The Strategic Impact of Resource Flexibility in Business Groups

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Abstract

We show that in business groups with efficient internal capital markets resources may be channelled to either more or less profitable units. Depending on the amount of internal resources, a group may exit a market in response to increased competition, or channel funds to the subsidiary operating in that market. This has important implications for the strategic impact of group membership. Affiliation to a monopolistic subsidiary can make a cash-rich (poor) stand-alone more (less) vulnerable to entry deterrence. Also, resource flexibility within a group makes subsidiaries’ reaction functions flatter, thus discouraging rivals’ strategic commitments when entry is accommodated.

Keywords: Business Groups, Financially Constrained Entry, Internal Capital Markets, Multimarket Competition.

JEL classification: G30, L13, L20

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1. Introduction

Business groups are a widespread organizational form in many countries. Groups often adopt a pyramidal structure, whereby individual subsidiaries are separate legal entities with limited liability and autonomous access to external capital markets. This marks a clear difference between business groups and multidivisional organizations.\(^1\) Yet, there is substantial evidence that groups establish internal capital markets just like multidivisional firms do.\(^2\) While many empirical studies of internal capital markets have looked at business groups, theoretical models have instead focussed on multidivisional firms. This paper is one of the first attempts to model the allocation of internal resources among group members.

The idea that business groups behave somehow differently in product markets is by no means new. In particular, competition authorities around the world since the Standard-Oil case have taken seriously the idea that firms’ access to a group’s deep pockets may be a source of market power.\(^3\) However, the exact mechanism through which the ability to shift resources across group members affects their competitive behavior has not been clarified yet. For instance, while some empirical studies suggest that groups do better than stand-alone firms in deterring entry (Lawrence, 1991), formal reasoning indicates that resource flexibility may well prevent a group from committing to provide a member firm with deep-pockets (Matsusaka and Nanda, 2002). This paper studies how internal resources are reallocated in response to changes in a group’s actual or prospective markets, and how this in turn affects its members’ competitive behavior. It thus provides a formal analysis of multimarket spillovers generated by internal capital market phenomena.

We study a business group operating in a monopoly and a duopoly market through two subsidiaries exerting unobservable R&D efforts. Group members, like stand-alone firms, raise funds on the external capital market to complement internal resources. However, while a stand-alone firm draws on its own internal funds, a subsidiary’s internal resources are pooled with the group’s resources and then reallocated. Hence, subsidiaries’ wealth is endogenously determined by the allocation decisions of the group’s headquarters. This is crucial in our model, in that the amount of internal resources determines the agency problem vis-à-vis outside investors, and thus the incentives of subsidiary managers to exert R&D efforts. Through this channel, the internal resource allocation affects a subsidiary’s product market strategy. Obviously, the headquarters would allocate resources strategically if its decisions were observable by product market competitors. Although our model rules out this possibility assuming that the allocation is not observable, the resource flexibility of business groups still has important strategic effects.

The central result of the paper is that in value-maximizing business groups resources may be channelled to either

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\(^1\) According to Claessens, Djankov, and Klapper (2000), the fraction of listed firms affiliated with business groups in East Asia and Chile ranges between 40 and 74 %. La Porta, Lopez-de-Silanes, and Shleifer (1999) and the European Corporate Governance Network (1997) provide evidence on the ubiquity of pyramids in various countries.

\(^2\) See for instance Houston, James, and Marcus (1997), Perotti and Gelfer (2001) and Samphantharak (2003). Studies of internal capital markets within groups are also more reliable, to the extent that data on assets and investments are better defined at the firm level than at the division level.

\(^3\) This concern has been recently expressed by the European Commission in support of its controversial decision to forbid the GE-Honeywell merger (Case no. COMP/M.2220, p 83-84, July 2001).
more or less profitable subsidiaries. In other words, both winner-picking and cross-subsidization may occur. In particular, a group may react to increased competition in a market by either exiting so as to focus on less competitive industries, or by channelling funds to the subsidiary operating in that market, depending on the resources it can draw on. In fact, unless total resources are scarce, additional internal assets are relatively more valuable for units facing a tougher competitive environment, as the latter suffer more serious agency problems vis-à-vis external investors when seeking funds. This result contrasts with previous theoretical work arguing that winner-picking always takes place in efficient internal capital markets (e.g. Stein, 1997), and is consistent with extensive evidence of cross-subsidization (see for instance Shin and Stulz, 1998 and Ferris, Kim, and Kitsabunnarat, 2003).

The paper then shows that resource flexibility in business groups has both strategic benefits and costs. First, group membership does not necessarily turn a firm into a better entrant in an oligopoly market. In fact, while the prospect of subsidization makes a cash-poor firm less sensitive to financial constraints and thus to entry deterrence, a cash-rich firm’s commitment to its home market is dramatically undermined by the possibility that its resources are diverted to more profitable affiliates; this may encourage rivals to adopt predatory practices. However, if the group decides to enter (stay) in the duopoly market, cross-subsidization is always optimal. Hence, group affiliation is a credible commitment to adopt a tougher R&D strategy in a duopoly market. This is in line with empirical findings that, upon entry in a market, group-affiliated firms compete more aggressively than stand-alone entities (Weinstein and Yafeh, 1995). A second implication of cross-subsidization is that a subsidiary’s strategic response to its rivals’ actions is partially counteracted by a resource reallocation response, so that group members have flatter reaction functions than stand-alone firms. Therefore, group membership is a good defensive strategy when entry is accommodated and rivals may adopt aggressive pre-commitments.4

This work promotes the understanding of business groups, which so far have been mostly the object of empirical investigation (see Khanna, 2000 for a survey), but also contributes to several other strands of literature. It is obviously related to the theoretical literature on internal capital markets (e.g., Stein, 1997; Fluck and Lynch, 1999); it contributes to the literature on corporate finance and product markets, dating back to Telser’s (1966) study of deep pockets and financial entry deterrence, exploring for the first time the interaction between internal capital markets and product market competition. The close relation to the literatures on conglomerate power and multimarket spillovers (in particular Bulow, Geanakoplos, and Klemperer, 1985) is also discussed at length in the paper.

The plan of this paper is as follows. Section 2 sets out the basic model with no strategic interaction, characterizes the optimal resource allocation, and explains why both cross-subsidization and winner-picking may occur within groups. Section 3 extends the analysis to the case where a group subsidiary competes in a duopoly market. Section 4 draws the strategic implications of the previous analysis; the relation to various strands of literature is then discussed. Section 5 concludes.

4However, group affiliation also discourages pro-collusive commitments that rivals may want to make when competition is in strategic complements. When this is the case, stand-alone entry is an optimal strategic choice.
2. Resource allocation within a business group

The basic model

We study a business group composed of two subsidiaries running independent projects. There are four agents in the model: subsidiary managers, corporate headquarters (HQ), outside investors and stand-alone firms. Each subsidiary needs to invest an amount $I$ in order to start or continue a project. The headquarters has control over corporate resources $A$: it allocates $A_1$ and $A_2$ to subsidiaries 1 and 2, provided $A_1 + A_2 = A$. We assume $A < 2I$: internal funds are not sufficient to start both projects. After a subsidiary manager is assigned $A_i \leq I$ by headquarters, she seeks the additional funds $I - A_i$ from outside investors. Investors are completely passive in the model (they just require to break even in order to finance a project) and behave competitively in the market for funds. Finally, stand-alone firms are identical to business subsidiaries, except that they have control over their own assets.

Projects. Each project is subject to moral hazard. After her project is financed, manager $i$ chooses a level of effort $e_i \in [0, 1]$. A simple interpretation is that $e_i$ is R&D effort exerted to develop a new technology.\(^5\) Neither headquarters nor external investors can observe (verify) the level of effort exerted. If the manager chooses a level of effort $e_i$, subsidiary $i$ gains a return $\pi_i$ with probability $e_i$; with probability $1 - e_i$, the project fails and the return is 0. $\pi_i$ thus represents the productivity of effort for firm $i$.

Preferences. All agents are risk-neutral. Effort $e_i$ imposes a private cost $\beta e_i^2$ on manager $i$. We assume $\beta \geq \pi_i$, to ensure that the first-best level of effort $e^{FB} \leq 1$. We also make the following assumption, to ensure that managers face a non-trivial fund raising problem (see Lemma 1 later):

Assumption 1. \[
\frac{\pi_1^2}{4\beta} < I \leq \frac{\pi_2^2}{2\beta}.
\]

Neither subsidiary managers nor the headquarters enjoy private benefits from running (controlling) extra projects. The headquarters’ resource allocation maximizes the group’s value. Note that in our model units could simply commit ex-ante to an optimal resource allocation policy. By having headquarters decide, we are implicitly assuming that affiliated firms relinquish control over assets to this third party and design its incentives so as to implement the optimal allocation rule.\(^6\)

Timing. The timing of events is as follows (see also Figure 1):

$t=0$ (Internal capital market allocation) Headquarters allocates total resources $A$, assigning $A_1$ and $A_2$ to subsidiaries 1 and 2.

\(^5\)The R&D interpretation is particularly appropriate, as $e_i$ is taken to be the strategic variable of a product market game in sections 3 and 4. As we will see, other dimensions of managerial effort (e.g. advertising effort) are consistent with our model, as long as increased effort in one firm reduces its rival’s expected profits.

\(^6\)The need to delegate control to headquarters could be endogenized by introducing in the model ex-ante uncertainty on the subsidiaries’ productivity or the degree of competition in each market. As the optimal resource reallocation is contingent on these variables, which may not be verifiable ex-post, it may be necessary to delegate the resource reallocation decision to a third party with the right incentives to maximize group value. However, modelling this stage lies beyond the scope of the paper.
t=1 (Financial contracting) Provided it is profitable to start a project, each subsidiary manager seeks $I - A_i$ ($i = 1, 2$) on the external capital market. She makes a contract offer to outside investors, who can accept or reject the offer.

$t=2$ (Moral Hazard) Each subsidiary manager chooses a level of effort.

$t=3$ Returns are realized and outside investors are paid according to financial contracts.

**Financial contracts.** When manager $i$ raises funds $I - A_i$ on the external capital market, she contracts on the outside investor’s share of returns ($\alpha_i$). As the investor can expect to be paid $\alpha_i \pi_i$ in case of success and 0 in case of failure, his claim can equivalently be interpreted as debt with face value $\alpha_i \pi_i$ or as an equity stake $\alpha_i$. We assume that the rest of the business group is not liable for a subsidiary’s financial obligations to its financiers.\(^7\)

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### Financial structure and governance in a business group

Business groups have been defined as collections of legally distinct firms that are partly or wholly owned by a single individual or family (here the “headquarters”) that controls the member firms’ assets. Many groups have a pyramidal ownership structure whereby the ultimate owner controls several firms holding only a fraction of their equity; there is ample evidence that pyramidal groups are widespread in Asian, Latin American and European economies.\(^8\)

Our model incorporates two features of corporate groups that make them different from multidivisional firms. First, subsidiaries routinely raise funds from and issue claims to external financiers; for instance, each company in a group can ask for a bank loan or issue its own shares. In contrast, conglomerate divisions do not have autonomous access to financial markets. Second, when group subsidiaries are partially owned, the ultimate owner is not responsible for the subsidiaries’ obligations to outside financiers.\(^9\)

There is substantial empirical evidence that groups establish internal capital markets.\(^10\) Even when subsidiaries are only partially owned, the headquarters often enjoys effective control rights and can thus redistribute the group’s

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\(^7\)Limited liability within the group rules out any diversification benefit of conglomerate à la Diamond (1984). In that paper, joint liability of unrelated projects vis à vis external investors increases their overall debt capacity (see also Li and Li, 1996), hence the financial advantage of internal capital markets rests upon centralized borrowing by conglomerate headquarters. Our paper shows that resource flexibility within business groups may bring benefits even when most of the funds are collected individually by affiliated firms and the latter have no joint liability. Hence, our limited liability assumption goes hand in hand with our focus on individual subsidiaries’ external finance rather than on centralized fund-raising. Limited liability is also a realistic assumption for partially-owned subsidiaries, which are very common in many European and Asian countries (see the next footnote).

\(^8\)In their sample of the 20 largest firms in 27 wealthy economies, La Porta, Lopez-de-Silanes, and Shleifer (1999) find that 26 % of the firms that have ultimate owners (31 % in countries with poor shareholder protection) are controlled through pyramidal structures. Bianchi, Bianco, and Enriques (1997) find that 56 % of Italian firms belong to a pyramidal group. Of these, 37 % are holding companies at the top of the pyramid, 31 % are wholly-owned subsidiaries, and 32 % are subsidiaries controlled by the ultimate owner with a less than 100 % stake. Pyramids are also common in other European countries, according to the European Corporate Governance Network (1997).

\(^9\)This rule is common to most legal systems, as argued by legal scholars (see Blumberg, 1989; Hadden, 1983; Antunes, 1994). Conversely, and with the notable exception of Bianco and Nicodano (2002) and Nicodano (2003), the corporate finance literature has failed so far to recognize limited liability as a salient feature of business groups as opposed to multidivisional firms.

\(^10\)Houston, James, and Marcus (1997) and Houston and James (1998) find that in bank holding companies subsidiaries’ lending activity is more closely tied to the cash flows and capital position of the holding company than it is to the bank’s own cash flow and capital position, suggesting that bank holding companies establish internal capital markets. Perotti and Gelfer (2001)
assets, to the extent that the voting rights of non-controlling shareholders are dispersed. One may argue that the legal protection of minority shareholders limits the scope for resource reallocation among partially-owned subsidiaries. In practice, however, the safeguard provided to minority shareholders is particularly weak in those countries where business groups are a main corporate form (see La Porta, Lopez-de-Silanes, and Shleifer, 1999).¹¹ In fact, the limited authority of minority shareholders in corporate groups has spurred a considerable amount of research concerning the conflict of interest between parent company and subsidiaries (see e.g. Bebchuk, Kraakman, and Triantis, 2000; Wolfenzon, 1999). As this conflict is not the focus of the present paper, we will assume here partially owned subsidiaries with completely passive minority shareholders.¹²

In our model - as in previous theoretical work on internal capital markets (Gertner, Scharfstein, and Stein, 1994; Stein, 1997; Matsusaka and Nanda, 2002) - business subsidiaries and stand-alone firms differ only in the way control over assets is allocated, whereas the nature of the agency problem vis-à-vis external investors is identical. In a business group, headquarters can transfer internal resources from one unit to another; once internal resources are allocated, business units seek funds on the external market in order to complement their own internal assets. This has the following consequence: as the amount of internal assets determines the financial contract with outside investors (and thus managerial incentives), a subsidiary’s managerial incentives and hence its value are endogenously determined by the headquarters’ allocation decision. Conversely, the value of a stand-alone firm depends solely on that firm’s resources, which are exogenous in the model.

To derive the optimal resource allocation, we characterize the subgame perfect Nash equilibrium of the game proceeding by backward induction. We first study the financial contracting problem of a business unit endowed with resources \( A_i \) (i.e. outside financial needs \( I - A_i \)). We then analyze how at \( t = 0 \) a group headquarters assigns resources \( A_i \) to affiliated business units, anticipating that this will affect contracts with outside investors.

**Outside finance, internal resources and business units’ agency problems**

The financial contracting subgame starting at \( t = 1 \) can itself be solved by backward induction. The manager’s effort choice at \( t = 2 \) solves:

\[
\max_{e_i \in [0,1]} e_i(1 - \alpha_i)\pi_i - \beta e_i^2 / 2 - A_i
\]

provide evidence of financial reallocation in Russian groups while Samphantharak (2003) finds that internal assets are extensively reallocated within Thai business groups. See also Claessens, Djankov, and Klapper (2000) for related evidence on East Asia and Chile.

¹¹Interestingly, in many countries the burden of proof for damages incurred by a subsidiary as a consequence of its parent’s decisions rests on the claimant, and poor information disclosure requirements make it almost impossible for the latter to offer conclusive evidence (see Rossi, 1996). Even in legal systems with strong minority protection laws, free-rider problems may actually hamper shareholder activism: absent a coordination mechanism, dispersed shareholders may never actually start a lawsuit for misappropriation against the parent company.

¹²Allowing for minority shareholders’ activism in the model would limit the extent of resource reallocation without changing the nature of our results. For instance, \( A \) might be reinterpreted as the amount of resources that the headquarters may safely reshuffle without triggering the expropriated shareholders’ reaction. See Samphantharak (2003) for evidence that (i) internal capital markets are not perfect when group affiliates are listed companies, due to stock exchange regulations restricting intra-group transfers; (ii) resource reallocation among public non-listed subsidiaries is not prevented by corporate laws and regulations.
which implies: \( e_i = \frac{(1-\alpha_i)\pi_i}{\beta} \) if an interior solution is assumed. At date 1, provided it is profitable to start a project, manager \( i \) makes a take-it-or-leave-it contract offer \( \alpha_i \) to outside investors to raise funds \( I - A_i \). The contract must satisfy the investors’ participation constraint:\(^{13}\)

\[
e_i\alpha_i\pi_i - (I - A_i) \geq 0
\]

(\( IR \))

Investors in fact anticipate that \( e_i = \frac{(1-\alpha_i)\pi_i}{\beta} \). This can be substituted into the manager’s objective function and into (\( IR \)), to obtain the manager’s financial contracting problem at \( t = 1 \):

\[
\max_{\alpha_i \in [0,1]} \left[ \frac{(1-\alpha_i)^2}{2\beta}\pi_i^2 - A_i \right]
\]

subject to:

\[
\frac{\alpha_i(1-\alpha_i)}{\beta}\pi_i^2 \geq I - A_i
\]

(\( IR \))

Clearly, the manager only seeks funds at \( t = 1 \) if the value of this program is positive. The equilibrium outcome of the financial contracting subgame is characterized in the following lemma:

**Lemma 1.** There exists a threshold level of internal resources \( \tilde{A}_i \in \left[ I - \frac{\pi_i^2}{4\beta}, I \right] \) such that:

- if \( A_i \geq \tilde{A}_i \): the business unit obtains outside finance and starts the project;
  \( \alpha^*_i \in [0, \frac{1}{2}] \) and is decreasing in \( A_i \); \( e^*_i \in \left[ \frac{\pi_i}{2\beta}, \frac{\pi_i}{\beta} \right] \) and is increasing in \( A_i \).

- if \( A_i < \tilde{A}_i \), the business unit is either unprofitable or cannot obtain outside funds, hence it is shut down.

**Proof.** Note first that the manager optimally sets \( \alpha_i \) so as to make (\( IR \)) bind: the investors’ pledgeable income \( \frac{\alpha_i(1-\alpha_i)}{\beta}\pi_i^2 \) must be equal to the funds provided, \( I - A_i \). Notice that \( \frac{\alpha_i(1-\alpha_i)}{\beta}\pi_i^2 \) is a concave function of \( \alpha_i \), and achieves its maximum in \( \alpha_i = \frac{1}{2} \). Hence, if \( \max_{\alpha_i} \frac{\alpha_i(1-\alpha_i)}{\beta}\pi_i^2 \equiv \frac{\pi_i^2}{4\beta} < I - A_i \), there is no level of \( \alpha_i \) that can satisfy the investors’ (\( IR \)) constraint, and the unit cannot be funded. This defines the threshold level of assets \( A^f_i \equiv I - \frac{\pi_i^2}{4\beta} \) below which the unit is financially constrained. By Assumption 1, \( A^f_i > 0 \).

If \( A_i \geq A^f_i \), the unit can raise funds. Suppose it does so: as the manager’s utility is decreasing in \( \alpha_i \), the optimal investors’ stake \( \alpha^*_i \) is the smallest solution to \( \frac{\alpha_i(1-\alpha_i)}{\beta}\pi_i^2 = I - A_i \). Hence, \( \alpha^*_i = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\beta(I-A_i)}{\pi_i^2}} \), which decreases

\(^{13}\)In our fixed investment model, where profitability does not depend on project size, the value of investors’ claims \( (e_i,\alpha_i,\pi_i) \) does not depend on whether the funds lent are then used within the subsidiary or partly redirected to a different unit at date 1. Thus, once internal funds are optimally allocated at \( t=0 \), the headquarters has no incentive to reshuffle external funds at \( t=1 \), and investors do not fear any expropriation from the potential tunneling of the funds lent. Our model therefore allows to abstract from the agency costs of tunneling (on this, see section 5).
monotonically from $\frac{1}{2}$ to 0 as $A_i$ increases from $A_i^f$ to $I$, while $e_i^* = \frac{(1-\alpha_i^*)\pi_i}{\beta}$ increases monotonically from $\frac{\pi_i}{2\beta}$ to $\frac{\pi_i}{\beta}$. The unit’s payoff $u_i(A_i) = e_i^*(A_i)\pi_i - \frac{\beta}{2}(e_i^*(A_i))^2 - I$, can be written as

$$u_i(A_i) = \frac{\pi_i^2}{\beta} \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\beta (I - A_i)}{\pi_i^2}} \right) \left( \frac{3}{4} - \frac{1}{2} \sqrt{\frac{1}{4} - \frac{\beta (I - A_i)}{\pi_i^2}} \right) - I$$

Note that $u_i(A_i)$ increases monotonically from $\frac{3\pi_i^2}{8\beta} - I$ to $\frac{\pi_i^2}{2\beta} - I$ as $A_i$ increases from $A_i^f$ to $I$. Two cases may then arise:

Case 1: $\frac{3\pi_i^2}{8\beta} - I \geq 0$. Hence, $u_i(A_i^f) \geq 0$, implying that whenever a business unit can raise funds it is optimal to do so: $\widetilde{A}_i = A_i^f$.

Case 2: $\frac{3\pi_i^2}{8\beta} - I < 0$. In this case, though a manager with $A_i^f$ might raise funds, her utility (net of effort costs) would be negative: $u_i(A_i^f) < 0$. Notice also that $u_i(I) = \frac{\pi_i^2}{2\beta} - I \geq 0$ by Assumption 1. Therefore, as $u_i(A_i)$ is continuous and strictly increasing in $A_i$ over $(A_i^f, I)$, there exists one $A_i^p \in (A_i^f, I)$ such that $u_i(A_i) > 0 \forall A_i > A_i^p$. Hence, $\widetilde{A}_i = A_i^p > A_i^f$. By solving $u_i(A_i) = 0$, one finds that $A_i^p = \frac{\beta I - \pi_i^2 + \sqrt{\pi_i^4 - 16\beta^2 I^2}}{2\beta}$.

Putting case 1 and 2 together, the threshold level of assets above which the project is started is: $\widetilde{A}_i \equiv \max \left\{ A_i^f, A_i^p \right\}$.

Q.E.D.

The value of each business unit as a function of internal resources $A_i$ can be written as:

$$V_i(A_i) \equiv \begin{cases} 
0 & \text{if } A_i < \widetilde{A}_i \\
\pi_i^2 (A_i) \pi_i - \frac{\beta}{2}(e_i^*(A_i))^2 - I & \text{if } A_i \geq \widetilde{A}_i 
\end{cases}$$

A business unit’s value is weakly increasing in the amount of its internal resources (see also Figure 2). If the unit is financially constrained or simply unprofitable ($A_i < \widetilde{A}_i$), the project is not started hence its value is zero. At $A_i = \widetilde{A}_i$ a discontinuity may exist as additional assets allow the unit to raise funds and start a profitable project.\footnote{Holmström and Tirole (1997) have already shown this result in a model with a binary effort decision.}

When $A_i \geq \widetilde{A}_i$, the unit’s value is still increasing in $A_i$: additional internal resources, implying smaller external financial needs, allow a reduction in the share of profits $\alpha_i$ to be left to outside investors, and thus an improvement in managerial incentives. Notice however that the marginal value of internal resources is decreasing: as $A_i$ approaches $I$, managerial effort gets closer to the first best, hence the role of additional internal funds in spurring incentives becomes less important. Formally, on $A_i \in \left[ \widetilde{A}_i, I \right]$ the value function is concave:

$$\frac{\partial^2 V_i}{\partial A_i^2} = \frac{\partial^2 e_i^*}{\partial A_i^2} \left[ \pi_i - \beta e_i^* \right] - \beta \left( \frac{\partial e_i^*}{\partial A_i} \right)^2 < 0,$$

as implied by $e_i^* \in \left[ \frac{\pi_i}{2\beta}, \frac{\pi_i}{\beta} \right]$, $\frac{\partial e_i^*}{\partial A_i} = \frac{1}{2be_i^* - \pi_i} > 0$ and $\frac{\partial^2 e_i^*}{\partial A_i^2} = \frac{-2\beta}{\left(2be_i^* - \pi_i\right)^2} \frac{\partial e_i^*}{\partial A_i} < 0$.

[Figure 2 about here]
The following lemma establishes whether additional internal assets are more valuable to more or less productive business units.\textsuperscript{15}

**Lemma 2.** Suppose two productive units have different returns if successful: \( \pi_1 < \pi_2 \). Then:

- unit 1 is more likely to be shut down: \( \tilde{A}_1 > \tilde{A}_2 \);
- unit 1’s value function is shifted downwards: \( V_1 < V_2 \);
- \( \frac{\partial V_i}{\partial A_i} > \frac{\partial V_j}{\partial A_j} \) for \( A_i \geq \tilde{A}_1, i = 1, 2 \), with \( \frac{\partial V_i}{\partial A_i} = \frac{\partial V_j}{\partial A_j} \) in \( A_i = I \).

**Proof.** See the Appendix.

The third result in Lemma 2 is central to our paper, implying that additional internal funds may well be more valuable to less productive business units. This is because the less productive unit, having (ceteris paribus) lower returns to pledge in case of success, is obliged to relinquish a larger share \( \alpha_i \) to outside investors. This in turn exacerbates its incentive problem with respect to the more productive unit. Additional internal funds allowing a reduction in the share \( \alpha_i \) are then more valuable to this unit. The result suggests that headquarters trying to maximize a group’s value need not necessarily concentrate resources on the most productive unit.

**Efficient resource allocation**

Consider productive units 1 and 2 affiliated to a group, with \( \pi_1 < \pi_2 \). We assume from now on that \( \pi_2 > \beta I (2 + \sqrt{2}) \) (see footnote 17 on this). At date 0, the headquarters chooses \( A_1 \) and \( A_2 \) so as to solve the problem:

\[
\max_{A_1, A_2} V(A_1, \pi_1) + V(A_2, \pi_2)
\]

subject to:

\[
A_1 + A_2 = A.
\]

The following proposition characterizes the optimal resource allocation:

**Proposition 1.** For any \( \pi_1 < \pi_2 \), there exists a threshold \( \overline{A}(\pi_1) \) such that, if \( A \geq \overline{A}(\pi_1) \), it is optimal to let both subsidiaries operate and assign relatively more resources to the weaker one. Hence, \( A_1^* \geq \overline{A}_1 \) and \( A_2^* \geq \overline{A}_2 \), with \( A_1^* > A_2^* \). If instead \( A < \overline{A}(\pi_1) \), all resources are diverted to the more productive subsidiary while unit 1 is shut down. The threshold \( \overline{A}(\pi_1) \) is strictly decreasing in \( \pi_1 \).

\textsuperscript{15}In this very simple model where project payoffs have a binary distribution, the subsidiary with higher returns in case of success is ceteris paribus more profitable. Although we sometimes refer to the subsidiary with a higher (lower) effort productivity as the more (less) profitable one, there is an obvious distinction between effort productivity \( \pi \), which of course affects profitability but is exogenous to the model) and firm profitability, which endogenously depends on internal liquidity and thus on resource reallocation within the group.
**Proof.** See the Appendix.

When internal resources are scarce or group subsidiaries have very different productivity levels, *winner-picking* maximizes the group value, namely, all resources are channelled to the more productive unit. Otherwise, *cross-subsidization* takes place from the more to the less productive unit. This result becomes intuitive once resource allocation is regarded as a two-stage decision process. We first ask whether it is optimal to keep both subsidiaries open (in which case each must receive at least \( \tilde{A}_i \)), rather than shut unit 1 down and divert all resources to 2. Before opting for the winner-picking solution, the increase in \( V_2 \) due to stronger incentives in unit 2 must be traded off with the discrete benefit of providing unit 1 with enough funds \( (A_1 \geq \tilde{A}_1) \) to let it start a profitable project. Diverting *all* resources to unit 2 cannot be efficient when \( A \) is very large, to the extent that internal funds have a decreasing marginal value for a unit. Hence, for high levels of \( A \), it is optimal to operate both subsidiaries.\(^{16}\) It is also fairly intuitive that winner-picking becomes less likely as the two units’ productivities get closer (i.e., when \( \pi_1 \) is increased).\(^{17}\)

Secondly, provided both units are allowed to operate, \( A \) must be optimally shared between the two. As agency problems are exacerbated for the less productive unit, additional internal resources are more valuable to it; hence, a cross-subsidization strategy is optimal in this case.

**Discussion and related literature**

We compare here the result in Proposition 1 to related work on internal capital markets. For this purpose, it is useful to think of the business group as the outcome of a “merger” between two stand-alone firms, each endowed with \( \frac{A}{2} \), and with productivity levels \( \pi_1 \) and \( \pi_2 \). Clearly, the ability to shift internal resources within the group creates value:

\[
\max_{A_1} V_1(A_1) + V_2(A - A_1) \geq V_1 \left( \frac{A}{2} \right) + V_2 \left( \frac{A}{2} \right).
\]

Before us, other papers have pointed at this “financial synergy” as a main motive for conglomeration (e.g. Stein 1997, Fluck and Lynch 1999, Matsusaka and Nanda 2002). The novel feature of our model is that the financial synergy takes very different forms, depending on the total amount of internal resources and the productivity differential between stronger and weaker units. In particular, three interesting cases may arise.

If \( \frac{A}{2} \in (\tilde{A}_2, \tilde{A}_1) \) but \( A \geq \tilde{A}(\pi_1) \), the internal capital market allows unit 1 to start a profitable project that would not be able to get funding as a stand-alone. Fluck and Lynch (1999) also propose a theory of conglomerate mergers allowing the financing of projects that would be denied funds as stand-alones. While this is the only financial

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\(^{16}\)One may wonder whether the opportunity cost of shifting resources from the more to the less productive subsidiary is not underestimated in our fixed investment model. We believe our qualitative results would still hold in a model where each unit has a continuum of projects of decreasing value to invest in (or a variable scale project with decreasing returns). In such a model, information problems would imply that larger internal resources allow a subsidiary to raise more funds and thus to run more projects (see e.g. Holmström and Tirole, 1997). The basic intuition of Proposition 1 would still be there: injecting additional liquidity in a high-productivity unit has a decreasing value, to the extent that (a) this unit has less trouble in raising external funds to finance further projects with respect to the weak unit; or (b) further projects have a decreasing value.

\(^{17}\)As shown in the Appendix, the assumption \( \pi_2^2 > \beta I \left( 2 + \sqrt{2} \right) \) ensures that for \( \pi_1 = \pi_2 \) winner-picking never takes place (even for low levels of \( A \)). We are thus ruling out the less interesting case where (for a cash-poor group) it is optimal to shut down unit 1 for \( \pi_1 = \pi_2 \), and thus for any \( \pi_1 < \pi_2 \).
advantage of conglomeration in their paper, the next two cases show that resource flexibility brings other benefits in our model.

A second case arises when $\frac{A}{2} \geq \bar{A}_1$ and $A \geq \bar{A}(\pi_1)$. Upon affiliation to a monopolistic firm, a relatively less productive firm receives a cash injection that alleviates its agency problems vis-à-vis outside investors. In other words, access to an internal capital market allows a project that can already be funded as a stand-alone to write a “better” contract with outside financiers. In a sense, this generalizes to a continuous framework Fluck and Lynch’s intuition that mergers serve to channel funds to marginally profitable units.\(^\text{18}\) This cross-subsidization result is in line with empirical papers documenting that both multidivisional firms and business groups redistribute resources away from more profitable units to units with worse investment opportunities.\(^\text{19}\) Various papers (e.g., Meyer, Milgrom, and Roberts, 1992; Scharfstein and Stein, 1998) argue that cross-subsidization represents the dark side of internal capital markets, and attribute the phenomenon to exacerbated agency problems and power struggles within conglomerates and groups. We showed here that cross-subsidization within cash-rich groups may be as well an efficient allocation decision, to the extent that it creates value by smoothing incentive problems across productive units.

Notice finally that for sufficiently low levels of $\pi_1$ it may be the case that $\frac{A}{2} \geq \bar{A}_1$ and yet $A < \bar{A}(\pi_1)$.\(^\text{20}\) Although a stand-alone firm, being committed to its home market, would start project 1, an efficient business group shuts unit 1 down and diverts all resources to the relatively more productive unit. Hence, the advantage of resource flexibility also rests on the group’s ability to engage in winner-picking when this is called for (i.e., when weak units have very low productivity levels). In a model of multidivisional firms where individual division do not raise external funds, Stein (1997) also finds that headquarters may give weaker projects less financing than they could obtain as stand-alones. However, in contrast to our paper, winner-picking always takes place in his model.

To conclude this discussion, it is worth noting that our basic model of internal capital markets focuses on the benefits of resource flexibility while neglecting its costs. First, in maximizing the group’s value, the internal resource allocation may well hurt the interests of individual subsidiaries’ shareholders. However, by taking as exogenous the group’s initial financial structure, we are ruling out any agency cost that may derive from minority shareholder expropriation.\(^\text{21}\) Most papers on pyramids so far have done the reverse: Bebchuk, Kraakman, and Triantis (2000) for

\(^{18}\)In Fluck and Lynch (1999) the nature of the agency problem is different from here: returns are not verifiable, hence a firm is funded provided the financial contract can induce managers to pay investors back in any period. The binary nature of the manager’s decision in their model (pay back or not) implies that, as long as the incentive constraint holds and the firm manages to get financed, the firm value is not affected by the amount of outside versus inside equity. Conversely, in our model managerial effort and firm value continuously increase with the amount of inside equity.

\(^{19}\)See for instance Ferris, Kim, and Kitsabunnarat (2003) on Korean chaebols and Perotti and Gelfer (2001) on Russian business groups. For evidence on multidivisional firms see Shin and Stultz (1998), Scharfstein (1998), and Rajan, Servaes, and Zingales (2000). However, see Chevalier (2000) for an argument that cross-subsidization results in the latter papers can be attributed to measurement errors and selection bias.

\(^{20}\)In the Appendix (Lemma A1) we characterize the threshold level of $\pi_1$ below which this case arises.

\(^{21}\)Of course one may wonder why in the first place minority shareholders are present in the group’s financial structure at date 0. This question would be easily addressed in a model with ex-ante uncertainty on the subsidiaries’ productivity levels. In such a model, the internal resource reallocation acts as an insurance mechanism against negative shocks that can hit individual subsidiaries. Delegating control to the group’s headquarters would thus be a credible commitment to a resource allocation that is ex-ante optimal.
instance stress the agency costs arising in pyramidal groups while neglecting any benefits that may rationalize their existence. Secondly, we are assuming that the group is free to redistribute assets at $t = 0$ but not date-2 profits. This rules out the possibility that the headquarters’ authority to engage in resource reallocation blunts managerial incentives in individual subsidiaries. We share this assumption with other models of internal capital markets (e.g., Matsusaka and Nanda, 2002; Stein, 1997), with the positive exception of Brusco and Panunzi (forthcoming), where diminished managerial incentives represent the cost of an ex-post efficient resource allocation.

In what follows, we show that resource allocation within a business group also responds to the competitive environments where subsidiaries operate. Hence, establishing an internal capital market may engender substantial strategic advantages or disadvantages. This is a further channel through which resource flexibility may affect a group’s value.

3. Resource allocation with product market competition

In this section we study how the internal resource allocation is affected by competitive conditions in the markets where a group operates. The assumptions are the same as in the basic model presented in section 2, except that now business units face imperfect competition in their respective markets. Subsidiary 1 and subsidiary 2 operate in separate product markets. Thus, they differ in that they may be faced with more or less aggressive competitors. To simplify the analysis, we assume that subsidiary 1 competes in a duopoly market (market 1) while subsidiary 2 is a monopolist in its own market. We denote by $R$ subsidiary 1’s rival.

The timing is the following. At $t = 0$, the headquarters allocates $A_1$ and $A_2$ to subsidiaries 1 and 2. At $t = 1$, each manager writes a contract $\{\alpha_i\}$ with outside investors to raise $I - A_i$ if needed. At $t = 2$ managers of all productive units simultaneously choose their levels of unverifiable effort $e_i$ ($i = 1, 2, R$). At $t = 3$, returns are realized.

Competition in market 1 is modeled as follows. A productive unit receives a return $\pi$ only if its project succeeds and the rival’s project does not, otherwise it earns zero. Under this assumption, unit $i$’s project (with $i = 1, R$) generates a return $\pi$ with probability $e_i (1 - e_j)$ where $i \neq j$ and $e_i$ is firm $i$’s R&D effort. R&D effort thus represents the strategic variable in market 1. All exogenous parameters ($A, I, \pi, \beta$), as well as competitive conditions in both markets, are common knowledge among competitors.

Finally, we make the following assumptions. First, financial contracts are not observed by product market rivals when R&D efforts are chosen. This rules out any commitment effect associated with the choice of contracts. Second, while investors obviously observe $I - A_i$ and thus $A_i$, product market rivals do not (or equivalently, headquarters cannot credibly commit to a given allocation). This assumption rules out the possibility of a strategic allocation of internal funds within the group. In spite of this, we will show that the allocation of internal resources within a corporate group does respond to product market factors.

\footnote{for all subsidiaries but that individual subsidiaries’ shareholders may dislike ex-post.}

\footnote{This is the case, for instance, when R&D for a new product is being carried out, and Bertrand competition takes place between two successful innovators.}

\footnote{Hence, our theory does not rely on Brander and Lewis’ (1986) hypothesis that financial contracts represent credible commitments (i.e., cannot be secretly renegotiated). To neutralize commitment effects, the non-observability of financial contracts by third
We now characterize unit 1’s best response (i.e., its entry decision and post-entry effort) to its rival’s contract, taking the rival’s contract, and thus its level of effort, as given. This best response is a function of internal assets $A_1$. Then we find the optimal resource allocation, which maximizes the group’s value taking the rival’s effort as given.

**Outside finance, internal resources and competition**

The financial contracting problem of unit 2 can be solved along the lines of Lemma 1, with $\pi$ replacing $\pi_2$. The financing problem of a productive unit operating in market 1 is also analogous to the one analyzed in section 2, except that now the unit’s expected payoff is $e_1(1 - e_R)\pi$.\(^{24}\) Hence $(1 - e_R)\pi$ must substitute $\pi_1$ throughout the model, so that increasing $e_R$ is equivalent to reducing $\pi_1$ in the model of section 2. Following directly from Lemma 1, Corollary 1 characterizes the financial contract and the equilibrium effort $e_1$ for any given pair $(e_R, A_1)$:

**Corollary 1.** For any level of the rival’s effort $e_R \in \left[0, 1 - \frac{\sqrt{2/3\pi}}{\pi}\right]$ there exists a threshold level of assets $\tilde{A}_1(e_R) \in \left[I - \frac{\pi^2(1-e_R)^2}{4\beta}, I\right]$ such that:

- if $A_1 \geq \tilde{A}_1(e_R)$ the unit can obtain outside finance and compete in market 1; $\alpha_1^*$ is decreasing in $A_1$; $e_1^*(A_1, e_R)$ is increasing in $A_1$ and decreasing in $e_R$.
- if $A_1 < \tilde{A}_1(e_R)$, the unit is either unprofitable or cannot obtain outside funds, hence it is shut down.

Being confronted with a tougher competitor makes it more difficult for a firm to obtain outside funds, to the extent that an increase in $e_R$ reduces the firm’s profitability and thus the income pledgeable to investors. Having a tougher competitor also weakens managerial incentives, thus reducing effort $e_1$; hence, competition in market 1 is in strategic substitutes. From Lemma 2, it is also immediate that an increase in $e_R$ reduces a business unit’s value $V_1(A_1, e_R)$, but for all $A_1 \geq \tilde{A}_1(e_R)$ – it increases its sensitivity to internal resources $\frac{\partial V_1}{\partial A_1}$.

The headquarters’ resource allocation problem in $t = 0$ is also analogous to the one solved in section 2. Since the allocation of resources is not observable to product market competitors, the headquarters chooses $A_1$ and $A_2$ taking the rival’s effort $e_R$ as given:

$$\max_{A_1, A_2} V_1(A_1, e_R) + V_2(A_2)$$

subject to:

$$A_1 + A_2 = A.$$
Corollary 2. For any $e_R \in \left[ 0, 1 - \frac{\sqrt{2\beta I}}{\pi} \right]$, there exists a threshold level of resources $\bar{A}(e_R)$ such that, if $A \geq \bar{A}(e_R)$, it is optimal for a business group to operate in both markets and assign relatively more resources to the subsidiary facing more intense competition. Hence, $A_1^*(e_R) > A_2^*$, with $A_1^*(e_R)$ increasing in $e_R$. If instead $A < \bar{A}(e_R)$, all resources are diverted to the subsidiary operating in the monopolistic market. The threshold $\bar{A}(e_R)$ is strictly increasing in $e_R$.

Conventional wisdom holds that in business groups cash-flows generated by monopolistic units are used to subsidize those units facing intense competition. Our result departs from this claim in two ways. First, resource flexibility may well encourage a group to swiftly exit a market where competition is toughening: a very high effort expected from competitors may increase the threshold $\bar{A}(e_R)$ well above the group’s resources $A$, which in turn makes it optimal to exit market 1. Hence, very intense competition makes winner-picking rather than cross-subsidization more likely. It is true however that if either competition is not too strong or the group’s assets are large enough, $A$ is larger than the threshold $\bar{A}(e_R)$ and subsidization of the unit facing competition becomes optimal. Yet, in contrast to standard deep-pocket arguments, our cross-subsidization result does not rely on strategic motivations: as the allocation of resources is not observable, unit 1 is not subsidized to affect its competitor’s behavior, but simply because it faces a more problematic access to outside finance and thus more serious incentive problems.

Financially constrained entry of business subsidiaries versus stand-alone firms

We are interested in studying how affiliation to a business group affects a firm’s behavior in the product market. To this aim, we take as a benchmark a stand-alone firm endowed with assets $A^*_2 < I$, and compare its entry (or symmetrically its exit) decision in market 1 with the behavior of a business-group endowed with resources $A < 2I$. From Corollary 1 we know that the stand-alone firm operates in market 1 provided $A_2^* \geq \tilde{A}_1(e_R)$, or: $A \geq 2\tilde{A}_1(e_R)$. Corollary 2 instead states that the group enters (or stays) in market 1 if and only if $A \geq \bar{A}(e_R)$. Both $2\tilde{A}_1(e_R)$ and $\bar{A}(e_R)$ are strictly increasing in $e_R$, i.e. increased competition makes entry less likely. However, the cut-off level of assets below which entry is prevented (exit triggered) displays a different sensitivity to the market’s competitive conditions for a business group with respect to a stand-alone firm. The implications of this are formally stated in the next lemma and illustrated in Figure 3.

Lemma 3. There exists a level of the rival’s effort $\hat{e}_R \in \left( 1 - \sqrt{\frac{8\beta I}{3\pi^2}}, 1 - \frac{3\sqrt{3\beta I}}{2\pi} \right)$, such that $\bar{A}(e_R) > 2\tilde{A}_1(e_R)$ if and only if $e_R > \hat{e}_R$.

Proof. See the Appendix.

[Figure 3 about here]

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25 See Bolton and Scharfstein (1990) for a model where internal resources and financial contracts, being observable, affect the firm’s ability to prey on rivals and to deter predation.

26 We thus assess the impact for a stand-alone unit of affiliation to a firm with equal endowment $\frac{I}{2}$ but facing a less competitive environment.
Resource flexibility within the group is the key to this result. A stand-alone firm exits a market when increased competition either makes it unable to raise funds or drives its net present value to zero. When competition is not intense, cross-subsidization occurs within the group, slackening the unit’s financial constraint, and hence making the entry decision less sensitive to competitive conditions. Thus, \( \bar{A}(e_R) < 2\bar{A}_1(e_R) \). However, while a stand-alone firm is committed to its home market a group’s assets are not, as they can easily be shifted to more profitable affiliates. When very intense competition makes unit 1 much less profitable than its monopolistic partner, such winner-picking effect dominates, accounting for an increased sensitivity of the cut-off \( \bar{A}(e_R) \) to competitive conditions. Thus, for high levels of \( e_R \), \( \bar{A}(e_R) \geq 2\bar{A}_1(e_R) \).

To summarize, resource flexibility need not make the group less prone to stay in market 1 with respect to a stand-alone firm. For all \( e_R < \hat{e}_R \), group membership, providing access to deeper pockets, makes entry more likely, while for \( e_R \geq \hat{e}_R \), resource flexibility translates into winner-picking, making entry in a highly competitive market less likely. In other words, resource flexibility makes a business group more prone to either swiftly exit a market in response to increased competition or to “stay and fight”, in line with recent empirical findings by Khanna and Tice (2001).

4. The strategic effects of business group affiliation

Conventional wisdom suggests that affiliation to a business group, providing access to the group’s deep pockets, makes a financially constrained firm less vulnerable to entry deterrence and predatory practices of rival firms. According to this view, the resource flexibility ensured in an internal capital market would represent an important strategic advantage of group affiliation. However, things are not so straightforward, as our formal analysis shows.

**Does group affiliation facilitate entry?**

Assume that firm \( R \) is the incumbent in market 1 and can commit to a higher R&D effort. For instance, firm \( R \) has deep pockets \( (A_R > I) \) and at \( t = 0 \) can make a costly verifiable investment in expertise to reduce its marginal cost of R&D effort and thus shift its reaction function upwards until entry is deterred. Of course, the more difficult is deterring entry, the less likely is firm \( R \) to adopt a costly commitment to a tough R&D strategy. Following the analysis of section 3, we define here the cut-off levels of the rival’s effort that deter entry in market 1. For a stand-alone firm endowed with assets \( \frac{4}{2} \), entry is deterred when the rival’s effort is expected to lie above \( \bar{e}_R \equiv \bar{A}_1^{-1} \left( \frac{4}{2} \right) \). For a business group with resources \( A \) and enjoying a monopoly in a second market, entry is deterred if the rival’s effort is expected to lie above \( \bar{e}_R \equiv \bar{A}^{-1}(A) \). Lemma 3 immediately implies the following result:

**Corollary 3.** There exists a level of assets \( \hat{A} \in (0, 2I) \) such that \( \bar{e}_R(A) > \bar{e}_R \left( \frac{4}{2} \right) \) if and only if \( A < \hat{A} \).

---

27Khanna and Tice (2001) study how multidivisional firms and stand-alone units in the discount department store business reacted to Wal-Mart’s entry into their markets between 1975 and 1996. They find that multidivisional firms differ in their response to new entry: compared to stand-alone firms, they “appear to be quicker in making the decision to either exit the discount business or stay and fight” (p. 1491) and, conditional on staying, invest more in the discount business than their focused counterparts.
This result implies that group affiliation may both discourage and invite entry deterrence by incumbent firms. A stand-alone firm with poor resources \( \left( \frac{A}{2} < \hat{A} \right) \) is vulnerable to entry deterrence making its financial constraints bind. Affiliation to a business group enjoying monopoly power in another market guarantees access to deep pockets, and hence reduces the sensitivity of the firm’s financing constraints to the rivals’ actions. In this case, the internal capital market acts as a credit line contract aimed at discouraging predation by competing firms. Hence, resource flexibility alleviates the difficulty of financially constrained entry for profitable but cash-poor stand-alones.

However, a cash-rich stand-alone firm does not benefit from affiliation to a group enjoying monopoly elsewhere. In fact, in this case winner-picking considerations may well induce the headquarters to channel the firm’s resources to its more profitable monopolistic affiliate and exit market 1 if competition is toughening there. Hence, resource flexibility encourages the rival’s entry deterrence behavior, whereas instead the stand-alone’s commitment of resources to its home market would discourage predatory practices. This result is closely related to Matsusaka and Nanda (2002), where the flexibility ensured by internal capital markets always entails a commitment cost for conglomerate divisions.28

**Group affiliation and R&D strategy**

To analyze the strategic effects of group affiliation we now consider the case where entry has occurred, and characterize a subsidiary’s R&D strategy as opposed to that of a stand-alone firm endowed with \( \frac{A}{2} \). In fact, to the extent that internal resources determine managerial incentives, the R&D strategy of the stand-alone firm may significantly change upon affiliation to a business group.

We know from Corollary 2 that - conditional upon entering market 1 - the business group always subsidizes unit 1, hence

\[
A_1^*(e_R) > \frac{A}{2} \quad \text{for all } e_R \in (0, \sigma_{R}(A)].
\]

This internal cash infusion in turn reduces firm 1’s need for outside finance and thus improves its effort incentives:

\[
e_1(A_1^*(e_R)) > e_1 \left( \frac{A}{2} \right).
\]

Therefore, affiliation to the group - provided entry occurs - shifts the firm’s reaction function upwards, and is thus a credible commitment to adopt a tougher R&D strategy. This is in line with existing empirical evidence: Dunne, Roberts, and Samuelson (1988) find that although conglomerates represent only 40% of new entrants, they account for 50% of the entrants’ output in all the industries analyzed in their sample.

[Figure 4 about here]

28In Matsusaka and Nanda (2002) cross-subsidization never takes place, hence it is always the case that fewer resources are assigned to a division when new competitors enter its market. The division is thus unable to credibly commit to an entry-deterring over-investment. This explains why, in stark contrast with our results, access to an internal capital market always brings a strategic disadvantage in that paper.

16
Indeed, group affiliation also affects the slope of a firm’s reaction function. By deriving a stand-alone’s optimal effort $e_1^*$ (characterized in Corollary 1) with respect to $e_R$ one can obtain the slope of the best reply function:

$$\frac{de_1}{de_R} = \frac{\partial e_1^*}{\partial e_R} \bigg|_{A_1} = \frac{A}{7} < 0$$

The subsidiary’s best reply function $e_1^*(e_R)$ can instead be obtained by plugging $A_1^*(e_R)$ into $e_1^*(A_1, e_R)$. Its slope is therefore:

$$\frac{de_1^*(e_R)}{de_R} = \frac{\partial e_1^*}{\partial e_R} \bigg|_{A_1=A_1^*(e_R)} + \frac{\partial e_1^*}{\partial A_1} \frac{\partial A_1^*(e_R)}{\partial e_R} < 0$$

The second term in the above expression captures the reallocation effect. If the rival gets tougher, the headquarters subsidizes unit 1 ($\frac{\partial A_1^*(e_R)}{\partial e_R} < 0$); the additional internal resources alleviate the manager’s incentive problem and induce a higher effort ($\frac{\partial e_1^*}{\partial A_1} > 0$). Note that $A_1^*(e_R) > \frac{A}{2}$ and $\frac{\partial^2 e_1^*}{\partial e_R \partial A_1} > 0$ imply:

$$\frac{de_1^*}{de_R} > \frac{de_1}{de_R}$$

The impact of business group affiliation on R&D strategy is summarized in the following proposition and illustrated in Figure 4.

**Proposition 2.** Affiliation to a business group - conditional upon its entry in market 1 - shifts a firm’s reaction function upwards and makes it flatter.

Both results justify why business group affiliates are often perceived as dangerous entrants in the product market.

**Accommodated entry and protection from strategic commitments**

As is well known, in case of accommodated entry an incumbent’s strategic incentives to commit to a high or a low effort depend on the nature of competition, that is on the slope of the entrant’s best reply function.\(^{29}\) If R&D efforts are strategic substitutes as is the case here, an incumbent firm has an incentive to commit to a high effort in order to make the entrant softer and increase its own profits. This strategic commitment is more valuable for the incumbent the larger is the strategic effect, i.e. the reduction in the entrant’s effort following the increase in its own effort. Having a flatter reaction function, a business group subsidiary discourages the incumbent from adopting such costly commitment to high R&D levels. Hence, when competition is in strategic substitutes, business-group affiliation is a defensive response to rivals threatening to take aggressive pre-commitments.

\(^{29}\)We refer to Fudenberg and Tirole (1984) for a taxonomy of the incumbent’s strategic incentives when the entrant firms are stand-alone. Their paper shows that a commitment strategy to be tough is adopted when the entrant’s reaction function is downward-sloping. Conversely, a commitment to be soft and thus make the incumbent non-aggressive is optimal when the entrant’s reaction function is upward-sloping and entry is accommodated.
**Strategic complementarity and business group affiliation.** One may alternatively ask whether business group affiliation is desirable when R&D efforts are strategic complements.\(^\text{30}\) This requires setting up a different model where strong R&D spillovers make a firm’s discovery more profitable when its rivals invest more in R&D.\(^\text{31}\) For instance, one might assume that expected R&D profits are:

\[
e_1 \pi + e_1 e_R \Delta \pi
\]

with \(\Delta \pi > 0\) capturing the effect of R&D spillovers, so that a firm’s productivity of effort \(\pi + e_R \Delta \pi\) is increasing in the rival’s effort. It is easy to see that in this model cross-subsidization would still make a subsidiary’s reaction function flatter with respect to a stand-alone’s, the reason being now that resources are drained from a unit whose rival is exerting a larger effort. This of course has a different strategic implication: an incumbent accommodating entry may want to commit to a high R&D effort so as to spur the entrant’s R&D; however, being less responsive to the rival’s effort, a business group subsidiary discourages such pro-collusive strategic move. This represents a strategic cost of business group affiliation when competition is in strategic complements.

**Discussion and related literature**

As already discussed in the introduction and in section 2, our results have an obvious relation with the large body of research on internal capital markets. In this section we draw the relation to other strands of literature.

**Financially Constrained Entry.** Our results on stand-alone entry versus business group entry draw on the prior intuition that when external capital markets are not perfect, deep pockets are a source of competitive advantage. Deep pockets allow a firm to engage in predatory practices to drive its rivals out of the market, or alternatively to withstand predation (Telser, 1966), whereas financial constraints undermine a firm’s ability to endure a price war (Benoit, 1984).\(^\text{32}\) Hence, a cash-poor firm may want to sign a credit line contract with a bank to limit the scope for predation (Bolton and Scharfstein, 1990). Our paper builds a bridge between these works and the literature on internal capital markets, and questions the informal claim that access to an internal capital market always makes a firm less vulnerable to entry deterrence and predatory practices. The model proposed predicts that while in general business groups tend to subsidize units faced with more intense competition, they are more prone than cash-rich stand-alones to exit a market where tough predatory practices are put in place. Hence, while affiliation to a group shields cash-poor firms from entry deterrence, it can only harm those firms with enough liquidity to withstand predatory practices as stand-alones.

\(^{30}\)In a previous version of this paper (Cestone and Fumagalli, 2001), we modelled R&D efforts along the lines of Aghion, Dewatripont, and Rey (1998) so that strategic substitutability and complementarity in R&D efforts endogenously arose within the model.

\(^{31}\)In a model where firms compete in the product market and R&D spillovers exist, D’Aspremont and Jacquemin (1988) find in fact that R&D investments may be either strategic substitutes or complements depending on how large R&D spillovers are.

\(^{32}\)This early papers have spurred a considerable amount of research on the interaction between financial structure and product market behavior. Poitevin (1989) has argued that entrants are more leveraged than incumbents, and thus more vulnerable to predation, because they bear more severe asymmetric information vis-à-vis investors. In Maksimovic (1988) and Spagnolo (2000), corporate financial policy can hamper or favor tacit collusion among competing firms. Cestone and White (2003) show instead that the financial contracts signed by incumbents can deter entry by affecting the credit market behavior of investors towards entrant firms.
One might expect that external finance is more difficult to obtain in sectors characterized by intangible asset investments and severe asymmetric information; an empirical implication of our model is thus that business group affiliation is particularly effective in favoring entry in R&D-intensive industries. This is in line with the stylized fact that multinational enterprises are often present in high-tech industries (see e.g. Caves, 1982), and is also consistent with Brock’s (1986) finding that the only method of entry into the computer industry in the 70s was by subsidization of the computer effort from other units of the corporation, as was the case for General Electric and RCA.

Conglomerate Power. Our paper also provides a formal framework to address concerns that conglomeration (here in the form of group affiliation) may be a source of market power. The competitive behavior of multimarket corporations has been the object of a literature dating back to at least Edwards (1955). Our results are obviously related to the idea that the anti-competitive potential of conglomeration relies on cross-subsidization, whereby a firm uses its profits from one market to support predatory practices in another market. Similar concerns have been expressed with respect to multinational enterprises, and more recently to privatized European utilities, which have been accused of financing aggressive pricing abroad using profits from protected home markets.

Probably the main lesson from our formal analysis is that one should be cautious about associating anti-competitive effects to business groups’ presence in a market. For instance, Corollary 3 gives conditions under which group affiliation allows a firm to overcome entry deterrence strategies of deep pocketed incumbents, and thus helps reduce industry concentration. Group membership can also represent the only means of entry in markets which are already dominated by business groups. Furthermore, entrant business groups may well promote competition by adopting more aggressive strategies than stand-alone firms would (see Proposition 2). Of course, once business groups have entered a market and enjoy monopoly rents elsewhere, they also bear an anti-competitive potential, to the extent that access to deep-pockets and commitment to tough strategies may soon turn them into dangerous predators driving rivals out of the market. It is thus less obvious whether the pro-competitive effect of group entry lasts for long. Which effect is the most relevant in practice is largely an empirical question, on which the available evidence is not conclusive.

Multimarket Spillovers. A main implication of our model is that internal resource flexibility within business groups generates multimarket spillovers, so that factors affecting competition in one market also affect the business group’s (and its rivals’) strategies in a second market. In fact, our paper is closely related to Bulow, Geanakoplos, and Klemperer’s (1985) seminal formalization of multimarket spillovers. In their model, a firm is a monopolist in one market

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33 See also Caves (1982), Teece (1982), as well as van Witteloostuijn (1984) for a broad survey of the topic.
34 We thank an anonymous referee for spurring these considerations.
35 For instance, Lawrence (1991) shows that imports and entry tend to be low in Japanese markets where keiretsu-affiliated firms have large market shares. Weinstein and Yafeh (1995) explain this evidence with their finding that keiretsu-affiliated firms compete more aggressively. However, legal scholars have argued that the pro-competitive effect of groups often prevails, and is indeed the reason why incumbents react to group entry by bringing complaints to competition authorities. For instance, in 1981 the US firm Zenith accused Matsushita and other Japanese firms of subsidizing predatory prices in the US television set market with profits from domestic sales, but eventually the Supreme Court ruled out a case for predation. Recent studies have confirmed ex-post that this decision was well-taken: Japanese firms never gained dominance of the US market, and the price of TV sets continued to decline in the post-entry period (see Elzinga, 1999).
(say, market 2) and competes in strategic substitutes (or complements) in another, oligopoly market (market 1). As production costs are interdependent, a change of conditions in the monopoly market leads the firm to reoptimize its overall product market strategies, and thus also affects the equilibrium in the oligopoly market. For instance, if there are joint economies of scope across markets, a positive demand shock in market 2 leads the firm to adopt a more aggressive strategy in market 1 as well, which may possibly drive its rivals out.

In our model, the sign of multimarket spillovers depends on the expected financial reaction of the business group to industry shocks. Following a shock in market 2, the group’s headquarters reoptimizes the internal resource allocation, which affects subsidiary 2 and subsidiary 1’s external financing problem, and thus their respective product market strategies. Knowing how resources are reallocated in response to changes in different markets is thus crucial to assess how multimarket effects work under the influence of internal capital market phenomena. This is where our paper makes a novel contribution to the understanding of multimarket competition.

5. Conclusion

Competition authorities have often pointed at financial factors and internal capital market phenomena as the source of the anti-competitive impact of business groups. Yet, antitrust decisions so far could not rely on a formal assessment of financially-driven multimarket effects. Our paper makes a first attempt at filling this gap. To this purpose, we have analyzed the functioning of internal capital markets in business groups, showing that both winner-picking and cross subsidization may occur in value maximizing groups. Secondly, we have studied the strategic effects of group membership, arguing that the resource flexibility brings both strategic benefits and costs.

There are some issues that our paper does not deal with. Our model does not aim at explaining why a pyramidal structure is put in place. Taking the group’s structure as exogenous, we study its internal resource allocation process, and the implications of this for the member firms’ competitive behavior. In fact, to focus on the strategic impact of resource flexibility, we have abstracted from the agency costs of finance that business groups may face. To this aim, we have assumed a pyramidal structure where existing minority shareholders are silent in the face of potential expropriation of internal assets at date 0, and new financiers have no reason to fear the tunneling of funds lent at date 1 (see footnote 13). As a consequence, in the non-strategic setting of section 2 internal capital markets only bring benefits, whereas the only costs of resource flexibility arise in the strategic environment of sections 3 and 4.

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36 Consider for instance the claim that an incumbent group can extend its monopoly power across industries. Indeed, the potential for anti-competitive spillovers greatly depends on the group’s expected financial reaction to entry: a rival will be discouraged from (encouraged to) entering if he expects cross-subsidization of (resource drainage from) the subsidiary facing his entry. Thus, an empirical prediction of our model is that business groups are better at deterring entry of small firms, as this is expected to spur a cross-subsidization response, but invite large scale entry, which rather leads the group to scale down its presence in a market.

37 For instance, recently the European Commission has maintained that following a merger with General Electric, Honeywell would have been able to adopt predatory practices relying on GE’s financial strength. Similar concerns have been expressed by the Italian competition authority with respect to Italy’s Enel using its monopoly electricity profits to buy its way into the telecoms sector (see The Economist, Special Report on Privatization in Europe, June 29th 2002, pp. 71-73).

38 Other papers (e.g., Bebchuk, Kraakman, and Triantis, 2000) have instead focussed on minority shareholders’ expropriation in pyramidal groups and stressed the implied agency costs, thus raising the puzzle of why pyramids exist in the first place.
To make predictions on when business groups arise as an optimal organization structure, one should consider the strategic impact of resource flexibility, but also take into account the potentially larger costs of finance that group subsidiaries may face. Our model could be extended in this direction by assuming that subsidiaries run projects of variable, rather than fixed, size. In this context, having a subsidiary raise external funds does not ensure that those are used to run a larger project in that subsidiary rather than be tunneled to a less productive unit. Outside investors thus fear expropriation, as the value of their claim in a subsidiary depends on the latter’s size. As a result, in such a model date-1 tunneling would be an issue, making external finance potentially more costly for group subsidiaries than for stand-alone firms. We leave this topic for future research (but see Brusco and Panunzi, forthcoming, for an analysis of the costs and benefits of a centralized control on resources that abstracts from strategic effects).

Our model is not one of diversification; hence, we cannot make predictions on the impact of diversification on groups’ internal capital markets and member firms’ product market behavior. Finally, we do not ask which factors lead entrepreneurs to put in place business groups rather than multidivisional firms; Wolfenzon (1999) and Nicodano (2003) are the first attempts at comparing these two organizational forms.
Appendix

Proof of Lemma 2.

(i) From Lemma 1 we know that $\bar{A}_i$ can take two values. In case 1, $\bar{A}_i \equiv \frac{\pi^2}{2\beta}$ hence $\frac{\partial \bar{A}_i}{\partial \pi_i} < 0$. In case 2, $\bar{A}_i \equiv A^0_i$ where $A^0_i \equiv \frac{\pi^2 - \beta \pi_i \sqrt{\frac{\pi^2}{2\beta} - \frac{\pi_i}{\beta}}}{\pi_i}$ solves $u_i(A_i) = 0$. Trivial algebra shows that $\frac{\partial A^0_i}{\partial \pi_i} < 0$.

(ii) We then prove that the value function is shifted upwards as $\pi_i$ increases. In the region $A_i > \bar{A}_i$:

$$dV_i = \frac{\partial V_i}{\partial \pi_i} + \frac{\partial e_i^*}{\partial \pi_i} \left[ \frac{\partial V_i}{\partial e_i} \right] = e_i^* + \frac{\partial e_i^*}{\partial \pi_i} \left[ \pi_i - \beta e_i^* \right] > 0$$

This is always true as $e_i^* \in \left( \frac{\pi}{2\beta}, \frac{\pi_i}{\beta} \right)$ implies $|\pi_i - \beta e_i^*| \geq 0$ and $\frac{\partial e_i^*}{\partial \pi_i} = \frac{e_i^*}{\beta e_i^* - \pi_i} > 0$. In the region $A_i \leq \bar{A}_i$, it can be easily checked that $V(\pi_1) \leq V(\pi_2) \forall \pi_1 < \pi_2$.

(iii) Last, we compare the slopes of $V_1$ and $V_2$ (with $\pi_1 < \pi_2$) when both units receive the same amount of internal resources: $A_1 = A_2 = A^0 \geq \bar{A}_1$. Remember that if $A^0 \in \left( \bar{A}_1, \bar{A}_2 \right)$, both value functions take the form: $V_i = e_i^* \pi_i - \frac{\beta}{2}(e_i^*)^2 - I$. One can then show:

$$\frac{\partial^2 V_i}{\partial A_i \partial \pi_i} = \frac{\partial^2 e_i^*}{\partial A_i \partial \pi_i} \left[ \pi_i - \beta e_i^* \right] + \frac{\partial e_i^*}{\partial A_i} \left[ 1 - \beta \frac{\partial e_i^*}{\partial \pi_i} \right]$$

$$= \frac{\partial^2 e_i^*}{\partial A_i \partial \pi_i} \left[ \frac{\partial V_i}{\partial e_i} \right] + \frac{\partial e_i^*}{\partial A_i} \left[ \frac{\partial^2 V_i}{\partial e_i \partial \pi_i} \right] \leq 0,$$

with $\frac{\partial^2 V_i}{\partial A_i \partial \pi_i} = 0$ only if $A_i = I$. The last inequality always holds as $e_i^* \in \left( \frac{\pi}{2\beta}, \frac{\pi_i}{\beta} \right)$ implies that: $\frac{\partial^2 e_i^*}{\partial A_i \partial \pi_i} = -\frac{\pi_i}{(2\beta e_i^* - \pi_i)^3} < 0$ and $\left[ 1 - \beta \frac{\partial e_i^*}{\partial \pi_i} \right] = \frac{\beta e_i^* - \pi_i}{2\beta e_i^* - \pi_i} \leq 0$. Note that a decrease in productivity has two effects on the slope of the value function. The first term represents the incentive effect: a smaller $\pi_i$ implies a more serious incentive problem, thus a higher (positive) impact of additional resources on effort $\left( \frac{\partial^2 e_i^*}{\partial A_i \partial \pi_i} < 0 \right)$ and hence on the unit’s value. The second term represents the convexity effect: the increase in effort due to additional resources $\left( \frac{\partial e_i^*}{\partial A_i} \right)$ has a stronger impact on the unit’s value when $\pi_i$ is smaller, as $\frac{\partial^2 V_i}{\partial e_i \partial \pi_i} < 0$.

Finally, in $A^0 = \bar{A}_1$, $\frac{\partial V_2}{\partial A_2}$ exists but $\frac{\partial V_1}{\partial A_1}$ does not. However, there exist $V_1^- \equiv 0$ and $V_1^+ > \frac{\partial V_2}{\partial A_2} \bigg|_{A_2 = A^0} > 0$.

All this implies:

$$\frac{\partial V_1}{\partial A_1} \bigg|_{A_1 = A^0} > \frac{\partial V_2}{\partial A_2} \bigg|_{A_2 = A^0} \forall \pi_1 < \pi_2$$

Q.E.D.

Proof of Proposition 1.

Let us assume first that $A \geq \bar{A}_2$. If not, total resources would not even allow the most productive unit to raise funds. Second, if $A < \bar{A}_1 + \bar{A}_2$, only one unit can be financed. Obviously, it is never optimal to shut the most productive unit, hence the solution is trivial: $A_1 = 0$ and $A_2 = A$. The rest of the proof thus focuses on the case $A \geq \bar{A}_1 + \bar{A}_2$.

The proof proceeds in three steps.
Step 1 – The headquarters must first decide whether both subsidiaries should operate, which requires setting \( A_1 \geq \tilde{A}_1 \) and \( A_2 \geq \tilde{A}_2 \), or whether the less productive subsidiary should be shut, in which case \( A_1 = 0 \) and \( A_2 = \). Step 2 – Suppose the headquarters decides to operate both subsidiaries. Conditional on this, the optimal allocation \( A_1^* (A_2^* = A - A_1) \) solves:

\[
\max_{A_1} V_1(A_1) + V_2(A - A_1)
\]

subject to:

\[
A_1 \geq \tilde{A}_1, A - A_1 \geq \tilde{A}_2
\]

Operating both subsidiaries is optimal provided \( WP(A) > 0 \). From the envelope theorem and the strict concavity of \( V_i(.) \) over \( [\tilde{A}_2, I] \), it follows that \( WP(A) \) is strictly decreasing in \( A \). In particular, for \( A = 2I \) it is always optimal to start both units, hence: \( WP(2I) \leq 0 \). Assume now that \( WP(\tilde{A}_1 + \tilde{A}_2) > 0 \). By continuity, there exists a threshold level of resources \( \overline{A} \in (\tilde{A}_1 + \tilde{A}_2, 2I] \) such that both subsidiaries are kept open if and only if \( A \geq \overline{A} \), while subsidiary 1 is shut for \( A < \overline{A} \). From the strict concavity of \( V_i(.) \) over \( [\tilde{A}_2, I] \) and \( \frac{\partial V_i}{\partial \pi_1} \bigg|_{A_1=A_1^*} > 0 \), it follows that \( \frac{\partial \pi}{\partial \pi_1} < 0 \).

Consider now the case where \( WP(\tilde{A}_1 + \tilde{A}_2) \leq 0 \). It is straightforward that in this case \( \overline{A} = \tilde{A}_1 + \tilde{A}_2 \), i.e. for all \( A \geq \tilde{A}_1 + \tilde{A}_2 \) it is never optimal to shut down subsidiary 1. It can be checked that: \( \frac{\partial \pi}{\partial \pi_1} < 0 \). Q.E.D.

Step 3 – We now investigate when it is indeed optimal to operate both subsidiaries (and have cross-subsidization) rather than shut the less productive one down (i.e., winner-picking). We define:

\[
WP(A) \equiv V_2(A) - [V_1(A_1^*) + V_2(A - A_1^*)]
\]

Lemma A1. There exists a threshold level of \( \pi_2 \), \( \tilde{\pi}_1 \in \left( \sqrt{\frac{8I}{3}}, \pi_2 \right) \), such that \( WP(\tilde{A}_1 + \tilde{A}_2) > 0 \), and hence \( \overline{A} \) is a threshold level of \( \pi_1 \), \( \tilde{\pi}_1 \), if and only if \( \pi_1 < \tilde{\pi}_1 \). In other words, for low levels of \( \pi_1 \) winner-picking occurs even if the group’s resources are sufficient to keep both units open.

Proof. By the assumption \( \pi_2 > \beta I (2 + \sqrt{2}) \), when \( \pi_1 = \pi_2 \) it is \( WP(\tilde{A}_2 + \tilde{A}_2) < 0 \). Second, recall that if \( \pi_1 = \sqrt{\frac{8I}{3}}, V_1(\tilde{A}_1) = 0 \). Hence, \( WP(\tilde{A}_1 + \tilde{A}_2) = V_2 \left( \tilde{A}_1 + \tilde{A}_2 \right) - V_2 \left( \tilde{A}_2 \right) > 0 \). As \( WP(\tilde{A}_1 + \tilde{A}_2) \) is continuous and strictly decreasing in \( \pi_1 \) over the interval \( \left[ \sqrt{\frac{8I}{3}}, \pi_2 \right] \), the result follows. Q.E.D.
Proof of Lemma 3.

We show first that the thresholds $\bar{A}(e_R)$ and $2\bar{A}_1(e_R)$ coincide at the extremes of the interval $\left[0, 1 - \frac{\sqrt{2}\beta I}{\pi}\right]$ over which $e_R$ varies. At $e_R = 0$, units 1 and 2 are identical, hence $\bar{A}_1(0) = \bar{A}_2 \equiv I - \frac{\pi}{\sqrt{2}\beta I}$. The assumption $\pi^2 > \beta I (2 + \sqrt{2})$ ensures that the group operates in both (monopoly) markets provided internal resources are enough to allow both units to raise external funds, hence: $\bar{A}(0) = 2\bar{A}_1(0) = 2 \left(1 - \frac{\pi^2}{\sqrt{2}\beta I}\right)$. At $e_R = 1 - \frac{\sqrt{2}\beta I}{\pi}$, competition is so intense that the value of a duopoly unit is 0 at the first best: $V_1(I) = 0$. Thus, the headquarters is indifferent between operating both units and diverting all resources to unit 2 provided $\bar{A} \geq 2I$: a stand-alone firm is indifferent between starting the project and shutting it down provided $\frac{\pi^2}{\sqrt{2}\beta I} \geq I$. Therefore, $\bar{A}(e_R)$ and $2\bar{A}_1(e_R)$ for all $e_R \in \left(0, 1 - \frac{\sqrt{2}\beta I}{\pi}\right)$. Lemma A1 implies that if $e_R \leq 1 - \frac{\pi}{\sqrt{2}\beta I}$, then it must be $\bar{A}(e_R) = \bar{A}_1(e_R) + \bar{A}_2$. In this case it is straightforward that $\bar{A}(e_R) < 2\bar{A}_1(e_R)$, with $\bar{A}(e_R)$ flatter than $2\bar{A}_1(e_R)$.

If instead $e_R > 1 - \frac{\pi}{\sqrt{2}\beta I}$, it is: $\bar{A}(e_R) > \bar{A}_1(e_R) + \bar{A}_2$. To investigate whether it is $\bar{A}(e_R) < 2\bar{A}_1(e_R)$ or $\bar{A}(e_R) \geq 2\bar{A}_1(e_R)$, we study the sign of $WP\left(2\bar{A}_1(e_R)\right)$. In fact, $\bar{A} > 2\bar{A}_1 \iff WP\left(2\bar{A}_1(e_R)\right) > 0$, whereas $\bar{A} \leq 2\bar{A}_1 \iff WP\left(2\bar{A}_1(e_R)\right) \leq 0$.

Three distinct cases arise:

1. If $e_R \in \left(1 - \frac{2\pi}{3\beta}, 1 - \frac{\sqrt{8\beta I}}{3\pi^2}\right)$, $\bar{A}_1(e_R) = A^*_1(e_R) = I - \frac{\beta I}{4\pi^2}$. Tedious calculations show that:

$$WP\left(2\bar{A}_1\right) = \frac{-\Pi^2 - 2\beta I - \pi \sqrt{\pi^2 + 4\beta I - 2\Pi^2} + \pi^2 \sqrt{1 - (1 - e_R)^2}}{4\beta} < 0,$$

where we have defined: $\Pi \equiv \pi (1 - e_R)$.

2. Consider now higher levels of $e_R$: $e_R \in \left[1 - \sqrt{\frac{8\beta I}{3\pi^2}}, 1 - \frac{3\sqrt{3}\beta I}{2\pi}\right]$. One can compute: $\bar{A}_1(e_R) = A^*_1(e_R) = \frac{\Pi^2 - \beta I + 2\Pi \sqrt{\Pi^2 - 2\beta I}}{\beta}$, and:

$$WP\left(2\bar{A}_1\right) = \frac{I/2 - \Pi^2/4\beta + \pi \sqrt{\pi^2 - 12\beta I + 8\Pi^2 - 8\sqrt{(\Pi^2 - 2\beta I)}}}{4\beta} - \frac{-\sqrt{(\pi^2 + \Pi^2)\left(\pi^2 + 9\Pi^2 - 16\beta I - 8\sqrt{(\Pi^2 - 2\beta I)}\right)}}{4\beta}.$$  

It can be checked that $WP\left(2\bar{A}_1\right) < 0$ in $e_R = 1 - \frac{\sqrt{8\beta I}}{3\pi^2}$ and $WP\left(2\bar{A}_1\right) > 0$ in $e_R = 1 - \frac{3\sqrt{3}\beta I}{2\pi}$ (using assumption 1 that $\pi^2 < 4\beta I$), with $WP\left(2\bar{A}_1\right)$ strictly concave in $e_R$. It follows that there exists a threshold $\bar{e}_R \in \left(1 - \sqrt{\frac{8\beta I}{3\pi^2}}, 1 - \frac{3\sqrt{3}\beta I}{2\pi}\right)$ such that $WP\left(2\bar{A}_1\right) > 0$ if and only if $e_R \in \left(\bar{e}_R, 1 - \frac{3\sqrt{3}\beta I}{2\pi}\right)$.

3. Finally, for $e_R \in \left(1 - \frac{3\sqrt{3}\beta I}{2\pi}, 1 - \frac{\sqrt{2}\beta I}{\pi}\right)$ it is:
\[ WP \left( 2\tilde{A}_1 \right) = \frac{\pi^2}{4\beta} - I + \frac{3\Pi^2}{4\beta} \frac{\Pi \sqrt{\Pi^2 - 2\beta I}}{\beta} - \frac{\sqrt{(\pi^2 + \Pi^2) \left( \pi^2 + 9\Pi^2 - 16\beta I - 8\Pi \sqrt{\Pi^2 - 2\beta I} \right)}}{4\beta} > 0. \]

From 1, 2, and 3, it follows that \( \tilde{A}(e_R) > 2\tilde{A}_1(e_R) \) if and only if \( e_R \in \left( e_R, 1 - \frac{\sqrt{2\beta I}}{\pi} \right) \). Q.E.D.
References


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Figure 1: Time line

0 1 2 3
Subsidiaries raise I-Ai on external capital market
Subsidiary managers choose effort ei
HQ allocates internal resources
Returns accrue
Figure 2: Value functions of subsidiaries 1 and 2
Figure 3: Entry thresholds of a business unit (thick curve) and a stand-alone (thin curve)
Figure 4: The dashed and continuous curves represent, respectively, the stand-alone’s and the subsidiary's R&D strategy.