, the bulk of it being in the 1990s and 2000s, depending upon the availability of the data. We have no missing observations.

B Definition of social transfers, OECD

Government expenditures net of transfers ($G$) were obtained by the following sums:

Total expenditures of General government (national currency, current prices, code: TLYCG:) –
(Subsidies (D3CG) +
Social transfers other than social transfers in-kind (D62CG) +
Other current transfers (D7CG) +
Capital transfers (D9CG))

The data was obtained from OECD Government expenditure by function. All estimates are constant 2000 dollars.

The share of education expenditures on $G$ was calculated as ratio of public expenditures to pre-primary and primary education (all types of transactions, all educational programmes) on $G$. $\gamma_{education}$ is obtained as $1 – \text{share of education expenditures in } G$. This data was obtained from UNESCO-OECD-Eurostat (UOE) data on education statistics, compiled on the basis of national administrative sources, reported by Ministries of Education or National Statistical Offices.

The share of health expenditures in $G$ was calculated as the ratio of public expenditures to health to $G$. $\gamma_{health}$ was obtained as $1 – \text{(share of education expenditures in } G)$. The data was obtained from OECD System of Health Accounts.