Choice by sequential procedures

Jose Apesteguia and Miguel A. Ballester

Universitat Pompeu Fabra and Universitat Autònoma de Barcelona
Barcelona GSE

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Introduction

- The traditional choice-theoretic approach takes behavior as rational if choice behavior can be explained as the outcome of maximizing a preference relation.

- However, over the last decades mounting evidence has been accumulated documenting systematic and predictable violations of this notion of rationality.
  - There are framing effects, menu effects, importance of reference points, cyclic choice patterns, choice overload effects, temporal inconsistencies, etc.
Here, we study an alternative model of choice: choice by sequential procedures

- It encompasses the standard model of choice as a special case.
- It is able to accommodate behavior often observed in empirical/experimental studies that the standard model of choice regards as irrational.
- It is testable: not all choice patterns can be explained as choice by sequential procedures.
Choice by sequential procedures:

- The DM applies a number of criteria (incomplete binary relations) in a fixed order of priority, gradually narrowing down the set of alternatives, until one is identified as the choice
  - Same set of criteria, applied in the same fixed order to every choice problem

Examples: individual and collective choice

- Buying a house: first location, then layout, and then price
- Social choice: first efficiency, then fairness
- Hiring a new professor: first area of research, then letters, then job market paper, then seminar and interviews
- Multiple selves, orderly applied
Concrete Examples

Let $X = \{x, y, z\}$ and

- $c(x, y, z) = x$
- $c(x, y) = y$
- $c(y, z) = z$
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Questions:

- Can we distinguish those choice functions that are SR, from those that are not?
- Can we find some property that characterizes SR, and that at the same time it is informative about the behavioral principles governing SR?
- Can we use such a property to establish the relation between SR and other models of choice?
Literature:

- Masatlioglu and Ok (2005, JET)
- Rubinstein and Salant (2006, Theoretical Economics)
- Xu and Zhou (2007, JET)
- Bernheim and Rangel (2007, 2008, AER, QJE)
- Masatlioglu and Nakajima (2009, WP)
- Eliaz and Spiegler (2009, WP)
- Cherepanov, Feddersen and Sandroni (2009, WP)
- Green and Hojman (2009, WP)

- Manzini and Mariotti (2007, AER)
Notation: choice

- $X$ finite set of alternatives
- $\mathcal{P}(X)$ collection of all non-empty subsets of $X$
- $c: \mathcal{P}(X) \rightarrow X$ with $c(A) \in A$
- $\mathcal{C}$ collection of all possible choice functions $c$ given $X$
Notation: rationales

- A rationale: an acyclic binary relation $P \subseteq X \times X$
- Maximal elements in $A \subseteq X$ according to $P$:
  $$M(A, P) = \{x \in A : (y, x) \in P \text{ for no } y \in A\}$$
- Given an ordered collection of rationales $\{P_1, \ldots, P_K\}$:
  $$M^K_1(A) = M(M(\ldots M(M(A, P_1), P_2), \ldots, P_{K-1}), P_K)$$
Sequential rationalizability: definition

Sequential Rationalizability (SR): A choice function \( c \) is sequentially rationalizable whenever there exists a non-empty ordered list \( \{P_1, \ldots, P_K\} \) of rationales on \( X \) such that

\[
c(A) = M^K_1(A) \text{ for all } A \subseteq X
\]
Characterization
Characterization: definitions

- A **binary selector** $f$ is a single-valued function that, for every choice problem $A$ with at least two alternatives, gives a binary problem in $A$.
- We say that the binary selector $f$ is **consistent** if it satisfies the Strong Axiom.
The classic IIA states that if an element $x$ is chosen from a set $A$, it should also be chosen from any subset of $A$ in which $x$ is present.
Characterization: property

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- Independence of Irrelevant Alternatives (IIA): For any consistent binary selector $f$ and any $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$ with $x^* = f(A) \setminus c(f(A))$. 
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**Independence of One Irrelevant Alternative (IOIA):** There is a consistent binary selector \( f \) such that, for any \( A \subseteq X \), \( c(A) = c(A \setminus \{x^*\}) \), with \( x^* = f(A) \setminus c(f(A)) \).
Theorem: $c$ is sequentially rationalizable $\iff c$ satisfies IOIA
Assessing whether a particular c is SR reduces to check whether there is a linear order over the binary sets such that, for every choice problem A and for the first binary problem $B \subseteq A$, the choice from A does not depend on the dominated alternative in $B$. 
Characterization: remarks

- **No Binary Cycles**: For all $x_1,\ldots,x_{r+1} \in X$, $c(x_j, x_{j+1}) = x_j$, $j = 1,\ldots,r$, implies that $c(x_1, x_{r+1}) = x_1$. 

**Lemma:** $c$ satisfies IIA if and only if $c$ satisfies IOIA and No Binary Cycles. 

IOIA can be understood as the interplay of a fully consistent component, the binary selector $f$, and a potentially irrational component, choices from binary problems.
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Characterization: applications

- Rationalizability by Game Trees (Xu and Zhou, JET 2007)
- Agenda Rationalizability (voting models; choice by elimination)
- Status Quo Bias Rationalizability (Masatlioglu and Ok, JET 2005)

Theorem: $C_{SQB} \subset C_{AR} \subset C_{RGT} \subset C_{SR}$
Our characterizing property IOIA can be used to study the relation of sequential rationalizability with other models:

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- Theorem: \( C^{SQB} \subset C^{AR} \subset C^{RGT} \subset C^{SR} \)
Razionalizability by game trees

- The choices of the DM are the equilibrium outcome of an extensive game with perfect information
- Consider the class of extensive games with perfect information \((G, P)\) such that:
  - The tree has alternatives of \(X\) as terminal nodes, each alternative appearing once and only once
  - Every node of the tree represents the decision of some agent \(i\), with an associated linear order \(P_i\)
- \(G|A\) is the reduced tree of \(G\) that retains all the branches of \(G\) leading to terminal nodes in \(A\)
- **Rationalizability by Game Trees:** A choice function \(c\) is rationalizable by game trees whenever there is a game tree \(G\) such that \(c(A) = \text{SPNE}(G|A; P)\) for all \(A \subseteq X\)
Rationalizability by game trees

- The relation between RGT and SR is not clear a priori:
  - The structure of rationales is richer in RGT (tree against linearity)
  - Rationales are more restrictive in RGT (linear orders)

\[ \text{C}^{\text{RGT}} \subseteq \text{C}^{\text{SR}} \]
Rationalizability by game trees

- The relation between RGT and SR is not clear a priori:
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  - Rationales are more restrictive in RGT (linear orders)
- Theorem:

\[ C^{RGT} \subset C^{SR} \]


Agenda rationalizability

- Alternatives linearly ordered (agenda): $1 < 2 < \cdots < n$
- Binary choice (a tournament) between 1 and 2. The winner faces 3, etc
- The final choice is the surviving alternative of this process: $e(<, T, A)$
- Related literature:
  - Individual choice: models of choice by ordered elimination: Rubinstein and Salant (TE, 2006), Salant and Rubinstein (REStud, 2008) or Masatlioglu and Nakajima (WP, 2007)
- Agenda Rationalizability: A choice function $c$ is agenda rationalizable whenever there exists a linear order $<$ over the set of alternatives (an agenda) and a tournament $T$ such that for every $A \in \mathcal{P}(X)$, $c(A) = e(<, T, A)$
Agenda rationalizability

Theorem

$\mathcal{C}^{AR} \subset \mathcal{C}^{SR}$
Agenda rationalizability

- **Theorem**
  \[ C^{AR} \subset C^{SR} \]

- **Indeed**, \( C^{AR} \subset C^{RGT} \subset C^{SR} \)
Status quo bias rationalizability

- Individuals often evaluate an alternative more highly when it is regarded as the status quo.
- Intense empirical and theoretical attention to this phenomenon.
- We adapt the axiomatization of Masatlioglu and Ok (2005, JET), to our setting:
  - There is a status quo $\bar{x} \in X$.
  - When the status quo is not present, the agent maximizes a multi attribute utility function over the set of alternatives.
  - If the status quo is present, the agent maximizes the utility function over the set of alternatives that dominate the status quo in every single dimension, if there is any.
  - Otherwise the agent sticks to the status quo.
A choice function $c$ is status-quo biased if there exists an element $\bar{x} \in X$, a positive integer $q$, an injective function $u : X \to \mathbb{R}^q$ and a strictly increasing map $h : u(X) \to \mathbb{R}$ such that:

1. For all $A \subseteq X$ with $\bar{x} \notin A$:

$$c(A) = \arg\max_{y \in A} h(u(x))$$

2. For all $A \subseteq X$ with $\bar{x} \in A$:

   - If $\hat{A} = A \cap \{x \in X : u(x) > u(\bar{x})\} = \emptyset$:
     $$c(A) = \bar{x}$$
   
   - If $\hat{A} \neq \emptyset$:
     $$c(A) = \arg\max_{y \in \hat{A}} h(u(y))$$
Status quo bias rationalizability

Theorem

\[ C^{SQB} \subset C^{SR} \]
Status quo bias rationalizability

- **Theorem**

  $$\mathcal{C}^{SQB} \subset \mathcal{C}^{SR}$$

- **Indeed,**

  $$\mathcal{C}^{SQB} \subset \mathcal{C}^{AR} \subset \mathcal{C}^{RGT} \subset \mathcal{C}^{SR}$$
Final remarks

- We study choice by sequential procedures
- We offer a behavioral characterization of sequential choice
- Our characterizing property IOIA can be used to establish the relation between SR and other models. In particular we have shown that SR subsumes a number of prominent models like:
  - Rationalizability by Game Trees (Xu and Zhou, JET 2007)
  - Agenda Rationalizability (voting models; choice by elimination)
  - Status Quo Bias Rationalizability (Masatlioglu and Ok, JET 2005)
- Future research: nature and manipulability of $f$