Educational Opportunity and Income Inequality

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Abstract

Since World War II, the United States government has made improved access to higher education a priority. This effort has substantially increased the number of people who complete college – generally thought to be a good thing. We show, however, that such policies can actually increase income inequality. The mechanism that drives our results is the ‘signaling’ role of education first explored by Spence (1973). We focus on government policies that reduce the effective interest rate on borrowing for education. When borrowing for education is difficult, lack of a college education could mean that one is either of low ability or high ability but has low financial resources. Wages and income reflect the presence of high ability individuals in the uneducated pool. When government programs make borrowing easier, high ability types get educated and leave the uneducated pool. We demonstrate our argument by solving for the

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relationship between the effective interest rate and income inequality in the steady state of a dynamic asymmetric information model.

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“[With the G.I. Bill, ] for the first time, the link between income and educational opportunity was broken.” — Diane Ravitch in The Troubled Crusade: American Education 1945-1980 (1983)

1 Introduction

Since World War II, the U.S. government has made improved access to higher education a priority, providing a package of grants, subsidized loans, subsidized ‘work-study’ jobs and other financial devices to make it possible for large numbers of Americans of modest means to get a post-secondary education. This effort has been largely successful; between 1947 and 1999, the percentage of people 25 years old and over who have completed 4 or more years of college has increased from 5.4% to 23.6%. Federal aid to postsecondary students in the 1998-1999 schoolyear amounted to almost $46 billion.1

Even still, many policy-makers continue to think that college tuition remains a substantial and possibly insurmountable financial burden for American families. Indeed, five bills directly addressing financial assistance for postsecondary education were proposed in Congress during 2001.2

In general, such policies are not all that controversial. While many liberal social policies are criticized for promoting equal outcomes, reducing financial constraints for education and thus allowing more Americans to go to college promotes equal opportunities. Without this type of help from the government, the poor would be locked in a cycle of poverty, in low-skill, low-pay jobs and unable to make the necessary investments in their children’s education.

However, not all the effects of increased access to education are benign. In this paper, we show that reducing the effective interest rate on borrowing for education can increase income inequality between skilled and unskilled workers. The mechanism that drives our results is the ‘signaling’ role of education first explored by Spence (1973). The financial cost of education generates pooling of types, reducing the value of education as a signal. Lack of education, in equilibrium, means either low ability or lack of financial resources. As financing becomes increasingly available, income inequality increases.

How important an effect is this? We ask the reader to think about the following question. Suppose you meet two people without any postsecondary education, one born in 1915 and the other born in 1975. Is your inference about their abilities relative to their cohorts the same? We argue that any inference about their lack of college education should be quite

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1See College Board (1999b) for the complete breakdown.
2See www.senate.gov.
different. In line with the model’s predictions, very little information is conveyed about the ability of the individual who grew up in a period with few opportunities, while a stigma is associated with a lack of education in more recent times.\(^3\)

We consider a world where individuals are characterized by their ability type (high or low) and the amount bequeathed to them. Higher education is costly both in money and in effort – the effort required is higher for low ability types. We assume sufficient conditions so that only high types want an education but must pay for it. The lower the initial endowment of an individual the more she must borrow to get an education. As a consequence, education is not appealing for individuals with low initial endowments. Thus an individual who has no postsecondary education could be either a high or low type. We then embed this decision problem in a dynamic model, endogenizing individual endowments of wealth through bequest behavior. This permits us to analyze the path of the economy as well as the specific mechanism the generates our results. As we reduce the interest rate, more and more of the poor get educated in the steady state, lowering the average quality of the low education types, and increasing income inequality. In addition, we find that the initial distribution of wealth affects the steady state distribution of educational attainment.

In the model, education performs a signaling role only. As in Spence’s original work, this is for expository purposes only. The effects we characterize manifest themselves even in richer models where education has direct productivity enhancing effects as well.

We take the position that signaling can play a significant role in educational decisions. Bedard (2001) also takes this position, examining the effect of increased access to universities on individuals’ incentives to drop out of high school. She provides empirical evidence that signaling has a role in determining behavior. Her work motivates an explicit consideration of how wealth constraints affect education decisions in a dynamic setting.

We find that the dynamics separate into two effects, one which consists of bequest transitions between generations and one which consists of wage determination within a generation. The combination leads to our main conclusions. Firstly, reducing the cost of loans should increase income inequality for the cohorts affected as well as for the steady state. Secondly, the magnitude of tuition reduction and interest rate subsidization determines how strong this effect is and will have asymmetric impacts on welfare.

\(^3\)In an article about the education gap, Stephen Rose, a senior economist at the Educational Testing Service underscored this point saying “You need a bachelor’s degree just to apply for the best jobs. That is how it should be for doctors, lawyers, scientists, engineers, computer specialists. But look at middle-level managers. In 1960, only 40 percent had a bachelor’s degree and today it is 80 percent” (New York Times, July 23, 2000).
Several theoretical papers in the development literature have characterized mechanisms in which imperfections in the credit market determine income distribution dynamics (e.g. Loury (1981), Banerjee and Newman (1993), and Aghion and Bolton (1997)), but none have taken into account signaling. Formally, our model draws from Galor and Zeira (1993). Adding to their basic framework enables us to develop a rich model: it is dynamic, allows for heterogeneity in types (wealth and ability), asymmetric information and mutations. It is also tractable: we get a closed form solution for the steady state. Galor and Moav (2000) consider a dynamic model of inequality with heterogeneous types with a focus on the interaction between technology adoption and skills. Fernandez and Rogerson (2001) also look at inequality in a model in which credit market imperfections affect educational attainment decisions, although education has no signaling properties in their model. Krugman (1998) examines the impact of signalling on income inequality in a static model.

The theory we present is consistent with empirical evidence over the last half-century. We discuss evidence below on the substantial expansion of government programs to reduce financial constraints on education spending. The increase in the wage gap has been extensively documented (see Katz and Murphy (1992), for example) and the model’s predictions are in line with the empirical literature.

The paper proceeds as follows. Section 2 lays out the basic economic environment in which individuals operate. Section 3 discusses the dynamics of bequests and our main results. Section 4 discusses related empirical evidence. Section 5 looks at extensions and welfare. Section 6 concludes.

## 2 Environment

We begin by describing the basic structure of the model. Time is discrete and extends to infinity. There exists a mass 1 of individuals at each time \( t \) who live for one period. There are two types of individuals in a given period: high ability \( (q^H) \) in proportion \( p \), and low ability \( (q^L) \) in proportion \( 1 - p \). Ability corresponds to one’s output when matched with a firm\(^4\). The expected ability of the population is \( \bar{q} = p q^H + (1 - p)q^L \).

Individuals must decide how to allocate their income between consumption for themselves and a bequest for their heir. Following Galor and Zeira (1993), we assume preferences are

\(^4\)We assume that abilities and education costs are constant over time. A more realistic framework would include: (1) growth in \( q^H \) and \( q^L \) to reflect changes in total factor productivity over time and (2) increases in tuition. However, the basic results of the model would not change.
characterized by a Cobb-Douglas utility function:

\[ u = \alpha \ln c + (1 - \alpha) \ln b \]  

(1)

where \( c \) is consumption, \( b \) is bequest size, and \( \alpha \) is a taste parameter between 0 and 1. The specification of utility essentially provides an allocation rule between goods for consumers\(^5\). It also allows for analytic tractability and intuitive interpretations due to the simplicity of the one dimensional taste parameter.

Individuals receive a non-negative bequest from the preceding generation, \( b_{t-1} \). In the first half of their lives they decide whether to invest in an education at tuition cost \( T \). In the second half, those who received an education earn wage\(^6\) \( w^e \) and those who do not earn wage \( w^n \), both of which are determined by employer behavior. Consumption then takes place and loans are repaid. The remaining income is bequeathed to the next generation (each individual has one child). Each chain of parents and children will be called a dynasty. At time \( t = 1 \), there exists a wealth distribution among individuals with wealth \( b_0 \) and the initial wealth distribution is characterized by two c.d.f.’s, \( G^H_0(b) \) and \( G^L_0(b) \), representing the distribution for high and low types, respectively.

The demand side of the labor market consists of many firms with constant returns to scale technology who compete Bertrand-style for workers by making simultaneous offers. Firms can’t observe types, bequests, or parental characteristics, but can observe whether a worker has received an education or not. In addition, firms know what types the population consists of, their preferences, and the distribution of bequests. In equilibrium, they offer two wages: \( w^e = E(q|\text{education}) \) and \( w^n = E(q|\text{no education}) \). We define a signaling equilibrium as choices of education based on skill and bequest level \( (\xi^*(q,b) \in \{e, n\}) \), beliefs by firms about type given education \( (\mu(H|\xi) \text{ for } \xi \in \{e,n\}) \), and wages as above. We employ the solution concept of Perfect Bayesian equilibrium and restrict the set of equilibria (see the next section) by using Cho and Kreps’ Intuitive Criterion.

Note that as in Spence (1973), education does not enhance productivity and only serves as a signal of type. For brevity, we abuse our terminology by often referring to postsecondary education as ‘education’ and someone who attends less school than university as ‘uneducated’.

\(^5\) Aghion and Bolton (1997) also assume a similar utility function. They point to evidence that parents gain utility from bequests independent of the actual utility gained by children (see Andreoni (1989)).

\(^6\) As we assume that labor supply is completely inelastic, we discuss ‘wages’ and ‘income’ interchangeably.
For those who receive a bequest lower than the cost of tuition, purchasing an education remains possible through borrowing. The world rate of interest is \( r \), and is fixed over time. Entry of lending institutions into the credit market is free. As in Galor and Zeira (1993), we assume that the credit market is imperfect due to the possibility of default, and that monitoring costs to the lender are linear in the size of the amount borrowed. The interest rate on borrowed funds \( i \) follows from the equation \( i = r + z \), where \( z \) is the monitoring cost per dollar loaned. Default risk\(^7\) is an important phenomenon in the market for education loans\(^8\). Such loans are very rarely made without some form of government subsidization due to the risk. This specification also admits simple transaction costs (i.e. \( z \) equals the cost of operation per dollar lent) and is therefore quite general. In terms of our model’s results, the important aspect of the credit market is that there exists a substantial wedge between the cost of borrowing and the return on lending. This makes education too costly for some individuals who would otherwise benefit from the wage premium it offers.

### 2.1 The Short Run

In the short run, we take as given the distribution of bequests \( b \), and find the optimal behavior of each individual given her type and bequest. The choices are whether to invest in human capital in the first half of her life, and how much to consume/bequest in her last half-period of existence. The optimal behavior of period \( t \) consumers generates \( G_t^H(b) \), the distribution of initial resources of time \( t + 1 \) consumers.

A worker who does not receive an education has the following available resources:

\[
y^n = w^n + b_{t-1}(1 + r)
\]

This person earns the uneducated wage and interest on her bequest, as she incurs no education or borrowing costs. A worker of type \( q \) who invests in education has an income of:

\[
y^e = w^e + b_{t-1} + \text{Max}\{b_{t-1} - T - k_q, 0\}r - \text{Max}\{T + k_q - b_{t-1}, 0\}i - T - k_q
\]

\(^7\)Aghion and Bolton (1997) model default risk explicitly as a hidden action problem. Using this they explain the development process through interest rate dynamics. We are more interested in the role of education in income dynamics and hence simplify the capital market interactions.

\(^8\)In 1990, U.S. expenditures on defaults amounted to about $3 billion dollars, which was approximately one fourth of the loan volume.
The positive part of income consists of wages, bequests, and interest on money saved. One can save when one’s bequest exceeds the costs of education. The negative part of income consists of interest payments on loans (paid when one’s bequest is smaller than the education costs), the tuition itself, and the indirect costs of education. We assume that $k_q$ represents the indirect monetary cost of studying to a type $q$ student and $k_L > k_H$. This implies that while colleges charge uniform prices, the level of indirect costs for a student with low ability is larger than that for a student of high ability. One might consider these as costs such as needing tutors, supplemental materials, or simply time costs.

Optimal behavior is given by choosing the maximum between $y^e$ and $y^n$, and subsequently maximizing the Cobb-Douglas preferences given in equation 1, which involves $c = \alpha y$ and $b = (1 - \alpha)y$. Hence the uneducated worker has an indirect utility of:

$$u^n(b_{t-1}) = \ln(w^n + b_{t-1}(1 + r)) + \gamma$$

where $\gamma = \alpha \ln\alpha + (1 - \alpha) \ln(1 - \alpha)$, and leaves a bequest:

$$b^n_t(b_{t-1}) = (1 - \alpha)(w^n + b_{t-1}(1 + r))$$

On the other hand, the educated individual gets a utility level that depends on whether she had to borrow to go to school, which in turn depends on her bequest:

$$u^e(b_{t-1}) = \begin{cases} 
\ln(w^e + (b_{t-1} - T - k_q)(1 + r)) + \gamma & \text{if } b_{t-1} \geq T + k_q \\
\ln(w^e - (T + k_q - b_{t-1})(1 + i)) + \gamma & \text{if } b_{t-1} < T + k_q 
\end{cases}$$

She leaves a bequest of:

$$b^e_t(b_{t-1}) = \begin{cases} 
(1 - \alpha)(w^e + (b_{t-1} - T - k_q)(1 + r)) & \text{if } b_{t-1} \geq T + k_q \\
(1 - \alpha)(w^e - (T + k_q - b_{t-1})(1 + i)) & \text{if } b_{t-1} < T + k_q 
\end{cases}$$

While the indirect cost of education serves a similar purpose as in Spence (1973), we make effort a monetary cost rather than an argument of the individual’s utility to simplify the presentation (without affecting the results).

Our assumption that both types have the same uniform bequest function (as a function of income) does not affect our results. One may think that a high type who knows her heirs are more likely to be high types than low types, and hence more likely to enjoy high income, would have a lower bequest function. To allow for that possibility we could assume they have different preferences represented by $\alpha^H$ and $\alpha^L$. Such a change shifts the bequest function without altering the functioning of the model, displacing only the convergent points.
Wealthy educated individuals (those who receive bequests above the tuition level) are able to save money and receive the interest rate \( r \) on their savings. This is reflected in both their utility and bequests. Less well off individuals must borrow at rate \( i \).

We want to study the role of financial market imperfections in a signaling framework. For this purpose we use Cho and Kreps’ Intuitive Criterion in order to restrict attention to parameters that yield separation of types in equilibrium. This generates the result that absent financial constraints, there is a unique equilibrium in which high types study and low types do not. The following conditions ensure separation of types:

\[
q^H - q^L < (T + k_L)(1 + r) \quad (2)
\]

\[
q^H - q > (T + k_H)(1 + r) \quad (3)
\]

Inequality 2 ensures that even the richest low type does not find it profitable to study. This guarantees existence of the separating equilibrium. What remains is to examine if any pooling equilibria exist. Pooling in education is not possible as equation 2 shows that low types would like to deviate. Pooling in no education can exist if high types do not want do deviate. Clearly there are off equilibrium path beliefs by firms that would keep high types from deviating (for example, if firms believe someone with an education is low skill), but using the Intuitive Criterion we can eliminate these beliefs as unreasonable. Since inequality 2 guarantees that low types would never choose education, the beliefs of firms upon seeing education must be that the individual is a high type. Lastly, we assume inequality 3 holds, which ensures that a rich high type finds it profitable to deviate from the pool and signal her ability through education.

Since this separating equilibrium does not involve any education on the part of low ability types, \( k_L \) will no longer be employed and we normalize \( k_H \) to zero for ease of presentation. In addition, we initially assume that all members of a dynasty are of the same type. We will relax this assumption in Section 5.

\[\footnote{With financial constraints, this will yield a unique equilibrium in which low types and poor high types don’t study.}

\[\footnote{An alternative set of assumptions is that all high types prefer to get an education and only some low types do. In this case, the mechanics do not reverse; they yield a result in which the steady state is unaffected by a change in the interest rate. Hence the signalling effect primarily plays a role when screening high types is important.}\]
2.2 Benchmark Case: Perfect Capital Markets

If capital markets were perfect, all individuals would have access to credit at rate $r$. All high ability individuals would get an education to signal their quality while low ability types would prefer not to go to college. Low ability types would be lenders and make bequests as in the previous section. High types may be borrowers (lenders) if they receive a bequest less (more) than $T$, and they bequeath:

$$b_t(b_{t-1}) = (1 - \alpha)(w^e + (b_t - T)(1 + r))$$

Since all high types and only high types get an education the uneducated wage must be $w^n = q^L$ and the educated wage is $w^e = q^H$. Then, the bequest levels converge to:

$$b^H = \frac{(1 - \alpha)(q^H - T(1 + r))}{1 - (1 - \alpha)(1 + r)}$$

and

$$b^L = \frac{(1 - \alpha)q^L}{1 - (1 - \alpha)(1 + r)}$$

In the steady state, all high types bequest $b^H$ and all low types bequest $b^L$ regardless of the initial distribution of wealth. It is clear that for existence of the steady state we need:

$$(1 - \alpha)(1 + r) < 1 \text{ or } (1 - \alpha)r < \alpha$$

which says that the weight to the bequest times the interest rate should not outweigh the preferences for current consumption. This makes sense intuitively - if most of a dynasty’s income is set aside for bequests, then as time passes, this wealth begins to explode.

2.3 Imperfect Capital Markets

Now we must characterize the decision to go to school when there is a differential in the interest rate between borrowing and lending. Individuals are endowed with two distinct features, a bequest and ability. The higher the initial wealth, the easier it is to attend school since the reliance on loans is lower. While no low type will get an education, high ability types wealthy enough find it optimal to study. For instance, equation 3 assures that high types with $b_{t-1} \geq T$ strictly prefer to get an education. Less wealthy high types may prefer not to study, depending on the interest rate wedge and how much they need to borrow. The wealth cut off below which no high type studies, $b^*_t$, is defined by the equality between $y^e$ and $y^n$:

$$w^e + (b^*_t - T)(1 + i) = w^n + b^*_t(1 + r)$$
which, after re-arranging becomes:

\[ b_{t-1}^* = \frac{T(1+i) - (w^e - w^n)}{i-r} = T - \frac{(w^e - w^n) - T(1+r)}{i-r} \] (5)

Inequality 3 assures the second term is positive. Therefore, assuming existence, the cut-off \( b_{t-1}^* \) monotonically increases in \( i \) up to \( T \). This is an intuitive conclusion, for as the cost of loans increase substantially (eliminating the loan market), only those with \( b_{t-1} \geq T \) can afford to send the signal to show they are high types.

We can now derive the equilibrium wages consistent with the optimal behavior of each individual in the economy. Since only high types study, the educated wage \( w^e = E(q|e) = q^H \). On the other hand, the group that attends no school includes low types as well as some high types that started with a low bequest. Hence:

\[ w^n_t = E(q|n) = \frac{q^L(1 - p) + q^H pG^H_t(b^*_{t-1})}{1 - p + pG^L_{t-1}(b^*_{t-1})} \]

We indicate the dependence of the uneducated wage on the shape of the bequest distribution by adding a time subscript. In addition, such dependence implies that there could be multiple solutions to equation 5 - i.e. multiple equilibria. For the time being, we will assume that a unique equilibrium exists. We will discuss the possible ramifications of more than one equilibria later in the text.

### 3 Bequest Dynamics

The short run behavior of a generation determines the initial wealth of the following generation. The long run distribution of wealth is determined by the short run transitions and the wage determination process.

The law of motion of wealth depends on the individuals’ types. For low ability types:

\[ b_t^L(b_{t-1}) = (1 - \alpha)(w^n_t + b_{t-1}(1 + r)) \] (6)

For high ability types:

\[ b_t^H(b_{t-1}) = \begin{cases} 
(1 - \alpha)(w^e + (b_{t-1} - T)(1 + r)) & \text{if } b_{t-1} \geq T \\
(1 - \alpha)(w^e - (T - b_{t-1})(1 + i)) & \text{if } b^*_{t-1} < b_{t-1} < T \\
(1 - \alpha)(w^n_t + b_{t-1}(1 + r)) & \text{if } b_{t-1} \leq b^*_{t-1} 
\end{cases} \] (7)
The pattern of bequests for high types segment them into three ranges: large bequests allow them to become educated and save, middle size bequests allow them to become educated at the cost of borrowing, and low bequests force them to skip education and save their inherited wealth. Those at the bottom receive the same pay as the lower skill population, which also forgoes education. Given these single stage patterns, we can now move to the dynamic solution. The dynamics are far from simple, as they must take into account a wage differential (and therefore cutoff level $b^*_t$) which is endogenous. We examine the solution in depth in the next section.

3.1 The Steady State

We start by defining points on the bequest distribution for high ability types that are critical to convergence. Subsequently we analyze the dynamics which lead to the steady state. As we can see from equation 7, high ability types break down into three groups. For those who can’t afford an education, we define the fixed point at time $t$ toward which they converge as $b_t$. This bequest level therefore solves the equation $b_t = (1 - \alpha)(w_t^n + b_t(1 + r))$, or:

$$b_t = \frac{(1 - \alpha)w_t^n}{1 - (1 + r)(1 - \alpha)}$$ (8)

This is the bequest level at which a worker leaves his heir the same bequest as he received.

Had the uneducated wage been exogenous, this would be a fixed point where the poorer high skilled population would end up. Even with the endogeneity of the uneducated wage, this point is focal for convergent dynamics. In fact, in a steady state, we expect it to become fixed and will focus on this shortly.

For the existence of $b_t$ and convergence of the model we need equation 4 to hold. In addition, we assume $(1 - \alpha)(1 + i) > 1$. This indicates that individuals’ preferences for dissaving (borrowing to invest in education) are sufficiently larger than their consumption preference. The implication is that if a well off dynasty was consuming too much, they would not be able to afford an education for too long. If the assumption was violated, the economy would converge trivially to a point where no one got an education.

We define the bequest level toward which high types with large amounts of income converge to as the fixed point $\bar{b}$:

$$\bar{b} = \frac{(1 - \alpha)(w^n - T(1 + r))}{1 - (1 + r)(1 - \alpha)}$$
Figure 1: Bequest dynamics

This expression, in contrast to the analogous one for low income individuals, does not depend on time\(^{13}\), since the high wage rate in the separating equilibrium remains fixed at the high productivity level \(q^H\).

Lastly, we define the point \(\tilde{b}\) as the point where the bequest that high skill workers who borrow to finance an education give equals the bequest that they receive. Formally,

\[
\tilde{b} = \frac{(1 - \alpha)(w_e - T(1 + i))}{1 - (1 - \alpha)(1 + i)}
\]

and like \(\bar{b}\), this expression is not time dependent.

The significance of this point can be seen in figure 1, in which it is denoted as point A. Depending on the initial distribution of wealth and the fundamental parameters, the model separates into two possible cases. The cases depend on whether the initial education cutoff

\(^{13}\)The level \(b\) exists if:

\[
(1 - \alpha)(w_e - k) > T \quad (9)
\]

If this inequality is violated, then the whole high skill population ends up giving up education and their wealth moves in a downward spiral from generation to generation. The reason is that a low \((1 - \alpha)\) implies a low level of bequest that makes education prohibitively expensive. This case is of little interest since the low levels of altruism destroy any role for education.
bequest level $b^*_0$ is smaller or larger than $\tilde{b}$. These are depicted in Figure 1 respectively as points B and C. Since both cases result in the same steady state and have dynamics that are exact opposites of each other, we will mainly focus on explaining the first case.

Suppose $b^*_0 < \tilde{b}$. Then those who receive bequests lower than $\tilde{b}$ (and above $b_t$) give lower and lower bequests and their dynasty’s wealth creeps downward. They must borrow to pay for their education although the payoff of a top job is not enough to permit them to pass larger bequests to their heirs. Educated dynasties on this path who receive a bequest slightly larger than $b^*_t$ will have heirs who decide not to go to college. On the other hand, those who receive bequests higher than $\tilde{b}$ (and lower than $\hat{b}$) start giving higher and higher bequests. Their initial endowments allows them to take advantage of the wage premium and pass on their wealth in increasing amounts. This starts off the dynamics of the model.

The solution depends very much on the endogeneity of the uneducated wage. The uneducated wage equals the expectation of the productivity of workers in the uneducated pool. The bequest dynamics increase the number of high skill workers entering this pool (as dynasties’ wealth creeps downward). As more high skill workers enter the pool, the uneducated wage rises, becoming more attractive and making education less desirable. With the rise in the uneducated wage, the wage premium for education decreases, and the cutoff level $b^*_t$ increases. This cycle keeps reinforcing itself\(^\text{14}\) until $b^*_t(b^*_{t-1}) = b^*_{t-1}$, which we already defined as the point $(\tilde{b}, \tilde{b})$, or point A in figure 1.

In some sense we may think of the driving forces here as the composition of two effects: a mechanical effect and a wage premium effect. The mechanical effect consists of a dynasty’s movement along the bequest distribution, which takes place between periods/lifetimes. The wage premium effect takes place in the first half of an individual’s life, where the decision to study is made based on the wage differential and bequest distribution.

These effects combine to make bequests converge to $\tilde{b}$ and $\hat{b}$. We offer a more formal presentation of this convergence in the appendix. The steady state would be the same had the initial conditions stated that $b^*_0 > \tilde{b}$. The dynamics for this condition have uneducated high ability dynasties getting wealthier (bequests are increasing) and deciding to get an education. This migration lowers the uneducated wage, decreasing the wealth of the uneducated until $b^*_t$ decreases to $\tilde{b}$.

Essentially, the dynamics when $b^*_0 > \tilde{b}$ ($b^*_0 < \hat{b}$) tell the story of an economy which increases (decreases) the amount of education and high wage employees by decreasing (increasing).

\(^{14}\)It should be noted that for some initial bequest distributions $G_0(b)$ this continuous reinforcing effect will ‘run out of steam’. We discuss this possibility in the appendix.
creasing) the well-being of the uneducated population. As we are interested in getting a greater understanding into how increasing education can lead to more wage inequality, we would like to understand when this is more likely to occur (i.e. when $b_0^* > \tilde{b}$ will occur). Straightforward calculation leads us to the following:

**Lemma 1** A lower cost of borrowing, lower tuition, and lower indirect costs of education increase the likelihood that the economy will follow a path of increasing education and while widening the wage gap.

### 3.2 The Effects of Imperfect Capital Markets

The steady state wages indicate that in the transition from this model to the benchmark with perfect capital markets, the wage gap expands. With the imperfection in the capital market, the premium becomes $\frac{q_H}{w}$, where $w$ is bounded above $q^L$ (given that $G^H_0(\tilde{b}) > 0$). In the benchmark, all high types can afford to signal their type, making the college premium $\frac{q_H}{q^L}$. From the steady state of our model we can also examine the effect of smaller reductions in the interest rate $i$ on the wage premium. The steady state wage $w$ is a function of $i$ through $\tilde{b}$. It is obvious that $\frac{dw}{db}$ is positive, and we can show using equation 9 that $\frac{db}{di}$ is positive, implying that as the interest rate wedge decreases, the college premium increases. This proves our main result:

**Proposition 2** In the steady state, reducing the cost of obtaining loans exacerbates wage inequality through the signaling mechanism.

In making the transition from imperfect to perfect capital markets both the supply and the wages of uneducated workers decreased as high ability workers switched sectors. The overall salary payments remain the same in both cases, meaning that the credit market imperfection had forced a redistribution from constrained high ability individuals to low ability individuals. Note that the steady state also depends on the initial distribution of wealth, as in Galor and Zeira (1993). However, we obtain a result that differs sharply with theirs:

**Corollary 3** For richer initial wealth distributions (i.e. as $G_0(\tilde{b})$ decreases), wage inequality increases.
The reason for this contrast is simple; in Galor and Zeira$^{15}$, the result is based on a human capital argument (and hence on more traditional supply and demand forces), while ours depends on signaling. Therefore, given a richer initial distribution where less dynasties move from being educated to being uneducated, the uneducated wage in Galor and Zeira actually increases.

Reducing the cost of education comes in many forms, and our model implies that the shape that such aid takes could be important. The model incorporates two possible types of subsidies: reducing the interest rate on borrowing and reducing tuition. Besides the main effect that we discuss in Proposition 1, a substantial reduction of the interest rate on borrowing could reduce the steady state from two points to one (when $(1 - \alpha)(1 + i) < 1$), where all high types purchase an education. A large decrease in tuition costs could also accomplish this. This decrease in tuition, however, could have two important side effects. First, the sole convergent point would be larger than when the interest rate is decreased, meaning that wealth would be higher. Second, large decreases in tuition could change the equilibrium from separating to one where low skill types pursue an education. This can be seen from assumption 2. It is also clear from assumption 2 that lowering the interest rate on borrowing would have no such effect.

Our theory provides a partial explanation of the wage premium based on signaling and credit constraints. In section 5, we demonstrate that these findings hold up when the dynamic model is generalized. Right now, we look at what implications of the model are corroborated by evidence on credit constraints and inequality.

4 Historical Background

Given the simple, yet strong predictions of the theory, we now discuss whether it is in accordance with evidence from the United States experience. We focus solely on the United States for several reasons. First, in most other countries, the cost of a university is highly subsidized to the point of being non-existent. As we saw earlier, a large reduction in tuition may have substantially different effects than reductions in the interest rate. One possibility is the violation of equation 2, which could force a switch to a pooling equilibrium, starting different dynamics than the ones we describe. Another potential effect of subsidizing tuition is inducing some form of rationing of educational resources. This may come in the form of entrance exams (which skew toward favoring those who are well off) or through personal

$^{15}$See Theorem 1, Galor and Zeira (1993).
connections\textsuperscript{16}. Rationing does not readily fit into our model of signaling. Secondly, even with universal access to universities, there are hidden costs to students: opportunity cost and living expenses. In many countries, it is difficult to find credit markets that cover these costs\textsuperscript{17}.

4.1 Federal Subsidies for Higher Education

The first major venture of the United States government into subsidizing university education came with the Serviceman’s Readjustment Act of 1944 (more commonly known as the G.I. Bill of Rights). This bill provided World War II veterans with a wide array of benefits, including (for an unmarried veteran) a $65 stipend per month and paid tuition up to $500\textsuperscript{18}. Over 2.2 million veterans attended university using these benefits, and at its peak, veterans comprised 49.2\% of enrolled students and 69.3\% of enrolled males\textsuperscript{19}. Estimates made at the time indicate that approximately 20\% of the enrolled veterans would never have gone to college without the subsidies. Gutek (1986, p.282) states, “Higher educational opportunities were made available to a larger and more varied socioeconomic group than ever before...the idea that higher education was the privilege of a well-born elite was finally shattered.” Subsequent G. I. bills for Korean and Vietnam War veterans proved less substantial.

In contrast, up until 1958, neither the public nor the private sector provided any loan or grant programs to the general U.S. public. That year the Perkins loans began under the auspices of the National Defense Education Act. They provided colleges with capital from the federal government so that the colleges could lend at low interest rates to students\textsuperscript{20}. In 1965, with the passage of the Higher Education Act, a grant program and an additional loan program (Guaranteed Student Loan Program, or GSLP) were created. These loans

\textsuperscript{16}A recent highly publicized illustration of the value of connections came from Britain, where a young woman from a public high school was denied acceptance to Oxford while at the same gaining admission to Harvard. In the ensuing scandal, the following fact surfaced: “Among pupils with the same qualifications, applicants from private schools are 25 times more likely to gain admission to [Oxford or Cambridge] than those from state schools” (New York Times, 6/3/00).

\textsuperscript{17}Many European countries have established funding for these expenses through grants and loans (see OECD (1990)). However, describing wage inequality in European countries also runs into difficulties because of the strength of unions and centralized wage setting mechanisms (see Gottschalk and Smeeding (1997)).

\textsuperscript{18}Behrman et. al (1989) emphasize the substantiveness of these benefits: “annual U.S. per capita disposable income in 1946 was $1,124, and the University of Pennsylvania’s undergraduate tuition plus general fee, which is comparable to Harvard’s, was $495.”

\textsuperscript{19}These statistics are from Olson (1974).

\textsuperscript{20}These loans and all subsequent loans all had limits on the amount that one can borrow. This does not qualitatively affect our theoretical results, but should shift $b$ downward. The loans required no collateral or credit history. Some are based on demonstrated ‘financial need’ and some are not.
were handled by banks and savings and loan institutions and were both guaranteed and subsidized by the federal government. Later subsumed under the title of Federal Family Education Loans (FFEL), these loans quickly eclipsed the Perkins loans in terms of volume. In 1976, the Pell grant, a need based grant, began and the other programs were strengthened. Ronald Reagan scaled back the programs somewhat in 1980. Until recently, there have been no major changes in these programs besides eligibility revisions and increases in amounts that students may borrow\textsuperscript{21,22}. In figure 2, we summarize the changes in borrowing. As one can see from the chart, the loan volume of FFEL loans increased from nothing in 1965 to over one quarter of 1\% of GDP (over $20 billion) in the mid 1990s. This offers a strong sense that the loans were less costly than outside opportunities. Over that same time frame, college entrance rates for recent high school graduates increased from 45\% (1959) to 58.9\% (1988)\textsuperscript{23}.

In figure 3, we depict the trade-off between borrowing versus saving. The interest rate

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart2.png}
\caption{Federal Family Education Loan Volume as a percentage of GDP: data from U.S. Department of Education and CEA}
\end{figure}

\textsuperscript{21}Direct lending programs began in the mid 1990s, but we do not examine their impact as our available data is only up until the mid 1990s.

\textsuperscript{22}An excellent summary of legislative activity and the history of student loans may be found in Mumper (1996).

\textsuperscript{23}This data was culled from the organization Postsecondary Education Opportunity (www.postsecondary.org).
depicted as the borrowing rate is that of the Stafford Loan, computed for each year X as the rate a person who took out a loan 4 years previously would have to pay in year X, i.e. the rate that influences their decision. The rate of interest on saving is shown as the return on a 6 month CD. For the most part, the wedge between borrowing and lending is apparent. The wedge between the return on a 6 month CD and commercial loans is substantially higher, indicating a high degree of subsidization by the government. The rigid low rates on Stafford Loans actually brought them substantially below the saving rate from 1979-1984. Around this time the government’s expenditures ballooned, which can be seen from the sharp increase in loan volume in figure 2. The loan volume also increased substantially in the 1990s, despite a relatively stable interest rate differential. This was motivated in part by increased loan limits, the introduction of unsubsidized loans and favorable changes in qualification standards.

The existence of a wedge between the rates for borrowing and saving makes a great deal of sense when one observes the massive amount of organization in place to prevent

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24 Note that individuals who take out Stafford Loans do not have to pay interest while in school. The Stafford Loans (called Guaranteed Student Loans prior to 1981) make up the largest part of FFEL loans. These loans had fixed rates up until 1993, when they became a function of the 91-day Treasury Bill. We thank Brian Smith of the Department of Education for his help in obtaining the data on Stafford Loans.

25 We also note that college tuition, in real terms, was very stable in the 1970s and did not begin growing substantially until 1982 (this data can be found in College Board (1999b)).
defaults. Many state guarantee agencies and secondary market loan associations26 exist solely to recover loans. Nevertheless, the prevalence of default is quite significant - in 1990 federal expenditures due to defaults amounted to approximately $3 billion, which was about one half of total federal expenditures on loans and one quarter of the loan volume in that year27.

4.2 The College Premium

We have argued that financial constraints affecting access to education impact the informational value of education and hence the returns to education28. How important an effect is this? A full empirical investigation of the significance of financial constraints on wage inequality is beyond the scope of this paper. However, we believe the mechanism discussed above has some interest given the patterns of inequality observed in the data29.

Researchers have explored many reasons why wage inequality has changed over time in the U.S. and in other countries. Most studies30 analyzing the college premium have focused on demand factors; technology-skill complementarities and international trade’s effect on skill composition are the most prominent explanations31. Acemoglu (1998, 1999) and Galor and Moav (2000) study how shifts in these two demand factors could arise endogenously from an increase in the proportion of high skill workers. We propose our theory as one additional factor to explain wage inequality32.

The basic facts about wage inequality are agreed upon. The college wage premium in the U.S. rose in the 1950s, flattened for the first half of the 1960s before rising again in the second half of that decade, fell in the 1970s, and began a very steep ascent around 1979 (for

26 These are organizations which specialize in student loans and purchase them from primary lenders, i.e. banks and savings and loan institutions. The largest of these is the Student Loan Marketing Association (better known as Sallie Mae).
27 These numbers are drawn from Mumper (1996) and the College Board (1999a).
28 Cameron and Taber (2000) show that there is no inequality in access to loans, addressing a different type of market failure than the one we discuss.
29 A careful reader will note that the source of the inequality in the model is that the wages of the skilled are constant while the wages of the unskilled decline. The constant wage of the skilled is contrary to evidence, although, as mentioned in footnote 4, we can change the model to include ability growth over time to capture this. The decline in the wages of the unskilled that our model displays is quite interesting; this decline is an empirical fact that most theoretical models are unable to capture (see Acemoglu (2002) for a review of the literature).
30 For surveys of the literature, see Levy and Murnane (1992) and Aghion et.al. (1999).
31 Katz and Murphy (1992) also point compellingly to the rate of change in the supply of college graduates as one explanation of the data.
32 Financial constraints are not the only exogenous factors that may affect educational attainment for cohorts. In the 1960s, for example, college education helped young people avoid military service.
graphs and analysis see Goldin and Margo (1992) or Katz and Murphy (1992)).

We point to two pieces of existing evidence that suggest that our theory may explain a non-trivial amount of variation in wage inequality. First, financial assistance for higher education has been an important factor in individual’s decisions since right after World War II. This started with the G.I. Bill providing substantial assistance for returning servicemen and continued with the various student loan and grant programs that offered more universal support. Over this same period, wage inequality has increased significantly. Second, our theory suggests that, all else equal, any permanent reduction in financial constraints should increase wage inequality for the cohorts receiving education now and for all subsequent cohorts. However, for cohorts who have already received education, the reduction in constraints should have no effect. As time progresses, a larger and larger fraction of cohorts take advantage of the lower constraints and overall wage inequality increases as well. This is consistent with evidence in Card and Lemieux (2001). They show that much of the sharp increase in the college premium since 1980 can be accounted for by the youngest cohorts.

5 Extension and Discussion

In this section we discuss extensions and welfare aspects of the basic model. We first examine relaxing the assumption that all members of a dynasty must be of the same type.

5.1 Mutations

Suppose that a child may not have the same ability type as his or her parent. This case involves a combination of the deterministic convergence presented in the previous section and a stochastic process followed by individuals’ types. We define the transition from generation to generation by the probability \( \lambda^H (\lambda^L) \), which represents the proportion of high (low) ability individuals who get a high (low) ability heir. Bequest dynamics are identical to those in the preceding section as long as subsequent generations remain in the same type category. These dynamics are altered when a mutation takes place, in which case the convergence process resumes at the same income level but following the law of motion of the new type.

\[\text{References:}\]

\[\text{Cameron and Heckman (1998) provide mixed evidence on the effectiveness of financial aid. Their work suggests that small changes in income have no effect on educational attainment, while large changes (i.e. permanent income shocks) boost attainment.}\]

\[\text{Card and Lemieux (2000), however, explain the cohort effect as a result of increasing educational attainment being outpaced by increased demand for high skilled workers.}\]

\[\text{In addition, there is a spike upwards in the wage premium from 1959 to 1970 for the youngest cohort, while the older cohort’s premium remains flat. See Card and Lemieux (2000), figure 1.}\]

21
In this case, the uneducated wage is defined as:

\[
w_t^u = \frac{q^L[N_{t-1}^L(1-\lambda^H)] + q^H[N_{t-1}^L(1-\lambda^L)G_{t-1}^L(b_{t-1})] + N_{t-1}^H(1-\lambda^H)G_{t-1}^H(b_{t-1})]}{1 - N_{t-1}^L(1-\lambda^L)(1 - G_{t-1}^L(b_{t-1})) - N_{t-1}^H(1-\lambda^H)(1 - G_{t-1}^H(b_{t-1}))}
\]

where \( N_{t-1}^q \) is the number of type \( q \) people at time \( t-1 \) and \( G_{t-1}^q(b) \) is the distribution of bequests from parents of type \( q \) to children living in time \( t \). The uneducated wage equals the expectation of ability of an uneducated worker given the transitions between types and the bequest distributions. Furthermore, we can solve the difference equations for transitions between types to find that \( N_{t}^L = -(\lambda^H + \lambda^L - 1)(p - \frac{1-\lambda^L}{2-\lambda^H-\lambda^L}) + \frac{1-\lambda^H}{2-\lambda^H-\lambda^L} \) and \( N_{t}^H = (\lambda^H + \lambda^L - 1)(p - \frac{1-\lambda^L}{2-\lambda^H-\lambda^L}) + \frac{1-\lambda^L}{2-\lambda^H-\lambda^L} \). Lastly, we define the education decision as before in equation 5. We make the following simplifying assumption in order examine properties of a solution to these type of dynamics: \( \lambda^H + \lambda^L = 1 \). We can call this the zero-correlation case, that is, the probability of having a type \( q \) child is the same for both types of parents. It leads us to the following proposition, which is proved in the appendix:

**Proposition 4** If \( \lambda^H + \lambda^L = 1 \), then there are 2 possible outcomes

i) (No convergence) \( b^H = \bar{b}, \) and \( b^L \) oscillates around \( \bar{b} \)

ii) (Steady state) \( b^H = b^L \leq \bar{b} \)

In this case, there are two possible outcomes. The no convergence outcome resembles our previous results, where there are a high wage and a low wage. Here the low bequest level hovers around \( \bar{b} \). We can show, as before, that \( \bar{b} \) increases with \( i \), and that at any time \( t \), a decrease in interest rate \( i \) (weakly) increases the wage premium. Hence, our results hold if the model does not converge. Convergence, however, implies complete equality with the wage equal to \( \bar{q} \). Convergence obviously depends very much on the original distribution of bequests, although it is difficult to provide general results about the nature of this dependence.

### 5.2 Welfare Analysis

The model’s potential policy implications suggest that considering a welfare analysis and policy tools are important. One clear implication has been that high ability workers getting trapped in uneducated jobs forced redistribution, meaning that a social welfare function that weighted the low ability types more would increase. The redistribution in itself, though, has no implications for the aggregate economy and growth. Overall production within this economy remains fixed over time. This stems from the assumption that education is not
productive. Consequently, one may actually view keeping high skill workers uneducated as positive news for the economy, not just because of redistribution, but because this forces wasteful education and loan monitoring costs to disappear. The tuition and effort costs are no longer deadweights upon the economy.

Since Spence’s seminal article in 1973, the debate over the validity of education as a signal versus its value in building human capital has been heated. It is therefore important to see how productive education can change the results of our dynamic model. Suppose that the augmentation in productivity makes individual i’s output $\rho q^i, \rho > 1$. Furthermore, assume that the conditions for a separating equilibrium still hold. Intuitively enough, none of the qualitative results change, but the welfare analysis may change substantially. If the productivity enhancement outweighs the cost of education (i.e. $(\rho - 1)q^i - T - k - I > 0$, where $I$ is the loan monitoring cost), the steady state of keeping high skilled individuals from an education may be viewed as a very negative result. Depending on the parameters, the steady state guarantees that a sizable amount of individuals will eventually choose not to purchase an education, stifling growth. Any policy to counteract this necessitates some type of permanent intervention. This lies in contrast to temporary pro-growth policies in other papers (Banerjee and Newman (1993) and Piketty (1997); for a review of the literature see Aghion, Caroli, and Garcia-Peñalosa (1999)).

Topel (1997) discusses the implications of wage inequality in a policy context. Using the argument that demand has substantially outpaced supply he recommends a policy of extensive investment in education to soak up excess demand. In the context of our model, which is outside of the standard supply and demand framework, such a policy would only increase inequality. While signaling is only one component of wage inequality, its effects certainly merit further investigation before such policies are fully supported.

A complicating factor in our analysis is the possibility of having multiple equilibria. This can come about in each period $t$ in the determination of the cutoff $b^*_{t-1}$ and depends on the shape of the bequest distribution. A sufficient condition for ruling out such equilibria is that the slope of the right hand side of equation 5 is less than one. It is possible that coordinating on one equilibrium from a set of multiple equilibria could reverse the dynamics of the model, but would keep our main results intact. Although we provide no explicit coordination mechanism for choosing an equilibria, it is interesting to consider the effects of the suggested policy interventions. Both decreasing the interest rate on borrowing or providing a price reduction for tuition can shift the equilibria or potentially eliminate them

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36 A review of the literature can be found in Fang (2000).
(the decrease increases the right hand side of equation 5).

6 Conclusion

In this paper, we argue that reducing financial constraints for postsecondary education can increase wage inequality. We use a dynamic approach based on job market signaling, a mechanism which has been unexplored in recent work on wage inequality. A reduction in the interest rate increases the number of the poor who get educated in steady state, lowering the average ability of the uneducated pool, and therefore increasing the wage gap. In addition, we find that the initial distribution of wealth affects both the steady state distribution of education and the trajectory that the economy follows. In the last 60 years, two events significantly reduced the cost of higher education (the G.I. Bill and the Higher Education Act), while many subsequent acts have added to increase its overall accessibility.

Our work suggests two natural directions for future research. First, researchers have identified many factors which in theory might affect the relationship between wages and education. One natural question to ask is how these other factors interact with the mechanism we have described here. Second, while we have provided some stylized facts which are consistent with the ideas in this paper, formal empirical tests can provide evidence on how changing financial opportunities have affected wage inequality.

7 Appendix

7.1 Dynamics

Here we discuss the conditions for convergence of the distribution of bequests for high skill workers to two points: \( \hat{b} \) and \( \bar{b} \). In the text, one condition has been already isolated that disrupts the convergence - if education is prohibitively expensive, the convergence is to a single bequest level where everyone forgoes an education.

Without this condition, all we must assume is uniqueness of \( b^*_{t-1} \) at all times \( t - 1 \). The bequest \( b^*_{t-1} \) is defined in equation 5 as \( b^*_{t-1} = F(G^H_{t-1}(b^*_{t-1})) \). It is clear that the uneducated wage increases with \( b^*_{t-1} \). We can then see that \( \frac{dF}{db_{t-1}} = \frac{dF}{dG^H_{t-1}} \frac{dG^H_{t-1}}{db_{t-1}} \) is positive. The separating equilibrium guarantees that \( F(0) > 0 \). Therefore, assuming uniqueness, we can draw the solution to the cutoff equation in Figure 4. For the case in which \( b^*_0 < \hat{b} \), it is clear that \( G^H_{t-1}(b^*_{t-1}) < G^H_t(b^*_{t-1}) \) from the laws of motion of bequests, and we can conclude
that $b_t^* > b_{t-1}^*$. This can continue until a time $t$ such that $b_{t-1}^* = \tilde{b}$. Similar reasoning can be applied in the case where $b_0^* > \tilde{b}$.

However, there are initial wealth distributions where these dynamics may run ‘run out of steam’, i.e. stop because not enough high skill dynasties are left to keep the process going. Consider the economy where $b_0^* < \tilde{b}$. If we suppose there are very few people with original wealth in $[b_0^*, \tilde{b}]$, for example, the number of low income educated individuals having heirs who choose to become uneducated may dry up. In that case the steady state may be a bequest level below that indicated, although the dynamics are the same. More formally, this will occur if the mass of types in $G_0(\tilde{b})$ all shift down while $b_{t-1}^* < \tilde{b}$ for some finite time $t-1$, or

$$G_0(\tilde{b}) < \left( \frac{A - q^L}{q^H - A} \right) \left( \frac{1 - p}{p} \right)$$

where $A = \left( q^H - k - T(1 + i) \right) \frac{1 - (1 - \alpha)(1 + r)}{1 - (1 - \alpha)(1 + i)}$.

Again, applying this type of reasoning to the case where $b_0^* > \tilde{b}$ gives us the condition where the steady state ends up at some $b_{t-1}^* > \tilde{b}$: $q^L < A$, where $A$ is defined as above.

### 7.2 Mutations

Since $\lambda^H + \lambda^L = 1$, $N_t^L = 1 - \lambda^H$ and $N_t^H = 1 - \lambda^L$ for all $t$ (this is evident from the text). Note that from equation 10, $w_t^n = \frac{q^n(\lambda^L + q^H Y)}{\lambda^L + Y}$ (where $Y = N^L(1 - \lambda^L)G_{t-1}^L(b_{t-1}^*) + N^H \lambda^H G_{t-1}^H(b_{t-1}^*)$).
This is increasing in $Y$ since $q^L < q^H$. We fix some time $t$, and examine the change in $Y$ at time $t+1$. Let $\Delta G^L = G^L_t(b^*_{t-1}) - G^L_{t-1}(b^*_{t-1})$ and analogously $\Delta G^H = G^H_t(b^*_{t-1}) - G^H_{t-1}(b^*_{t-1})$. Taking into account all of the inflows and outflows:

$$\Delta G^H = \frac{1}{\lambda^H}\{-N^H(1 - \lambda^H)G^H_{t-1}(b^*_{t-1}) + N^L(1 - \lambda^L)G^L_{t-1}(b^*_{t-1} + \varepsilon_1) + N^H\lambda^H(G^H_{t-1}(b^*_{t-1} + \varepsilon_2) - G^H_{t-1}(b^*_{t-1}))\}$$

$$\Delta G^L = \frac{1}{\lambda^L}\{-N^L(1 - \lambda^L)G^L_{t-1}(b^*_{t-1}) + N^H(1 - \lambda^H)G^H_{t-1}(b^*_{t-1} + \varepsilon_1) + N^L\lambda^L(G^L_{t-1}(b^*_{t-1} + \varepsilon_1) - G^L_{t-1}(b^*_{t-1}))\}$$

First assume that $b^*_{t-1} < \bar{b}$. Then $\varepsilon_1 = \frac{b^*_{t-1}(1-(1-\alpha)(1+r))}{(1-\alpha)(1+i)} - \frac{w^n}{1+r} > 0$ and $\varepsilon_2 = \frac{b^*_{t-1}(1-(1-\alpha)(1+i))}{(1-\alpha)(1+i)} - \frac{w^n}{1+r} + T > 0$, since more people are shifting downwards towards $b^*_{t-1}$. Define $\Delta Y = N^L(1 - \lambda^L)\Delta G^L + N^H\lambda^H\Delta G^H$. It is then straightforward to show that given $\lambda^H + \lambda^L = 1$, $\Delta Y > 0$. This implies that $w^n(b^*_{t-1}) > w^n_{t-1}$, which, using the logic from the no mutations case (and again assuming a unique equilibrium), implies that $b^*_{t} > b^*_{t-1}$.

Next, assume that $b^*_{t-1} > \tilde{b}$. Now $\varepsilon_1 < 0$ and $\varepsilon_2 < 0$, since people are shifting away from $b^*_{t-1}$. This produces the opposite effect, such that $\Delta Y < 0$ and hence $b^*_{t} < b^*_{t-1}$.

Lastly, assume that $b^*_{t-1} = \tilde{b}$. At this point, $\varepsilon_2 = 0$, since no high types will flow above or below $b^*_{t-1}$. Low types still flow downwards however, making $\varepsilon_1 > 0$. In this case $\Delta Y > 0$ and $b^*_{t} > b^*_{t-1}$.

We can now discuss the results listed in the proposition. If $b^*_{t-1} \leq \tilde{b}$ then the pressure from low skilled individuals bequeathing less to the next generation forces more high skilled individuals in the subsequent generation into the uneducated pool, raising the uneducated wage and the bequest cutoff. If $b^*_{t-1} > \tilde{b}$ larger bequests deplete the uneducated pool of high skill workers, decreasing the uneducated wage and the bequest cutoff.

A steady state may exist if all high bequest individuals have been pushed down the bequest distribution, i.e. $G^H_{t-1}(b^*_{t-1}) = G^L_{t-1}(b^*_{t-1}) = 1$ at some time $t$. At this point $\Delta Y = 0$, and the wage and cutoff will stay the same ($w^n = \bar{q}$). This can happen at any point where the period before the steady state begins, the last high bequest individuals inch down the distribution, i.e. where $b \leq \tilde{b}$.

This type of convergence may not occur, for if there still remain individuals at the higher end of the bequest distribution, we have shown that there is no bequest $b$ that is stable. In addition all of the movement is directed towards $\tilde{b}$, but $\tilde{b}$ is not a stable point.
References


